Robust Power Optimization for Device-to-Device Communication in a Multi-cell Network under Partial CSI

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***Abstract*—For device-to-device (D2D) underlaid cellular networks, the perfect channel state information (CSI) may not be available at the base station (BS). In this work, under an assumption of partial CSI, we study the problem of maximizing the expected sum rate for a cellular user (CU) and a D2D pair, with receive beamforming at the BS, subject to minimum SINR requirements for both the CU and D2D pair, per-node maximum power, and inter-cell interference constraints in multiple neighboring cells. We solve this non-convex joint optimization problem in two steps. We first consider the D2D admissibility problem to determine whether the D2D pair can reuse the channel resource of the CU. We then propose a robust power control algorithm using a ratio-of-expectation approximation to maximize the expected sum rate. For benchmarking, we further provide an upper bound on the maximum expected sum rate. Simulation results show that our proposed solution gives performance close to the upper bound.**

I. Introduction

To meet the growing demand of high data rate and spectrum efficiency in cellular networks, device-to-device (D2D) communication has been developed, where nearby users can establish a direct communication link to transmit data to each other without going through the backhaul network [1], [2]. D2D communication can improve the overall network utilization due to resource reuse by both the cellular users (CUs) and the D2D pairs.

For a D2D underlaid cellular network, the D2D pair reuses the channel resource of a CU, *i.e.,* they cause intra-cell interference to each other. Furthermore, their transmissions generate inter-cell interference (ICI) to neighboring cells. To meet the quality-of-service requirements for CUs and D2D pairs and to limit the ICI to neighboring cells, various power allocation algorithms have been studied in the literature. In [3], sum rate maximization has been studied for a cellular network with one CU and one D2D pair with rate constraints and a minimum SINR requirement for the CU. For a single-antenna system, optimal power allocation for sum rate maximization of a D2D pair and a CU has been obtained in [4] without considering the ICI. In [5], we have jointly optimized the power of a CU and a D2D pair for their sum rate maximization, while satisfying minimum SINR requirements and a worst-case ICI limit in a neighboring cell.

The vast majority of the existing literature on D2D communication is focused on power allocation when perfect channel state information (CSI) is available at the scheduling base station (BS). However, this assumption imposes substantial signaling overhead due to the requirement of CSI feedback for channels that are away from the BS. In practical scenarios, the BS may not have perfect CSI knowledge. With only partial CSI knowledge of the interfering link from a CU to a D2D pair, probabilistic access control has been proposed in [6] for the D2D pair to maximize the expected sum rate for uplink resource sharing. In [7], D2D sum rate maximization is studied with channel uncertainty under SINR requirement of a CU by relaxing the objective and constraints. A channel assignment algorithm based on dynamic programming is proposed in [8] to maximize the network utility with partial CSI. Despite the results in [6]–[8], the effect of ICI due to D2D communication has not been investigated in the existing literature with partial CSI.

In this paper, we consider a multi-cell uplink scenario, where both the CU and D2D pair may generate significant ICI at multiple neighboring BSs. The CU and D2D users are each equipped with a single antenna, and unlike the simplifying assumption in [6]–[8] of a single-antenna BS, we consider the more practical scenario where the BS is equipped with multiple antennas. We assume perfect CSI only for the direct channels from the CU and D2D to the BS. For other channels, only partial CSI is available. We jointly optimize receive beamforming for the CU and the transmit powers of the CU and D2D transmitter. Our objective is to maximize the expected uplink sum rate of the CU and D2D pair under minimum CU and D2D SINR requirements, as well as per-node maximum power and ICI constraints in multiple neighboring cells.

The uncertain CSI, the non-convex expected sum rate, and the various power, interference, and SINR constraints, lead to a challenging optimization problem. We first study D2D admissibility and obtain a simple feasibility test. Assuming the D2D pair is admissible, we propose an efficient robust power control algorithm based on a ratio-of-expectation (ROE) approximation to maximize the expected sum rate. For performance benchmarking, we also develop an upper bound on the maximum expected sum rate. Simulation shows that the proposed ROE algorithm gives performance that is close to the upper bound, and it substantially outperforms a CU-priority heuristic.

The rest of this paper is organized as follows. In Section II, the system model is described and the expected sum rate maximization problem is formulated. In Section III, the necessary and sufficient condition for admissibility of the D2D pair is obtained, the power control solution is developed, and

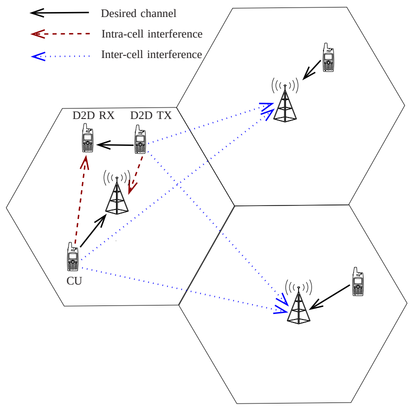

Fig. 1. System model.

the upper bound is given. Numerical results are presented in Section IV, and conclusions are drawn in Section V.

II. System Model and Problem Formulation

A. System Model

Consider a cellular system where the D2D pairs reuse the spectrum resource already assigned to the CUs for uplink communication. We follow the conventional assumption of orthogonal spectrum resource allocation among CUs in a cell. Thus, these CUs do not interfere with each other. When a D2D pair communicates using the channel of a CU, they both cause intra-cell interference to each other. Due to orthogonal channelization within each cell, we may focus on one CU and one D2D pair as shown in Fig 1. We assume that all users are equipped with a single antenna and the BS is equipped with N antennas. The BS coordinates the transmission of the CU and D2D pair.

