Synchronization and Transient Stability in Power Networks and Non-Uniform Kuramoto Oscillators

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Outline

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Introduction and Motivation

- The electrical grid is the largest and most complex machine ever made. [2]
  - Large-scale, complex and rich nonlinear dynamic behaviors.
  - Prone to instabilities, which can ultimately lead to power blackouts.
- The blackout happened in 2003 affected a wide swath of territory in the U.S. and Canada.
Introduction and Motivation

• Expected developments in **Smart Grid**
  – Large number of distributed power sources.
  – Large-scale heterogeneous networks with stochastic disturbances.
  – The detection and rejection of instability mechanisms will be one of the major challenges.

**Transient Stability**: the ability of the generators maintain synchronism when subjected to a severe transient disturbance such as a fault on transmission facilities or loss of a large load.
Other existing methods

- Treat the transient stability problem as a special case of the more general synchronization problems, which considers a possibly longer time horizon, drifting generator rotor angles, and local excitation controllers aiming to restore synchronism.
- Do not result in simple conditions to check if a power system synchronizes for a given network state and parameters.

It is an outstanding problem to relate synchronization and transient stability of a power network to the underlying network parameters, state, and topology.
Transient Stability Analysis in Power Networks

• Previous work
  – Structure-preserving DAE power network model
  – Network-reduced ODE power network model
    • Network reduced to active nodes (generators) by using kron-reduction technique.
    • Admittance matrix $Y_{\text{red}}$ induces complete all-to-all coupling graph

Fig 1. Network Reduction Procedure for New England 39-bus test system [3]
Transient Stability Analysis in Power Networks

- Classic interconnected swing equations

\[
\frac{M_i}{\pi f_0} \ddot{\theta}_i = -D_i \dot{\theta}_i + P_{\text{mech.in},i} - P_{\text{electr.out},i}
\]

\[
\frac{M_i}{\pi f_0} \ddot{\theta}_i = -D_i \dot{\theta}_i + \omega_i - \sum_{j=1}^{n} P_{ij} \sin(\theta_i - \theta_j + \varphi_{ij})
\]

- \( P_{ij} = |V_i| |V_j| |Y_{\text{red},ij}| > 0 \) max power transferred between \( i \) & \( j \)

- \( \varphi_{ij} = \arctan(\text{Re}(Y_{\text{red},ij}) / \text{Im}(Y_{\text{red},ij})) \in [0, \pi / 2) \) line loss between \( i \) & \( j \)

- \( \omega_i = P_{m,i} - |V_i|^2 \text{Re}(Y_{\text{red},ii}) \) effective power input of \( i \)
Transient Stability Analysis in Power Networks

- Transient stability and synchronization
  - Frequency equilibrium:
    \((\dot{\theta}_i, \ddot{\theta}_i) = (0, 0)\) for all \(i\).
  - Synchronous equilibrium:
    \(|\theta_i - \theta_j| \leq \gamma\) and \(\dot{\theta}_i(t) - \dot{\theta}_j(t) \to 0\) for all \((i, j)\) as \(t \to \infty\).

- Classic analysis tools: Hamiltonian arguments

\[
\frac{M_i}{\pi f_0} \ddot{\theta}_i = -D_i \dot{\theta}_i - \nabla_i U(\theta)^T
\]

where

\[
U(\theta) = -\sum_{i=1}^{n} \omega_i \theta_i + \sum_{j=1}^{n} P_{ij} (1 - \cos(\theta_i - \theta_j))
\]

Dimension-reduced gradient flow analysis:

\[
\theta_i = -\nabla_i U(\theta)^T \quad \Rightarrow \quad \text{hyerbolic type-k equilibrium } (\theta^*, 0)
\]
Transient Stability Analysis in Power Networks

• Classic analysis tools: Hamiltonian arguments
  – Shortcomings:
    • Over simplify the model: e.g. assuming $\varphi_{ij} = 0$
    • Implement the numerical procedures rather than induce concise conditions: e.g. only indicating “sufficiently small” transfer conductances without quantifying the smallness.
    • Not setup the relationship between the synchronization & transient stability in power networks and the underlying network state, parameters & topology.
Singular Perturbation Analysis and Main Synchronization Results

\[ \frac{M_i}{\pi f_0} \ddot{\theta}_i = -D_i \dot{\theta}_i + \omega_i - \sum_{j=1}^{n} P_{ij} \sin(\theta_i - \theta_j + \varphi_{ij}) \]

