Online Downlink MIMO Wireless Network Virtualization in Fading Environments

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***Abstract*—We consider downlink multiple-input multiple-output (MIMO) wireless network virtualization (WNV) in a fading environment, via a base station (BS) precoding design. The BS is owned by an infrastructure provider (InP) and is shared by several service providers (SPs) who are oblivious to each other. The SPs realize their virtual-cell transmissions via MIMO precoding provided by the InP. We aim to minimize the time-averaged expected deviation of the precoding provided by the InP from the SPs' virtualization demands, considering both long-term and short-term transmit power limits at the BS. We propose an online MIMO WNV algorithm to provide a precoding solution through Lyapunov optimization. Our online precoding solution only requires the current channel state information, and it has a semi-closed form with low computational complexity. We provide an upper bound on the performance of the proposed algorithm, showing that it can be arbitrarily close to the optimum over any given time horizon. Simulation results validate the performance of our proposed algorithm under typical urban micro-cell settings.**

I. Introduction

Spectrum scarcity is a major concern for current and future wireless networks. To address this issue, the concept of wireless network virtualization (WNV) has been proposed as an extension of wired network virtualization to the wireless domain. By abstracting and slicing the physical resources, WNV reduces the capital and operational expenses of wireless networks [1]. A virtualized wireless network generally consists of an infrastructure provider (InP) that creates virtual slices of the physical infrastructure and radio resources, and service providers (SPs) that lease these virtual slices and provide services to their subscribing users under their own management and requirements, unaware of the underlying physical architecture [2]. Compared with wired network virtualization, WNV concerns the sharing of both the wireless hardware and the radio spectrum, which brings new challenges that do not exist in a wired network. Due to the broadcast and random nature of the wireless medium, guaranteeing the isolation among virtual networks is a difficult task [3].

In this work, we focus on downlink WNV of a multiple-input multiple-output (MIMO) system in a fading environment, where several SPs share one InP-owned base station (BS) to serve their subscribing users. Most existing works on WNV in MIMO systems enforce strict physical isolation [4]–[9], which is inherited from wired network virtualization [10]. The system throughput and energy efficiency maximization problems for WNV in OFDM-based massive MIMO systems have been investigated in [4] and [5]. Orthogonal sub-carriers are allocated among the SPs through a two-level hierarchical auction architecture in [6]. Antenna allocation through pricing for virtualized massive MIMO systems has been studied in [7]. The resource allocation problems for uplink MIMO WNV combining the cloud radio networks and non-orthogonal multiple access techniques have been studied in [8] and [9]. In contrast to strict physical isolation, stochastic robust precoding for downlink massive MIMO WNV has been investigated in [11] to allow simultaneous sharing of both antennas and spectrum among the SPs.

Note that all existing works on MIMO WNV have focused on one-shot optimization problems subject to an instantaneous transmit power constraint. Since the long-term average transmit power is an important indicator of energy usage [12], in this work, we consider an online optimization framework that incorporates the long-term average transmit power constraint. We adopt the precoding-based MIMO WNV approach of [11], where each SP makes individual precoding demands to serve its own users ignoring the other SPs, and the InP uses global downlink precoding to serve all users simultaneously, with implicit mitigation of the inter-SP interference. We aim to minimize the long-term time-averaged expected deviation of the received signals due to the InP's actual precoding from those demanded by the SPs. Note that the considered precoding design is stochastic in nature due to channel fading over time. Furthermore, the long-term average transmit power constraint introduces correlation among the precoding solutions over time. This is a challenging problem even in the offline scenario where the channel states are known over the entire time horizon. In practice, future channel states are unknown, which leads to a more complicated online optimization problem. To the best of our knowledge, this is the first work to achieve online MIMO WNV that allows simultaneous sharing of all antennas and spectrum resources among the SPs.

The main contributions of this paper are summarized below:

- We formulate the above downlink MIMO WNV as a precoding problem to allow simultaneous sharing of all antennas and spectrum resources among the SPs for efficient resource allocation. In each time slot, each SP is allowed to demand its own precoder without the need to be aware of the other SPs. The InP designs the global precoder to mitigate the inter-SP interference, and minimizes the deviation of the actual received signals from the desired ones by the SPs' demands at their users.

We accommodate both long-term and short-term transmit power constraints.

- We propose, to the best of our knowledge, the first online MIMO WNV algorithm based on the framework of Lyapunov optimization. Our proposed algorithm determines the downlink precoding only based on the current channel state, and the online precoding solution is in semi-closed form. Our analysis shows that the performance of our proposed algorithm can be arbitrarily close to the optimum over any given time horizon.
- Simulation results demonstrate the fast convergence of the proposed algorithm. Performance studies under typical urban micro-cell network settings validate the proposed algorithm, and demonstrate the performance advantage of the virtualized network enabled by the proposed algorithm over non-virtualized networks.

II. System Model and Problem Formulation

A. System Model

We consider a virtualized MIMO cellular network that is formed by one InP and M SPs. In each cell, the InP owns the BS and performs virtualization. The SPs, oblivious to each other, serve their own subscribing users. Other parts of the networks, including the core network and computational resources, are assumed to be already virtualized.