Let P_D and P_C denote the transmit power of the D2D pair and the CU, respectively. The uplink received SINR at the BS from the CU is given by

$$\gamma_C = \frac{P_C|\mathbf{w}^H\mathbf{h}_C|^2}{\sigma^2 + P_D|\mathbf{w}^H\mathbf{g}_D|^2} \tag{1}$$

where $\mathbf{h}_C \in \mathbb{C}^{N\times 1}$ is the channel between the CU and the BS, $\mathbf{g}_D \in \mathbb{C}^{N\times 1}$ is the interference channel between the D2D transmitter and the BS, $\mathbf{w}$ is the receive beam vector at the BS with unit norm, *i.e.,* $\|\mathbf{w}\|^2 = 1$, and σ^2 is the noise variance at the BS. The SINR at the D2D receiver is given by

$$\gamma_D = \frac{P_D|h_D|^2}{\sigma_D^2 + P_C|g_C|^2} \tag{2}$$

where $h_D \in \mathbb{C}$ is the channel between the D2D pair, $g_C \in \mathbb{C}$ is the interference channel between the CU and the D2D receiver, and σ_D^2 is the noise variance at the D2D receiver.

In a multi-cell network, both D2D and CU transmissions cause ICI in a neighboring cell. In this work, we consider ICI for uplink transmission at b neighboring BSs. However, our approach can be applied also to ICI in the downlink scenario. Let $\mathbf{f}_{C,j} \in \mathbb{C}^{N\times 1}$ and $\mathbf{f}_{D,j} \in \mathbb{C}^{N\times 1}$ denote the ICI channels from the CU and the D2D transmitter to neighboring BS j, respectively. Since the beam vector at neighboring BS j is typically unknown to the CU and D2D pair, we consider the worst-case ICI given by[1]

$$P_{\mathcal{I},j} = P_C\|\mathbf{f}_{C,j}\|^2 + P_D\|\mathbf{f}_{D,j}\|^2. \tag{3}$$

We assume perfect instantaneous CSI is available only for $\{\mathbf{h}_C, \mathbf{g}_D\}$, *i.e.,* the direct channels from the CU and D2D to the BS in Fig. 1. However, only partial CSI is available for h_D, g_C, $\{\mathbf{f}_{D,j}\}_{j=1}^b$, and $\{\mathbf{f}_{C,j}\}_{j=1}^b$. In particular, only distance-based statistical knowledge is available at the BS scheduler. We assume $|h_D|^2 \sim \exp(\eta_1)$ and $|g_C|^2 \sim \exp(\eta_2)$, which is a common assumption corresponding to Rayleigh fading, but the distributions of $\{\mathbf{f}_{D,j}\}_{j=1}^b$ and $\{\mathbf{f}_{C,j}\}_{j=1}^b$ can be general. Instead of instantaneous CSI, we assume η_1 and η_2 are known at the BS. Note that measuring and transmitting these statistical parameters is much easier than the instantaneous CSI [9]. This substantially reduces the signaling overhead due to D2D communication. For the ICI channels, we assume $\mathbb{E}[|\mathbf{f}_{D,j}|^2] = \lambda_{D,j}$ and $\mathbb{E}[|\mathbf{f}_{C,j}|^2] = \lambda_{C,j}$ for $j = 1, \cdots, b$, where only $\{\lambda_{D,j}\}_{j=1}^b$ and $\{\lambda_{D,j}\}_{j=1}^b$ are known at the BS scheduler. These statistical parameters can be estimated in neighboring BSs and shared with the BS in the desired cell through the wired backhaul.

B. Problem Formulation

Let $P_C^{\max}$ and $P_D^{\max}$ denote the maximum transmit power at the CU and D2D transmitters, respectively. Our goal is to maximize the expected sum rate of the D2D pair and the CU uplink transmission by optimizing the transmit powers $\{P_D, P_C\}$ and the beam vector $\mathbf{w}$, under per-node power and ICI constraints, as well as SINR requirements for both the CU and the D2D pair. Due to the partial CSI assumptions explained in Section II-A, the D2D SINR and ICI at each neighboring BS are random variables. For the D2D pair's SINR requirement, we consider a probabilistic constraint limiting the outage probability of the D2D pair. We also limit the expected worst-case ICI. Thus, the expected sum rate maximization problem is given by

$$\text{P1:} \quad \max_{(P_D, P_C, \mathbf{w})} \Big(\log_2(1+\gamma_C) + \mathbb{E}[\log_2(1+\gamma_D)]\Big)$$
$$\text{subject to } \gamma_C \geq \tilde{\gamma}_C, \tag{4}$$
$$\Pr\{\gamma_D \leq \tilde{\gamma}_D\} \leq \epsilon, \tag{5}$$
$$P_C \leq P_C^{\max}, \; P_D \leq P_D^{\max}, \tag{6}$$
$$\mathbb{E}[P_{\mathcal{I},j}] \leq \tilde{\mathcal{I}}, \; j = 1, \cdots, b \tag{7}$$

where $\tilde{\gamma}_C$ is the minimum SINR requirement for the CU, ϵ is the maximum probability of the D2D SINR dropping below a certain threshold $\tilde{\gamma}_D$, and $\tilde{\mathcal{I}}$ is the expected maximum ICI power in the neighboring cells.

III. Admissibility Test and Power Allocation

In this section, we solve the expected sum rate maximization problem P1. This problem is non-convex, since the objective function is non-convex. We solve P1 in two steps. First, we

[1]Note that $P_{\mathcal{I},j}$ in (3) is an an upper bound of the actual ICI. Let $\tilde{\mathbf{w}}_j$ denote the beam vector at neighboring BS j. If $\tilde{\mathbf{w}}_j$ is known, then we can consider the actual ICI through replacing $\|\mathbf{f}_j\|$ by $|\tilde{\mathbf{w}}_j^H\mathbf{f}_j|$ in (3).

need to ensure whether the D2D pair can be admitted to reuse the CU's assigned channel. Then, if the D2D pair is admissible, we attempt to optimize the powers and beam vector to maximize the expected sum rate.

