Optimal Cooperative Relay Beamforming for Interference Minimization

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***Abstract*—We consider a wireless cellular network with multiple amplify-and-forward (AF) relays in each cell, assisting the communication of multiple source-destination pairs with relay transmission beamforming. Our objective is to minimize the maximum interference power among all active receivers in a neighboring cell subject to per-relay power and minimum received SNR constraints. We propose an efficient algorithm to obtain the *optimal* relay beamforming vectors. We show that even though the optimization problem is non-convex, it has zero Lagrange duality gap and can be converted to a semi-definite programming problem. The performance of the proposed algorithm is studied numerically, both for the case where the interference channel information is exactly known and for the case of inaccurate channel information due to either limited feedback or channel estimation error. It is demonstrated that the min-max interference approach substantially outperforms the alternative where we simply minimize the maximum relay transmission power.**

I. Introduction

Modern cellular systems suffer from inter-cell interference due to the small frequency reuse factor in a cell [1]. The other types of interference, intra-cell interference and co-antenna interference, can be avoided by orthogonal transmission of users in a cell such as time-division multiple access and orthogonal frequency-division-multiple-access (OFDMA). Hence, we focus on inter-cell interference in this paper. In particular, we study how to use multiple relays in beamforming to reduce such interference.

Wireless relaying has been a subject of many studies in the literature and is specified in standards such as LTE-Advanced [2] and WiMax [3]. The design of relay cooperative networks in interference limited environments has been considered under various criteria such as capacity, throughput, area spectral efficiency, and received signal-to-interference-plus-noise ratio (SINR) [4]–[8]. The objectives of these works do not include inter-cell interference reduction. Inter-cell interference mitigation techniques for relay networks with orthogonal-based transmission have been studied in [9]–[13]. The authors of [9] have proposed a radio resource management strategy for relay-user association, resource allocation, and power control. In [10], the performance of different relay strategies has been studied in interference-limited cellular systems. In [11], a joint subcarrier allocation, scheduling, and power control scheme has been proposed for OFDMA-based relay inter-cell interference limited networks. For relay-aided cellular OFDMA systems, the authors of [12] have proposed an interference coordination heuristic scheme. In [13], a game theoretic framework has been developed to mitigate interference in OFDMA relay networks. However, none of these works aims to directly minimize the inter-cell interference. Furthermore, none of them considers relay beamforming, which can lead to a complicated optimization problem.

In this work, we present a novel approach to optimally design relay beamforming in order to minimize inter-cell interference. We consider a cellular network where each cell has multiple single-antenna amplify-and-forward (AF) relays collaborating for the communication of multiple independent sources and destinations using orthogonal spectrum resource. The goal of this paper is finding the optimum relay beamforming in order to minimize the maximum received interference at the receivers in a neighboring cell. Our numerical results show that substantial reduction of interference can be achieved by using a moderate number of relays.

We first formulate the relay beamforming problem in order to minimize the maximum interference under minimum received SNR and per-relay power constraints. Then, the original non-convex problem is recast as a second-order-conic programming (SOCP) problem, through which we show that the original problem has zero Lagrange duality gap and hence the Lagrange dual method can be applied. We transform the dual problem into a semi-definite-programming (SDP) problem with much fewer variables and constraints compared to the original optimization problem. Hence, the computation complexity in finding the optimal beamweights is reduced significantly. Expressed in the SDP form, the dual problem can be solved by the interior-point methods having polynomial complexity. Three cases for the optimum dual variables are identified and the optimal relay beamweights are obtained accordingly. Furthermore, observing a form of the uplink-downlink duality [14]–[16], we derive a semi-closed form expression for the optimal relay beamweights. We evaluate the performance of min-max interference, both when true interference CSI is available and when there is only limited channel feedback or channel estimation error.

The rest of this paper is organized as follows: In Section II, the system model is described. The min-max interference problem is solved in Section III. Numerical results are presented in Section IV, and conclusions are drawn in Section V.

Notation: We use $\|\cdot\|$ to denote the Euclidean norm of a vector, and $\odot$ to denote element wise multiplication. We use $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^\dagger$ to denote transpose, Hermitian, and matrix pseudo-inverse, respectively. The conjugate is represented by

$(\cdot)^*$. The notation $\mathrm{diag}(\mathbf{A})$ represents a vector consisting of the diagonal elements of a matrix $\mathbf{A}$. We use $\mathbf{I}$ to denote an $N \times N$ identity matrix, and $\mathbf{Y} \succeq \mathbf{Z}$ to indicate that $\mathbf{Y} - \mathbf{Z}$ is a positive semi-definite matrix.

II. System Model and Problem Formulation

A. System Model

We consider a cellular system where each cell contains M source-destination pairs and N relays, and all nodes are equipped with a single antenna. A multichannel communication system (*e.g.,* OFDMA) consisting of M subchannels is used in each cell. Each source transmits data to its destination through the relays using a separate subchannel, and transmission among different pairs are orthogonal. We assume that the half-duplex AF protocol is used for relaying, and the direct path is ignored. Different subchannels of each relay may be assigned to multiple source-destination pairs. In this work, we study the interference caused by N relays in one cell (desired) to the M destinations in its neighboring cell.

