Fair Bandwidth Allocation

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What is fair ?

Max-Min Fair Allocation

Set of flows: $\mathcal{N}$, Link rate: C

r_i	requested rate of flow i
a_i	allocated rate of flow i
f	**fair share**

1. If $\sum_j r_j \leq C$, then $a_i = r_i$ for each flows i.
2. If $\sum_j r_j > C$, then $a_i = \min\{r_i,\, f\}$ for i, where f is selected such that $\sum_j a_i = C$.

Goal: Find fair share f such that $\boxed{C = \sum_j \min\{r_j,\, f\}\,.}$

Finding the fair share

$M_{\mathrm{sat}} := \{j \mid r_j \leq f\}$ (set of satisfied flows)

$$C = \sum_{j \in M_{\mathrm{sat}}} r_j + \sum_{j \notin M_{\mathrm{sat}}} f$$

$$\boxed{f = \frac{C - \sum_{j \in M_{\mathrm{sat}}} r_j}{|\mathcal{N} \setminus M_{\mathrm{sat}}|}}$$

Notation:
$\mathcal{N} \setminus M_{\mathrm{sat}} = \{x \in \mathcal{N} \mid x \notin M_{\mathrm{sat}}\}$
$|X|$: number of elements in set X (=cardinality of set X)

Max-min fair allocation

The following yield equivalent allocations:

1. $a_i = \min\{r_i, f\}$ with

$$\boxed{f = \frac{C - \sum_{j \in M_{\text{sat}}} r_j}{|\mathcal{N} \setminus M_{\text{sat}}|}} \quad = \quad \max_{M \subset \mathcal{N}} \frac{C - \sum_{j \in M} r_j}{|\mathcal{N} \setminus M|}$$

2. $a_i = \min\{r_i, f_i\}$ with

$$\boxed{f_i = \max_{M \subseteq \mathcal{N} \setminus \{i\}} \frac{C - \sum_{j \in M} r_j}{|\mathcal{N} \setminus M|}}$$

3. If $a_i < r_i$ then

$$a_i \geq a_j \quad \forall j \in \mathcal{N}\,, \qquad \text{and} \qquad \sum_{j \in \mathcal{N}} a_j = C\,.$$

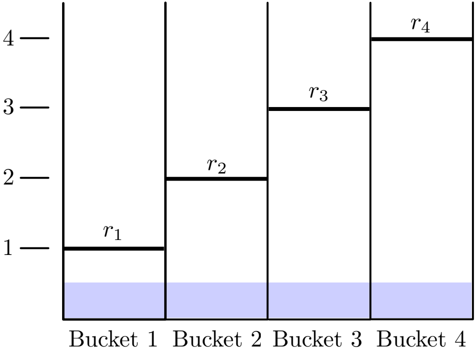
Algorithm: Fair Share Calculation

Input: N flows with request $r_i \geq 0$ for flow i, link capacity C
Output: Fair share f

$f_o \leftarrow 0$
$n \leftarrow 0$
repeat
  $n \leftarrow n + 1$
  $U_n \leftarrow \{j \mid r_j \leq f_{n-1}\}$
  $O_n \leftarrow \{j \mid r_j > f_{n-1}\}$
  $f_n \leftarrow \dfrac{C - \sum_{i \in U_n} r_i}{|O_n|}$
until $f_n = f_{n-1}$
return $f \leftarrow f_n$

Waterfilling ($C = 8$)

$r_1 = 1, r_2 = 2, r_3 = 3, r_4 = 4$



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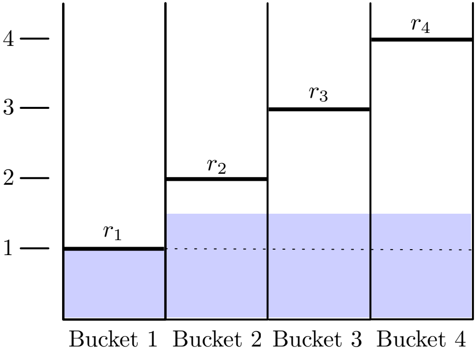

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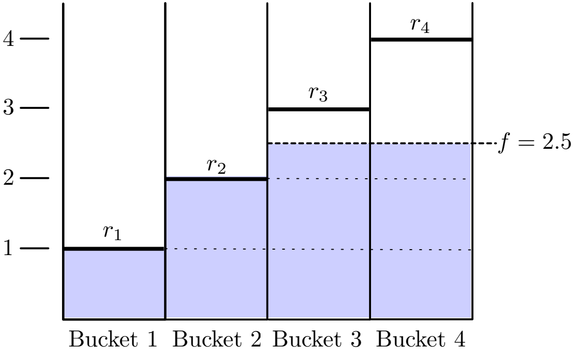


Weighted Max-Min Fair Allocation

Set of flows: $\mathcal{N}$, Link rate: C

r_i	requested rate of flow i
a_i	allocated rate of flow i
ϕ_i	weight of flow i
f	fair share

1. If $\sum_j r_j \leq C$, then $a_i = r_i$ for each flows i.
2. If $\sum_j r_j > C$, then $a_i = \min\{r_i, \phi_i f\}$ for flow i, where f is selected such that $\sum_j a_i = C$.