A. The Admissibility Test

Given the power constraints, SINR requirements, and ICI constraints, the admissibility of the D2D pair can be determined by solving the feasibility test given by

$$\begin{aligned} &\text{find } \{P_D, P_C, \mathbf{w}\} \\ &\text{subject to } (4),(5),(6),(7). \end{aligned} \tag{8}$$

Following a similar argument in [5], we first obtain the optimal beam vector $\mathbf{w}$ in terms of $\{P_C, P_D\}$ that maximizes γ_C at the left-hand side of constraint (4). For a given set of $\{P_C, P_D\}$, the optimal beam vector is given by $\mathbf{w}^o = \frac{\mathbf{\Lambda}_D^{-1}\mathbf{h}_C}{\|\mathbf{\Lambda}_D^{-1}\mathbf{h}_C\|}$ where $\mathbf{\Lambda}_D \triangleq \sigma^2\mathbf{I} + P_D\mathbf{g}_D\mathbf{g}_D^H$. Substituting $\mathbf{w}^o$ into (1), the SINR constraint (4) is given by

$$\frac{P_C\|\mathbf{h}_C\|^2}{\sigma^2}\Big(1 - \frac{\rho^2 P_D\|\mathbf{g}_D\|^2}{P_D\|\mathbf{g}_D\|^2 + \sigma^2}\Big) \geq \tilde{\gamma}_C \tag{9}$$

where $\rho \triangleq \frac{|\mathbf{h}_C^H\mathbf{g}_D|}{\|\mathbf{h}_C\|\|\mathbf{g}_D\|}$.

For notation simplicity, in the following, we denote $x \triangleq P_D$ and $y \triangleq P_C$. The D2D admissibility condition is given in the following lemma.

Lemma 1: The necessary and sufficient condition for the D2D pair to be admissible is given by

$$0 < x_{\mathcal{I}} \leq P_D^{\max}, \tag{10}$$

$$0 < y_{\mathcal{I}} \leq P_C^{\max}, \tag{11}$$

$$c_{1,j}y_{\mathcal{I}} + c_{2,j}x_{\mathcal{I}} \leq 1,\ j = 1, \cdots, b \tag{12}$$

where $c_{1,j} \triangleq \lambda_{C,j}/\tilde{\mathcal{I}}$ and $c_{2,j} \triangleq \lambda_{D,j}/\tilde{\mathcal{I}}$ and $\{x_{\mathcal{I}}, y_{\mathcal{I}}\}$ is the unique power solution of the following system of equations

$$y = \alpha\left(1 - \frac{K_1}{1 + K_2/x}\right)^{-1} \tag{13}$$

$$y = l_1 x\left(\frac{\exp(-l_2/x)}{1-\epsilon} - 1\right) \tag{14}$$

where $K_1 \triangleq \rho^2$, $K_2 \triangleq \frac{\sigma^2}{\|\mathbf{g}_D\|^2}$, $l_1 = \frac{\eta_2}{\eta_1\tilde{\gamma}_D}$ and $l_2 = \eta_1\sigma_D^2\tilde{\gamma}_D$.

Proof: We first obtain the cumulative distribution function for random variable $Z = \frac{X}{\sigma_D^2 + Y}$ where $X \sim \exp(\eta_1/x)$ and $Y \sim \exp(\eta_2/y)$.

$$\begin{aligned} F_Z(z) &= \Pr\left\{\frac{X}{\sigma_D^2 + Y} \leq z\right\} = \int_0^\infty f_Y(t)F_X(z(\sigma_D^2 + t))\,\mathrm{d}t \\ &= \int_0^\infty \frac{\eta_2\exp(-\eta_2 t/y)}{y}\left(1 - \exp(-\eta_1 z(\sigma_D^2 + t)/x)\right)\mathrm{d}t \\ &= 1 - \frac{\eta_2/y}{\eta_1 z/x + \eta_2/y}\exp(-\eta_1 z\sigma_D^2/x). \end{aligned} \tag{15}$$

The constraint (5) can be written as $F_Z(\tilde{\gamma}_D) \leq \epsilon$, *i.e.,*

$$y \leq l_1 x\left(\frac{\exp(-l_2/x)}{1-\epsilon} - 1\right). \tag{16}$$

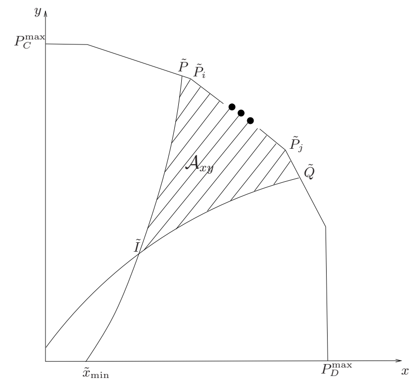

Fig. 2. An example of feasible region.

It is not difficult to show that $g(x) = l_1 x\left(\frac{\exp(-l_2/x)}{1-\epsilon} - 1\right)$ is a convex and increasing function of x. Furthermore, the D2D SINR requirement (16) can be satisfied only if

$$x \geq x_{\min} \triangleq \frac{-\eta_1\sigma_d^2\tilde{\gamma}_D}{\ln(1-\epsilon)}. \tag{17}$$

Considering both (9) and (16) with equality, the unique power solution is given by $\{x_{\mathcal{I}}, y_{\mathcal{I}}\}$. Note that $x_{\mathcal{I}}$ is the solution of $\alpha\left(1 - \frac{K_1}{1+K_2/x}\right)^{-1} = l_1 x\left(\frac{\exp(-l_2/x)}{1-\epsilon} - 1\right)$, which can be obtained efficiently using a bisection search algorithm within the range $x_{\min} \leq x \leq P_D^{\max}$. ■

We note that constraints (10) and (11) ensure the maximum powers at the D2D and CU are enough to satisfy both SINR requirements. Constraint (12) guarantees the ICI limits can be met.