Consensus protocol in \( \mathbb{R}^n \)

\[
\begin{align*}
\dot{x}_i &= -\sum_{j=1}^{n} a_{ij} (x_i - x_j) \\
\dot{x} &= -L(a_{i,j}) x
\end{align*}
\]

- \( n \) identical agents with state variable
- **Objective**: state agreement: \( x(t) \to x_c \) as \( t \to \infty \)

Kuramoto model in \( \mathbb{T}^n \)

\[
\dot{\theta}_i = \omega_i - \frac{K}{n} \sum_{j=1}^{n} \sin(\theta_i - \theta_j)
\]

- \( n \) non-identical oscillators with phase \( \theta_i \in \mathbb{T} \)
- **Objective**
  - phase locking: \( |\theta_i(t) - \theta_i(t)| < \gamma \)
  - frequency entrainment: \( \dot{\theta}_i(t) = \dot{\theta}_j(t) \)
  - phase synchronization: \( \theta_i(t) = \theta_j(t) \)
Singular Perturbation Analysis

- Singular perturbation analysis
  - Time-scale separation in power network model
    \[ \frac{M_i}{\pi f_0} \dot{\theta}_i = -D_i \dot{\theta}_i + \omega_i - \sum_{j=1}^{n} P_{ij} \sin(\theta_i - \theta_j + \varphi_{ij}) \]
  - Singular perturbation parameter: \( \epsilon = \frac{M_{\text{max}}}{\pi f_0 D_{\text{min}}} \)
  - Reduced dynamics on slow time-scale (for \( \epsilon<<1 \))
    non-uniform Kuramoto model
    \[ D_i \dot{\theta}_i = \omega_i - \sum_{j=1}^{n} P_{ij} \sin(\theta_i - \theta_j + \varphi_{ij}) \]
  - Assume the non-uniform Kuramoto model synchronizes exponentially, \( \forall (\theta(0), \dot{\theta}(0)) \), \( \exists \epsilon^*>0 \), s.t. \( \forall \epsilon<\epsilon^* \) and \( \forall t \geq 0 \): \( \theta_i(t) \) power network - \( \theta_i(t) \) non-uniform kuramoto = \( O(\epsilon) \). For \( \epsilon \) and the network losses \( \varphi_{ij} \) sufficiently small, \( O(\epsilon) \) converges to 0 asymptotically.
Singular Perturbation Analysis

• Discussion of the assumption $\epsilon$ is sufficiently small.
  – Generator internal control effects (e.g. local excitation controllers) imply $\epsilon \in O(0.1)$
  – Topological equivalence independent of $\epsilon$: 1st-order and 2nd-order models have the same equilibria, the Jacobians have the same inertia, and the regions of attractions are bounded by the same separatrices.
  – Simulation studies show accurate approximation even for large $\epsilon$.
  – Non-uniform Kuramoto corresponds to reduced gradient system $\dot{\theta}_i = -\nabla_i U(\theta)^T$ used successfully in academia and industry since 1978.
Synchronization of Non-Uniform Kuramoto Oscillators

non-uniform Kuramoto model

\[ D_i \dot{\theta}_i = \omega_i - \sum_{j=1}^{n} P_{ij} \sin(\theta_i - \theta_j + \varphi_{ij}) \]

- Non-uniformity in network: \( D_i, \omega_i, P_{ij}, \varphi_{ij} \)
- Phase shift \( \varphi_{ij} \) induces lossless and lossy coupling

\[ \dot{\theta}_i = \frac{\omega_i}{D_i} - \sum_{j=1,j \neq i}^{n} \left( \frac{P_{ij}}{D_i} \cos(\varphi_{ij}) \sin(\theta_i - \theta_j) + \frac{P_{ij}}{D_i} \sin(\varphi_{ij}) \cos(\theta_i - \theta_j) \right) \]