Consider downlink transmissions in a virtualized cell, where the InP-owned BS is equipped with N antennas. The M SPs share the N antennas at the BS and the spectrum resources provided by the InP. Each SP m has K_m subscribing users. Let $\mathcal{N} = \{1, \dots, N\}$, $\mathcal{M} = \{1, \dots, M\}$, and $\mathcal{K}_m = \{1, \dots, K_m\}$. There is a total of $K = \sum_{m\in\mathcal{M}} K_m$ users in the cell. Let $\mathcal{K} = \{1, \dots, K\}$. We assume $K \leq N$.

We consider a time-slotted system with time indexed by t. Let $\mathbf{H}(t) \in \mathbb{C}^{K\times N}$ denote the MIMO channel state between the BS and all K users at time t. We assume a block fading channel model, so that $\{\mathbf{H}(t)\}$ over time t is independent and identically distributed (i.i.d.). The distribution of $\mathbf{H}(t)$ is unknown and can be arbitrary. We assume that at any time t, the channel gain is bounded by a constant B, given by

$$\|\mathbf{H}(t)\|_F \leq B, \forall t \tag{1}$$

where $\|\cdot\|_F$ denotes the Frobenius norm.

We adopt the spatial virtualization approach first proposed in [11], which is illustrated in Fig. 1. Let $\mathbf{H}_m(t) \in \mathbb{C}^{K_m\times N}$ denote the channel state between the BS and the K_m users of SP m. At each time t, the InP shares $\mathbf{H}_m(t)$ with SP m and allocates transmit power P_m to the SP. Using $\mathbf{H}_m(t)$, each SP m designs its own precoding matrix $\mathbf{W}_m(t) \in \mathbb{C}^{N\times K_m}$, subject to the transmit power limit $\|\mathbf{W}_m(t)\|_F^2 \leq P_m$. SP m then sends $\mathbf{W}_m(t)$ to the InP as a virtual precoding matrix by the SP. For SP m, with $\mathbf{W}_m(t)$, the *desired* received signal vector $\mathbf{y}'_m$ (at its K_m users) in the absent of noise is given by

$$\mathbf{y}'_m(t) = \mathbf{H}_m(t)\mathbf{W}_m(t)\mathbf{x}_m(t), m \in \mathcal{M}$$

where $\mathbf{x}_m(t)$ is the transmitted signal vector for the K_m users. Define $\mathbf{y}'(t) \triangleq [\mathbf{y}'^H_1(t), \dots, \mathbf{y}'^H_M(t)]^H$ as the desired received

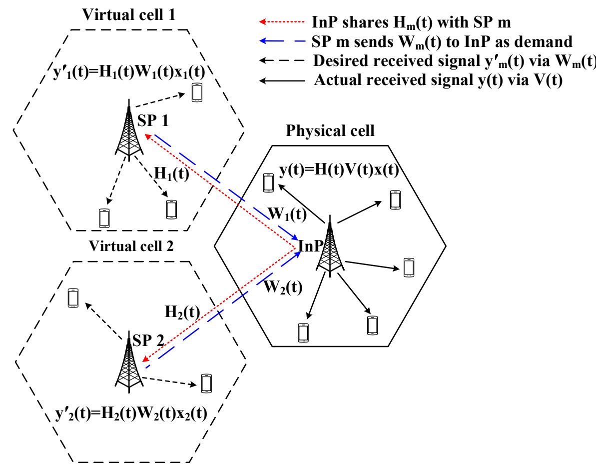

Fig. 1. An illustration of MIMO virtualization in a cell with one InP and two SPs serving its own users in a virtual cell.

signal vector at all K users, we have $\mathbf{y}'(t) = \mathbf{D}(t)\mathbf{x}(t)$ where $\mathbf{D}(t) \triangleq \text{blkdiag}\{\mathbf{H}_1(t)\mathbf{W}_1(t), \dots, \mathbf{H}_M(t)\mathbf{W}_M(t)\} \in \mathbb{C}^{K\times K}$ can be viewed as the virtualization demand made by all SPs, and $\mathbf{x}(t) \triangleq [\mathbf{x}_1^H(t), \dots, \mathbf{x}_M^H(t)]^H$. We assume that at each time t, the transmitted signal to each user is zero-mean with unit power and uncorrelated to each other, *i.e.,* $\mathbb{E}\{\mathbf{x}(t)\} = \mathbf{0}$ and $\mathbb{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{I}, \forall t$.

At each time t, the InP designs the actual downlink precoding matrix $\mathbf{V}(t) \triangleq [\mathbf{V}_1(t), \dots, \mathbf{V}_M(t)] \in \mathbb{C}^{N\times K}$, where $\mathbf{V}_m(t) \in \mathbb{C}^{N\times K_m}$ is the actual downlink precoding matrix for SP m. The *actual* received signal vector $\mathbf{y}_m(t)$ (excluding noise) at the K_m users of SP m is given by

$$\mathbf{y}_m(t) = \mathbf{H}_m(t)\mathbf{V}_m(t)\mathbf{x}_m(t) + \sum_{i\in\mathcal{M}, i\neq m} \mathbf{H}_m(t)\mathbf{V}_i(t)\mathbf{x}_i(t), \forall m \in \mathcal{M}$$

where the second term is the inter-SP interference from the other SPs to the users of SP m. The actual received signal vector $\mathbf{y}(t) \triangleq [\mathbf{y}_1^H(t), \dots, \mathbf{y}_M^H(t)]^H$ at all K users is given by $\mathbf{y}(t) = \mathbf{H}(t)\mathbf{V}(t)\mathbf{x}(t)$.