B. Proposed Power Control Algorithm

Assuming the D2D pair is admissible, we now solve the optimal power allocation problem to maximize P1. After substituting $\mathbf{w}^o$ into (1), we need to solve P1 by optimizing (x, y), *i.e.,*

$$\text{P2:} \quad \max_{(x,y)} \mathcal{R}(x, y)$$

$$\text{subject to } y\Big(1 - \frac{K_1 x}{K_2 + x}\Big)l_3 \geq \tilde{\gamma}_C, \tag{18}$$

$$y \leq l_1 x\left(\frac{\exp(-l_2/x)}{1-\epsilon} - 1\right), \tag{19}$$

$$y \leq P_C^{\max},\ x \leq P_D^{\max}, \tag{20}$$

$$c_{1,j}y + c_{2,j}x \leq 1,\ j = 1, \cdots, b \tag{21}$$

where $\mathcal{R}(x, y) = \log_2\left((1 + y(1 - \frac{K_1 x}{K_2 + x})b)\right) + \mathbb{E}[\log_2(1 + \gamma_D)]$ and $l_3 \triangleq \|\mathbf{h}_C\|^2/\sigma^2$. Let $\mathcal{A}_{xy}$ denote the feasible solution region of problem P2. An example of $\mathcal{A}_{xy}$ for a specific scenario is shown in Fig. 2.

Two properties of the objective function in P2 are provided in the following lemmas.

Lemma 2: The optimal power solution pair (x^o, y^o) is at the vertical, horizontal, or a tilted boundary of $\mathcal{A}_{xy}$, given by $x = P_D^{\max}$, $y = P_C^{\max}$, or $c_{1,j}y + c_{2,j}x = 1$ for some j, respectively.

Proof: We omit the details of the proof. It is similar to [5, Lemma 1]. ■

Lemma 3: The expected D2D rate is given by

$$\overline{\mathcal{R}}_D = \frac{\eta_2 x \log_2(e)}{\eta_1 y - \eta_2 x}\big(E'(\eta_2\sigma_D^2/y) - E'(\eta_1\sigma_D^2/x)\big) \qquad (22)$$

where $E'(x) = \exp(x)E_1(x)$ and $E_1(x) = \int_0^\infty \frac{\exp(-t)}{t}\,\mathrm{d}t$.

Proof: Since $|h_D|^2 \sim \exp(\eta_1)$ and $|g_C|^2 \sim \exp(\eta_2)$, we can obtain $\mathbb{E}[\log_2(1+\gamma_D)]$ by taking a double integral. We omit the details due to page limitation. ■

By Lemma 2, the optimal power solution pair (x^o, y^o) to maximize P2 is given in one of two cases: 1) A corner point of the horizontal, vertical, or tilted boundary line segment(s) of $\mathcal{A}_{xy}$; or 2) an interior point of the horizontal, vertical, or tilted boundary line segment(s) of $\mathcal{A}_{xy}$. In Case 2, we need to solve a numerical equation to find the set of candidate power pairs over each boundary line. Unfortunately, there is no closed-form solution or efficient algorithm to solve those equations. In order to tackle this issue, we propose to obtain the powers through approximating the objective function as follows.

We propose to replace $\mathbb{E}[\log_2(1+\gamma_D)]$ in the objective of P2 by the ratio of expectation, *i.e.,* $\log_2\left(1 + \frac{x\mathbb{E}[|h_D|^2]}{\sigma_D^2 + y\mathbb{E}[|g_C|^2]}\right)$. We observed through simulation that we can do so with very little performance degradation. In other words, we propose to solve the following problem

$$\text{P3:} \qquad \max_{(x,y)} \tilde{\mathcal{R}}(x,y)$$
$$\text{subject to } (18)-(21)$$

where $\tilde{\mathcal{R}}(x,y) = \log_2\left(\left(1 + y(1-\frac{K_1 x}{K_2+x})b\right)\left(1 + \frac{x/\eta_1}{\sigma_D^2 + y/\eta_2}\right)\right)$.

We can now solve P3 by generalizing our algorithm in [5], which was proposed for the perfect CSI scenario and a single ICI constraint, to a scenario with multiple ICI constraints. Following similar arguments in [5], we first summarize the properties of the optimal power pair (x^o, y^o): The optimal power solution pair is at the vertical, horizontal, or a tilted boundary of $\mathcal{A}_{xy}$. If the boundaries of the feasible region $\mathcal{A}_{xy}$ do not include any tilted boundary line segment, then the optimal power pair is at one corner point of the vertical or horizontal boundary. If the boundaries of the feasible region $\mathcal{A}_{xy}$ include $c_{1,j}y + c_{2,j}x = 1$ for some j, then the optimal power pair is given in one of two cases: 1) A corner point of the horizontal, vertical, or tilted boundary line segment(s) of $\mathcal{A}_{xy}$; or 2) an interior point of the tilted boundary line segment(s) of $\mathcal{A}_{xy}$, whose x-coordinate is one of the roots of the following quartic equation

$$e_4x^4 + e_3x^3 + e_2x^2 + e_1x + e_0 = 0 \qquad (23)$$

where $\{e_i\}_{i=0}^4$ are given in [5].

