Efficient Power Allocation in Cooperative OFDM System with Channel Variation

Morteza Ibrahimi and Ben Liang
Dept. Electrical and Computer Engineering, University of Toronto
Email: {mibrahimi, liang}@comm.utoronto.ca

***Abstract*—Cooperative communication is emerging as an effective approach for realizing efficient wireless networks. Performance of these networks has been shown to be enhanced significantly by dynamic resource allocation, especially in orthogonal frequency division multiplexing (OFDM) systems, where there are more degrees of freedom. On the other hand, dynamic resource allocation imposes signalling and computational overhead on the system. In this paper, a multi-relay OFDM system is considered, where the cooperation gain of distributed antenna array is exploited. First we introduce the optimal power allocation problem and discuss the signaling overhead for implementing the optimal solution. We then propose suboptimal schemes with considerably less overhead and study the conditions under which they perform close to the optimal scheme. Furthermore, we investigate how imperfect implementation of these scheme results in performance degradation. We also analyze how much feedback is needed to implement this scheme with a desirable accuracy.**

I. Introduction

Resource allocation has been proven effective in improving the performance of communication systems in many cases. Resources available to be allocated are traditionally bandwidth, time, and power. The recent introduction of cooperative communication has opened a new front in resource allocation. In these networks a group of nodes cooperate to communicate with one or many destinations. Since in this setting many entities participate to accomplish a shared goal, the resource allocation problem arises naturally. A number of strategies for cooperation can be found in the literature, but essentially a node can cooperate as a relay in two ways [1]. A relay can try to decode the data it receives and retransmit the re-encoded data to the destination. This strategy is known as decode and forward. Alternatively, a relay can retransmit an amplified version of it's received signal. This method is known as amplify and forward.

In this paper we investigate into power allocation (PA) and its signaling overhead for a cooperative OFDM system with multiple relays and coherent reception. We isolate one communication session and consider a source, a destination, and many relays cooperating using the amplify-and-forward strategy. We also assume phase and frequency synchronization leading to coherent reception of relay signals at the destination. While this assumption is still out of reach for practical systems, the considerable amount of research on this topic suggests that it can be possible to implement this scheme, see for example [2] and [3] and the references there in. All communications are assumed to be done using the orthogonal frequency division multiplexing (OFDM) technique, and power needs to be allocated among subchannels at the source and among all relays.

This work was supported in part by a grant from LG Electronics.

The system under consideration is a broadband fixed or slowly varying system where feedback information can be obtain within a reasonable time and cost but at the same time cannot be perfect and costless. A good example is IEEE 802.16. We consider OFDM as the underlying communication technique since it has emerged as a major candidate for the future broadband wireless systems and also it has been shown that optimal resource allocation can result in significant performance improvement in these systems. At the same time, however, using OFDM adds another dimension to our problem since optimal PA needs to also determine power distribution among all subchannels.

We consider the optimal PA problem to maximize the sum of achievable data-rate from the source to the destination, given a power budget at the source and a sum power budget for relays. In a practical system, the sum power budget for relays cannot be very high due to the excessive interference it causes to the other source-relay-destination clusters in the network when all relays transmit at the same time. Furthermore, the gain of increasing the relaying power budget also decreases as the relaying power increases because of the noise amplification effect of the amplify-and-forward strategy. Therefore, it is unlikely that a relay reaches its power limit while the system maintains the total relaying power constraint.

Noting that the optimal PA problem is non-convex in general and its solution requires excessive control signalling, we propose a suboptimal PA scheme that has less overhead and implementation complexity than but close performance to the optimal solution. We call it *Cooperative Channel Equalization* (CCE), since PA in this scheme is aimed at improving the equivalent channel between the source and the destination. More specifically, in this scheme PA among subchannels at the source is uniform, and *information exchange is necessary only between the destination and the relays*. We further propose a variant of CCE with subchannel selection (CCE-S) to trade-off signaling overhead for better performance. We then compare the relative performance, overhead, and channel estimation error resilience of CCE and CCE-S against other optimal and suboptimal PA schemes.

The rest of this paper is organized as follow. In the next section we briefly present some related work. Section

III contains the system description and problem statement. In section IV, the optimal PA problem is formulated and suboptimal schemes are presented and analyzed. Section V is devoted to an investigation into signaling implementation of these schemes. Section VI contains numerical and comparative results and section VII concludes the paper.

II. Related Works

There is a vast amount of literature on optimal resource allocation. This problem has been of greater interest in OFDM systems since considerable performance improvement can be obtained by optimal resource allocation in these systems, see for example [4]. On the other hand, the amount of feedback needed has always been an obstacle in realizing optimal resource allocation especially in OFDM systems when there are many subchannels. To cope with this problem, the authors in [5] proposed a limited feedback scheme and showed that it can achieve a great proportion of the benefits of the optimal resource allocation in OFDM systems.