B. Problem Formulation

For downlink MIMO WNV, the InP designs precoding matrix $\mathbf{V}(t)$ to perform virtualization. Note that at each time t, each SP m designs its own virtual precoding matrix $\mathbf{W}_m(t)$ without considering the inter-SP interference, while the InP designs the actual downlink precoding matrix $\mathbf{V}(t)$ to mitigate the inter-SP interference, in order to meet the virtualization demand $\mathbf{D}(t)$ gathered from the SPs.

With the InP's actual precoding matrix $\mathbf{V}(t)$ and each SP m's virtual precoding matrix $\mathbf{W}_m(t)$, the expected deviation of the received signal vector via the InP's actual precoding from that via the SPs' virtual precoding, is given by

$$\begin{aligned}\mathbb{E}\{\|\mathbf{y}(t) - \mathbf{y}'(t)\|_2^2\} &= \mathbb{E}\{\|[\mathbf{H}(t)\mathbf{V}(t) - \mathbf{D}(t)]\mathbf{x}(t)\|_2^2\} \\ &= \mathbb{E}\{\|\mathbf{H}(t)\mathbf{V}(t) - \mathbf{D}(t)\|_F^2\}.\end{aligned}$$

The goal of the InP is to optimize MIMO precoding to minimize the long-term time-averaged expected precoding deviation from the virtualization demand, subject to both long-term and short-term transmit power constraints. The optimization problem is formulated as follows:

$$\begin{aligned}\mathbf{P1}: \quad \min_{\{\mathbf{V}(t)\}} \quad & \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{\|\mathbf{H}(t)\mathbf{V}(t)-\mathbf{D}(t)\|_F^2\} \\ \text{s.t.} \quad & \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\|\mathbf{V}(t)\|_F^2 \le \bar{P}, \qquad (2) \\ & \|\mathbf{V}(t)\|_F^2 \le P_{\max} \qquad (3)\end{aligned}$$

where $\bar{P}$ is the long-term transmit power limit, and $P_{\max}$ is the maximum transmit power limit. Both power limits are set by the InP, and we assume $\bar{P} \le P_{\max}$ to avoid triviality. With random channel state $\mathbf{H}(t)$, **P1** is a stochastic optimization problem. It is challenging to solve, especially when the distribution of $\mathbf{H}(t)$ is unknown.[1]

With only the instantaneous channel state $\mathbf{H}(t)$ available at each time t, in this work, we aim to develop an online MIMO WNV algorithm based on $\mathbf{H}(t)$ and $\mathbf{D}(t)$ for a precoding solution $\{\mathbf{V}(t)\}$ to **P1**.

III. Online MIMO Virtualization Algorithm

In this section, we develop an online MIMO WNV algorithm by exploring Lyapunov optimization technique [13].

A. Online Optimization Formulation

To design an online algorithm to solve **P1**, we introduce a virtual queue $Z(t)$ for the long-term average transmit power constraint (2) with the updating rule given by

$$Z(t+1) = \max\{Z(t) + \|\mathbf{V}(t)\|_F^2 - \bar{P}, 0\}. \qquad (4)$$

Define $L(t) \triangleq \frac{1}{2}Z^2(t)$ as the quadratic Lyapunov function and $\Delta(t) \triangleq L(t+1) - L(t)$ as the corresponding Lyapunov drift at time t. Instead of minimizing the objective in **P1** directly under the long-term average constraint, we minimize the objective while stabilize the virtual queue through minimizing a drift-plus-penalty (DPP) metric [13], defined as $\mathbb{E}\{\Delta(t)|Z(t)\} + U\mathbb{E}\{\rho(t)|Z(t)\}$, where $\rho(t) = \|\mathbf{H}(t)\mathbf{V}(t) - \mathbf{D}(t)\|_F^2$ and $U > 0$ is the relative weight. The DPP metric is a weighted sum of the conditional expectation on the Lyapunov drift $\Delta(t)$ and the penalty $\rho(t)$ on precoding deviation, given the current virtual queue length $Z(t)$. We first provide an upper bound for the DPP metric in the following Lemma. The proof follows standard Lyapunov optimization techniques [13] and is omitted due to space constraint.

Lemma 1. At each time t, for any precoding design of $\mathbf{V}(t)$, the DPP metric is upper bounded for all $Z(t)$ and $U > 0$ as

$$\begin{aligned}&\mathbb{E}\{\Delta(t)|Z(t)\} + U\mathbb{E}\{\rho(t)|Z(t)\} \\ &\le S + U\mathbb{E}\{\rho(t)|Z(t)\} + Z(t)\mathbb{E}\{\|\mathbf{V}(t)\|_F^2 - \bar{P}|Z(t)\} \qquad (5)\end{aligned}$$

where $S \triangleq \frac{1}{2}\max\{(P_{\max} - \bar{P})^2, \bar{P}^2\}$.

[1]If the channel distribution is known, it is possible to solve **P1** through Dynamic Programming (DP). However, the DP method faces the curse of dimensionality in computational complexity and is impractical for real systems.

Algorithm 1 Online MIMO WNV Algorithm

1: Choose an appropriate constant $U > 0$ and let $Z(0) = 0$. At each time t, observe $\mathbf{H}(t)$ and $Z(t)$, and then do the following:
2: Solve the per-slot problem **P2** for $\mathbf{V}^\star(t)$ (see Section III-B).
3: Update $Z(t+1) = \max\{Z(t) + \|\mathbf{V}^\star(t)\|_F^2 - \bar{P}, 0\}$.

