Optimal Channel Assignment and Power Allocation for Dual-Hop Multi-channel Multi-user Relaying

Mahdi Hajiaghayi*, Min Dong†, and Ben Liang*
*Dept. of Electrical and Computer Engineering, University of Toronto, Canada
†Faculty of Engineering and Applied Science, University of Ontario Institute of Technology, Canada
Email: {mahdih, liang}@comm.utoronto.ca, min.dong@uoit.ca

***Abstract*—We consider the problem of jointly optimizing channel pairing, channel-user assignment, and power allocation in a single-relay multiple-access system. The optimization objective is to maximize the weighted sum-rate under total and individual power constraints on the transmitters. By observing the special structure of a three-dimensional assignment problem derived from the original problem, we propose a polynomial-time algorithm based on continuity relaxation and dual minimization. The proposed method is shown to be optimal for all relaying strategies that give a concave rate function in terms of power constraints.**

I. Introduction

In cooperative relaying with multiple frequency channels, the relay can exploit the additional frequency dimension to process incoming signals adaptively based on the diversity in channel strength. For example, *subcarrier pairing*, which devises a matching of incoming and outgoing subcarriers in OFDM relaying, was first proposed independently in [1] and [2] for single-user relaying[1]. Furthermore, in a multi-user communication environment, both incoming and outgoing channels at the relay are shared among all users. Since the channel condition can vary drastically for different users, judicious channel-user assignment, which allocates channels to users, can potentially lead to significant improvement in spectral efficiency.

There is strong correlation between channel pairing, channel-user assignment, and power allocation. Therefore, optimal system performance requires joint consideration of these three problems. However, the combinatorial nature of channel pairing and assignment entails a mixed integer programming problem, whose solution often bears prohibitive computational complexity. Previous attempts to optimize the performance of dual-hop multi-channel multi-user relaying often either consider only a subset of the three problems [3] - [10] or adopt a suboptimal approach [11].

For a single-user amplify-and-forward (AF) OFDM relaying system, [3] showed that the *sorted subcarrier pairing* scheme, which matches the incoming and outgoing subcarriers according to the sorted order of their SNRs for given power allocation, is sum-rate optimal when the direct source-destination link is unavailable. The authors of [4] and [5], for AF and decode-and-forward (DF) respectively, showed that joint power allocation and subcarrier pairing are separable for sum-rate optimization in single-user OFDM relaying, without the direct source-destination link. This separation was also found for the multi-hop case in [12]. The authors of [6] and [7], for AF and DF respectively, considered joint power allocation and subcarrier pairing in a single-user OFDM system with the direct source-destination link available. The joint optimization problems were formulated as mixed integer programs and solved in the Lagrangian dual domain. Although strict optimality was not established, the proposed solutions are optimal in the limiting case as the number of subcarriers approaches infinity based on the time-sharing argument [13]. For given power allocation, [8] and [9] sought an optimal subcarrier-user assignment to maximize the end-to-end rate. Using continuous relaxation and allowing partial subcarrier assignment, [10] provided an asymptotically optimal solution to the problem of joint power and subcarrier-user assignment for DF relaying. The authors of [11] considered all three problems in DF relaying with a total power constraint and *without* the direct source-destination link. They showed that it is optimal to separately apply subcarrier-user assignment by channel gain, sorted subcarrier pairing, and water-filling power allocation. However, this approach is suboptimal for the general case where the direct link is available or when individual power constraints are considered.

In this work, we show that there is an efficient method to jointly optimize channel pairing, channel-user assignment, and power allocation in a general dual-hop multi-user relaying network, which allows direct source-destination links and both total and individual power constraints. The proposed solution framework is built upon continuity relaxation and Lagrangian dual minimization, but at its core is a special incidence of the class of three-dimensional assignment problems, which is NP-hard in general but has polynomial-time solution in this case. In Section V, we further illustrate with numerical data that there is a large performance gap between the separate optimization approach proposed in [11] and the jointly optimal solution.

II. System Model and Problem Formulation

We consider a communication scenario where a source communicates with K users via a single relay as illustrated in Fig. 1. The available radio spectrum is divided into N equal-bandwidth channels, accessible by all nodes. We denote by

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[1]Since a vast majority of multi-channel relaying systems in the literature are based on OFDM, we use it as an illustrative example in this work, so that the terms "channel" and "subcarrier" are synonymous.