The idea of using relays can be traced to the information theoretic work of van der Meulen [6]. It was greatly improved by the work of Cover and El Gamal [1]. The idea was then almost forgotten about until recently when practical cooperation strategies and their performance analysis have been introduced [7][8][9][10]. In term of optimal PA for relay networks there are numerous works reported in the publications. These works usually quantify the gain of optimal PA in different cases and given different performance criteria. For example, the authors in [11] assumed a three node configuration and introduced capacity bounds and then showed that optimal PA in the presence of full CSI leads to higher rates especially for half-duplex relaying in time division mode. In [12] the authors considered a regenerative channel with Rayleigh fading. Again they considered the single relay case and compared performance in the full and partial CSI case with uniform power loading.

There is also reported research that focuses particularly on relaying in OFDM systems. In [13] a single relay is considered. Then, given a fixed power distribution at the source (or relay), water-filling among subchannels is shown to be optimal at the relay (or source). An iterative approaches is suggested to utilize these two interrelated solutions and its convergence is shown through numerical investigation. In [14], a multi-relay network is considered and the capacity is maximized over the relays' gain distribution using the dominant eigenvalue technique. A closed form for relaying gains is presented in this work.

The authors in [15] proposed the problem of jointly optimizing power distribution among the subchannels at the relay and source (base station), relay selection, and relaying strategy selection. They argued that the duality gap is zero for the optimization problem they stated and then broke down the problem into subproblems using the dual method. The subproblems are correlated by dual variables and a hierarchical algorithm is proposed to solve the global problem. See [15] and the references there for a more detailed discussion of the previous works.

The main differences between our work and the works mentioned above is that here we consider an OFDM system and jointly optimized PA among the source, relays and all subchannels in a cooperative network with multiple relays and coherent reception. Further, we propose efficient suboptimal PA schemes and discuss the feedback requirement and the effect of imperfection in the implementation of these PA scheme on the system performance.

III. System Description and Problem Statement

The model we consider is an OFDM wireless system where a source wants to communicate with a destination and also there are some nodes who can listen to this transmission and act as relays. In an OFDM system, the channel can be considered as a set of orthogonal subchannels, such that the received signal in each subchannel is independent of the signals of other subchannels [16]. Following the common approach, for practical reasons the system is assumed to be half-duplex, which means that nodes cannot receive and transmit at the same time and in the same frequency band. The relays implement the amplify-and-forward strategy in a time division fashion: a symbol duration is divided into two time slots. In the first time slot, the source transmits its message, and the destination and relays listen. In the second time slot, all the relays synchronously transmit an amplified version of their received signals, and the destination receives the sum of the signals from all relays.

Let R and S represent the set of available subchannels and relays respectively, and let $K = |R|$ and $N = |S|$ be the number of relays and subchannels. The channel is considered to be flat for each subchannel, and its channel gain can be represented by a complex number. The source-relay channels are represented by an $N \times K$ matrix $H = \{h_{ij}\}$ where h_{ij} is subchannel i's gain between the source and relay j. The destination is represented as node 0 throughout this paper. We define $T_{N\times K} = \{t_{ij}\}$ in the same way as H for channels between the relays and the destination. Let $Z_{N\times K} = \{z_{ij}\}$ be the noise matrix where z_{ij} is zero-mean white Gaussian noise which adds to the signal at relay j in subchannel i. Also, let $z_{io}^{(1)}$ and $z_{io}^{(2)}$ be zero-mean white Gaussian noise which adds to the signal at the destination in subchannel i in the first and second time slots respectively.

The received and transmited signals of relay j in subchannel i are respectively

$$\begin{aligned} r_{ij} &= h_{ij} s_i + z_{ij} \\ w_{ij} &= \beta_{ij} r_{ij} \ . \end{aligned} \tag{1}$$

The above equation states that relay j amplifies its received signal in subchannel i with an amplification factor β_{ij} and transmits the resulting signal, w_{ij}. The destination combines the signals from all relays and the signal it received in the first time slot from the source and decodes the resulting signal. Let $y_i^{(1)}$ and $y_i^{(2)}$ be the destination's received signals at the first and second time slot respectively. Then

$$\begin{aligned} y_i^{(1)} &= h_{i0}s_i + z_{i0}^{(1)} \\ y_i^{(2)} &= \sum_{j\in R} t_{ij}w_{ij} + z_{i0}^{(2)} \ . \end{aligned} \tag{2}$$

We also define $g_{ij} = t_{ij}\beta_{ij}$ as the effective gain of subchannel i through relay j and $G_{N\times K} = \{g_{ij}\}$. Then the signal that the destination receives at subchannel i in the second time slot can be written in matrix form as

$$y_i^{(2)} = (G_i H_i^T)s_i + G_i Z_i^T + z_{i0}^{(2)} \ . \tag{3}$$

where G_i, H_i and Z_i are $i'th$ row of matrices G, H and Z respectively.