Minimizing the DPP metric directly is still difficult due to the dynamics involved in the Lyapunov drift $\Delta(t)$. Instead, we minimize its upper bound given in Lemma 1 which is no longer a function of $\Delta(t)$. Specifically, given $\mathbf{H}(t)$ at each time t, we consider the per-slot version of the upper bound in (5) by removing the conditional expectation. Further removing the constant terms in the upper bound, the resulting per-slot optimization problem is given as follows:

$$\begin{aligned}\mathbf{P2}: \quad \min_{\mathbf{V}(t)} \quad & U\|\mathbf{H}(t)\mathbf{V}(t) - \mathbf{D}(t)\|_F^2 + Z(t)\|\mathbf{V}(t)\|_F^2 \\ \text{s.t.} \quad & \|\mathbf{V}(t)\|_F^2 \le P_{\max}. \qquad (6)\end{aligned}$$

Note that **P2** is a per-slot precoding optimization problem under the current channel state $\mathbf{H}(t)$ and virtual queue length $Z(t)$, subject to the short-term transmit power constraint only. Compared with the original **P1**, the long-term time-averaged expected objective is modified to the per-slot version of the DPP metric in **P2**, where the long-term average constraint (2) is converted to the queue stability in $Z(t)$ as part of the DPP metric. Solving **P2**, we obtain the optimal precoding matrix $\mathbf{V}^\star(t)$ for **P2** at each time t, which we use to update the virtual queue $Z(t)$ according to (4). The online MIMO WNV algorithm is outlined in Algorithm 1.

B. Online Precoding Solution to P2

Now we show how to solve **P2** to obtain the optimal $\mathbf{V}^\star(t)$. Since **P2** is a per-slot optimization problem at time t, without causing ambiguity, in the following, we omit time index t for notation simplicity. Since **P2** is a convex optimization problem satisfying the Slater's condition, the strong duality holds. We solve **P2** by studying the KKT conditions [14]. The Lagrange function for **P2** is given by

$$L(\mathbf{V}, \lambda) = U\|\mathbf{H}\mathbf{V} - \mathbf{D}\|_F^2 + Z\|\mathbf{V}\|_F^2 + \lambda(\|\mathbf{V}\|_F^2 - P_{\max})$$

where λ is the Lagrangian multiplier associated with constraint (6). Taking partial derivative of $L(\mathbf{V}, \lambda)$ with respect to $\mathbf{V}^*$ to $\mathbf{0}$, we have

$$\frac{\partial L(\mathbf{V},\lambda)}{\partial \mathbf{V}^*} = U(\mathbf{H}^H\mathbf{H}\mathbf{V} - \mathbf{H}^H\mathbf{D}) + (Z+\lambda)\mathbf{V} = \mathbf{0} \qquad (7)$$

which follows from $\|\mathbf{A}\|_F^2 = \mathrm{tr}\{\mathbf{A}\mathbf{A}^H\}$, $\frac{\partial\,\mathrm{tr}\{\mathbf{A}\mathbf{B}^H\}}{\partial \mathbf{B}^*} = \mathbf{A}$, and $\frac{\partial\,\mathrm{tr}\{\mathbf{A}\mathbf{B}\}}{\partial \mathbf{B}^*} = \mathbf{0}$. The KKT conditions for $(\mathbf{V}^\star, \lambda^\star)$ being globally optimal are given by

$$\left(\mathbf{H}^H\mathbf{H} + \frac{Z+\lambda^\star}{U}\mathbf{I}\right)\mathbf{V}^\star = \mathbf{H}^H\mathbf{D}, \qquad (8)$$

$$\|\mathbf{V}^\star\|_F^2 - P_{\max} \le 0, \qquad (9)$$

$$\lambda^\star \ge 0, \qquad (10)$$

$$\lambda^\star(\|\mathbf{V}^\star\|_F^2 - P_{\max}) = 0 \qquad (11)$$

where (8) is derived from (7). We now discuss conditions (8)-(11) to find the optimal solution. Note that $Z \geq 0$ by (4). We have the following cases.

1) $Z + \lambda^\star > 0$: From (8), $\mathbf{H}^H\mathbf{H} + \frac{Z+\lambda^\star}{U}\mathbf{I} \succ \mathbf{0}$, which implies that it is invertible, and we have

$$\mathbf{V}^\star = \left(\mathbf{H}^H\mathbf{H} + \frac{Z+\lambda^\star}{U}\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{D}. \tag{12}$$

We discuss the solution in two subcases: 1.i) If $Z > 0$: By (9), $\mathbf{V}^\star$ in (12) is the optimal solution with $\lambda^\star = 0$, if $\|(\mathbf{H}^H\mathbf{H} + \frac{Z}{U}\mathbf{I})^{-1}\mathbf{H}^H\mathbf{D}\|_F^2 \leq P_{\max}$. Otherwise, by (11), $\mathbf{V}^\star$ in (12) is the optimal solution with $\lambda^\star > 0$ such that $\|(\mathbf{H}^H\mathbf{H} + \frac{Z+\lambda^\star}{U}\mathbf{I})^{-1}\mathbf{H}^H\mathbf{D}\|_F^2 = P_{\max}$. 1.ii) If $Z = 0$: It follows that $\lambda^\star > 0$. By (11), we have $\lambda^\star > 0$ such that $\|(\mathbf{H}^H\mathbf{H} + \frac{\lambda^\star}{U}\mathbf{I})^{-1}\mathbf{H}^H\mathbf{D}\|_F^2 = P_{\max}$, and $\mathbf{V}^\star$ in (12) with $Z = 0$ is the optimal solution.