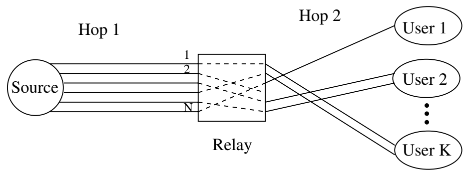

Fig. 1. Illustration of dual-hop multi-channel relaying; direct links from source to users may be present but are not shown.

h_i^{sr}, h_i^{rk}, and h_i^{sk} the state of channel i, for $1 \leq i \leq N$, over the first hop between the source and the relay, over the second hop between the relay and user k, and over the direct link between the source and user k, respectively. The additive noise on a channel at the relay and user k are modeled as i.i.d. zero-mean Gaussian random variables with variances σ_r^2 and σ_k^2, respectively. We constrain our study to practical half-duplex transmission.

Channel Assignment. The relay transmits a processed version of the incoming data to its intended user using a specific relay strategy. The relay also conducts channel pairing and channel-user assignment. Clearly, channel-pairing choices are closely connected with how the channels are assigned to the users. We term the joint decision on channel pairing and channel-user assignment the *channel assignment* problem.

We say a path $\mathcal{P}(m,n,k)$ is selected, if first-hop channel m is paired with second-hop channel n, and the pair of channels (m,n) is assigned to user k. We define indicator functions for channel assignment as follows:

$$\phi_{mnk} = \begin{cases} 1, & \text{if } \mathcal{P}(m,n,k) \text{ is selected,} \\ 0, & \text{otherwise .} \end{cases} \quad (1)$$

A user may be assigned multiple channel pairs, but a channel pair cannot be assigned to multiple users. Hence,

$$\sum_{n=1}^{N}\sum_{k=1}^{K} \phi_{mnk} = 1, \forall m, \quad \sum_{m=1}^{N}\sum_{k=1}^{K} \phi_{mnk} = 1, \forall n. \quad (2)$$

Power Allocation. Along any path $\mathcal{P}(m,n,k)$, the source and relay transmission powers are denoted by P_{mnk}^s and P_{mnk}^r, respectively. Then the total power allocated to path $\mathcal{P}(m,n,k)$ is $P_{mnk} \triangleq P_{mnk}^s + P_{mnk}^r$. For clarity of presentation given the page limitation, we initially focus on the *total power constraint*, expressed by

$$\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} \phi_{mnk} P_{mnk} \leq P_t \ . \quad (3)$$

We show later in Section IV how to also accommodate *individual power constraints*, similarly expressed with maximum allowed powers P_t^s and P_t^r at the source and the relay, respectively.

Relaying Strategy. We initially focus on DF relaying but will later show how the proposed method can be applied to other relaying schemes. In DF, each transmission time frame is divided into two equal slots. In the first slot, the source transmits an information block in each channel, which is received by both the relay and the intended user. In the second slot, the relay attempts to decode the received message from each incoming channel (first hop), and forwards a version of the decoded message on an outgoing channel (second hop) to the intended user. The intended user applies maximum ratio combining on the received signals in both slots and decodes the message.

Consider the simple repetition-coding based DF relaying, the maximum source-destination achievable rate on path $\mathcal{P}(m,n,k)$ is given by [14]

$$\begin{aligned} R(m,n,k) =& \frac{1}{2}\min\{\log(1 + a_m P_{mnk}^s), \\ & \log(1 + b_{nk} P_{mnk}^r + c_{mk} P_{mnk}^s)\} \ , \end{aligned} \quad (4)$$

where $a_m = \frac{|h_m^{sr}|^2}{\sigma_r^2}$, $b_{nk} = \frac{|h_n^{rk}|^2}{\sigma_k^2}$, $c_{mk} = \frac{|h_m^{sk}|^2}{\sigma_k^2}$, and the base of logarithm is 2.