The transmission power of relay j in subchannel i, p_{ij}, is

$$p_{ij} = \beta_{ij}^2(|h_{ij}|^2 p_i^B + \sigma_{ij}^2) \tag{4}$$

where p_i^B is the source's transmission power at subchannel i and $\sigma_{ij}^2 = E\{z_{ij}^2\}$ is the noise power of relay j at subchannel i. The noise power is assumed to be equal for the destination and all relays in each subchannel, which means $\sigma_{ij}^2 = \sigma_i^2$ for all j. Let $p_i = \sum_{j\in R} p_{ij}$ be the total relaying power at subchannel i.

As previously discussed, the performance measure considered here is the maximum achievable aggregate data-rate, which we call data-rate for short from now on, defined as

$$c_{total} = \sum_{i\in S} c_i = \sum_{i\in S} \log(1 + SNR_i) \tag{5}$$

where S is the set of subchannels utilized and logarithm is taken with basis e so data-rate is in nut/symbol/hertz. Using (2) and (3) and assuming coherent reception of the relays' signals in the second time slot and maximal ratio combining of the signals in the first and second time slots, SNR_i can be written as

$$SNR_i = SNR_{y_i^{(1)}} + SNR_{y_i^{(2)}} \tag{6}$$

$$= \frac{p_i^s|h_{i0}|^2}{\sigma_i^2} + \frac{p_i^s|G_iH_i^T|^2}{\sigma_i^2(G_iG_i^T + 1)} \tag{7}$$

$$= \frac{p_i^s}{\sigma_i^2}\left(|h_{0i}|^2 + \frac{\left|\sum\limits_{j\in R} t_{ij}\beta_{ij}h_{ij}\right|^2}{1 + \sum\limits_{j\in R}|t_{ij}\beta_{ij}|^2} \right) \tag{8}$$

$$= \frac{p_i^s}{\sigma_{eff,i}^2}|h_{eff,i}|^2 \tag{9}$$

where

$$\begin{aligned} |h_{eff,i}|^2 &= |G_iH_i^T|^2 + |h_{i0}|^2(G_iG_i^T + 1) \\ \sigma_{eff,i}^2 &= \sigma_i^2(G_iG_i^T + 1) \end{aligned} \tag{10}$$

are the effective power gain and noise power of subchannel i. Then the aggregate data-rate can be rewritten as

$$c_{total} = \sum_{i\in S} \log\left(1 + \frac{p_i^s|h_{eff,i}|^2}{\sigma_{eff,i}^2}\right) \tag{11}$$

Furthermore, since we assumed perfect phase and frequency synchronization, we can drop the channel phase and work with their magnitude from now on.

IV. Problem Formulation and Suboptimal Solution

In this section we introduce the optimal PA problem that maximizes the aggregate data-rate of the system given power constraints at the source and a total power constraint for the relays. Assume p^s and p^r to be the power budget of the source and the relays respectively. The optimization problem can be stated as

$$\begin{aligned} \max_{p_i^s, p_{ij}} \quad & c_{total} \\ s.t. \quad & \sum_{i\in S, j\in R} p_{ij} = p^r \\ & \sum_{i\in S} p_i^s = p^s \\ & p_i^s \geq 0 \quad \forall i \in S \\ & p_{ij} \geq 0 \quad \forall i \in S, j \in R \end{aligned} \quad . \tag{12}$$

Because of the coherent reception at the receiver, this problem may not be convex in general, and therefore its solution involves considerable complexity. Moreover, it requires a centralized solution, since all channel information are needed to be available at a central point where the optimal solution will be found and then fed back to all transmitters.

In a single-carrier system, it has been shown that a strategy that uses only the *best* relay usually performs close to the optimal solution [17]. However, in a multi-carrier system, due to different channel variation across subchannels at the relays, the *best* relay cannot be defined as before. Therefore, other suboptimal schemes are necessary to decrease overhead and complexity while maintaining an acceptable portion of the optimal PA gain.

