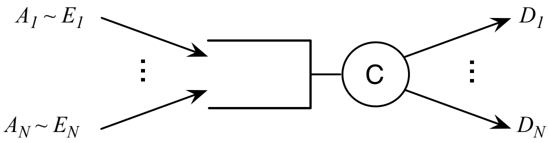
Delay Bounds for Scheduling Algorithms

First-In-First-Out (FIFO)



- $\mathcal{N} = \{1, 2, \ldots, N\}$ is set of flows
- Each flow j has a traffic envelope E_j ($A_j \sim E_j$)
- Goal: Compute a condition so that the delay of any arrival does not exceed d.

First-In-First-Out (FIFO)

- If we set

$$A(t) := \sum_{k=1}^{N} A_k(t) \ , \qquad E(t) := \sum_{k=1}^{N} E_k(t) \ ,$$

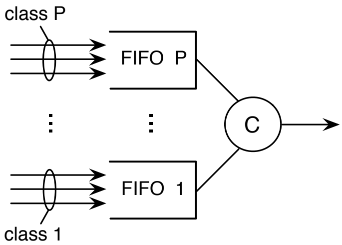
  we obtain a buffered link with a single arrival flow.
- We have computed this before:

$$\forall s \geq 0 \ : E(s-d) \leq Cs$$

- Substitute $s - d \to s$:

$$\boxed{d \geq \sup_{s>0} \left\{ \frac{E(s) - Cs}{C} \right\}}$$

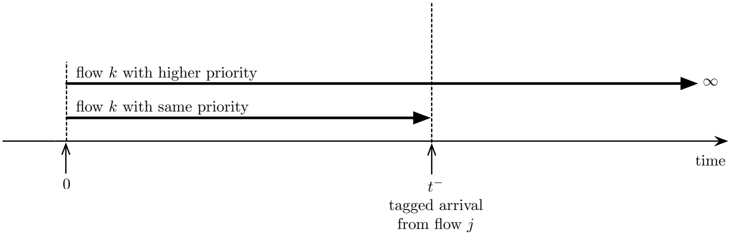
Static Priority (SP)



- P priority classes ($1=$ lowest, $P=$ highest)
- $\mathcal{N}_p$ is the set of flows with priority p
- Each flow j has a traffic envelope E_j ($A_j \sim E_j$)
- Ignore fact that packet transmission cannot be preempted.

- Goal: Compute a condition so that the delay of a **tagged arrival** at time from flow $j \in \mathcal{N}_p$ at time t^- has a delay less than d_p.

Static Priority



- $t - x_p$: last time before t that the SP scheduler does not have any backlog from priority-p or higher.
  - From $t - x_p$, until tagged arrival leaves, scheduler only transmits traffic from priority p or higher.
  - $A_j(t - x_p) = D_j(t - x_p)$

SP Analysis (1)

Notation: $A_j(s,t) = A_j(t) - A_j(s)$, $D_j(s,t) = D - j(t) - D_j(s)$

Delay of tagged arrival:

$$
\begin{aligned}
W_j(t) &= \inf\left\{ y > 0 \,|\, D_j(t+y) \geq A_j(t) \right\} \\
&= \inf\left\{ y > 0 \,|\, D_j(t - x_p, t+y) \geq A_j(t - x_p, t) \right\} \ ,
\end{aligned}
$$

$$
\begin{aligned}
D_j(t - x_p, t + y) \leq \Big[&\Big\{ \text{Total traffic transmitted in } [t - x_p, t + y) \Big\} \\
&- \Big\{ \text{Arrivals in } [t - x_p, t + y) \text{ with higher precedence} \Big\} \Big]^+ \\
= \Big[C(y + x_p) - &\sum_{k \in \mathcal{N}_p, k \neq j} A_k(t - x_p, t) - \sum_{q=p+1}^{P} \sum_{k \in \mathcal{N}_q} A_k(t - x_p, t + y) \Big]^+
\end{aligned}
$$

SP Analysis (2)

- Replace D_j in expression for W_j by its bound:

$$W_j(t) \leq \inf \left\{ y > 0 \,|\, C(y + x_p) - \sum_{k \in \mathcal{N}_p, k \neq j} A_k(t - x_p, t) \right.$$
$$\left. - \sum_{q=p+1}^{P} \sum_{k \in \mathcal{N}_q} A_k(t - x_p, t + y) \geq A_j(t - x_p, t) \right\} .$$

- If d_p is a delay bound, then $W_j(t) \leq d_p$. So:

$$C(d_p + x_p) - \sum_{k \in \mathcal{N}_p, k \neq j} A_k(t - x_p, t)$$
$$- \sum_{q=p+1}^{P} \sum_{k \in \mathcal{N}_q} A_k(t - x_p, t + d_p) \geq A_j(t - x_p, t) .$$

SP Analysis (3)

- Re-arrange terms and allow x_j to be any value $s \geq 0$:

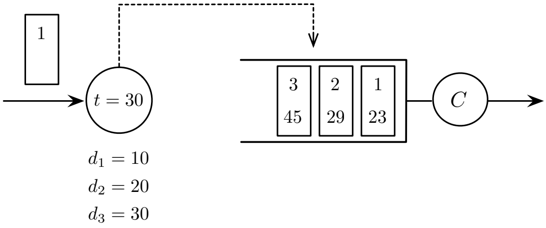
$$C d_p \geq \sup_{s \geq 0} \left\{ \sum_{k \in \mathcal{N}_p} A_k(t-s, t) + \sum_{q=p+1}^{P} \sum_{k \in \mathcal{N}_q} A_k(t-s, t+d_p) - Cs \right\} .$$

- Relax expression
  - Set $s = x_p$
  - Use that $A_k(s,t) \leq E_k(t-s)$
- Simplify: $E_q(t) = \sum_{j \in \mathcal{N}_q} E_j(t)$

$$\boxed{d_p \geq \sup_{s \geq 0} \frac{1}{C} \left\{ E_p(s) + \sum_{q=p+1}^{P} E_q(s + d_p) - Cs \right\}}$$

If condition is satisfied, tagged arrival has delay no more than d_p.