Optimization Objective. We mainly focus on the weighted sum-rate in this paper, but the proposed method can be similarly applied to other objectives. Given fixed total power P_{mnk}, it can be shown that the maximum of $R(m,n,k)$ over (P_{mnk}^s, P_{mnk}^r) is $\frac{1}{2}\log(1 + a_{mnk} P_{mnk})$, where a_{mnk} is the equivalent channel gain on path $\mathcal{P}(m,n,k)$, given by $\min\{a_m, c_{mk}\}$ if $b_{nk} < c_{mk}$, and $\frac{a_m b_{nk}}{a_m + b_{nk} - c_{mk}}$ otherwise. Denoting by w_k the relative weight for user k, such that $\sum_{k=1}^{K} w_k = 1$, our optimization objective is

$$\max_{\mathbf{\Phi},\mathbf{P}} \ \frac{1}{2}\sum_{k=1}^{K} w_k \sum_{m=1}^{N}\sum_{n=1}^{N} \phi_{mnk} \log(1 + a_{mnk} P_{mnk}) \ , \quad (5)$$

where $\mathbf{P} \triangleq [P_{mnk}]_{N\times N\times K}$ and $\mathbf{\Phi} \triangleq [\phi_{mnk}]_{N\times N\times K}$, subject to the constraints on $\mathbf{P}$ and $\mathbf{\Phi}$ as stated earlier in this section.

III. Weighted Sum-Rate Maximization for DF with Total Power Constraint

To illustrate the basic idea of our approach, we first consider DF under a total power constraint. In the section that follows, we will explain how the proposed solution accommodates individual power constraints and other relaying strategies.

A. Continuous Relaxation and Convex Reformulation

We relax the constraints by allowing ϕ_{mnk} to take any value in the interval $[0,1]$. Moreover, we introduce a new variable $S_{mnk} = \phi_{mnk} P_{mnk}$. Then, the relaxed optimization problem in terms of $\mathbf{\Phi}$ and $\mathbf{S} \triangleq [S_{mnk}]_{N\times N\times K}$ can be rewritten as

$$\begin{aligned} \max_{\mathbf{\Phi},\mathbf{S}} \ & \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} w_k \phi_{mnk} \log(1 + a_{mnk}\frac{S_{mnk}}{\phi_{mnk}}) \\ s.t. \quad & (2), \quad \sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} S_{mnk} \leq P_t, \\ & 0 \leq \phi_{mnk} \leq 1, \quad S_{mnk} \geq 0, \quad \forall m,n,k. \end{aligned} \quad (6)$$

In the objective function above, each function $\phi_{mnk}\log(1 + a_{mnk}\frac{S_{mnk}}{\phi_{mnk}})$ is concave in (ϕ_{mnk}, S_{mnk}), since it is the perspective function [15] of the concave function $\log(1 + a_{mnk} S_{mnk})$. Therefore, (6) is a convex optimization problem. Furthermore, since all constraints are affine, Slater's condition is satisfied [15]. Hence, (6) has zero duality gap, so that a globally optimal solution can be found in the Lagrangian dual domain.

However, all globally optimal solutions to (6) do not necessarily give a binary $\mathbf{\Phi}$, which is required in (5). Next, we show that there always exists a globally optimal solution with binary $\mathbf{\Phi}$, while we present a modified Lagrangian dual method to ensure that such an optimal solution can be found.

B. Maximization of Lagrange Function with Binary $\mathbf{\Phi}$

Consider the Lagrange function for (6)

$$\mathcal{L}(\mathbf{\Phi}, \mathbf{S}, \lambda) = \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} w_k \phi_{mnk} \log\left(1 + a_{mnk}\frac{S_{mnk}}{\phi_{mnk}}\right) - \lambda\left(\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} S_{mnk} - P_t\right), \quad (7)$$

with λ being the Lagrange multiplier associated with the power constraint on $\mathbf{S}$ in (6). The dual function is therefore

$$g(\lambda) = \max_{\mathbf{\Phi},\mathbf{S}} \mathcal{L}(\mathbf{\Phi}, \mathbf{S}, \lambda) \quad (8)$$
$$s.t. \quad (2), \quad 0 \leq \phi_{mnk} \leq 1, \quad S_{mnk} \geq 0, \quad \forall m, n, k.$$

The above maximization of the Lagrange function can be carried out in the following two steps.

