Synchronization and Transient Stability in Power Networks and Non-Uniform Kuramoto Oscillators

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Outline

- Introduction and Motivation
- Transient Stability Analysis in Power Networks
- Singular Perturbation Analysis and Main Synchronization Results
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- My Assessments

Introduction and Motivation

- *The electrical grid is the largest and most complex machine ever made.* [2]
  - Large-scale, complex and rich nonlinear dynamic behaviors.
  - Prone to instabilities, which can ultimately lead to power blackouts.
    - The blackout happened in 2003 affected a wide swath of territory in the U.S. and Canada.

Introduction and Motivation

- Expected developments in **Smart Grid**
  - Large number of distributed power sources.
  - Large-scale heterogeneous networks with stochastic disturbances.
  - The detection and rejection of instability mechanisms will be one of the major challenges.

Transient Stability: the ability of the generators maintain synchronism when subjected to a severe transient disturbance such as a fault on transmission facilities or loss of a large load.

Introduction and Motivation

- Other existing methods
  - Treat the transient stability problem as a special case of the more general synchronization problems, which considers a possibly longer time horizon, drifting generator rotor angles, and local excitation controllers aiming to restore synchronism.
  - Do not result in simple conditions to check if a power system synchronizes for a given network state and parameters.

It is an outstanding problem to relate synchronization and transient stability of a power network to the underlying network parameters, state, and topology.

Transient Stability Analysis in Power Networks

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- Previous work
  - Structure-preserving DAE power network model
  - Network-reduced ODE power network model
    - Network reduced to active nodes (generators) by using kron-reduction technique.
    - Admittance matrix Y_{red} induces complete all-to-all coupling graph

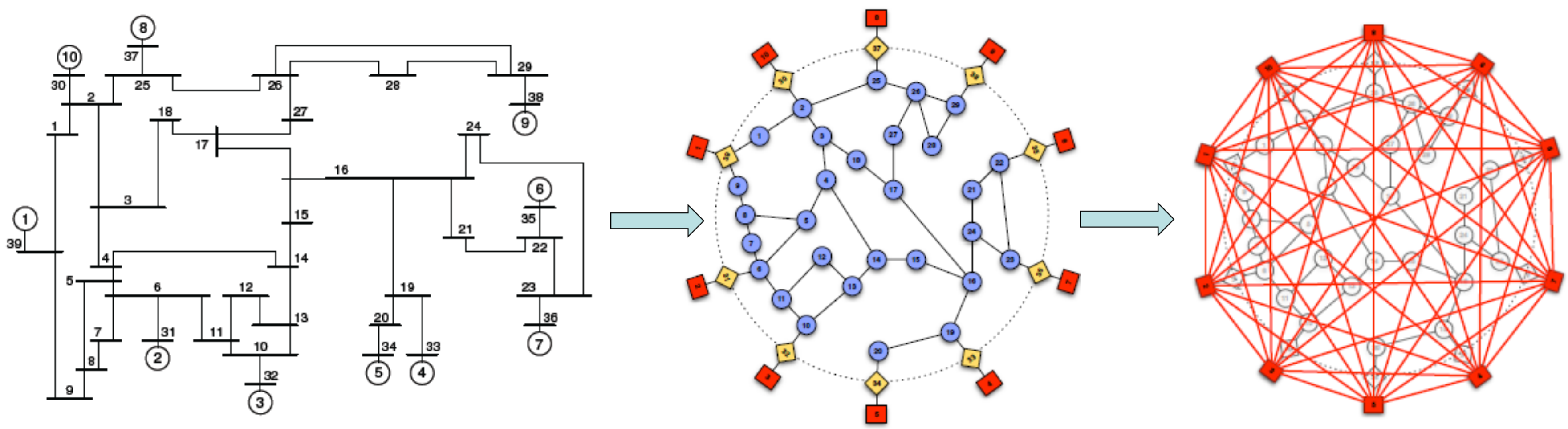

Fig 1. Network Reduction Procedure for New England 39-bus test system [3]

Transient Stability Analysis in Power Networks

- Classic interconnected swing equations

$$\frac{M_i}{\pi f_0}\ddot{\theta}_i = -D_i\dot{\theta}_i + P_{\text{mech.in},i} - P_{\text{electr.out},i}$$

$$\frac{M_i}{\pi f_0}\ddot{\theta}_i = -D_i\dot{\theta}_i + \omega_i - \sum\nolimits_{j=1}^{n} P_{ij}\sin(\theta_i - \theta_j + \varphi_{ij})$$

  - $P_{ij} = |\mathbf{V}_i||\mathbf{V}_j||\mathbf{Y}_{red,ij}| > 0$ max power transferred between *i* & *j*
  - $\varphi_{ij} = \arctan(\mathrm{Re}(Y_{red,ij}) / \mathrm{Im}(Y_{red,ij})) \in [0, \pi/2)$ line loss between *i* & *j*
  - $\omega_i = P_{m,i} - |\mathbf{V}_i|^2\,\mathrm{Re}(\mathbf{Y}_{red,ii})$ effective power input of i