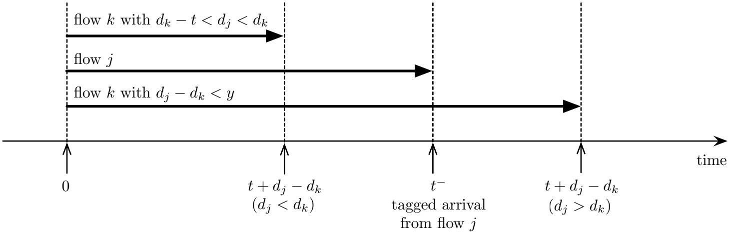
Earliest Deadline First (EDF)



- Each flow j has a delay index d_j.
- Each flow j has a traffic envelope E_j ($A_j \sim E_j$)
- Goal: Compute a condition so that the delay for a **tagged arrival** at time from flow $j \in \mathcal{N}_p$ at time t^- does not exceed d_j ($\rightarrow$ Tagged arrival meets its deadline!)

Earliest Deadline First (EDF)

Q: Which traffic has higher precedence than tagged arrival?
A: All traffic with a deadline before $t + d_j$!



- for flow j: All arrivals before t
- for any flow k: All arrivals before $t + d_j - d_k$

If the tagged arrival has not departed by time $t + y$, traffic from flow k with deadline $\leq t + d_j$ or earlier, are the arrivals until time $t + \min\{y, d_j - d_k\}$.

Earliest Deadline First (EDF)

- Define $t - x_j$: last time before t that the EDF scheduler does not have any backlog with deadline $t + d_j$ or earlier
    - From $t - x_j$, until tagged arrival leaves, scheduler only transmits traffic with deadline $\leq t + d_j$
    - $A_j(t - x_j) = D_j(t - x_j)$

EDF Analysis (1)

- Delay of tagged arrival:

$$\begin{aligned} W_j(t) &= \inf\left\{y > 0 \,|\, D_j(t+y) \geq A_j(t)\right\} \\ &= \inf\left\{y > 0 \,|\, D_j(t-x_j, t+y) \geq A_j(t-x_j, t)\right\} \ . \end{aligned}$$

- If tagged arrival has not left by time $t + y$:

$$\begin{aligned} D_j(t-x_j, t+y) &\leq \Big[\big\{\text{Total traffic transmitted in } [t-x_j, t+y)\big\} \\ &\quad - \big\{\text{Arrivals from flow } j \text{ in } [t-x_j, t+\min\{y, d_j-d_k\})\ \big\}\Big]^+ \\ &= \Big[C(y+x_j) - \sum_{k\in\mathcal{N}, k\neq j} A_k(t-x_j, t+\min\{y, d_j-d_k\})\Big]^+ \ . \end{aligned}$$

EDF Analysis (2)

- Replace D_j in expression for W_j by its bound:

$$W_j(t) \leq \inf \Big\{ y > 0 \,|\, C(y + x_j) - \sum_{k \in \mathcal{N}, k \neq j} A_k(t - x_j, t + \min\{y, d_j - d_k\}) \geq A_j(t - x_j, t) \Big\}$$

- The delay index d_j is a delay bound, if $W_j(t) \leq d_j$. So:

$$C(d_j + x_j) - \sum_{k \in \mathcal{N}, k \neq j} A_k(t - x_j, t + \min\{d_j, d_j - d_k\}) \geq A_j(t - x_j, t) \,.$$

EDF Analysis (3)

- Re-arrange terms and allow x_j to be any value $s \geq 0$:

$$\sup_{s \geq 0} \left\{ \sum_{k \in \mathcal{N}} A_k(t-s, t+d_j-d_k) - C(d_j+s) \right\} \leq 0 \quad .$$

- Use that $A_k(s,t) \leq E_k(t-s)$

$$\sup_{s \geq 0} \left\{ \sum_{k \in \mathcal{N}} E_k(s+d_j-d_k) - C(s+d_j) \right\} \leq 0 \ .$$

- Substitute $s + d_j \to s$

$$\boxed{\sup_{s \geq 0} \left\{ \sum_{k \in \mathcal{N}} E_k(s-d_k) - Cs \right\} \leq 0}$$

If condition is satisfied, tagged arrival departs before its deadline $t + d_j$.

Schedulability Conditions

- The importance of a delay bound analysis for link schedulers it can be used to devise tests that determine whether a scheduler can satisfy given delay requirements for a set of flows at a link scheduler with capacity C.

- Conditions that verify whether a set of flows is schedulable are called *schedulability conditions*.
- The derived inequalities are schedulability conditions for FIFO, SP, and EDF.
  - Conditions are 'tight', i.e., violation of the conditions may result in violation of delay bound.