1) Power Allocation: Optimizing over $\mathbf{S}$ by employing the KKT conditions for a given $\mathbf{\Phi}$ yields S^*_{mnk} in terms of λ as

$$S^*_{mnk}(\lambda) = \left[\frac{w_k}{\alpha\lambda} - \frac{1}{a_{mnk}}\right]^+ \phi_{mnk}(\lambda), \quad (9)$$

where $[x]^+ \stackrel{\Delta}{=} \max(x, 0)$ and $\alpha = 2\ln 2$. This is one instance of the water-filling solution to optimal power allocation.

2) Channel Assignment: We insert (9) into (8) and define

$$A_{mnk}(\lambda) = \frac{1}{2} w_k \log\left(1 + a_{mnk}\left[\frac{w_k}{\alpha\lambda} - \frac{1}{a_{mnk}}\right]^+\right) - \lambda\left[\frac{w_k}{\alpha\lambda} - \frac{1}{a_{mnk}}\right]^+ . \quad (10)$$

Then, (8) is equivalent to the following optimization over $\mathbf{\Phi}$:

$$g(\lambda) = \max_{\mathbf{\Phi}} \sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} \phi_{mnk} A_{mnk}(\lambda) + \lambda P_t \quad (11)$$
$$s.t. \quad (2), \quad 0 \leq \phi_{mnk} \leq 1, \quad \forall m, n, k.$$

Lemma 1: Any matrix $\mathbf{\Phi} = [\phi_{mnk}]_{N\times N\times K}$ with $0 \leq \phi_{mnk} \leq 1$ and satisfying (2) can be decomposed into one matrix $\mathbf{X} = [x_{mn}]_{N\times N}$ and MN vectors $\mathbf{y}^{mn} = [y^{mn}_k]_{1\times K}$, such that $\phi_{mnk} = x_{mn} y^{mn}_k, \forall m, n, k$, with $0 \leq x_{mn} \leq 1$ and $0 \leq y^{mn}_k \leq 1$, satisfying $\sum_{n=1}^{N} x_{mn} = 1, \forall m$, $\sum_{m=1}^{N} x_{mn} = 1, \forall n$, and $\sum_{k=1}^{K} y^{mn}_k = 1, \forall m, n$. Furthermore, any such matrix $\mathbf{X}$ and vectors $\mathbf{y}^{mn}$ uniquely determine a matrix $\mathbf{\Phi}$ that satisfies both equations in (2).

The proof to Lemma 1 is omitted here. Note that the mapping from $\mathbf{\Phi}$ to $(\mathbf{X}, \{\mathbf{y}^{mn}_k\})$ is one-to-many, which is different from the binary case. Lemma 1 implies that any optimization over $(\mathbf{X}, \{\mathbf{y}^{mn}_k\})$ also optimizes $\mathbf{\Phi}$ for the same objective. Hence, we can equivalently seek a solution to

$$\max_{\mathbf{X},\{\mathbf{y}^{mn}\}} \sum_{m=1}^{N}\sum_{n=1}^{N} x_{mn} \sum_{k=1}^{K} y^{mn}_k A_{mnk}(\lambda) , \quad (12)$$

subject to the constraints on $(\mathbf{X}, \{\mathbf{y}^{mn}_k\})$ stated in Lemma 1.

We solve (12) over two stages. First, the inner-sum term is maximized over y^{mn}_k for each (m, n) pair, *i.e.,*

$$A'_{mn}(\lambda) = \max_{\mathbf{y}^{mn}} \sum_{k=1}^{K} y^{mn}_k A_{mnk}(\lambda) \quad (13)$$
$$s.t. \quad \sum_{k=1}^{K} y^{mn}_k = 1, \quad 0 \leq y^{mn}_k \leq 1, \quad \forall k.$$

The optimal solution to (13) is readily obtained as

$$y^{mn*}_k = \begin{cases} 1, & \text{if } k = \arg\max_{1\leq l\leq K} A_{mnl}(\lambda) \\ 0, & \text{otherwise} \end{cases} . \quad (14)$$

In the above maximization, arbitrary tie-breaking can be performed if necessary.