Assume that the m-th source-destination pair communicate through N relays over subchannel m. In the desired cell, the received signal at relay i is given by $y_{m,i} = \sqrt{P_0} h_{m,i} s_m + n_{r,m,i}$, where $h_{m,i}$ is the subchannel m between source m and relay i, s_m is the transmitted symbol with $\mathbb{E}[|s_m|^2] = 1$, P_0 is the transmission power, and $n_{r,m,i}$ is the AWGN with variance σ_r^2. Next, the relay i multiplies the received signal over subchannel m with a complex beamweight, denoted by $w_{m,i}$, for forwarding. Let $g_{m,i}$ denote the subchannel m from relay i to destination m. Then, the received signal at destination m from all relays is given by

$$r_m = \sqrt{P_0} \mathbf{g}_m^T \mathbf{W}_m \mathbf{h}_m s_m + \mathbf{g}_m^T \mathbf{W}_m \mathbf{n}_{r,m} + n_m \tag{1}$$

where $\mathbf{h}_m \triangleq [h_{m,1}, \cdots, h_{m,N}]^T$, $\mathbf{g}_m \triangleq [g_{m,1}, \cdots, g_{m,N}]^T$, $\mathbf{W}_m \triangleq \mathrm{diag}(w_{m,1}, \cdots, w_{m,N})$, and $\mathbf{n}_{r,m} \triangleq [n_{r,m,1}, \cdots, n_{r,m,N}]^T$ are the channel vectors at the first hop and second hop, the beamweight matrix, and the noise vector, through all relays for the m-th source-destination pair, respectively. In addition, n_m is the AWGN at destination m with variance σ_d^2. The received SNR at destination m is given by

$$\mathrm{SNR}_m = \frac{P_0 \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma_d^2} \tag{2}$$

where $\mathbf{w}_m \triangleq \mathrm{diag}(\mathbf{W}_m)$, $\mathbf{F}_m \triangleq (\mathbf{f}_m \mathbf{f}_m^H)^*$ with $\mathbf{f}_m = \mathbf{g}_m \odot \mathbf{h}_m$, and $\mathbf{G}_m \triangleq \sigma_r^2 \mathrm{diag}((\mathbf{g}_m \mathbf{g}_m^H)^*)$.

Each transmitting relay causes interference to its neighboring cell. We focus on the interference from N relays in one cell to the M destinations in the neighboring cell. Let $\tilde{\mathbf{g}}_m$ denote the corresponding interfering channel vector over subchannel m from N relays of the desired cell to destination m of its neighboring cell. The received interference at destination m of the neighboring cell is given by $\tilde{r}_m = \tilde{\mathbf{g}}_m^T \mathbf{W}_m (\sqrt{P_0} \mathbf{h}_m s_m + \mathbf{n}_r)$. The corresponding received interference power is given by $\mathcal{I}_m \triangleq P_0 \mathbf{w}_m^H \tilde{\mathbf{F}}_m \mathbf{w}_m + \mathbf{w}_m^H \tilde{\mathbf{G}}_m \mathbf{w}_m$, where $\tilde{\mathbf{F}}_m \triangleq (\tilde{\mathbf{f}}_m \tilde{\mathbf{f}}_m^H)^*$, $\tilde{\mathbf{f}}_m \triangleq \tilde{\mathbf{g}}_m \odot \mathbf{h}_m$ and $\tilde{\mathbf{G}}_m \triangleq \sigma_r^2 \mathrm{diag}((\tilde{\mathbf{g}}_m \tilde{\mathbf{g}}_m^H)^*)$ are the interference corresponding to the forwarded signal and the amplified noise from the relays.

Let P_r denote the total power available at each relay. Power allocation over each subchannel at relay i should satisfy $\sum_{m=1}^M |w_{m,i}|^2 [\mathbf{R}_{y,m}]_{i,i} \leq P_r$, where $\mathbf{R}_{y,m} \triangleq P_0 \mathbf{h}_m \mathbf{h}_m^H + \sigma_r^2 \mathbf{I}$.

We assume perfect knowledge of CSI including the interfering channels in designing the relay beamweights of the desired cell. In Section IV, we further study through simulation the case where the interfering CSI is imperfect.

B. Problem Formulation

Define $\mathbf{R}_m \triangleq \mathrm{diag}([\mathbf{R}_{y,m}]_{1,1}, \cdots, [\mathbf{R}_{y,m}]_{N,N})$, and let $\mathbf{D}_i$ denote the $N \times N$ diagonal matrix with 1 in the i-th diagonal and zero otherwise. Rewrite the interference power $\mathcal{I}_m = \mathbf{w}_m^H \tilde{\mathbf{B}}_m \mathbf{w}_m$, where $\tilde{\mathbf{B}}_m \triangleq P_0 \tilde{\mathbf{F}}_m + \tilde{\mathbf{G}}_m$, for $m = 1, \cdots, M$. Our goal is to design the relay beamweights of the desired cell to minimize the maximum interference to its neighboring cell, subject to per-relay power constraint and minimum SNR guarantee.[1] The optimization problem is given by

$$\min_{\mathbf{w}_1, \cdots, \mathbf{w}_M, \tilde{\theta}} \tilde{\theta} \tag{3}$$

$$\text{subject to } \mathbf{w}_m^H \tilde{\mathbf{B}}_m \mathbf{w}_m \leq \tilde{\theta}, \; m = 1, \cdots, M, \tag{4}$$

$$\sum_{m=1}^M \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m \leq P_r, \; i = 1, \cdots, N, \tag{5}$$

$$\frac{P_0 \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma^2} \geq \gamma_m, \; m = 1, \cdots, M. \tag{6}$$

We use $\tilde{\theta}^o$ to denote the minimum objective under the optimal solution.