A. Cooperative Channel Equalization

To overcome the complexity and overhead issues, we propose a suboptimal PA scheme in which power is uniformly allocated at the source, i.e.,

$$p_i^s = \frac{p^s}{N} \ , \tag{13}$$

and the total relaying power is set to be equal for all subchannel, which means

$$p_i = \frac{p^r}{N} \ . \tag{14}$$

Power allocated to each subchannel is then distributed optimally among the relays. Note that this scheme is not equivalent to optimally allocating power among the relays given uniform power allocation among the subchannels at each relay. In the resulting PA, the transmission power at different subchannels would not be equal for each relay. Rather, the sum of transmission power of all relays at each subchannel is constant and equal to the total relaying power budget divided by the number of subchannels. We call this scheme *Cooperative Channel*

Equalization (CCE) since it aims at using relays to *equalize* the channel, in contrast to PA in non-cooperative OFDM systems where the optimal PA scheme tries to efficiently utilize the *difference* between subchannels.

To find the optimal PA in this scheme, the dominant eigenvector optimization technique is utilized as in [14]. First we split g_{ij} into two parts

$$\begin{aligned} g_{ij} &= a_{ij} b_{ij} \\ a_{ij} &= \sqrt{p_{ij}} \\ b_{ij} &= \frac{t_{ij}}{\sqrt{p_i^B |h_{ij}|^2 + \sigma_i}} \end{aligned} \tag{15}$$

and define $A = \{a_{ij}\}$ $B = \{b_{ij}\}$. Now $y_i^{(2)}$ from (3) can be written as

$$y_i^{(2)} = (A_i H_i^{(d)} B_i^T) s_i + A_i B_i^{(d)} Z_i^T + z_{i0}^2 \tag{16}$$

where A_i, B_i and H_i are the i'th row of A, B and H respectively. $X^{(d)}$ denotes a diagonal matrix formed by elements of vector X on it's diagonal. The SNR corresponding to $y_i^{(2)}$ is equal to

$$\begin{aligned} SNR_{y_i^{(2)}} &= \frac{p_i^s |A_i H_i^{(d)} B_i^T|^2}{\sigma_i^2 (1 + |A_i B_i^{(d)T}|^2)} \\ &= \frac{p_i^s A_i H_i^{(d)} B_i^T B_i H_i^{(d)T} A_i^T}{\sigma_i^2 A_i \left(\frac{1}{p_i} I + B_i^{(d)T} B_i^{(d)} \right) A_i^T} \ . \\ &\triangleq \frac{p_i^s A_i Q_{in} A_i^T}{\sigma_i^2 A_i Q_{id} A_i^T} \end{aligned} \tag{17}$$

Our goal is to maximize the data-rate of subchannel i given the power constraint. Since $SNR_{y_i^{(1)}}$ does not depend on the PA among relays, this is equal to choosing A_i such that it satisfies $A_i A_i^T = p_i$ and maximizes $SNR_{y_i^{(2)}}$. This is a dominant eigenvector problem with the solution for A_i that satisfies

$$Q_{in} A_i^T = \lambda_i^{max} Q_{id} A_i^T \ . \tag{18}$$

It is easy to see that Q_{id} is a diagonal matrix with strictly positive elements on its diagonal. Therefore it is invertible and (18) can be written as

$$Q_{id}^{-1} Q_{in} A_i^T = \lambda_i^{max} A_i^T \tag{19}$$

where λ_i^{max} is the largest eigenvalue of $Q_{id}^{-1} Q_{in}$ corresponding to the dominant eigenvector A_i. By multiplying (18) by A_i from the left and putting $A_i A_i^T = p_i$, λ_i^{max} can be written as

$$\lambda_i^{max} = \frac{1}{p_i} A_i Q_{id}^{-1} Q_{in} A_i^T \tag{20}$$

Therefore from (17) and (19) the maximum value of $SNR_{y_i^{(2)}}$ given the power constraint is

$$\max_{A_i} SNR_{y_i^{(2)}} = \frac{p_i^s \lambda_{max}}{\sigma_i^2} \ . \tag{21}$$

Furthermore, since $B_i^T B_i$ has rank one, Q_{in} has rank one, and A can be found as

$$a_{ij} = \alpha_i \left(\frac{1}{p_i} + |b_{ij}|^2 \right)^{-1} b_{ij} h_{ij} \tag{22}$$

where α_i is a multiplication factor that ensures that $A_i A_i^T = p_i$. Having a_{ij}, the optimal PA among subchannels for each relay is $p_{ij} = |a_{ij}|^2$. In addition, from (17) λ_i^{max} is

$$\lambda_i^{max} = \frac{\left| \sum_{j \in R} a_{ij} b_{ij} h_{ij} \right|^2}{1 + \sum_{j \in R} |a_{ij} b_{ij}|^2} \ . \tag{23}$$

Then the aggregate data-rate can be found as

$$\begin{aligned} c_{total} &= \sum_{i \in S} \log(1 + SNR_i) \\ &\sum_{i \in S} \log(1 + \frac{p_i^s}{\sigma_i^2} \left(|h_{0i}|^2 + \lambda_i^{max} \right)) \ . \end{aligned} \tag{24}$$

Equation (21) is worth some more discussion. Essentially it says that the received signal from the relays in the second time slot has the same SNR as if the source has transmitted in the second time slot with the same power as in the first time slot and through a channel with power gain $|h_i^r|^2 = \lambda_i^{max}$. An interesting point here is that λ_i^{max} also implicitly expresses the effect of noise amplification at relays.