Transient Stability Analysis in Power Networks

- Transient stability and synchronization
  - Frequency equilibrium:
    $(\dot{\theta}_i, \ddot{\theta}_i)$=(0,0) for all i.
  - Synchronous equilibrium:
    $|\theta_i - \theta_j| \leq \gamma$ and $\dot{\theta}_i(t) - \dot{\theta}_j(t) \to 0$ for all (*i*, *j*) as $t \to \infty$.
- Classic analysis tools: Hamiltonian arguments

$$\frac{M_i}{\pi f_0}\ddot{\theta}_i = -D_i\dot{\theta}_i - \nabla_i U(\theta)^T$$

where $U(\theta) = -\sum_{i=1}^{n} \omega_i\theta_i + \sum_{j=1}^{n} P_{ij}\left(1 - \cos(\theta_i - \theta_j)\right)$

Dimension-reduced gradient flow analysis: $\theta_i = -\nabla_i U(\theta)^T$

⇨ hyperbolic type-k equilibium $(\theta^*, \mathbf{0})$

Transient Stability Analysis in Power Networks

- Classic analysis tools: Hamiltonian arguments
  - Shortcomings:
    - Over simplify the model: e.g. assuming $\varphi_{ij}=0$
    - Implement the numerical procedures rather than induce concise conditions: e.g. only indicating "sufficiently small" transfer conductances without quantifying the smallness.
    - Not setup the relationship between the synchronization & transient stability in power networks and the underlying network state, parameters & topology.

Singular Perturbation Analysis and Main Synchronization Results

$$\frac{M_i}{\pi f_0}\ddot{\theta}_i = -D_i\dot{\theta}_i + \omega_i - \sum_{j=1}^{n} P_{ij}\sin(\theta_i - \theta_j + \varphi_{ij})$$

Consensus protocol in $\mathbb{R}^n$

$$\dot{x}_i = -\sum_{j=1}^{n} a_{ij}(x_i - x_j)$$

$$\dot{x} = -L(a_{ij})x$$

- *n* identical agents with state variable
- **Objective**: state agreement: $\mathbf{x}_i(t) \to \mathbf{x}_\infty$ as $t \to \infty$

Kuramoto model in $\mathbb{T}^n$

$$\dot{\theta}_i = \omega_i - \frac{K}{n}\sum_{j=1}^{n}\sin(\theta_i - \theta_j)$$

- *n* non-identical oscillators with phase $\theta_i \in \mathbf{T}$
- **Objective**
  - phase locking: $|\theta_i(t) - \theta_i(t)| < \gamma$
  - frequency entrainment: $\dot{\theta}_i(t) = \dot{\theta}_j(t)$
  - phase synchronization: $\theta_i(t) = \theta_j(t)$

Singular Perturbation Analysis

- Singular perturbation analysis
  - Time-scale separation in power network model

$$\frac{M_i}{\pi f_0}\ddot{\theta}_i = -D_i\dot{\theta}_i + \omega_i - \sum\nolimits_{j=1}^{n} P_{ij}\sin(\theta_i - \theta_j + \varphi_{ij})$$

  - Singular perturbation parameter: $\epsilon = \dfrac{M_{\mathrm{max}}}{\pi f_0 D_{\mathrm{min}}}$
  - Reduced dynamics on slow time-scale (for ϵ<<1)

    non-uniform Kuramoto model

$$D_i\,\dot{\theta}_i = \omega_i - \sum\nolimits_{j=1}^{n} P_{ij}\sin(\theta_i - \theta_j + \varphi_{ij})$$

  - Assume the non-uniform Kuramoto model synchronizes exponentially, $\forall\left(\theta(\mathbf{0})£\ \dot{\theta}(\boldsymbol{\theta})\right)$, $\exists\ \epsilon^*>0$, s.t. $\forall\ \epsilon<\epsilon^*$ and $\forall t \geq \mathbf{0}$: $\theta_i(t)_{\text{power network}} - \theta_i(t)_{\text{non-uniform kuramoto}} = O(\epsilon)$. For ϵ and the network losses φ_{ij} sufficiently small, $O(\epsilon)$ converges to 0 asymptotically.

Singular Perturbation Analysis

- Discussion of the assumption ϵ is sufficiently small.
  - Generator internal control effects (e.g. local excitation controllers) imply $\epsilon \in \mathcal{O}(0.1)$
  - Topological equivalence independent of ϵ: 1st-order and 2nd-order models have the same equilibria, the Jacobians have the same inertia, and the regions of attractions are bounded by the same separatrices.
  - simulation studies show accurate approximation even for large ϵ.
  - Non-uniform Kuramoto corresponds to reduced gradient system $\dot{\theta}_i = -\nabla_i U(\theta)^T$ used successfully in academia and industry since 1978.