2) $Z = \lambda^\star = 0$: From (8), the optimal solution must satisfy

$$\mathbf{H}^H\mathbf{H}\mathbf{V}^\star = \mathbf{H}^H\mathbf{D}. \tag{13}$$

Recall that we assume $K \leq N$. We categorize (13) in two subcases: 2.i) If $K < N$: $\mathbf{H}^H\mathbf{H} \in \mathbb{C}^{N\times N}$ is a rank deficient matrix, and there are infinitely many solutions for $\mathbf{V}^\star$. We choose $\mathbf{V}^\star$ to minimize $\|\mathbf{V}^\star\|_F^2$ subject to (13), which is an under-determined least square problem with a closed-form solution:

$$\mathbf{V}^\star = \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\mathbf{D}. \tag{14}$$

By (9), if $\|\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\mathbf{D}\|_F^2 \leq P_{\max}$, then $\mathbf{V}^\star$ in (14) is the optimal solution. 2.ii) If $K = N$: $\mathbf{H}^H\mathbf{H} \in \mathbb{C}^{N\times N}$ is of full rank[2], and we have a unique solution:

$$\mathbf{V}^\star = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{D}. \tag{15}$$

Again, If $\|\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{D}\|_F^2 \leq P_{\max}$, then $\mathbf{V}^\star$ in (15) is the optimal solution. For both subcases 2.i) and 2.ii), $\mathbf{V}^\star$ in (14) and (15) cannot satisfy (9), which means the condition in Case 2) does not hold at optimality, *i.e.,* $\lambda^\star > 0$, and the optimal solution is given by Case 1).

Note that if $\lambda^\star = 0$ at optimality, we have a closed-form solution for $\mathbf{V}^\star(t)$ in (14) or (15). Otherwise, we have a semi-closed form solution for $\mathbf{V}^\star(t)$ in (12), where $\lambda^\star > 0$ can be obtained by bi-section search to satisfy (11).

IV. Performance Analysis

In this section, we provide performance bounds for our proposed online MIMO WNV algorithm. Note that Algorithm 1 is applicable to any precoding schemes adopted by the SPs. In the following, we focus on the maximum ratio transmission (MRT) precoding scheme. The analysis can be extended to more general precoding schemes as well.

[2]Since the channels from the BS to users are independent, $\mathbf{H}(t) \in \mathbb{C}^{K\times N}$ is of full rank at each time t. The independent channel assumption is typically satisfied in practice for users at different locations.

We assume each SP m uses the following MRT precoding matrix to maximize the received signal-to-noise ratio (SNR) at each time t:

$$\mathbf{W}_m(t) = \alpha_m(t)\mathbf{H}_m^H(t) \tag{16}$$

where $\alpha_m(t) = \sqrt{\frac{P_m}{\|\mathbf{H}_m(t)\|_F^2}}$ is a power normalizing factor.

We first show in the following lemma that in Algorithm 1, the virtual queue $Z(t)$ at each time t is upper bounded.

Lemma 2. By Algorithm 1, $Z(t)$ at each time t is upper bounded by

$$Z(t) \leq UB^2\xi + P_{\max} - \bar{P} \tag{17}$$

where $\xi \triangleq \sqrt{\frac{N}{\bar{P}}\sum_{m\in\mathcal{M}} P_m}$.

Proof: See Appendix A.

For channel state $\mathbf{H}(t)$ being i.i.d. over time t, there exists a stationary randomized optimal precoding solution to **P1**, which depends only on the distribution of $\mathbf{H}(t)$, and achieves the minimum objective value of **P1** denoted as ρ^{opt} [13]. Leveraging the key results in Lemma 1 and Lemma 2, the following theorem provides performance bounds for Algorithm 1 over any given time horizon T.

Theorem 3. Given any $\epsilon > 0$, set $U = \frac{S}{\epsilon}$ in Algorithm 1, where constant S is defined below (5). For any $T > 0$, the following bounds hold regardless of the distribution of channel state $\mathbf{H}(t)$:

$$\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left\{\|\mathbf{H}(t)\mathbf{V}^\star(t) - \mathbf{D}(t)\|_F^2\right\} \leq \rho^{\text{opt}} + \epsilon, \tag{18}$$

$$\frac{1}{T}\sum_{t=0}^{T-1}\|\mathbf{V}^\star(t)\|_F^2 \leq \bar{P} + \frac{SB^2\xi + \epsilon(P_{\max} - \bar{P})}{\epsilon T}. \tag{19}$$

Proof: The proof of (18) utilizes the Lyapunov optimization techniques [13] and key results in Lemma 1. Details are omitted due to space constraint. Note that (18) only holds if $Z(0) = 0$. The proof of (19) is as follows. For any time t, we have $Z(t+1) \geq Z(t) + \|\mathbf{V}^\star(t)\|_F^2 - \bar{P}$ from (4). Rearranging terms as $\|\mathbf{V}^\star(t)\|_F^2 \leq \bar{P} + Z(t+1) - Z(t)$, summing over t and dividing by T gives $\frac{1}{T}\sum_{t=0}^{T-1}\|\mathbf{V}^\star(t)\|_F^2 \leq \bar{P} + \frac{Z(T)-Z(0)}{T} \overset{(a)}{=} \bar{P} + \frac{Z(T)}{T}$, where (a) follows from $Z(0) = 0$. We complete the proof of (19) by substituting S, U, and the virtual queue upper bound in Lemma 2 into the above inequality. ■