B. CCE with Subchannel Selection

We observe that some subchannels are assigned zero power in the optimal power allocation scheme. We further observe that this phenomenon is more pronounced in the low power regime, which can lead to poor performance by CCE. To overcome the poor performance of CCE in low power we introduce an improvement to CCE by first performing subchannel selection and then applying CCE to the set of selected subchannels. We call it *CCE with Subchannel Selection* (CCE-S). For the purpose of comparison we also consider subchannel selection by itself in the numerical results section in which once the subchannels are selected, power is assigned uniformly to all subchannels at the source and relays.

A simple criterion is used to select subchannels. This criterion can best be stated in terms of the following algorithmic implementation.

Algorithm 1:

1) Set $p_i^s = \frac{p^s}{N}$ and $p_i = \frac{p^r}{N}$
2) Calculate λ_i^{max} from (23) with a_{ij} from (22)
3) Set the normalized noise power for subchannel i equal to $\frac{\sigma_i^2}{\sqrt{|h_{i0}|^2 + \lambda_i^{max}}}$ and find the water-filling solution for a total power of p^s
4) Select subchannels to which a positive power is assigned in the previous step.

We will show, through numerical analysis, that CCE-S performs very close to the optimal scheme for a wide range of available power. However, the cost of this performance improvement is the increase in the signaling overhead and computational complexity of the PA scheme. In addition to original signaling requirement for CCE, in CCE-S the source also needs to be informed of the set of selected subchannels. Also while the solution of CCE is available in closed form as a function of channel parameters, as shown in Section IV.A, CCE-S requires numerical computation to find the set of selected subchannels.

V. Signaling Implementation

How much signaling is needed to implement these schemes? With CCE, first, power distribution is uniform at the source and no communication is necessary with the source regarding PA. Second, if the destination feeds backs the relay-destination channel gains, t_{ij}, to the relays, PA can be determined up to a normalization factor at the relays (Equation (22)). These points leads us to the following signaling implementation model.

Every time PA needs to be updated, the destination feeds back the relay-destination channel information to the corresponding relays. Each relay then calculates its power allocation without the normalization factor ξ_i, adjusts its transmission power using the previous value of ξ_i, and starts sending data. The destination then measures its received power and calculate the ratio $\frac{\xi_i^{new}}{\xi_i^{old}}$, which will be broadcasted to all relays. The relays then adjust their transmission power according to this factor. Therefore the amount of feedback necessary to implement CCE is $KN\theta_c + N\theta_\xi$, where θ_c is the number of bits used to represent channel parameters and θ_ξ is the number of bits used to update $\frac{\xi_i^{new}}{\xi_i^{old}}$.

Note that signaling is limited to the feedback from the destination to the relays and does not include any communication with the source. This is particularly interesting since the whole cooperating process is hidden from the source in this scheme. In a downlink scenario this has an attractive meaning: a set of nodes can start cooperating and exploit the performance improvement of optimal PA without any coordination with the access point. These nodes are very likely a cluster of nodes relatively close to each other. In that case the communication among them can be considered as *local signaling*, which can be done with lower cost than communicating with the access point.

To implement CCE-S, λ_i and therefore the channel information from all relays is required. Therefore, for an implementation of the CCE-S algorithm at the destination, $KN\theta_c$ bits is required to send the source-relay channel information. In addition to this, less than or equal to $KN\theta_p$ bits of feedback to the relays and N bits to the source is necessary. Hence, a total of $KN(\theta_c+\theta_p)$ and N bits of information exchange are required between the source and relays, and the source and destination, respectively, to implement CCE-S.