III. Minimizing Maximum Per-Subchannel Interference

The solution of the min-max interference problem (3) is provided in this section. We reformulate the problem which leads to a semi-closed form solution in the Lagrange dual domain. In order to obtain the optimal $\{\mathbf{w}_1, \cdots, \mathbf{w}_M\}$, an SDP-based algorithm with polynomial worst-case complexity is proposed. Then three cases for the dual variables are studied and the optimal $\{\mathbf{w}_1, \cdots, \mathbf{w}_M\}$ are determined accordingly.

A. Strong Duality

Since the SNR constraint (6) is not convex, the problem (3) is non-convex. In the following, we show that (3) has zero duality gap and can be solved in the Lagrange dual domain.

Proposition 1: Strong duality holds for the min-max interference problem (3).

Proof: We omit the details and provide an outline of the proof. We first show that the problem (3) can be reformulated as a second-order conic programming (SOCP) problem. It is

[1] We assume that the beamweights of the neighboring cell are also optimized such that the maximum interference in the desired cell is minimized. As future work, we may study how to set a pre-determined maximum interference power for the cells such that the minimum received SINR at the receivers is maximized.

known that the SOCP has zero *conic* duality gap [17]. Then we show that the Lagrange dual of the problem (3) and the Lagrange dual of the SOCP are equivalent. ■

Note that the problem (3) could be solved numerically by solving the above equivalent SOCP with $MN+1$ variables and $2M+N$ constraints. To gain insight into the structure of the beamweights, an efficient algorithm using the Lagrange dual domain is proposed. In the following, we provide a semi-closed form solution of (3) using SDP. Through the proposed algorithm, the structure of beamweights is derived and the computational complexity is reduced.

B. The Semi-Closed Form Solution

Using the results of Proposition 1, we can obtain the optimum solution of (3) through the dual problem. Let $\boldsymbol{\mu} \triangleq [\mu_1, \cdots, \mu_M]^T$, $\boldsymbol{\lambda} \triangleq [\lambda_1, \cdots, \lambda_N]^T$, and $\boldsymbol{\alpha} \triangleq [\alpha_1, \cdots, \alpha_M]^T$ denote the Lagrange multipliers associated with the max interference constraint (4), per relay power constraint (5), and SNR constraint (6), respectively. The Lagrangian of (3) is given by $L(\{\mathbf{w}_m\}, \tilde{\theta}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}) = \sum_{m=1}^{M} \alpha_m \sigma^2 + \tilde{\theta}(1 - \sum_{m=1}^{M} \mu_m) - P_r(\sum_{i=1}^{N} \lambda_i) + \sum_{m=1}^{M} \mathbf{w}_m^H \big(\mathbf{K}_m - \frac{\alpha_m P_0}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H\big) \mathbf{w}_m$, where

$$\mathbf{K}_m \triangleq \mathbf{R}_m \mathbf{D}_{\boldsymbol{\lambda}} + \mu_m \tilde{\mathbf{B}}_m + \alpha_m \mathbf{G}_m \tag{7}$$

and $\mathbf{D}_{\boldsymbol{\lambda}} \triangleq \mathrm{diag}(\lambda_1, \cdots, \lambda_N)$.

The dual problem of the problem (3) is given by

$$\max_{\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\alpha}} \ \min_{\mathbf{w}_m, \tilde{\theta}} L(\{\mathbf{w}_m\}, \tilde{\theta}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}) \tag{8}$$

$$\text{subject to } \boldsymbol{\lambda} \succeq 0, \boldsymbol{\mu} \succeq 0, \boldsymbol{\alpha} \succeq 0. \tag{9}$$

We observe that, after the inner minimization of (8), the dual problem (8) is equivalent to

$$\max_{\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\alpha}} \sum_{m=1}^{M} \alpha_m \sigma^2 - P_r(\sum_{i=1}^{N} \lambda_i) \tag{10}$$

$$\text{subject to } \mathbf{K}_m \succeq \frac{\alpha_m P_0}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H, \ m = 1, \cdots, M \tag{11}$$

$$\sum_{m=1}^{M} \mu_m \leq 1, \tag{12}$$

and (9).

This is because the constraints (11) and (12) are implicit in the optimal solution of the problem (8). To see this, suppose one of the constraints (11) or (12) is not satisfied. Then there is some $\{\mathbf{w}_m, \tilde{\theta}\}$ such that the inner minimization of (8) leads to $L(\{\mathbf{w}_m\}, \tilde{\theta}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}) = -\infty$, which is not an optimum solution of (8).

In order to solve the dual problem (10), we first discuss the feasibility of constraint (11) in the following lemma.

Lemma 1: If either $\mu_m > 0$ or $\boldsymbol{\lambda} \succ 0$, then $\alpha_m > 0$, *i.e.,* the Lagrange dual variable associated with the SNR constraint (6) is strictly positive.