Goal: Find fair share f such that $\boxed{C = \sum_j \min\{r_j, \phi_j f\}.}$

Finding the fair share

$M_{\text{sat}} := \{j \mid r_j \leq \phi_j f\}$ (set of satisfied flows)

$$C = \sum_{j \in M_{\text{sat}}} r_j + \sum_{j \notin M_{\text{sat}}} \phi_j \cdot f$$

$$\boxed{f = \frac{C - \sum_{j \in M_{\text{sat}}} r_j}{\sum_{j \notin M_{\text{sat}}} \phi_j}}$$

Weighted max-min fair allocation

The following yield equivalent allocations:

1. $a_i = \min\{r_i, \phi_i f\}$ with

$$\boxed{f = \frac{C - \sum_{j \in M_{\text{sat}}} r_j}{\sum_{j \notin M_{\text{sat}}} \phi_j}} \quad = \quad \max_{M \subset \mathcal{N}} \frac{C - \sum_{j \in M} r_j}{\sum_{j \notin M} \phi_j}$$

2. $a_i = \min\{r_i, \phi_i f_i\}$ with

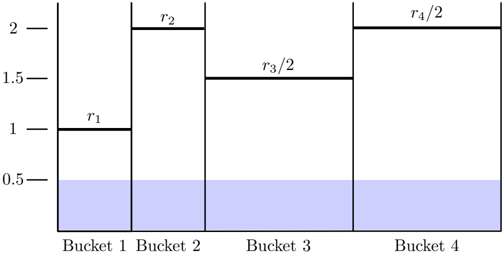
$$\boxed{f_i = \max_{M \subseteq \mathcal{N} \setminus \{i\}} \frac{C - \sum_{j \in M} r_j}{\sum_{j \notin M} \phi_j}}$$

3. If $a_i < r_i$ then

$$\frac{a_i}{\phi_i} \geq \frac{a_j}{\phi_j} \quad \forall j \in \mathcal{N}, \qquad \text{and} \qquad \sum_{j \in \mathcal{N}} a_j = C\,.$$

Waterfilling ($C = 8$)

$\phi_1 = \phi_2 = 1$ and $\phi_3 = \phi_4 = 2$
$r_1 = 1, r_2 = 2, r_3 = 3, r_4 = 4$



Waterfilling

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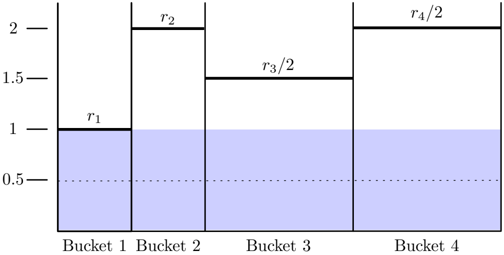

Waterfilling

$\phi_1 = \phi_2 = 1$ and $\phi_3 = \phi_4 = 2$

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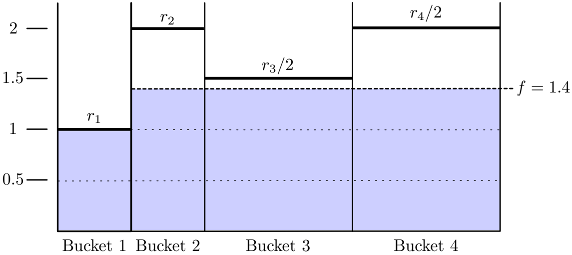

Fluid-flow Weighted Fair Queueing (Fluid-flow WFQ)

A *fluid-flow Weighted Fair Queueing* scheduler is a workconserving scheduling algorithm for a link with rate C, which ensures that for any $[s, t)$ where flow i is backlogged,

$$\forall j \in \mathcal{N} : \qquad \frac{D_i(s,t)}{D_j(s,t)} \geq \frac{\phi_i}{\phi_j}$$

where $D_j(s, t)$ are the departures of flow $j \in \mathcal{N}$.

$\Rightarrow$ If a flow is backlogged in a time interval, its service in this time interval is at least proportional (with respect to the weights) to the service to any other flows.

Important properties of Weighted Fair Queueing

Property 1: When packet-level WFQ selects a packet p for transmission before some other backlogged packet q, then the departure time of p is also less than that of q ($d_p \leq d_q$) under fluid-flow WFQ.

d_p	departure time of packet p under fluid-flow WFQ
$\hat{d}_p$	departure time of packet p under packet-level WFQ
C	Rate of link
L_{max}	max. packet size

Property 2: For any packet p, it holds that

$$\hat{d}_p \leq d_p + \frac{L_{\mathsf{max}}}{C}\,.$$