VI. Numerical Results

In this section we provide numerical results to quantify the system performance as well as to obtain insights about the behavior of the system. The setup consists of a source-destination pair located at a distance of one from each other. There are three relays in the network. Three scenarios are

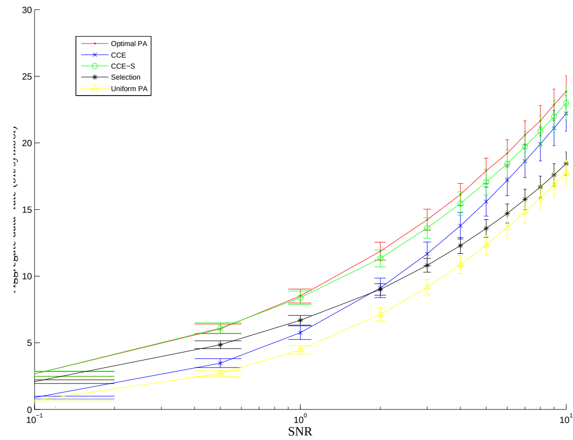

Fig. 1. Aggregate data-rate (bit/symbol) vs. SNR for distributed relay placement

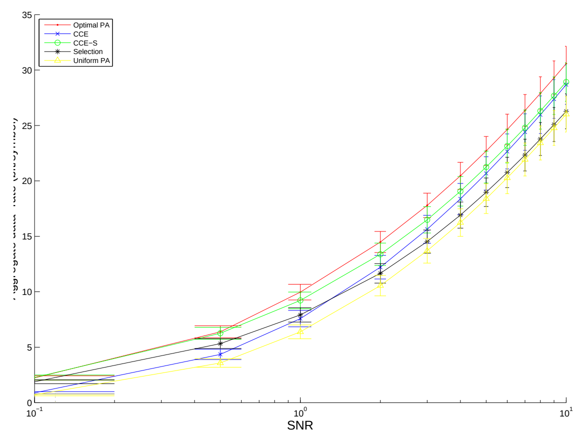

Fig. 2. Aggregate data-rate (bit/symbol) vs. SNR for source-centered relay placement

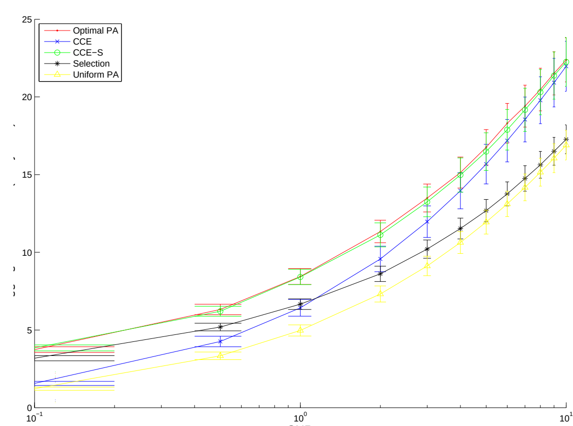

Fig. 3. Aggregate data-rate (bit/symbol) vs. SNR for destination-centered relay placement

considered for relay placement. In the *distributed* placement, the relay locations are chosen uniformly from a unit circle with its center in the mid point between the source and the destination, while in the *source-centered* and *destination-centered* placements, the relays are located uniformly in a circle with radius $\frac{1}{2}$ centered at the source and destination, respectively. The amplitude of channel gains are derived from Rayleigh distributed random variables with average channel gain $d^{-\kappa}$ where d is the distance between the transmitter and receiver and κ is set to be equal to 3. The noise power for each subchannel is derived from a uniform random variable with unit average power. Note that this also sets the unit of power in this model. A total of 32 subchannels are considered in the system. To make the results independent of a specific network topology, we consider 30 different realization sof node placement and show all data with their 90% confidence interval.

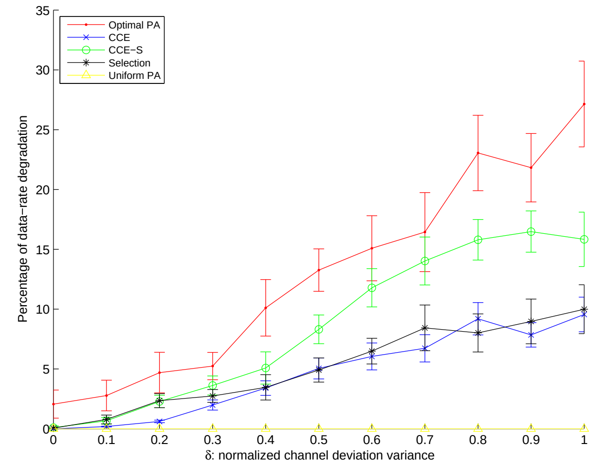

Fig. 4. Percentage of data-rate degradation vs. normalized channel deviation variance for distributed relay placement

A. Aggregate Data-rate

We investigate the aggregate data-rate in bits per symbol versus SNR. Figs. 1, 2, and 3 show the aggregate data-rate for distributed, source-centered, and destination-centered relay placements respectively, with the relays to source power ratio equal to 0.5. In these figures, "optimal PA" refers to the PA scheme computed by brute-force optimization based on (11), "selection" refers to simple subchannel selection as described in Section IV.B, and "uniform PA" refers to distributing power equally among all subchannels and all relays. The first point that can be observed from these figures is consistence behavior of these schemes for different node placement realizations as can be seen from the confidence intervals.An especially interesting point is that the behavior of CCE and CCE-S is different in different power regimes and depends on the relay placement.