To solve P3, we propose an iterative algorithm by analyzing the feasible region when a new tilted line is considered one at a time (*i.e.,* when a new ICI constraint is considered in a neighboring cell). We define two matrices: $\mathbf{C}$, which includes the corner points of the feasible region by considering constraints (20) and (21), *i.e.,* ignoring the SINR requirements for the CU and D2D pair; and $\mathbf{A}$, which specifies the line segment connecting any two consecutive corner points in $\mathbf{C}$. In particular, $\mathbf{C}$ is initially constructed considering only constraint (20). The first and last columns of the initial $\mathbf{C}$ are $[0,0]^T$, *i.e.,* the origin coordinates. The other corner points are in the columns of the initial $\mathbf{C}$ in a clock-wise order. For matrix $\mathbf{A}$, the column $\mathbf{A}_{:,i}$ is $[\mathbf{A}_{i1}, \mathbf{A}_{i2}, \mathbf{A}_{i3}]^T$ when the line segment between $\mathbf{C}_{:,i}$ and $\mathbf{C}_{:,i+1}$ is $\mathbf{A}_{i1}x + \mathbf{A}_{i2}y = \mathbf{A}_{i3}$.

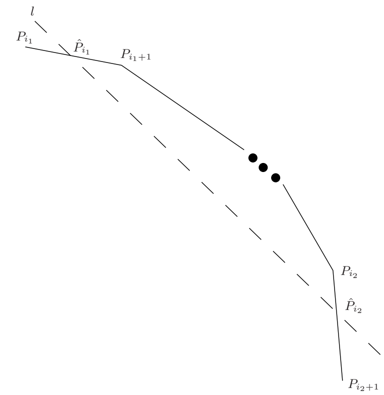

Fig. 3. Intersection of a new tilted line.

Let $\mathcal{A}_{xy,j}$ denote the updated feasible region after considering ICI constraint j. Then we add a new tilted line l due to ICI constraint $j+1$ as shown in Fig. 3. Note that l intersects $\mathcal{A}_{xy,j}$ in exactly two points if there is any intersection at all. We denote $\{P_{i_1}, P_{i_1+1}, \cdots, P_{i_2}, P_{i_2+1}\} \subset \mathcal{A}_{xy,j}$ a set of corner points in $\mathcal{A}_{xy,j}$ such that the intersections of l with $\mathcal{A}_{xy,j}$ are on the lines specified by $\mathbf{A}_{:,i_1}$ and $\mathbf{A}_{:,i_2}$. Since $\hat{P}_{i_1}$ and $\hat{P}_{i_2}$ are the corner points of the new feasible region $\mathcal{A}_{xy,j+1}$, we update $\mathbf{C}$ by keeping the corner points $\{P_{i_1}, P_{i_2+1}\}$ and removing $\{P_{i_1+1}, \cdots, P_{i_2}\}$, *i.e.,* all the middle points. The new feasible region $\mathcal{A}_{xy,j+1}$ includes $\{P_{i_1}, \hat{P}_{i_1}, \hat{P}_{i_2}, P_{i_2+1}\}$. Accordingly, we update the matrices $\mathbf{C}$ and $\mathbf{A}$.

In order to implement the admissibility test presented in Lemma 1, we consider the intersection of $\mathcal{A}_{xy,b}$ with the curves associated with minimum SINR requirements (18) and (19). A necessary and sufficient condition for the D2D pair to be admissible is that the solution $\mathbf{s}_{\mathcal{I}} \triangleq [x_{\mathcal{I}}, y_{\mathcal{I}}]^T$ in Lemma 1 satisfies $\boldsymbol{\Delta} \cdot \mathbf{s}_{\mathcal{I}} \preceq \boldsymbol{\delta}$ where $\boldsymbol{\Delta}$ and $\boldsymbol{\delta}$ are obtained iteratively through the algorithm. Let $\tilde{P}$ and $\tilde{Q}$ denote the points where the curves $I-\tilde{P}$ and $I-\tilde{Q}$ intersect $\mathcal{A}_{xy,b}$ as shown in Fig. 2. The feasible region is specified as the shaded area in this figure. The feasible region may include some tilted, horizontal, or vertical boundary line segments.

In order to solve problem P3, we need to find all candidates to be an optimal solution. As discussed earlier, the optimal power pair (x^o, y^o) can be one of Points $\{\tilde{P}, P_i, \cdots, P_j, \tilde{Q}\}$. Let $\tilde{\mathbf{C}}$ denote the set of all feasible corner points. We find all roots of (23), whose x-coordinates are within the range of two consecutive corner points in $\tilde{\mathbf{C}}$. Let $\mathbf{S}_j$ denote the set of roots that meet the range constraint for $c_{1,j}y + c_{2,j}x = 1$. Then the set of candidate points on the interior of line segment $c_{1,j}y + c_{2,j}x = 1$ is given by $\mathbf{Z}_j \triangleq \{(x, (1-c_{2,j}x)/c_{1,j}) : x \in \mathbf{S}_j\}$. Thus, the set of candidate pairs for (x^o, y^o) is given by $\mathbf{P}^o = \tilde{\mathbf{C}} \bigcup_{j=1}^{b} \mathbf{Z}_j$.

The steps to solve problem P3 are summarized in Algorithm 1, *i.e.,* the ROE approximation algorithm. In this

Algorithm 1 Maximizing the objective of problem P3

Input: α, K_1, K_2, l_1, l_2, x_H, $\{c_{1,j}\}_{j=1}^{b}$, $\{c_{2,j}\}_{j=1}^{b}$, $P_C^{\max}$, $P_D^{\max}$
Output: P_C^o, P_D^o, and $\mathbf{w}^o$