- Synchronization analysis
  - Phase locking: \( |\theta_i - \theta_j| \leq \gamma \)
  - Frequency entrainment: \( \dot{\theta}_i(t) - \dot{\theta}_j(t) \to 0 \)
  - Phase synchronization: \( |\theta_i(t) - \theta_j(t)| \to 0 \)
Synchronization of Non-Uniform Kuramoto Oscillators

• Synchronization condition:

\[ n \frac{P_{\text{min}}}{D_{\text{max}}} \cos(\varphi_{\text{max}}) > \max_{\{i,j\}} \left( \frac{\omega_i}{D_i} - \frac{\omega_j}{D_j} \right) + \max_i \sum_{j=1}^{n} \frac{P_{ij}}{D_i} \sin(\varphi_{ij}) \]

Worst lossless coupling \quad Worst non-uniformity \quad Worst lossy coupling

The network connectivity has to dominate the network’s non-uniformity, the network’s losses, and the lack of phase locking
Synchronization of Non-Uniform Kuramoto Oscillators

• Synchronization condition:
\[ n \frac{P_{\text{min}}}{D_{\text{max}}} \cos(\varphi_{\text{max}}) > \max_{\{i,j\}} \left( \frac{\omega_i}{D_i} - \frac{\omega_j}{D_j} \right) + \max_i \sum_{j=1}^{n} \frac{P_{ij}}{D_i} \sin(\varphi_{ij}) \]

• Phase Locking
\[ \arcsin(\cos(\varphi_{\text{max}}) \frac{\text{RHS}}{\text{LHS}}) \leq \gamma \leq \frac{\pi}{2} - \varphi_{\text{max}} \]

\[ \gamma_{\text{min}} \leq \gamma \leq \gamma_{\text{max}} \]

• Frequency entrainment
  – From all initial conditions in a \( \gamma_{\text{max}} \) arc, exponential frequency synchronization.
  – \( \lambda_{fe} = -\lambda_2(L(P_{ij})) \cos(\gamma) \cos(\angle(D1, 1))^{2}/D_{\text{max}} \).
Synchronization of Non-Uniform Kuramoto Oscillators

• Main proof ideas
  – An analogous result guarantees synchronization of the non-uniform Kuramoto oscillators whenever the graph induced by $\mathbf{P}$ has a globally reachable node.

\[
\frac{d}{dt} \dot{\theta}_i = - \sum_{j=1}^{n} a_{ij}(\theta(t))(\dot{\theta}_i - \dot{\theta}_j), \quad i \in \{1, \ldots, n\}
\]

where $a_{ij}(\theta(t)) = (P_{ij}/D_i) \cos(\theta_i(t) - \theta_j(t) + \varphi_{ij})$
Synchronization of Non-Uniform Kuramoto Oscillators

- Alternative synchronization condition
  - Non-uniform Kuramoto model:

\[
\dot{\theta}_i = \frac{\omega_i}{D_i} - \sum_{j=1,j\neq i}^{n} \left( \frac{P_{ij}}{D_i} \cos(\varphi_{ij}) \sin(\theta_i - \theta_j) + \frac{P_{ij}}{D_i} \sin(\varphi_{ij}) \cos(\theta_i - \theta_j) \right)
\]

- Condition:

\[
\lambda_2(L(P_{ij} \cos(\varphi_{ij}))) > \lambda_{\text{critical}} := \frac{\|HD^{-1}\omega\|_2 + \sqrt{\lambda_{\text{max}}(L)}\|\cdots, \sum_{j=1}^{n} \frac{P_{ij}}{D_i} \sin(\varphi_{ij}), \cdots\|_2}{\cos(\varphi_{\text{max}})(\kappa/n)\mu \min\{i,j\} \{D \neq \{i,j\}\}}
\]
Conclusions

• Study the synchronization and transient stability problem for a network-reduction model of a power system.
• Provide the conditions depending on network parameters and initial phase differences suffice for the synchronization of non-uniform Kuramoto oscillators as well as the transient stability of the power network model.
My Assessments

• Suggestions on future works
  – Specify the synchronization frequency.
  – Specify the exponential rate of the frequency synchronization.
  – Develop the stochastic analysis instead of the worst-case analysis.
  – Study the more general power network (e.g. the underlying graph is not complete).
  – Provide the simulations on practical power test system.
References