Next, inserting $A'_{mn}(\lambda)$ into (12), we reduce (12) to the following linear optimization problem

$$\max_{\mathbf{X}} \sum_{m=1}^{N}\sum_{n=1}^{N} x_{mn} A'_{mn}(\lambda) \quad (15)$$
$$s.t. \quad \sum_{n=1}^{N} x_{mn} = 1, \forall m, \quad \sum_{m=1}^{N} x_{mn} = 1, \forall n,$$
$$0 \leq x_{mn} \leq 1, \quad \forall m, n.$$

It can be shown that there always exists a binary solution $\mathbf{X}^* \in \{0, 1\}^{N\times N}$ to this problem. Furthermore, with binary constraints on $\mathbf{X}$, this is known as the *two-dimensional assignment problem*. Efficient algorithms, such as the Hungarian Algorithm [16], exist to produce the binary solution with complexity that is a polynomial function in N.

Finally, the optimal ϕ^*_{mnk} is obtained as the product of x^*_{mn} and y^{mn*}_k. Since both x^*_{mn} and y^{mn*}_k are binary, ϕ^*_{mnk} is binary. This shows that there exists at least one binary optimal solution to the Lagrangian maximization in (11).

Note that (11) with *integer* constraints $\phi_{mnk} \in \{0, 1\}$ can be regarded as a special case of the *axillary three-dimensional assignment problems* [17]. It is well known that the general form of this family of problems is NP hard and cannot be solved by continuous relaxation on ϕ_{mnk}. In our case, the availability of an efficient solution to (11) is a direct consequence of the specific constraints on $\mathbf{\Phi}$. It is also worth noting that in general, given any λ value, there may exist non-integer optimal solutions to (11). However, the proposed procedure above finds only the required binary optimal solution.

C. Subgradient Updating

Next, the standard Lagrangian dual approach calls for minimizing the dual function $g(\lambda)$ subject to $\lambda \geq 0$. This dual problem can be solved using the subgradient method [18]. It is easy to see that a subgradient at the point λ is

$$\theta(\lambda) = P_t - \sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} S^*_{mnk}(\lambda) \ . \tag{16}$$

This is used to update λ through $\lambda^{(l+1)} = \lambda^{(l)} - \theta(\lambda^{(l)})\nu^{(l)}$ where $\nu^{(l)}$ is the step size at the lth iteration. Several step-size rules have been proven to guarantee convergence under some general conditions [18], [19]. We may further impose a constraint $\lambda^{(l)} > \lambda_{min}$, where λ_{min} is chosen properly to guarantee some required computation complexity as shown in Section III-D.

Once convergence to λ^* is achieved, the optimal primal solution $\mathbf{S}^*$ and $\mathbf{\Phi}^*$ are also determined. The optimal power allocation $\mathbf{P}^*$ is then recovered by $P^*_{mnk} = S^*_{mnk}, \forall m, n, k$.

D. Optimality and Complexity

Proposition 1: The pair of channel assignment and power allocation matrices ($\mathbf{\Phi}^*$, $\mathbf{P}^*$) found in the previous subsection is a globally optimal solution to the original problem (5).

Proof: Since (6) is a constraint-relaxed version of (5), ($\mathbf{\Phi}^*$, $\mathbf{P}^*$) gives an upper bound to the objective of (5). However, since $\mathbf{\Phi}^*(\lambda)$ satisfies the binary constraints in (5) at each iteration of the subgradient algorithm, ($\mathbf{\Phi}^*$, $\mathbf{P}^*$) satisfies all constraints in (5) and hence also gives a lower bound. ■

We note that using conventional convex optimization software packages directly on the relaxed problem (6) is not sufficient to solve (5). This is because there is no guarantee that they will return a binary $\mathbf{\Phi}^*(\lambda)$, and furthermore due to complicated three-dimensional dependencies among ϕ_{mnk}'s, there is no readily available method to transform a fractional $\mathbf{\Phi}^*(\lambda)$ to the desired binary solution.

Proposition 2: For DF relaying with a total power constraint, to achieve sum-rate within an arbitrary $\epsilon > 0$ neighborhood of the optimum $g(\lambda^*)$, using either a constant step size or a constant step length in subgradient updating, the proposed algorithm has polynomial computation complexity in N and K.