Set $\tilde{i}=0$, $\mathbf{C}=\begin{bmatrix}0 & 0 & P_D^{\max} & P_D^{\max} & 0\\ 0 & P_C^{\max} & P_C^{\max} & 0 & 0\end{bmatrix}$, $\boldsymbol{\delta}=\begin{bmatrix}0\\0\\P_D^{\max}\\P_C^{\max}\end{bmatrix}$, $\mathbf{A}=\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 0 & P_C^{\max} & P_D^{\max} & 0\end{bmatrix}$, and $\boldsymbol{\Delta}=\begin{bmatrix}-1 & 0\\ 0 & -1\\ 1 & 0\\ 0 & 1\end{bmatrix}$.
for $j=1:b$ **do**
  for $i=1:n_{\mathbf{A}}$ **do**
    Compute $\mathbf{s}=[\tilde{x},\tilde{y}]^T$ where $\tilde{x}=\frac{\mathbf{A}_{i2}-c_{1,j}\mathbf{A}_{i3}}{c_{2,j}\mathbf{A}_{i2}-c_{1,j}\mathbf{A}_{i1}}$ and $\tilde{y}=\frac{c_{2,j}\mathbf{A}_{i3}-\mathbf{A}_{i1}}{c_{2,j}\mathbf{A}_{i2}-c_{1,j}\mathbf{A}_{i1}}$.
    if $\boldsymbol{\Delta}\cdot\mathbf{s}\preceq\boldsymbol{\delta}$ and $\tilde{i}==0$ **then**
      Set $i_1=i$, $\tilde{i}=1$, and $\mathbf{s}_1=\mathbf{s}$.
    else if $\boldsymbol{\Delta}\cdot\mathbf{s}\preceq\boldsymbol{\delta}$ and $\tilde{i}==1$ **then**
      Set $i_2=i$ and $\mathbf{s}_2=\mathbf{s}$.
    end if
  end for
  if $\tilde{i}>0$ **then**
    Set $\mathbf{C}_1\triangleq\mathbf{C}_{:,1:i_1}$, $\mathbf{C}_2\triangleq\mathbf{C}_{:,i_2+1:n_{\mathbf{C}}}$, $\mathbf{A}_1\triangleq\mathbf{A}_{:,1:i_1}$, and $\mathbf{A}_2\triangleq\mathbf{A}_{:,i_2:n_{\mathbf{A}}}$.
    Update $\mathbf{C}=[\mathbf{C}_1,\mathbf{s}_1,\mathbf{s}_2,\mathbf{C}_2]$, $\mathbf{A}=[\mathbf{A}_1,\mathbf{c},\mathbf{A}_2]$ where $\mathbf{c}\triangleq[c_{2,j},c_{1,j},1]^T$.
    Update $\boldsymbol{\Delta}=[\boldsymbol{\Delta}^T,\mathbf{c}_{1:2}^T]^T$ and $\boldsymbol{\delta}=[\boldsymbol{\delta}^T,1]^T$.
  end if
end for
Check the feasibility $\boldsymbol{\Delta}\cdot\mathbf{s}_{\mathcal{I}}\preceq\boldsymbol{\delta}$.
Set $i_s=2$ and $i_f=n_{\mathbf{A}}-1$.
if $\mathbf{A}_{1:2,i_s}==[0,1]^T$ **then**
  Set $i_s=3$, $\mathbf{Q}_{:,1}=\left[K_2\left(\frac{K_1}{1-\alpha/P_C^{\max}}-1\right)^{-1},P_C^{\max}\right]^T$, and $\mathbf{T}_{:,1}=\left[x_H,P_C^{\max}\right]^T$.
else if $\mathbf{A}_{1:2,i_f}==[1,0]^T$ **then**
  Set $i_f=n_{\mathbf{A}}-2$, $\mathbf{Q}_{:,n_{\mathbf{A}}-2}=\left[P_D^{\max},\alpha\left(1-\frac{K_1}{1+K_2/P_D^{\max}}\right)^{-1}\right]^T$, and $\mathbf{T}_{:,n_{\mathbf{A}}-2}=\left[P_D^{\max},l_1P_D^{\max}\left(\frac{\exp(-l_2/P_D^{\max})}{1-\epsilon}-1\right)\right]^T$.
end if
for $j=i_s:i_f$ **do**
  Set $\mathbf{T}_{:,j-1}=[\psi_2,\frac{1-c_2\psi_2}{c_1}]^T$ and $\mathbf{Q}_{:,j-1}=[\psi_1,\frac{1-c_2\psi_1}{c_1}]^T$ with $c_1=\mathbf{A}_{j2}$ and $c_2=\mathbf{A}_{j1}$.
end for
Find the indexes j_1 and j_2 such that $\boldsymbol{\Delta}\cdot\mathbf{T}_{:,j_1}\preceq\boldsymbol{\delta}$ and $\boldsymbol{\Delta}\cdot\mathbf{Q}_{:,j_2}\preceq\boldsymbol{\delta}$.
Define $\tilde{\mathbf{C}}=\left\{\mathbf{T}_{:,j_1},\mathbf{C}_{:,j_1+2:j_2+1},\mathbf{Q}_{:,j_2}\right\}$ and set $\mathbf{P}^o=\tilde{\mathbf{C}}$.
for $k=1:n_{\tilde{\mathbf{C}}}-1$ **do**
  if $\mathbf{A}_{1:2,k+j_1}==[1,0]^T$ or $\mathbf{A}_{1:2,k+j_1}==[0,1]^T$ **then return**
  else
    Compute $\mathbf{z}=[x_r,\frac{1-\mathbf{A}_{1,k+j_1}x_r}{\mathbf{A}_{2,k+j_1}}]^T$ where x_r is the root of (23) with $\tilde{\mathbf{C}}_{1,k}\le x_r\le\tilde{\mathbf{C}}_{1,k+1}$.
    Update $\mathbf{P}^o=\mathbf{P}^o\bigcup\{\mathbf{z}\}$.
  end if
end for
Enumerate among candidate solution set $\mathbf{P}^o$ to find the optimal solution.
Obtain the optimal beam vector.

algorithm, $\alpha\triangleq\tilde{\gamma}_C/l_3$, $\psi_1\triangleq\frac{\theta+\sqrt{\theta^2-4c_2(1-K_1)K_2(\alpha c_1-1)}}{2c_2(1-K_1)}$ with $\theta\triangleq 1-K_1-c_2K_2-\alpha c_1$. In addition, x_H and ψ_2 are given by solving $l_1x_H\left(\frac{\exp(-l_2/x_H)}{1-\epsilon}-1\right)=P_C^{\max}$ and $l_1\psi_2\left(\frac{\exp(-l_2/\psi_2)}{1-\epsilon}-1\right)=\frac{1-c_2\psi_2}{c_1}$, respectively, using bisection.