In Theorem 3, (18) provides a performance bound of the objective value in **P1**, *i.e.,* the time-averaged expected precoding deviation from the virtualization demand. It indicates that, for any given time horizon T, the performance of Algorithm 1 can be arbitrarily close to the optimum, where the performance gap ϵ is a controllable constant by our design and can be set arbitrarily small. Furthermore, (19) provides a performance bound of the time-averaged transmit power over any given T. The bound indicates for all $T \geq \frac{1}{\epsilon^2}$, Algorithm 1 guarantees that the deviation from the long-term transmit power limit $\bar{P}$ is within $O(\epsilon)$. Finally, notice that as $T \to \infty$, (19) becomes the long-term average transmit power constraint (2).

V. Simulation Results

We consider an InP that owns a BS equipped with $N = 30$ antennas at the center of an urban hexagon micro-cell of 500 m radius. It serves $M = 4$ SPs, each with 2 to 5 users uniformly distributed in the cell. We set $P_{\text{max}} = 16$ dBm, noise spectral density $N_0 = -174$ dBm/Hz, noise figure $N_F = 10$ dB, and channel bandwidth $B_W = 10$ kHz as default system parameters. The fading channel from the BS to user k is modeled as $\mathbf{h}_k = \sqrt{\beta_k}\mathbf{g}_k$, where $\mathbf{g}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$, and $\beta_k[\text{dB}] = -31.54 - 33\log_{10}(d_k) - \psi_k$ captures path-loss and shadowing, with d_k being the distance from the BS to user k and $\psi_k \sim \mathcal{CN}(0, \sigma_\phi^2)$ being the shadowing with $\sigma_\phi = 8$ dB. We set time slot duration to be 1 sec. Note that Algorithm 1 is not limited to a specific channel model as we only assume the channel gain is bounded in (1). We set $B = 1.645\sqrt{N\sum_{k\in\mathcal{K}}\beta_k}$, which gives a Chernoff upper bound of 3.8×10^{-9} for the probability of bound violation $\mathbb{P}\{\|\mathbf{H}(t)\|_F > B\}$.

For performance study of Algorithm 1, we define the normalized time-averaged precoding deviation from the virtualization demand as

$$\bar{\rho}(T) \triangleq \frac{\frac{1}{T}\sum_{t=0}^{T-1}\|\mathbf{H}(t)\mathbf{V}^\star(t) - \mathbf{D}(t)\|_F^2}{\frac{1}{T}\sum_{t=0}^{T-1}\|\mathbf{D}(t)\|_F^2}$$

and the time-averaged downlink transmit power as

$$\bar{P}(T) \triangleq \frac{1}{T}\sum_{t=0}^{T-1}\|\mathbf{V}^\star(t)\|_F^2.$$

We assume each SP m is allocated with $P_m = \frac{P_{\text{max}}}{M}$ transmit power such that $\sum_{m\in\mathcal{M}} P_m = P_{\text{max}}$ in our simulation.

We first study the effect of weighting factor $U = \frac{S}{\epsilon}$ in Algorithm 1. We set $\epsilon = \theta B^2 P_{\text{max}}$ since $\|\mathbf{D}(t)\|_F^2 \le \sum_{m\in\mathcal{M}}\|\mathbf{H}_m\|_F^2\|\mathbf{W}_m\|_F^2 \le B^2 P_{\text{max}}, \forall t$, and study the performance dependency on U by varying θ. Fig. 2 shows the time trajectory of $\bar{\rho}(T)$ and $\bar{P}(T)$ under different values of θ with $\bar{P} = 14$ dBm. We observe fast convergence of our proposed algorithm (within 100 time slots). As θ decreases, U is larger, more emphasis is on the precoding deviation $\rho(t)$, and less on the Lyapunov drift $\Delta(t)$ in the DPP metric. As a result, it takes a longer time for the virtual queue to be stabilized, and the performance to reach steady state. Furthermore, at steady state, $\bar{\rho}(T)$ decreases as θ decreases, and $\bar{P}(T)$ converges to $\bar{P}$. These are consistent with Theorems 3. For the remaining simulation results, $\theta = 0.1\%$ is used as the default value.

Fig. 3 shows $\bar{\rho}(T)$ and $\bar{P}(T)$ under different long-term transmit power limit $\bar{P}$. The case of $\bar{P} = \infty$ refers to removing the long-term average transmit power constraint from **P1**. At steady state, $\bar{\rho}(T)$ is around 0.5% when $\bar{P} = \infty$. The steady-state value of $\bar{\rho}(T)$ is only around 2% when $\bar{P} = 14$ dBm. Note that there is a natural trade-off between $\bar{P}$ and $\bar{\rho}(T)$, which allows the InP to balance the power consumption and the deviation of actual precoding from the virtualization demand.

We further compare the performance between virtualized and non-virtualized networks. All users share the channel

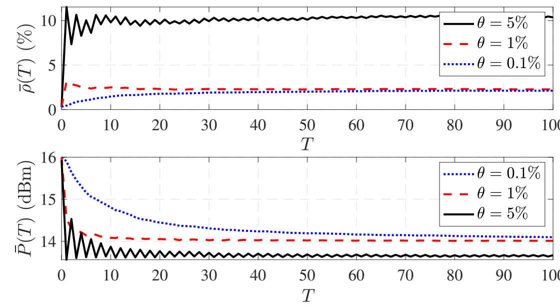

Fig. 2. $\bar{\rho}(T)$ and $\bar{P}(T)$ vs. T under different θ.