Proof: We can show that the constraint (11) is equivalent to $\mathbf{R}_m \mathbf{D}_{\boldsymbol{\lambda}} + \mu_m \tilde{\mathbf{B}}_m + \alpha_m \big(\mathbf{G}_m - \frac{P_0}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H\big) \succeq 0$ and $\mathbf{G}_m - \frac{\alpha_m P_0}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H$ is an indefinite matrix. ■

The above lemma provides the condition under which the SNR constraint (6) for the m-th source-destination pair is met with equality. In the following, we provide the solution assuming $\boldsymbol{\alpha} \succ 0$, *i.e.,* the SNR constraint (6) is met with equality for all m. The solution for other cases is obtained similarly and is presented in Section III-D.

Theorem 1: If $\boldsymbol{\alpha} \succ 0$, the optimum beamforming vector $\mathbf{w}_m^o$ of the min-max interference problem (3) is given by

$$\mathbf{w}_m^o = \zeta_m {\mathbf{K}_m^o}^{\dagger} \mathbf{f}_m \tag{13}$$

where

$$\zeta_m \triangleq \sigma \Big[\frac{P_0}{\gamma_m} |\mathbf{f}_m^H {\mathbf{K}_m^o}^{\dagger} \mathbf{f}_m|^2 - \mathbf{f}_m^H {\mathbf{K}_m^o}^{\dagger} \mathbf{G}_m {\mathbf{K}_m^o}^{\dagger} \mathbf{f}_m \Big]^{\frac{-1}{2}} \tag{14}$$

with $\mathbf{K}_m^o$ obtained by substituting the optimum dual variables into (7).

Proof: See Appendix A. ■

Note that $\mathbf{w}_m^o$ in (13) is a semi-closed form solution, because it still depends on the optimum dual variables $\{\boldsymbol{\lambda}^o, \boldsymbol{\mu}^o, \boldsymbol{\alpha}^o\}$. In the next section, we provide an SDP-based numerical solution to find the dual variables.

C. The Optimal Dual Variables Through SDP

To determine the optimum $\{\boldsymbol{\lambda}^o, \boldsymbol{\mu}^o, \boldsymbol{\alpha}^o\}$, instead of solving the dual problem (10) directly, we reformulate it into an SDP problem.

Proposition 2: Denote $\mathbf{x} \triangleq [\boldsymbol{\alpha}^T, \boldsymbol{\lambda}^T, \boldsymbol{\mu}^T]^T$, $\mathbf{a} \triangleq [-\sigma^2 \mathbf{1}_{M\times 1}^T, P_r \mathbf{1}_{N\times 1}^T, \mathbf{0}_{M\times 1}^T]^T$ and $\mathbf{b} \triangleq [\mathbf{0}_{(M+N)\times 1}^T, \mathbf{1}_{M\times 1}^T]^T$. The dual problem (10) can be re-expressed as the following SDP

$$\min_{\mathbf{x}} \ \mathbf{a}^T \mathbf{x} \tag{15}$$

$$\text{subject to } \sum_{i=1}^{2M+N} x_i \boldsymbol{\Psi}_{m,i} \preceq \mathbf{0}, \ m = 1, \cdots, M, \tag{16}$$

$$\mathbf{x} \succeq \mathbf{0}, \ \mathbf{b}^T \mathbf{x} \leq 1 \tag{17}$$

where $\boldsymbol{\Psi}_{m,m} = \frac{P_0}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H - \mathbf{G}_m$, $\boldsymbol{\Psi}_{m,M+j} = -\mathbf{R}_m \mathbf{D}_j$ for $j = 1, \cdots, N$, $\boldsymbol{\Psi}_{m,M+N+m} = -\tilde{\mathbf{B}}_m$ for $m = 1, \cdots, M$, and all other $\boldsymbol{\Psi}$ are zeros.

Standard SDP softwares such as SeDuMi can be used to solve (15). Note that the original problem (3) with $2M+N$ constraints and $MN+1$ variables is converted to an SDP problem with $M+2$ constraints and $2M+N$ variables. In addition to reducing the computation complexity, the semi-closed form solution (13) shows the structure of optimum beamweights.

D. Three Cases of Dual Variables

In the following, we partition the set of optimum dual variables in the dual problem (10) into three cases and propose an algorithm to obtain the the optimum beamforming vectors (if existent) for each case.

1) Case 1: If $\mu_m^o = 0$, for $m = 1, \cdots, M$, there is no solution for the original problem (3). In other words, there should be at least one active constraint (4). This case happens due to the infeasibility of (3), *i.e.,* either the minimum SNR guarantees (6) cannot be achieved or per relay power exceeds the given threshold in (5).

In the following, we assume $\mu_m^o > 0$ for some m.

2) Case 2: If $\forall m \in \{1, \cdots, M\}$, $\mu_m^o > 0$ or $\boldsymbol{\lambda}^o \succ 0$, we have $\alpha_m^o > 0$ for $m = 1, \cdots, M$ in (10). In other words, if $\mathbf{K}_m^o - \alpha_m^o \mathbf{G}_m \succ 0$, then $\alpha_m^o > 0$, for all m, and the solution is given by Theorem 1.