C. An Upper Bound to the Maximum Objective of P2

In the previous section, we have presented an algorithm to solve the ROE approximation problem P3 exactly. Let (x^*,y^*) denote the optimal solution of P3. Substituting (x^*,y^*) into the objective of P2, we have $\mathcal{R}(x^*,y^*)\le\mathcal{R}^o$ where $\mathcal{R}^o$ denotes the optimal value of the objective in P2, as well as the original problem P1. Since it is difficult to compute $\mathcal{R}^o$, to evaluate the gap between $\mathcal{R}(x^*,y^*)$ and $\mathcal{R}^o$, we next propose an upper bound on $\mathcal{R}^o$.

Proposition 1: An upper bound on the optimal objective of P2 can be obtained by solving the problem

$$\text{P4:}\qquad \max_{(x,y)}\ \hat{\mathcal{R}}(x,y)\qquad \text{subject to } (18)-(21)$$

where $\hat{\mathcal{R}}(x,y)=\log_2\left(\left(1+y(1-\frac{K_1x}{K_2+x})b\right)\left(1+G\frac{x/\eta_1}{\sigma_D^2+y/\eta_2}\right)\right)$ and $G=(1+\sigma_D^2\eta_2/P_C^{\max})E'(\sigma_D^2\eta_2/P_C^{\max})>1$.

Proof: We consider two random variables $Z_1\triangleq x\mathbb{E}[|h_D|^2$ and $Z_2\triangleq\sigma_D^2+y\mathbb{E}[|g_C|^2]$ for notation simplicity. First, we show the following inequality holds:

$$\frac{\mathbb{E}[Z_1]}{\mathbb{E}[Z_2]}\le\mathbb{E}\left[\frac{Z_1}{Z_2}\right]=\frac{\mathbb{E}[Z_1]}{\mathbb{E}[Z_2]}\left(1+\frac{\sigma^2\eta_2}{y}\right)E'\left(\frac{\sigma^2\eta_2}{y}\right).\tag{24}$$

Note that Z_1 and Z_2 are independent random variables. Hence, we have

$$\mathbb{E}\left[\frac{Z_1}{Z_2}\right]=\mathbb{E}[Z_1]\mathbb{E}\left[\frac{1}{Z_2}\right]\ge\frac{\mathbb{E}[Z_1]}{\mathbb{E}[Z_2]}\tag{25}$$

by Jensen's inequality, since $f(y)=1/(y)$ is a convex function.

Now, we show that $\varphi(t)=(1+t)E'(t)$ is a strictly decreasing function of t. The continued fraction expansion of $E_1(t)$ is given by [10]

$$E'(t)=\frac{1}{t+\frac{1}{1+\frac{1}{t+\cdots}}}.\tag{26}$$

Ignoring high order terms in (26), we have

$$E'(t)<\frac{t+1}{t(t+2)}\text{ for all }t.\tag{27}$$

Using the inequality (27) and taking the first order derivative of $\varphi(t)$, we have

$$\frac{\mathrm{d}\varphi(t)}{\mathrm{d}t}=(2+t)E'(t)-1-\frac{1}{t}<0.\tag{28}$$

Since $\varphi(t)$ is a strictly decreasing function, the right-hand side of (24) is maximized by substituting $y=P_C^{\max}$, *i.e.,*

$$\mathbb{E}\left[\frac{Z_1}{Z_2}\right]<G\frac{\mathbb{E}[Z_1]}{\mathbb{E}[Z_2]}\text{ for all }(x,y).\tag{29}$$

Finally, we note that $\mathbb{E}[\log_2(1+Z_1/Z_2)]\le\log_2(1+\mathbb{E}[Z_1/Z_2])$ for any given (x,y) due to Jensen's inequality. Hence, the optimal objective of P4 is always an upper bound on the objective of P2 under the optimal solution. ■

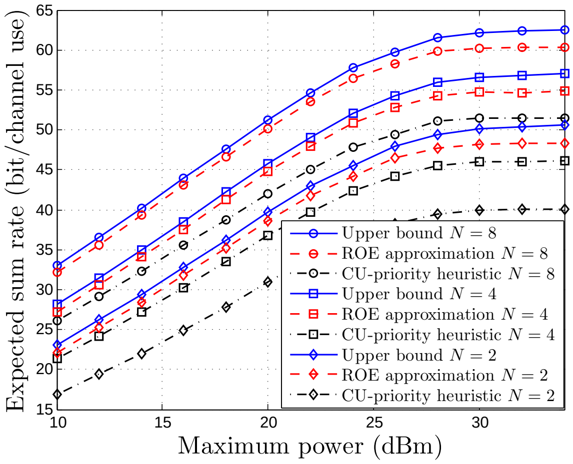

Fig. 4. The expected sum rate with $d_D/d_0 = 0.1$ for $N = 2, 4, 8$.