These figures show that, in low power, allocating uniform power to *bad* subchannels and *good* ones, as done in CCE, is an inefficient utilization of power, which results in performance well below the optimal case and close to the uniform PA scheme. As power increases, all subchannels should almost always be utilized, and optimally allocating power among relays become more crucial, and therefore the performance of CCE becomes closer to the optimal case.

While CCE-S has close performance to the optimal case at low powers for all relay placement scenarios, its behavior in the high power regime depends on relay placement. In the source-centered relay placement, as SNR increases, the gap between the optimal data-rate and the data-rate of CCE-S increases. CCE-S performs close to CCE for the high power regime, achieving roughly half of the optimal PA gain. In a destination-centered relay placement setup, CCE-S continues to perform close to optimal for high powers. In contrast to the previous case, here CCE performs better as SNR increases and its performance becomes close to the optimal case and that of CCE-S.

B. Effect of Channel Variation

Next, we present numerical results to analyze the system behavior in the presence of channel variation. Our analysis does not include the phase variation, and therefore by channel variation we mean variation in the amplitude of the channel gains. The SNR of the received signal is set to be one (zero dB). We look at the percentage of data-rate loss versus normalized maximum channel variation Δ, where the new value for channel x at subchannel i, $x_i^{(1)}$, is derived from the original value, $x_i^{(0)}$, as

$$x_i^{(1)} = [x_i^{(0)} + \mathbb{N}(0, \delta x_i^{(0)})]^+ \tag{25}$$

in which $N(0, \delta x_i^{(0)})$ is a zero-mean Gaussian random variable with variance $\delta x_i^{(0)}$. the data-rate loss is defined as

$$\frac{c_t(\mathbb{P}^{(1)}) - c_t(\mathbb{P}^{(0)})}{c_t(\mathbb{P}^{(1)})} \tag{26}$$

where $c_t(\mathbb{P}^{(0)})$ is the aggregate data-rate after channel variation with original PA and $c_t(\mathbb{P}^{(1)})$ is the aggregate data-rate after channel variation with accordingly updated PA.

We investigate how data-rate degrades when all channels (source-destination, source-relay, and relay-destination) vary according to (25). For the optimal PA scheme, up to 40% of the aggregate data-rate can be lost when variance of the variation is equal to the original channel gain. Refereing to Figs. 1, 2, and 3, it can be seen that this is almost all the PA gain. Interestingly, when the relays are distributed uniformly, the system is more resilient to the channel variation. We also observe that Subchannel Selection and CCE are the most resilient schemes with about 5% maximum data-rate loss in the distributed relay placement scenario. The resilence of CCE-S is between the optimal scheme and CCE.

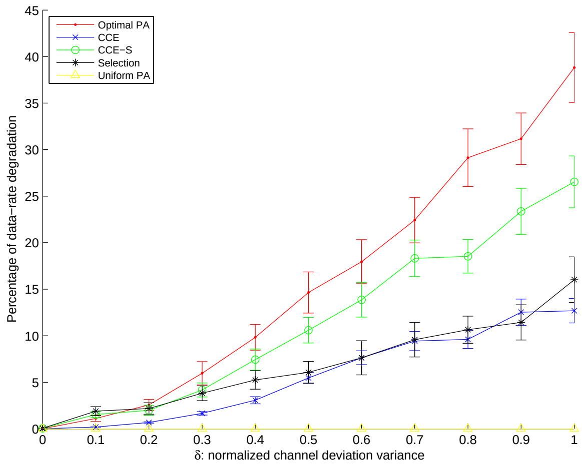

Fig. 5. Percentage of data-rate degradation vs. normalized channel deviation variance for source-centered relay placement

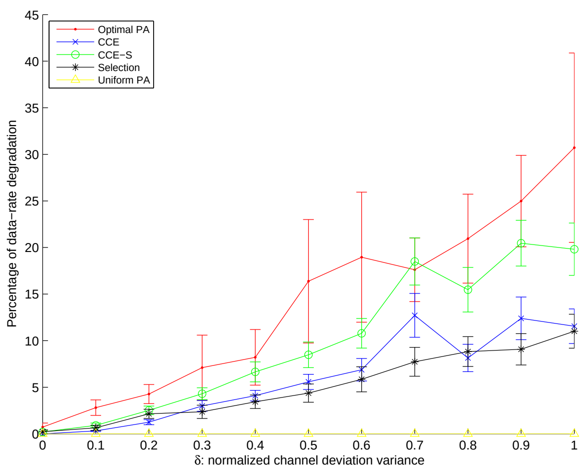

Fig. 6. Percentage of data-rate degradation vs. normalized channel deviation variance for destination-centered relay placement

VII. Conclusion

In this paper we presented the optimal PA problem in a cooperative OFDM system with multiple relays. We proposed two suboptimal schemes for power allocation, namely, Cooperative Channel Equalization and Cooperative Channel Equalization with Subchannel Selection. We have compared the performance of these two suboptimal schemes with each other and with the optimal scheme, simple subchannel selection, and uniform PA.