3) Case 3: If $\alpha_m^o = 0$ for some m, *i.e.,* there exists m such that $\mu_m^o = 0$ and $\boldsymbol{\lambda}^o \nsucc 0$ as shown in Lemma 1, we cannot use (13) for $m = 1, \cdots, M$. Let $\tilde{m}$ denote the pair with $\mu_{\tilde{m}}^o > 0$. For simplicity, suppose $\mu_m^o = 0$ for $m \neq \tilde{m}$ and $\lambda_i^o = 0$ for some i. Suppose that $\alpha_{\tilde{m}}^o > 0$ and $\alpha_m^o = 0$ for $m \neq \tilde{m}$. We can simply extend our solution to the case in which $\alpha_m^o > 0$ for arbitrary m's. Similar to the proof of Theorem 1, we have $\frac{\alpha_{\tilde{m}}^o P_0}{\gamma_{\tilde{m}}} \mathbf{f}_{\tilde{m}}^H {\mathbf{K}_{\tilde{m}}^o}^\dagger \mathbf{f}_{\tilde{m}} = 1$. Hence, we can use the solution (13) to obtain the beamforming vector of $\tilde{m}$. Then assuming the original problem (3) is feasible, we have $\tilde{\theta}^o = \alpha_{\tilde{m}}^o \sigma^2 - P_r(\sum_{i=1}^N \lambda_i^o)$. Let $\mathcal{M} \triangleq \{1, \cdots, M\} \backslash \{\tilde{m}\}$. In order to obtain the beamforming vectors for $m \neq \tilde{m}$, we need to solve the following feasibility problem

$$\text{find } \mathbf{w}_1, \cdots, \mathbf{w}_{\tilde{m}-1}, \mathbf{w}_{\tilde{m}+1}, \cdots, \mathbf{w}_M \tag{18}$$

$$\text{subject to } \mathbf{w}_m^H \tilde{\mathbf{B}}_m \mathbf{w}_m < \tilde{\theta}^o, m \in \mathcal{M},$$

$$\sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m \leq P_r, i = 1, \cdots, N, \tag{19}$$

$$\frac{P_0 \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma^2} \geq \gamma_m, m \in \mathcal{M}. \tag{20}$$

Note that we can always scale $\mathbf{w}_m$ such that (20) meets with equality for $m \neq \tilde{m}$. Among the infinite set of possible solutions of $\mathbf{w}_m$ for $m \neq \tilde{m}$, we propose to extract one using the following algorithm. The essence of this algorithm is to remove the $\tilde{m}$-th pair from consideration and solve the resultant min-max interference problem to find the optimum beamweights associated with the other pairs.

Denote ${\mathbf{w}_{\tilde{m}}^o}^H \mathbf{R}_{\tilde{m}} \mathbf{D}_i \mathbf{w}_{\tilde{m}}^o \triangleq e_i$ for $i = 1, \cdots, N$. We can solve the following problem

$$\min_{\mathbf{w}_1, \cdots, \mathbf{w}_M, \delta} \delta \tag{21}$$

$$\text{subject to } \mathbf{w}_m^H \tilde{\mathbf{B}}_m \mathbf{w}_m \leq \delta, m \in \mathcal{M}, \tag{22}$$

$$\sum_{m=1, m \neq \tilde{m}}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m \leq P_r - e_i, \forall i, \tag{23}$$

$$\frac{P_0 \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma^2} \geq \gamma_m, m \in \mathcal{M}. \tag{24}$$

Let δ^o denote the optimum value of (21), and suppose $\alpha_{\tilde{m}}^o > 0$. If $\delta^o < \tilde{\theta}^o$, then we can find $\mathbf{w}_{\tilde{m}}^o$. If $\delta^o \geq \tilde{\theta}^o$, then (3) is infeasible. In the following, the SDP to obtain the optimum dual variables of (21) is summarized. Define

Algorithm 1 Minimizing the maximum interference

Solve the SDP problem (15) finding $\boldsymbol{\alpha}^o, \boldsymbol{\mu}^o, \boldsymbol{\lambda}^o$
Obtain $\Upsilon \triangleq \{m \mid \alpha_m^o > 0\}$
if $\Upsilon == \{1, \cdots, M\}$ **then**
  Compute $\mathbf{K}_m^o$ (7)
  Compute the coefficient ζ_m (14) and $\mathbf{w}_m^o$ (13), $\forall m$
else
  Find $\mathbf{w}_m^o$ (13) for all $m \in \Upsilon$
  Update $\bar{\mathbf{a}}$ (25), $\bar{\mathbf{b}}$ (26), and $\bar{\boldsymbol{\Psi}}_{m,i}$
  Solve (27) finding $l \in \{1, \cdots, M\} \setminus \Upsilon$ with $\alpha_l^o > 0$
  Compute ζ_l, $\mathbf{w}_l^o$ (13), and update $\Upsilon = \Upsilon \bigcup \{l\}$
end if

$$\bar{\mathbf{a}} \triangleq [\hat{\mathbf{a}}_1^T, P_r - e_1, \cdots, P_r - e_N, \hat{\mathbf{a}}_2^T]^T, \tag{25}$$

$$\bar{\mathbf{b}} \triangleq [\mathbf{0}_{(M+N)\times 1}^T, \hat{\mathbf{b}}_1^T]^T \tag{26}$$

where $\hat{\mathbf{a}}_1 \in \mathbb{R}^{M\times 1}$, $\hat{\mathbf{a}}_2 \in \mathbb{R}^{M\times 1}$, and $\hat{\mathbf{b}}_1 \in \mathbb{R}^{M\times 1}$ are obtained by substituting $\mathbf{a}(\tilde{m}) = 1$, $\mathbf{a}(M+N+\tilde{m}) = 1$ (or any arbitrary positive value), and $\mathbf{b}(M+N+\tilde{m}) = 0$, respectively.