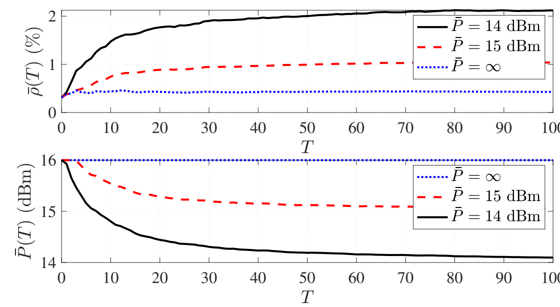

Fig. 3. $\bar{\rho}(T)$ and $\bar{P}(T)$ vs. T under different $\bar{P}$.

bandwidth B_W simultaneously in the non-virtualized network. We assume the InP directly serves all users and optimizes the transmit power of MRT precoding to maximize the long-term time-averaged expected rate subject to both long-term and short-term transmit power constraints as follows:

$$\begin{aligned}\textbf{P3}: \quad \min_{\{\alpha(t)\}} \quad & \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left\{-\sum_{k\in\mathcal{K}} R_k(t)\right\} \\ \text{s.t.} \quad & \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\|\alpha(t)\mathbf{W}(t)\|_F^2 \le \bar{P}, \\ & \|\alpha(t)\mathbf{W}(t)\|_F^2 \le P_{\text{max}}\end{aligned}$$

where $R_k(t) = \log_2\left(1 + \frac{\alpha^2(t)|\mathbf{h}_k^T(t)\mathbf{w}_k(t)|^2}{\sigma_n^2 + \alpha^2(t)\sum_{k'\in\mathcal{K}, k'\neq k}|\mathbf{h}_{k'}^T(t)\mathbf{w}_{k'}(t)|^2}\right)$, and $\mathbf{W}(t) = \mathbf{H}^H(t)$. The solution to **P3** is omitted due to space constraint.

Fig. 4 shows the time-averaged rate per user $\bar{R}(T) \triangleq \frac{1}{K}\sum_{k\in\mathcal{K}}\log_2\left(1 + \frac{|\mathbf{h}_k^T(t)\mathbf{v}_k(t)|^2}{\sigma_n^2 + \sum_{k'\in\mathcal{K}, k'\neq k}|\mathbf{h}_{k'}^T(t)\mathbf{v}_{k'}(t)|^2}\right)$ achieved by the virtualized and non-virtualized networks with $\bar{P} = 15$ dBm. Note that the rate demand from the SPs is calculated by $\frac{1}{K}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}_m}\log_2\left(1 + \frac{|\mathbf{h}_{m,k}^T(t)\mathbf{w}_{m,k}(t)|^2}{\sigma_n^2 + \sum_{k'\in\mathcal{K}_m, k'\neq k}|\mathbf{h}_{m,k'}^T(t)\mathbf{w}_{m,k'}(t)|^2}\right)$. We observe that it is higher than the actual rate achieved, since the SPs request maximum transmit power $P_{\text{max}} = 16$ dBm and design their virtual precoding matrices without considering the inter-SP interference.

Compared with the non-virtualized network, a virtualized network using the proposed algorithm achieves higher rate. This is because our downlink precoding minimizes the precoding deviation from the virtualization demand, while implicitly mitigating the inter-SP interference. This indirectly increases the rates of all SPs.

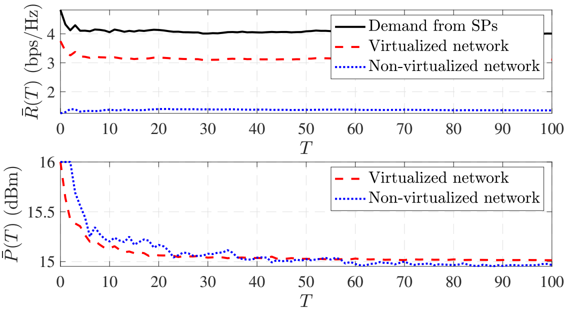

Fig. 4. Comparison of $\bar{R}(T)$ between virtualized and non-virtualized networks.

VI. Conclusions

This paper has considered the precoding design for downlink MIMO virtualization in a fading environment. We have proposed an online precoding algorithm to minimize the long-term time-averaged expected deviation of the actual precoding solution by the InP from the virtualization demands set by the SPs, subject to both long-term and short-term transmit power constraints. The proposed algorithm only depends on the current channel state, without requiring knowledge of the channel distribution, and the online precoding solution is in a semi-closed form. Our analysis have shown that the performance of the proposed algorithm can be arbitrarily close to the optimum over any given time horizon. Simulation results under typical urban micro-cell settings have validated the performance of the proposed algorithm.