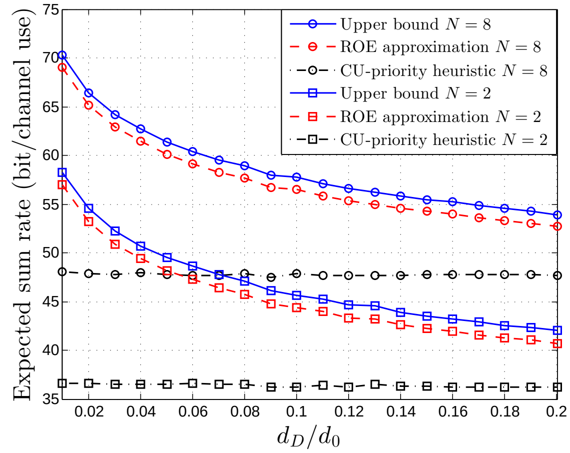

Fig. 5. The expected sum rate with $P^{\max} = 24$ dBm for $N = 2, 8$.

Note that $G \to 1$ as $P_C^{\max} \to 0$, which can be shown using the following inequalities [10]:

$$\frac{1}{2}\ln\left(1+\frac{2}{t}\right) < E'(t) < \ln\left(1+\frac{1}{t}\right) \quad \text{for all } t > 0. \quad (30)$$

This suggests that the solution of the ROE approximation algorithm is optimal when $P_C^{\max}$ is small enough.

We note that, since the objectives of problems P3 and P4 have similar structures, we can simply modify Algorithm 1 to solve P4.

IV. Numerical Results

We provide numerical results to evaluate the performance of the proposed ROE algorithm as described in Algorithm 1, with respect to the original problem P1. We set the number of neighboring cells as $b = 6$. We consider 5 CUs and one D2D pair that are randomly dropped in the cell of interest. The BS coordinates D2D communication by associating the D2D pair with a CU to achieve the maximum sum rate. The cell radius is $d_0 = 0.5$ km and the D2D distance is denoted by d_D. We assume Rayleigh fading for each channel with path loss $128.1 + 37.6\log_{10}(d)$. We set $\sigma^2 = \sigma_D^2 = -103$ dBm, $\tilde{\gamma}_C = \tilde{\gamma}_D = 3$ dB, $P_C^{\max} = P_D^{\max} = P^{\max}$, $\tilde{\mathcal{I}} = NI_0$ where I_0 is the ICI threshold reference, and $I_0/\sigma^2 = 5$ dB. We use 5000 channel realizations to evaluate the average performance. For performance comparison, we consider both the upper bound developed in Section III-C and a *CU-priority heuristic* algorithm: it selects a corner point of the feasible region to maximize the SINR of the CU.

The expected sum rate versus the maximum power $P^{\max}$ for $N = 2, 4, 8$ is shown in Fig. 4. It can be seen that the proposed ROE algorithm significantly outperforms the CU-priority heuristic in for all values of N. Furthermore, the gap between the algorithm and the upper bound is small, at less than 5% of the optimal expected sum rate.

The expected sum rate versus the normalized D2D distance d_D/d_0 for $N = 2, 8$ is shown in Fig. 5. We observe that, when the D2D channel is strong, *i.e.,* the D2D distance is small, significant expected sum rate is achievable while knowing only partial CSI.

V. Conclusion

In this paper, under the partial CSI assumption, we have considered the maximization of the expected sum rate of one CU and one D2D pair with receive beamforming at the BS, subject to minimum SINR requirements, per-node maximum power, and maximum ICI constraints in multiple neighboring cells. An efficient robust power control algorithm based on ROE approximation has been proposed to obtain the transmit powers of the CU and D2D transmitter, along with an algorithm to compute an upper bound of the maximum expected sum rate. Simulation results demonstrate that the proposed algorithm is close to optimal.

References

[1] K. Doppler, M. Rinne, C. Wijting, C. Ribeiro, and K. Hugl, "Device-to-device communication as an underlay to LTE-Advanced networks," *IEEE Commun. Mag.*, vol. 47, pp. 42–49, Dec. 2009.

[2] G. Fodor, E. Dahlman, G. Mildh, S. Parkvall, N. Reider, G. Miklós, and Z. Turányi, "Design aspects of network assisted device-to-device communications," *IEEE Commun. Mag.*, vol. 50, pp. 170–177, Mar. 2012.

[3] C.-H. Yu, K. Doppler, C. Ribeiro, and O. Tirkkonen, "Resource sharing optimization for device-to-device communication underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 2752–2763, Aug. 2011.

[4] D. Feng, L. Lu, Y. Yuan-Wu, G. Y. Li, G. Feng, and S. Li, "Device-to-device communications underlaying cellular networks," *IEEE Trans. Commun.*, vol. 61, pp. 3541–3551, Aug. 2013.

[5] A. Ramezani-Kebrya, M. Dong, B. Liang, G. Boudreau, and S. H. Seyedmehdi, "Optimal power allocation in device-to-device communication with SIMO uplink beamforming," in *Proc. IEEE SPAWC*, Stockholm, Sweden, June 2015, pp. 425–429.

[6] D. Feng, L. Lu, Y. W. Yi, G. Y. Li, G. Feng, and S. Li, "QoS-aware resource allocation for device-to-device communications with channel uncertainty," *IEEE Trans. Veh. Technol.*, vol. 65, pp. 6051–6062, Aug. 2016.

[7] Q.-T. Thieu and H.-Y. Hsieh, "Outage protection for cellular-mode users in device-to-device communications through stochastic optimization," in *Proc. IEEE VTC*, Glasgow, Scotland, May 2015, pp. 1–5.

[8] R. Wang, J. Zhang, S. H. Song, and K. B. Letaief, "Optimal QoS-aware channel assignment in D2D communications with partial CSI," *IEEE Trans. Wireless Commun.*, vol. 15, pp. 7594–7609, Nov. 2016.

[9] A. Ramezani-Kebrya, I.-M. Kim, F. Chan, R. Inkol, and H.-K. Song, "Detection for an AF cooperative diversity network in the presence of interference," *IEEE Commun. Lett.*, vol. 17, pp. 653–656, Apr. 2013.

[10] M. Abramovitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. New York, USA: Dover, 1972.