The dual problem is equivalent to

$$\min_{\mathbf{x}} \bar{\mathbf{a}}^T \mathbf{x} \tag{27}$$

$$\text{subject to } \sum_{i=1}^{2M+N} x_i \bar{\boldsymbol{\Psi}}_{m,i} \preceq \mathbf{0}, m = 1, \cdots, M.$$

$$\mathbf{x} \succeq 0, \bar{\mathbf{b}}^T \mathbf{x} \leq 1$$

where $\bar{\boldsymbol{\Psi}}_{\tilde{m},i} = \mathbf{0}$ and $\bar{\boldsymbol{\Psi}}_{m,i} = \boldsymbol{\Psi}_{m,i}$ for $m \in \mathcal{M}$ and $i = 1, \cdots, 2M+N$. By the definition of $\bar{\mathbf{a}}$ in the objective (27), any positive $\alpha_{\tilde{m}}$ or $\mu_{\tilde{m}}$ would be penalized. Solving (27) is equivalent to removing the terms associated with the $\tilde{m}$-th pair in both the objective and constraints of (3), where only the maximum per-relay power upper-bounds are updated according to the power consumed by the m-th pair.

The steps required to solve the min-max interference problem (3) are summarized in Algorithm 1.

IV. Numerical Results

In this section, we provide simulations results to evaluate the performance of the proposed min-max interference algorithm. In the simulations, we set $\sigma_r^2 = \sigma_d^2 = 1$, $P_0/\sigma_r^2 = 10$ dB, and $P_r/\sigma_d^2 = 20$ dB. The number of feasible realizations is set to 500. The minimum SNR targets are set to $\gamma_m = 5$ dB for $m = 1, \cdots, M$. The channel vectors $\mathbf{h}_m$ and $\{\mathbf{g}_m, \tilde{\mathbf{g}}_m\}$ are assumed i.i.d. zero-mean Gaussian with variance 1.

To study the behavior of the maximum interference as the number N of relays increases, we plot the CDF of the maximum interference power under various N in Fig. 1. Also shown in Fig. 1 is the maximum interference under the optimization problem where the objective is to minimize the maximum transmission power over all relays while meeting the minimum SNR guarantees. The min-max relay power problem can be solved using a similar technique to the one proposed in this paper. The number of antennas are chosen

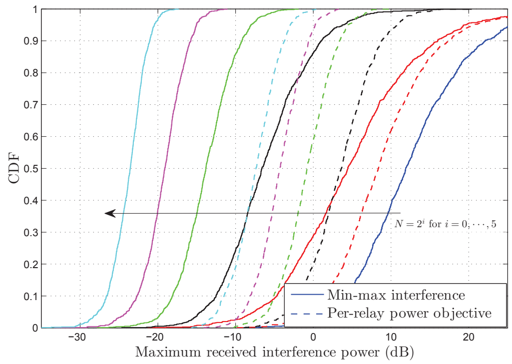

Fig. 1. CDF of maximum received interference power over subchannels when $M = 2$.

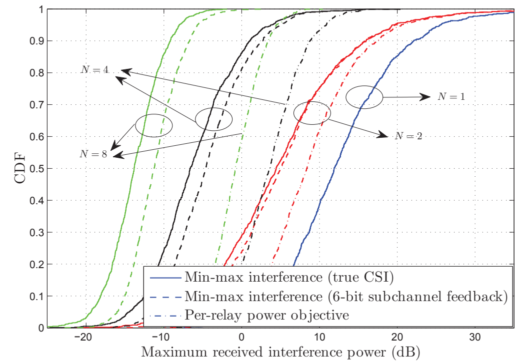

Fig. 2. Comparing the empirical CDF of maximum received interference power under true interference CSI with the limited FB case based on equiprobable quantization.

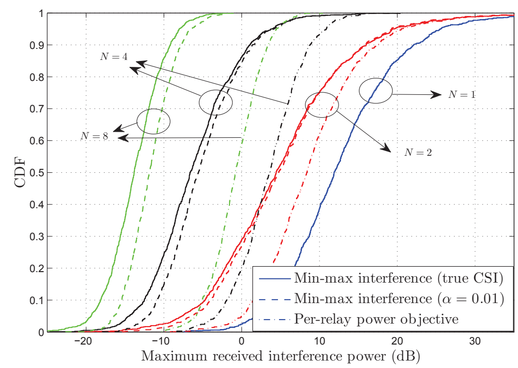

Fig. 3. Comparing the empirical CDF of maximum received interference power under true interference CSI with the case of Gaussian estimation error ($\alpha = 0.01$) for $M = 2$.

as $N = 2^i$ for $i \in \{0, \cdots, 5\}$. It can be noticed that as N increases, the maximum interference CDF curves are shifted to the left for both optimization objectives. It can be seen that the curves for the min-max interference do not converge as N becomes very large. In fact, those curves are uniformly shifted to the left. Note that the min-max interference approach outperforms the per-relay power approach for each N, and the performance gap increases as N increases.

So far, true interference CSI is assumed to be known perfectly at the relays. In practice, obtaining such interference CSI may not be possible. In order to observe how robust the proposed algorithm is with respect to imperfect CSI, we consider the following scenarios with two types of imperfect CSI, *i.e.,* limited number of CSI feedback (FB) bits and channel estimation error.

In Scenario 1, the receiver knows the interference CSI perfectly. However, the FB bits to the relays are limited. We consider equiprobable quantization of channel values. Let B denote the number of available FB bits. Every real and imaginary part of a channel is quantized with equal probability according to the channel.