Appendix A
Proof of Lemma 2

We omit time index t for notation simplicity. Let $\mathbf{H}_m^H\mathbf{H}_m = \mathbf{U}_m\mathbf{\Sigma}_m\mathbf{U}_m^H$, where $\mathbf{U}_m$ is an unitary matrix and $\mathbf{\Sigma}_m = \text{diag}(\sigma_{m,1},\dots,\sigma_{m,N})$. Let $\mathbf{H}^H\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H$, where $\mathbf{U}$ is an unitary matrix, and $\mathbf{\Sigma} = \text{diag}(\sigma_1,\dots,\sigma_N)$. It follows that $\mathbf{H}^H\mathbf{H} + \frac{Z+\lambda^\star}{U}\mathbf{I} = \mathbf{U}\mathbf{\Sigma}'\mathbf{U}^H$, where $\mathbf{\Sigma}' = \text{diag}(\sigma'_1,\dots,\sigma'_N)$ and $\sigma'_j = \sigma_j + \frac{Z+\lambda^\star}{U}, \forall j \in \mathcal{N}$. If $Z > 0$, $\mathbf{V}^\star$ is given by (12) in Section III-B, and we have

$$\begin{aligned}
\|\mathbf{V}^\star\|_F^2 &= \text{tr}\left\{\mathbf{H}^H\mathbf{D}\mathbf{D}^H\mathbf{H}\left(\mathbf{H}^H\mathbf{H} + \frac{Z+\lambda^\star}{U}\mathbf{I}\right)^{-2}\right\} \\
&\overset{(a)}{=} \sum_{m\in\mathcal{M}} \text{tr}\left\{\alpha_m^2\mathbf{U}_m\mathbf{\Sigma}_m^3\mathbf{U}_m^H\mathbf{U}\mathbf{\Sigma}'^{-2}\mathbf{U}^H\right\} \\
&\overset{(b)}{=} \sum_{m\in\mathcal{M}} \text{tr}\left\{\sum_{i\in\mathcal{N}}\alpha_m^2\sigma_{m,i}^3\mathbf{u}_{m,i}\mathbf{u}_{m,i}^H\sum_{j\in\mathcal{N}}\sigma'^{-2}_j\mathbf{u}_j\mathbf{u}_j^H\right\} \\
&= \sum_{m\in\mathcal{M}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\sigma'^{-2}_j\alpha_m^2\sigma_{m,i}^3(\mathbf{u}_{m,i}^H\mathbf{u}_j)(\mathbf{u}_j^H\mathbf{u}_{m,i}) \\
&\overset{(c)}{\le} \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{N}}\sigma'^{-2}_j\frac{P_m}{\sum_{i\in\mathcal{N}}\sigma_{m,i}}\sum_{i\in\mathcal{N}}\sigma_{m,i}^3 \\
&\overset{(d)}{\le} \sum_{m\in\mathcal{M}}\sum_{j\in\mathcal{N}}\sigma'^{-2}_j\frac{P_m}{\sum_{i\in\mathcal{N}}\sigma_{m,i}}B^4\sum_{i\in\mathcal{N}}\sigma_{m,i} \\
&\overset{(e)}{\le} \frac{U^2B^4N\sum_{m\in\mathcal{M}}P_m}{Z^2}
\end{aligned} \tag{20}$$

where (a) follows from (16) and

$$\mathbf{H}^H\mathbf{D}\mathbf{D}^H\mathbf{H} = \sum_{m\in\mathcal{M}}\alpha_m^2(\mathbf{H}_m^H\mathbf{H}_m)^3 = \sum_{m\in\mathcal{M}}\alpha_m^2\mathbf{U}_m\mathbf{\Sigma}_m^3\mathbf{U}_m^H;$$

in (b), $\mathbf{u}_{m,i}$ is the i-th column vector of $\mathbf{U}_m$ and $\mathbf{u}_j$ is the j-th column vector of $\mathbf{U}$; (c) follows from $\alpha_m^2 = \frac{P_m}{\|\mathbf{H}_m\|_F^2} = \frac{P_m}{\sum_{i\in\mathcal{N}}\sigma_{m,i}}, \forall m \in \mathcal{M}$, the Cauchy Schwartz inequality $\mathbf{u}_{m,i}^H\mathbf{u}_j \le \sqrt{\|\mathbf{u}_{m,i}\|_2^2\|\mathbf{u}_j\|_2^2}$, and $\|\mathbf{u}_{m,i}\|_2 = \|\mathbf{u}_j\|_2 = 1, \forall j \in \mathcal{N}, \forall m \in \mathcal{M}$; (d) follows from $\sigma_{m,i} \le \sum_{m\in\mathcal{M}}\text{tr}\{\mathbf{\Sigma}_m\} = \sum_{m\in\mathcal{M}}\|\mathbf{H}_m\|_F^2 \le B^2$; and (e) is because $\sigma'^{-2}_j = (\sigma_j + \frac{Z+\lambda^\star}{U})^{-2} \le \frac{U^2}{Z^2}$, $\forall j \in \mathcal{N}$, due to $\sigma_j \ge 0$ and $\lambda^\star \ge 0$. From (20), we have the following sufficient condition for $Z(t)$ to ensure $\|\mathbf{V}^\star(t)\|_F^2 \le \bar{P}$ for any time t:

$$Z(t) \ge UB^2\xi. \tag{21}$$

If (21) holds, $\|\mathbf{V}^\star(t)\|_F^2 \le \bar{P}$, and by (4), the virtual queue decreases, *i.e.,* $Z(t+1) \le Z(t)$. Otherwise, the maximum increase of virtual queue is $P_{\text{max}} - \bar{P}$, *i.e.,* $Z(t+1) \le Z(t) + P_{\text{max}} - \bar{P}$. Thus, the virtual queue is upper bounded as in (17) at any time t. ∎

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