Proof: It is easy to see that the computation complexity at each iteration of subgradient updating is polynomial in N and K, so it remains to show that the number of iterations is not more than polynomial in N or K.

Since $\phi^*_{mnk}(\lambda^*) \in \{0, 1\}$, we have

$$\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} S^*_{mnk}(\lambda^*) \leq \sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} \frac{w_k}{\alpha\lambda^*} \leq \frac{N^2}{\alpha\lambda^*} \ . \tag{17}$$

Furthermore, clearly there exists at least one index vector (m', n', k') such that $\phi^*_{m'n'k'}(\lambda^*) = 1$, $w_{k'} > 0$, and $a_{m'n'k'} > 0$. Hence, we have

$$\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} S^*_{mnk}(\lambda^*) \geq \frac{w'_k}{\alpha\lambda^*} - \frac{1}{a_{m'n'k'}} \ . \tag{18}$$

Since the total power budget is necessarily fully utilized to achieve a maximum sum-rate, the power constraint (3) is satisfied with equality, which combines with (17) and (18) to give

$$\lambda_{min} \triangleq \frac{\min_{k, w_k > 0} w_k}{\alpha(P_t + \max_{\substack{m,n,k,\\ a_{m,n,k}>0}} a^{-1}_{m,n,k})} \leq \lambda^* \leq \frac{N^2}{\alpha P_t} \triangleq \lambda_{max} \ . \tag{19}$$

The above suggests we can limit the subgradient updating to $\lambda \geq \lambda_{min} > 0$. Substituting this into (16), it can be shown that

$$-\frac{N^2}{\alpha\lambda_{min}} = -\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{K} \frac{w_k}{\alpha\lambda_{min}} \leq \theta(\lambda^{(l)}) \leq P_t \ . \tag{20}$$

Hence, $|\theta(\lambda^{(l)})|$ is upper bounded by $\Theta = O(N^2)$.

If we choose $\lambda^{(0)}$ in the interval $[\lambda_{min}, \lambda_{max}]$, then the distance between $\lambda^{(0)}$ and λ^* is upper bounded by λ_{max}. Then, it can be shown that, at the lth iteration, the distance between the current best objective to the optimum objective $g(\lambda^*)$ is upper bounded, by $\frac{\lambda^2_{max}+\nu^2\Theta^2 l}{2\nu l}$ if a constant step size is used (i.e., $\nu^{(l)} = \nu$), or by $\frac{\lambda_{max}\Theta+\nu^2\Theta l}{2\nu l}$ if a constant step length is used (i.e., $\nu^{(l)} = \nu/\theta(\lambda^{(l)})$) [18], [19]. For these two bounds, if we set $\nu = \epsilon/\Theta^2$ and $\nu = \epsilon/\Theta$ respectively, it is easy to see that both are upper bounded by ϵ when $l > \lambda^2_{max}\Theta^2/\epsilon^2 = O(N^2)$. Hence, the number of required iterations until convergence is polynomial in N and independent of K. ■

IV. Extensions to General Scenarios

A. Simultaneous Total and Individual Power Constraints

In this case, the individual power constraints on P^s_{mnk} and P^r_{mnk}, similar to (3), are added to the optimization problem in (5). We again relax the binary constraint on ϕ_{mnk} and rewrite (5) in a concave form in terms of $(\mathbf{S}^s, \mathbf{S}^r, \mathbf{\Phi})$. The Lagrange dual is then a function of the multipliers (μ_1, μ_2, λ), corresponding to the two individual power constraints and the total power constraint. Closed-form expression for the optimal power allocation vector $(S^{s*}_{mnk}(\mu_1, \mu_2, \lambda), S^{r*}_{mnk}(\mu_1, \mu_2, \lambda))$ can be similarly found, albeit with more tedious derivation, and binary-constrained maximization can be carried out over $\mathbf{\Phi}$. From here, the subgradient method can be applied to minimize the Lagrange dual. The optimality of this approach can be similarly justified, and the computation complexity remains polynomial in N and K.