In Scenario 2, the channels are estimated at the receiver with error and the exact estimated channel is fed back to the relays. In order to model the channel estimation error, let us define $\hat{h} = h + \alpha\tilde{h}$, where h is the true channel coefficient, $\hat{h}$ is the estimated channel coefficient used for optimization, and $\tilde{h} \sim \mathcal{CN}(0, 1)$ is the error. The coefficient α is set to adjust the variance of error.

In Fig. 2, the CDF of the maximum received interference under true interference CSI is compared with that of Scenario 1 with $B = 6$. It can be seen the performance of the maximum received interference in Scenario 1 is very close to the that of true CSI even for $N = 8$. As expected, the performance gap between the limited FB case and the true CSI case increases as N increases. In addition, the min-max interference under limited FB still outperforms the min-max per-relay objective in terms of maximum received interference.

Finally, Fig. 3 shows the CDF of the maximum received interference under true interference CSI as compared with that of Scenario 2 with estimation error for $\alpha = 0.01$. The performance under Scenario 2 is very close to that of true CSI. As expected, performance degrades as α increases, and is very poor when $\alpha = 0.5$, *i.e.,* estimation error's variance is half of the variance of true CSI. In addition, we see that the performance gap between Scenario 2 and the true CSI case increases as N increases.

V. Conclusion

In this paper, we have considered a multi-relay cellular network, where each cell has multiple source-destination pairs communicating in orthogonal channels with assistance from the relays. In order to manage inter-cell interference, we have formulated the min-max interference problem, under per-relay power and guaranteed received SNR constraints. We have shown that the strong duality property holds for this non-convex problem. Using the Lagrange dual domain, an efficient SDP-based algorithm has been proposed to find the optimum relay beamweights.

Appendix A
Proof of Theorem 1

Since $\boldsymbol{\alpha} \succ 0$, we have $\mathbf{K}_m \succ 0$ for $m = 1, \cdots, M$. Using [15, Lemma 1] and rewriting the expression of the matrix inequality (11), the dual problem (10) can be expressed as

$$\max_{\boldsymbol{\lambda},\boldsymbol{\mu}} \max_{\boldsymbol{\alpha}} \sum_{m=1}^{M} \alpha_m \sigma^2 - P_r(\sum_{i=1}^{N} \lambda_i) \quad (28)$$

$$\text{subject to } \frac{\alpha_m P_0}{\gamma_m} \mathbf{f}_m^H \mathbf{K}_m^{-1} \mathbf{f}_m \leq 1,\ m = 1, \cdots, M \quad (29)$$

$$(12),\ (9).$$

Note that $(\boldsymbol{\mu}, \boldsymbol{\lambda})$ satisfies the condition in Lemma 1, *i.e.,* the optimum solution of (28) remains in the feasible set defined by Lemma 1. For the rest of the proof, we use the general approach in [15] by establishing the duality between (28) and SIMO beamforming problem. Let us consider the following problem

$$\max_{\boldsymbol{\lambda},\boldsymbol{\mu}} \min_{\boldsymbol{\alpha}} \sum_{m=1}^{M} \alpha_m \sigma^2 - P_r(\sum_{i=1}^{N} \lambda_i) \quad (30)$$

$$\text{subject to } \frac{\alpha_m P_0}{\gamma_m} \mathbf{f}_m^H \mathbf{K}_m^{\dagger} \mathbf{f}_m \geq 1,\ m = 1, \cdots, M \quad (31)$$

$$(12),\ (9).$$

There are two differences between (28) and (30). The inner maximization becomes minimization and the SNR inequality is reversed. For a given $(\boldsymbol{\lambda}, \boldsymbol{\mu})$, we show that $\Phi(\alpha_m) \triangleq \frac{\alpha_m P_0}{\gamma_m} \mathbf{f}_m^H \mathbf{K}_m^{\dagger} \mathbf{f}_m$ is a monotonically increasing function of $\alpha_m > 0$ for $m = 1, \cdots, M$. Substituting (7) into $\Phi(\alpha_m)$, we have $\Phi_m(\alpha_m) = \frac{P_0}{\gamma_m} \mathbf{f}_m^H \big(\frac{1}{\alpha_m}(\mathbf{R}_m \mathbf{D}_{\boldsymbol{\lambda}} + \mu_m \tilde{\mathbf{B}}_m) + \mathbf{G}_m\big)^{\dagger} \mathbf{f}_m$, which is a monotonically increasing function of α_m. Hence, both (29) and (31) are met with equality at optimality, and the solution of both (28) and (30) is the same α_m^o that satisfies $\Phi_m(\alpha_m^o) = 1$ for $m = 1, \cdots, M$. This implies that the optimization problems (28) and (30) are equivalent. Note that (30) is actually obtained by substituting $\bar{\mathbf{w}}_m = \frac{\alpha_m P_0}{\sum_{m=1}^{M} \alpha_m \sigma^2 - P_r(\sum_{i=1}^{N} \lambda_i)} \mathbf{K}_m^{\dagger} \mathbf{f}_m$ into

$$\max_{\boldsymbol{\lambda},\boldsymbol{\mu}} \min_{\mathbf{w}_m,\boldsymbol{\alpha}} \sum_{m=1}^{M} \alpha_m \sigma^2 - P_r(\sum_{i=1}^{N} \lambda_i) \quad (32)$$