Synchronization of Non-Uniform Kuramoto Oscillators

non-uniform Kuramoto model

$$D_i \dot{\theta}_i = \omega_i - \sum_{j=1}^{n} P_{ij} \sin(\theta_i - \theta_j + \varphi_{ij})$$

- Non-uniformity in network: D_i, ω_i, P_{ij}, φ_{ij}
- Phase shift φ_{ij} induces lossless and lossy coupling

$$\dot{\theta}_i = \frac{\omega_i}{D_i} - \sum_{j=1, j\neq i}^{n} \left(\frac{P_{ij}}{D_i} \cos(\varphi_{ij}) \sin(\theta_i - \theta_j) + \frac{P_{ij}}{D_i} \sin(\varphi_{ij}) \cos(\theta_i - \theta_j) \right)$$

- Synchronization analysis
  - Phase locking: $|\theta_i - \theta_j| \leq \gamma$
  - Frequency entrainment: $\dot{\theta}_i(t) - \dot{\theta}_j(t) \rightarrow 0$
  - Phase synchronization: $|\theta_i(t) - \theta_j(t)| \rightarrow 0$

Synchronization of Non-Uniform Kuramoto Oscillators

- Synchronization condition:

$$\underbrace{n\frac{P_{\min}}{D_{\max}}\cos(\varphi_{\max})}_{\text{Worst lossless coupling}} > \underbrace{\max_{\{i,j\}}\left(\frac{\omega_i}{D_i}-\frac{\omega_j}{D_j}\right)}_{\text{Worst non-uniformity}} + \underbrace{\max_i\sum_{j=1}^{n}\frac{P_{ij}}{D_i}\sin(\varphi_{ij})}_{\text{Worst lossy coupling}}$$

The network connectivity has to dominate the network's non-uniformity, the network's losses, and the lack of phase locking

Synchronization of Non-Uniform Kuramoto Oscillators

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- Synchronization condition:

$$n\frac{P_{\min}}{D_{\max}}\cos(\varphi_{\max}) > \max_{\{i,j\}}\left(\frac{\omega_i}{D_i}-\frac{\omega_j}{D_j}\right)+\max_i\sum_{j=1}^{n}\frac{P_{ij}}{D_i}\sin(\varphi_{ij})$$

- Phase Locking

$$\underbrace{\arcsin\left(\cos(\varphi_{\max})\frac{RHS}{LHS}\right)}_{\gamma_{\min}} \le \gamma \le \underbrace{\frac{\pi}{2}-\varphi_{\max}}_{\gamma_{\max}}$$

- Frequency entrainment
  - From all initial conditions in a $\gamma_{\max}$ arc, exponential frequency synchronization.
  - $\lambda_{\mathrm{fe}} = -\lambda_2(L(P_{ij}))\cos(\gamma)\cos(\angle(D1,1))^2/D_{\max}$.

Synchronization of Non-Uniform Kuramoto Oscillators

- Main proof ideas
  - An analogous result guarantees synchronization of the non-uniform Kuramoto oscillators whenever the graph induced by **P** has a globally reachable node.

frequency entrainment in $\Delta\gamma$ $\Longleftrightarrow$ consensus protocol in R^n

$$\frac{d}{dt}\dot{\theta}_i = -\sum_{j=1}^{n} a_{ij}(\theta(t))(\dot{\theta}_i - \dot{\theta}_j), \quad i \in \{1, \ldots, n\}$$

where $a_{ij}(\theta(t)) = (P_{ij}/D_i)\cos(\theta_i(t) - \theta_j(t) + \varphi_{ij})$

Synchronization of Non-Uniform Kuramoto Oscillators

- Alternative synchronization condition
  - Non-uniform Kuramoto model:

$$\dot{\theta}_i = \frac{\omega_i}{D_i} - \sum_{j=1, j\neq i}^{n} \left(\frac{P_{ij}}{D_i} \cos(\varphi_{ij}) \sin(\theta_i - \theta_j) + \frac{P_{ij}}{D_i} \sin(\varphi_{ij}) \cos(\theta_i - \theta_j) \right)$$

  - Condition:

$$\lambda_2(L(P_{ij}\cos(\varphi_{ij}))) > \lambda_{\text{critical}} :=$$

$$\frac{\left\| H D^{-1} \omega \right\|_2 + \sqrt{\lambda_{\max}(L)} \left\| \left[\ldots, \sum_{j=1}^{n} \frac{P_{ij}}{D_i} \sin(\varphi_{ij}), \ldots \right] \right\|_2}{\cos(\varphi_{\max})(\kappa/n)\mu \min_{\{i,j\}}\{D_{\neq\{i,j\}}\}}$$

Conclusions

- Study the synchronization and transient stability problem for a network-reduction model of a power system.
- Provide the conditions depending on network parameters and initial phase differences suffice for the synchronization of non-uniform Kuramoto oscillators as well as the transient stability of the power network model.

My Assessments

- Suggestions on future works
  - Specify the synchronization frequency.
  - Specify the exponential rate of the frequency synchronization.
  - Develop the stochastic analysis instead of the worst-case analysis.
  - Study the more general power network (e.g. the underlying graph is not complete).
  - Provide the simulations on practical power test system.

References

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