Through numerical analysis, we have shown that the behavior of these schemes depends on the geometry of the network. CCE is shown to perform better in term of data-rate when the relays are concentrated around the destination. We have also shown that the relative data-rate of different suboptimal schemes depends on the power constraints at the relays and the source. CCE is shown to perform better at higher power levels while the opposite trend is observed for simple subchannel selection. With higher implementation complexity and signaling overhead, CCE-S exploits the advantages of both schemes and has better performance than both for a wide range of power constraint.

We have further investigated into the data-rate degradation effect of out-dated PA under channel variation. CCE and simple subchannel selection are shown to be relatively resilient to channel variation, while the optimal scheme can perform even worse than uniform PA when the channels deviate significantly from the original values based on which PA was calculated. Therefore, CCE provides an excellent balance between data-rate performance, signaling overhead, and channel estimation error resilience.

References

[1] T. Cover and A. Gamal, “Capacity theorems for the relay channel,” *IEEE Transactions on Information Theory*, vol. 25, no. 5, pp. 572–584, 1979.

[2] R. Mudumbai, G. Barriac, and U. Madhow, “On the feasibility of distributed beamforming in wireless networks,” *IEEE Transction on Wireless Communication*, vol. 6, no. 4, pp. 1–10, 2007.

[3] Y. Tu and G. Pottie, “Coherent cooperative transmission from multiple adjacent antennas to a distant stationary antenna through AWGN channels,” in *Proc. IEEE 55th Vehicular Technology Conference (VTC)*, vol. 1, 2002.

[4] J. Jang and K. Lee, “Transmit power adaptation for multiuser OFDM systems,” *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 2, pp. 171–178, 2003.

[5] S. Sanayei, A. Nosratinia, and N. Aldhahir, “Opportunistic dynamic subchannel allocation in multiuser OFDM networks with limited feedback,” in *Proc. IEEE Information Theory Workshop*, 2004, pp. 182–186.

[6] E. van der Meulen, “Three-terminal communication channels,” *Adv. Appl. Prob*, vol. 3, pp. 120–154, 1971.

[7] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity. Part I. System description,” *IEEE Transactions on Communications*, vol. 51, no. 11, pp. 1927–1938, 2003.

[8] ——, “User cooperation diversity. Part II. Implementation aspects and performance analysis,” *IEEE Transactions on Communications*, vol. 51, no. 11, pp. 1939–1948, 2003.

[9] J. Laneman and G. Wornell, “Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks,” *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2415–2425, 2003.

[10] J. Laneman, D. Tse, and G. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, 2004.

[11] A. Host-Madsen and J. Zhang, “Capacity bounds and power allocation for wireless relay channels,” *Information Theory, IEEE Transactions on*, vol. 51, no. 6, pp. 2020–2040, 2005.

[12] Z. Qi, Z. Jingmei, S. Chunju, W. Ying, Z. Ping, and H. Rong, “Power allocation for regenerative relay channel with Rayleigh fading,” in *Proc. IEEE 59th Conference on Vehicular Technology (VTC 2004-Spring)*, vol. 2, 2004.

[13] I. Hammerstrom and A. Wittneben, “On the optimal power allocation for nonregenerative OFDM relay links,” in *Proc. IEEE International Conference on Communications (ICC)*, ser. 10, 2006.

[14] I. Hammerstrom, M. Kuhn, and A. Wittneben, “Impact of relay gain allocation on the performance of cooperative diversity networks,” in *Proc. IEEE 60th Conference on Vehicular Technology (VTC)*, vol. 3, Fall 2004.

[15] T. Ng and W. Yu, “Joint Optimization of Relay Strategies and Resource Allocations in Cooperative Cellular Networks,” *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 328–339, 2007.

[16] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*. Artech House, Inc. Norwood, MA, USA, 2000.

[17] Y. Zhao, R. S. Adve, and T. Lim, “Beamforming with limited feedback in amplify-and-forward cooperative networks,” in *Proc. IEEE Globecom*, 2007, pp. 3457–3461.