B. General Relaying Strategies

For any relaying strategy in which information sent through different communication paths $\mathcal{P}(m, n, k)$ is independent and the achievable rates $R(m, n, k)$ is a concave function in transmission powers (S^s_{mnk}, S^r_{mnk}), the proposed solution approach gives jointly optimal channel assignment and power allocation for weighted sum-rate maximization. To see this, we first note that any concave rate function would lead to convex programming for the relaxed and reformulated problem, which satisfies Slater's condition and hence has zero duality gap. Furthermore, the maximization of the Lagrange function can

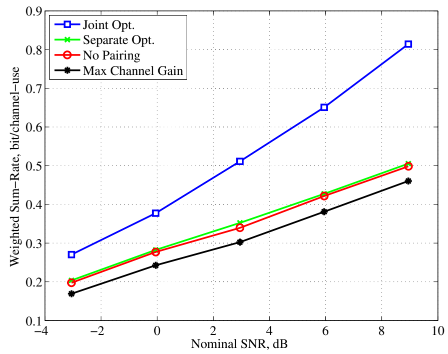

Fig. 2. Comparison of weighted sum-rate vs. nominal SNR.

be reduced to $N \times N \times K$ independent subproblems, which are in general in the following form:

$$\max_{S^s_{mnk} \geq 0, S^r_{mnk} \geq 0} w_k \phi_{mnk} R(\frac{S^s_{mnk}}{\phi_{mnk}}, \frac{S^r_{mnk}}{\phi_{mnk}}) - \mu_1 S^s_{mnk} - \mu_2 S^r_{mnk} .$$

It can be shown that we always have S^{s*}_{mnk} and S^{r*}_{mnk} as the product of ϕ_{mnk} and a non-negative factor. This leads to a maximization problem of the form in (11) with some extra constant terms in the objective function. Therefore, this problem similarly admits a binary optimal solution. Example relaying strategies that have concave achievable rates include selective DF and some variants of Compress-and-Forward [14]. For AF, near optimal solutions can be obtained by using either concave bounds or the time-sharing property [13].

V. Optimization Performance Gain

We compare the performance of jointly optimal channel assignment and power allocation with that of the following suboptimal schemes: (1) *Separate Optimization*: the three-stage solution proposed in [11]; (2) *No Pairing*: channel-user assignment by maximum channel-gain and power allocation by water-filling, but no channel pairing; (3) *Max Channel Gain*: channel-user assignment by maximum channel-gain, with uniform power allocation and no channel pairing. We consider an OFDMA system with $K = 4$ users and $N = 16$ subcarriers. An L-tab frequency-selective fading channel is assumed for each hop with $L = 4$. The source and relay are placed on a horizontal line, and the K users are placed on a vertical line centered at the horizontal line. Distances between the relay and users are $\mathbf{d}_{rd} = [23, 23, 40, 40]$, between the source and relay $d_{sr} = 10$, and between the source and users $\mathbf{d}_{sd} = [32, 32, 45, 45]$. A total power constraint P_t is assumed, along with individual power constraints $P^s_t = P^r_t = \frac{2}{3} P_t$.

Fig. 2 shows the weighted sum-rate vs. the nominal SNR for DF relaying with weight vector $\mathbf{w} = [.15, .15, .35, .35]$. The nominal SNR is defined as $\frac{P_t(\bar{d}_{sd})^{-\kappa}}{2N\sigma^2}$, where $\kappa = 3$ denotes the pathloss exponent, σ^2 the noise power per channel, and $\bar{d}_{sd}$ the average distance between the source and users. We observe that the jointly optimal scheme substantially outperforms other suboptimal schemes. For instance, at 20 dB nominal SNR, it nearly doubles the weighted sum-rate compared with the other schemes. We further observe that the gap between the optimal and suboptimal schemes widens as the SNR increases.

VI. Conclusion

We have studied the problem of jointly optimizing channel pairing, channel-user assignment, and power allocation, to maximize the weighted sum-rate in a general single-relay multi-channel multi-user system. Although such joint optimization naturally leads to a mixed-integer programming problem, we show that there is an efficient solution. The proposed approach is built on convex relaxation and transforms the original problem into a specially structured three-dimensional assignment problem, which leads to polynomial-time computation complexity. Our numerical results demonstrate significant improvement of system performance by the jointly optimal solution over suboptimal alternatives.

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