$$\text{subject to } \frac{\alpha_m P_0 |\mathbf{w}_m^H \mathbf{f}_m|^2}{\|\mathbf{K}_m^{\frac{1}{2}} \mathbf{w}_m\|^2} \geq \gamma_m,\ m = 1, \cdots, M \quad (33)$$

$$(12),\ (9).$$

The inner minimization of (32) is the SIMO beamforming problem where the M receivers each are equipped with N antennas. For the m-th receiver, the noise covariance matrix is $\bar{\mathbf{K}}_m \triangleq \frac{\sum_{m=1}^{M} \alpha_m \sigma^2 - P_r(\sum_{i=1}^{N} \lambda_i)}{\alpha_m P_0} \mathbf{K}_m$, and the transmit power is $\sum_{m=1}^{M} \alpha_m \sigma^2 - P_r(\sum_{i=1}^{N} \lambda_i)$. The solution of the SIMO beamforming problem, *i.e.,* the inner minimization of (32), is given by $\bar{\mathbf{w}}_m^o = \bar{\mathbf{K}}_m^{\dagger} \mathbf{f}_m$. Substituting $\bar{\mathbf{w}}_m^o$ into (32), (30) is derived. Note that the optimum $\bar{\mathbf{w}}_m^o$ can be scaled by any non-zero coefficient ξ such that $\xi \bar{\mathbf{w}}_m^o$ is also an optimum solution. The dual problem (10) is equivalent to the SIMO beamforming problem (32). Due to the zero duality gap shown in Proposition 1 and the fact that $\bar{\mathbf{w}}^o$ is unique up to a scale factor, the optimum solution of (3) is given by $\mathbf{w}_m^o = \zeta_m \mathbf{K}_m^o \mathbf{f}_m$. Since $\alpha_m^o > 0$, the SNR constraint (6) is met with equality based on the slackness condition. Substituting $\mathbf{w}_m^o$ into the equation $\frac{P_0 \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma^2} = \gamma_m$ and after some manipulations, (14) is obtained and the proof is complete.

REFERENCES

[1] J. Andrews, W. Choi, and R. W. Heath, Jr., "Overcoming interference in spatial multiplexing MIMO cellular networks," *IEEE Wireless Commun. Mag.*, vol. 14, pp. 95–104, Dec. 2007.

[2] 3GPP TS 36.211 V8.2.0 3rd Generation Prtnership Project; Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation (Release 8), Mar. 2008.

[3] IEEE standard for local and metropolitan area networks part 16: Air interface for fixed broadband wireless access systems, *IEEE Std 802.16- 2004*, pp. 1C857, 2004.

[4] I. Krikidis, J. S. Thompson, S. Mclaughlin, and N. Goertz, "Max-min relay selection for legacy amplify-and-forward systems with interference," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 3016–3027, June 2009.

[5] S. Ren and M. van der Schaar, "Distributed power allocation in multiuser multi-channel cellular relay networks," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 1952–1964, June 2010.

[6] R. W. Heath, Jr., T. Wu, Y. Kwon, and A. Soong, "Multiuser MIMO in distributed antenna systems with out-of-cell interference," *IEEE Trans. Signal Process.*, vol. 59, pp. 4885–4899, Oct. 2011.

[7] T. Ahmad, R. Gohary, H. Yanikomeroglu, S. Al-Ahmadi, and G. Boudreau, "Coordinated port selection and beam steering optimization in a multi-cell distributed antenna system using semidefinite relaxation," *IEEE Trans. Wireless Commun.*, vol. 11, pp. 1861–1871, May 2012.

[8] A. Ramezani-Kebrya, I.-M. Kim, F. Chan, R. Inkol, and H.-K. Song, "Detection for an AF cooperative diversity network in the presence of interference," *IEEE Commun. Lett.*, vol. 17, pp. 653–656, Apr. 2013.

[9] M. Pischella and J. Belfiore, "Power control in distributed cooperative OFDMA cellular networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 1900–1906, May 2008.

[10] S. W. Peters, A. Y. Panah, K. T. Truong, and R. W. Heath, Jr., "Relaying architectures for 3GPP LTE-Advanced," *EURASIP J. Adv. Signal Process*, vol. 28, pp. 1455–1468, 2009.

[11] Y. Hua, Q. Zhang, and Z. Niu, "Resource allocation in multi-cell OFDMA-based relay networks," in *Proc. IEEE INFOCOM*, pp. 2133–2141, Mar. 2010.

[12] L. Liang, G. Feng, and Y. Zhang, "Integrated interference coordination for relay-aided cellular OFDM system," in *Proc. IEEE ICC*, pp. 1–5, June 2011.

[13] L. Liang and G. Feng, "A game-theoretic framework for interference coordination in OFDMA relay networks," *IEEE Trans. Veh. Tech.*, vol. 61, pp. 321–332, Jan. 2012.

[14] M. Dong, B. Liang, and Q. Xiao, "Unicast multi-antenna relay beamforming with per-antenna power control: optimization and duality," *IEEE Trans. Signal Process.*, vol. 61, pp. 6076–6090, Dec. 2013.

[15] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 55, pp. 2646–2606, June 2007.

[16] A. Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, pp. 161–176, Jan. 2006.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge, U.K.: Cambridge Univ. Press, Mar. 2004.