Robust Online Learning against Malicious Manipulation and Feedback Delay with Application to Network Flow Classification

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***Abstract*—Malicious data manipulation reduces the effectiveness of machine learning techniques, which rely on accurate knowledge of the input data. Motivated by real-world applications in network flow classification, we address the problem of robust online learning with delayed feedback in the presence of malicious data generators that attempt to gain favorable classification outcome by manipulating the data features. When the feedback delay is static, we propose online algorithms termed `ROLC-NC` and `ROLC-C` when the malicious data generators are non-clairvoyant and clairvoyant, respectively. We then consider the dynamic delay case, for which we propose online algorithms termed `ROLC-NC-D` and `ROLC-C-D` when the malicious data generators are non-clairvoyant and clairvoyant, respectively. We derive regret bounds for these four algorithms and show that they are sub-linear under mild conditions. We further evaluate the proposed algorithms in network flow classification via extensive experiments using real-world data traces. Our experimental results demonstrate that the proposed algorithms can approach the performance of an optimal static offline classifier that is not under attack, while outperforming the same offline classifier when tested with a mixture of normal and manipulated data.**

I. Introduction

We consider the problem of robust online learning against malicious manipulation with delayed feedback. In this problem, each data sample belongs to a specific class, and the task of an online classifier is to estimate the classes of a sequence of data samples that arrive over time. Malicious data generators may exist, which can manipulate their data features to best respond to the classification model used by the classifier while incurring some manipulation cost, e.g., we will see in Sec. VI that malicious flow generators can manipulate their flow features to increase the likelihood of a certain classification outcome so as to increase their own utility. Our objective is to decide a sequence of classification models over time, given some delayed feedback on the true class labels of the data samples, such that the classification accuracy is maximized, even in the presence of malicious data generators.

Many real-world applications motivate this problem, e.g., traffic flow classification in computer networking and credit card fraud in banking. We take network flow classification as an example, which is essential to modern network management [2]. The success of machine learning (ML) techniques for flow classification depends on having reliable knowledge of the flow feature values [2]–[4]. However, malicious network flow generators may manipulate their flow features to game the classifier, or to evade detection [5]. Here we consider the scenario where each traffic flow has a specific required quality of service (QoS) level [6]. The task is to classify flows that arrive online into multiple classes corresponding to different QoS requirements. The classifier inspects the sequence of flows one-by-one as they arrive and decides a classification model in each round. When a flow arrives, it observes the presented features and assigns to the arriving flow a predicted label denoting its QoS level. A flow generator may be an application, a user, or some other entity that is concerned about the QoS level of the flow. Each flow is allocated network resource based on the classifier's estimation of its QoS level. *Malicious flow generators* may exist, which can manipulate their flow features to best respond to the classification model to increase the likelihood of a certain outcome so as to increase their own utility, e.g., to be prioritized for network resource allocation [7]–[10]. Such malicious behavior can render the conventional statistics-based methods ineffective.

To counter the attacks against ML systems, defensive strategies have been proposed, generally termed adversarial learning [11]. Most of them are offline strategies [12]. However, in many applications the training data are generated over time (e.g., traffic flows in computer networks), so online learning is often desirable [13]. Furthermore, since the malicious data generators can manipulate the data features to best respond to the latest classification model employed by the classifier, it is important to update the classification model over time to counter such malicious attacks [14], [15]. Finally, online learning can limit the amount of memory required to store the training data, and it allows dynamic adaptation to new malicious behavior patterns.

It is challenging to address the problem of robust online learning against malicious feature manipulation. First, online optimization implies that the algorithms make decision with no information about data in the future. Second, the feature manipulation by a malicious data generator may be the best response to the classification model, so that the feature presented might be a function of the model, which further complicates the design space. Third, the classifier depends on the feedback of the true labels to adjust the classification model

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TABLE I: Comparison of Proposed Algorithms

Proposed Algorithms	Data Generator	Feedback Delay
ROLC-NC (Sec. IV-A)	Non-Clairvoyant	Static
ROLC-C (Sec. IV-B)	Clairvoyant	Static
ROLC-NC-D (Sec. V-A)	Non-Clairvoyant	Dynamic
ROLC-C-D (Sec. V-B)	Clairvoyant	Dynamic

to dynamically adapt to malicious behavior patterns as well as the normal ones. However, such feedback often is delayed, making learning difficult.

In this work, we propose online classification algorithms for the cases where the malicious generators are *non-clairvoyant* and *clairvoyant*. A non-clairvoyant data generator is one that can observe the classification model committed by the classifier only after some delay, while, representing the worst-case challenge, a clairvoyant data generator is one that can observe the classification model as soon as it is changed. For each of these cases of the malicious generators, we consider both *static* and *dynamic* feedback delay over time. To evaluate the performance of these four algorithms, we analyze their *regret*. The regret is the cumulative cost difference between the considered algorithm and an offline oracle that chooses the best static classification model given all information of the future [16]. Specifically, the major contributions of our work are summarized as follows:

- We study the problem of robust online learning against malicious feature manipulation with delayed feedback. We consider a detailed system model of online linear classification, which captures practical issues including the delay for the data generators to observe the classifier's classification model (denoted by ξ) and the delay for the classifier to receive the feedback of the true label, which is either static (denoted by τ) or dynamic (denoted by τ_t for round t).
- When the feedback delay is static, we propose a robust online classification algorithm termed ROLC-NC when the malicious data generators are non-clairvoyant (i.e., $\xi > 0$) and another algorithm termed ROLC-C when the malicious data generators are clairvoyant (i.e., $\xi = 0$). ROLC-NC requires knowledge only of the observed features of arriving data samples (regardless whether they are manipulated) and the delayed feedback of true labels, while ROLC-C requires further knowledge of the cost function of the malicious data generators. We prove that both algorithms have a regret bound that is *sub-linear in time* under mild conditions, which implies that the largest possible difference between the *average* costs of the classifier and the offline oracle vanishes as the time horizon goes to infinity. We further consider a more general scenario where the feedback delay is dynamic. We extend the above results to this scenario and propose two additional online classification algorithms termed ROLC-NC-D and ROLC-C-D. We develop a new delay aggregation method to derive *sub-linear* regret bounds for these two algorithms as well. Table I summarizes the characteristics of the proposed algorithms.
- We consider the application of network flow classification and conduct extensive experiments with real-world data traces. Our results demonstrate the effectiveness of the proposed algorithms. For example, with 50% clairvoyant malicious flow generators and feedback delay $\tau = 10$, ROLC-NC can achieve 0.91 accuracy and F_1 score, while ROLC-C can achieve 0.93 accuracy and F_1 score, in steady state. Keeping the same setting, and choosing the dynamic τ_t uniformly randomly from $\{0, 1, \cdots, 20\}$, ROLC-NC-D can achieve 0.88 accuracy and F_1 score, while ROLC-C-D can achieve 0.93 accuracy and F_1 score, in steady state. Furthermore, under a wide range of experiment settings, our algorithms can approach the performance of an optimal static offline classifier that is not under attack (Offline-Norm), and they outperform the same offline classifier when tested with a mixture of normal and manipulated flows (Offline). Our evaluation also sheds light on how to choose an appropriate loss function and tune the parameters in the classification model.

The rest of the paper is organized as follows. We first discuss the related work in Sec. II. We propose the system model and formulate the robust online classification problem in Sec. III. We develop ROLC-NC and ROLC-C and derive their regret bounds in Sec. IV. Then, we extend the problem to the dynamic delay case, and develop ROLC-NC-D and ROLC-C-D and derive their regret bounds in Sec. V. This is followed by performance evaluation with an application to robust online network flow classification based on real-world data traces in Sec. VI. Finally, we make concluding remarks in Sec. VII.

II. Related Work

A. Robust Online Learning

There are three main types of adversarial attacks in ML: data poisoning (DP), reverse engineering (RE), and test-time evasion (TTE) [17]–[22]. The malicious feature manipulation in our work belongs to the family of TTE attacks. Most TTE attacks seek to simply degrade a classifier's accuracy without additional benefit to the data generator. Correspondingly, robust classification aims to correctly classify the samples generated under TTE, which is usually achieved by modifying the training process, through, for example, feature obfuscation [23], [24] and robust training [25], [26]. Anomaly detection of the TTE attacks is also a common defense strategy [27]. All of the above works study offline strategies, while our work considers an online setting.

There is a paucity of prior art on online adversarial learning. Abramson [14] discussed, in general, the scenarios where an attacker seeks to evade a classifier or modify an online learning algorithm. This position paper did not present a formulated

problem or a detailed solution approach. Barreno et al. [15] introduced a DP attack in online learning with an aim to alter the training process through influence over the training data on the fly. They did not propose any defense against such attacks. Kloft and Laskov [28] studied online centroid anomaly detection in the presence of DP attacks, where the goal of the considered attack was to force the learning model to accept a set of targeted data samples as normal. Different from [28], we consider a kind of TTE attack. Specifically, the malicious data generators, after observing the committed learning model, manipulates the data features aiming at increase the likelihood of certain classification outcome. Sethi and Kantardzic [29] proposed an unsupervised drift detection approach against TTE through adversarial drift in streaming data. The main idea of their approach was to detect the drift according to the disagreements between two orthogonal classifiers, each trained on a disjoint subset of the features of the data. Different from [29], our proposed methods require no detection of malicious data before classifying the data sample. Furthermore, in addition to the differences in problem setting, our proposed algorithms have provable performance guarantee in terms of their regret bounds, which is not available in [28] or [29].

In [1], we considered only the case of static delay for the online classifier to receive the feedback of the true label of a data sample. In this extension, we generalize the problem to one with dynamic feedback delay to accommodate more practical scenarios. We propose two additional online classification algorithms `ROLC-NC-D` and `ROLC-C-D` for this problem. Further, we develop a new analytical approach to that both algorithms have provable performance guarantee in terms of a sub-linear regret bound. Finally, we conduct additional trace-driven experiments to evaluate the performance of these algorithms.

There is another line of research using the word "adversarial" to mean an internal non-malicious mechanism to challenge and improve the design of a learning system, e.g., Generative Adversarial Networks (GANs) [30] and other works related to online learning [31], [32]. These works have no relevance to our work.

B. Network Flow Classification

Network traffic classification enables proper assignment of network resource (e.g., bandwidth) to applications with different service requirements (e.g., delay). Thus, it is essential to modern network management [2].

Due to the ineffectiveness of traditional port-based or payload-based approaches, especially for encrypted traffic, statistics-based approaches have emerged [2]–[4], [7], [33]–[42]. These methods usually employ machine learning techniques, which can be categorized as supervised, such as CNN [6], [35], unsupervised, such as K-means [39] and DBSCAN [43], and semi-supervised [40]. However, these methods are offline, based on prior collection of training data, rendering them unsuitable for our online flow classification problem.

Online flow classification methods have been proposed to generate timely networking decisions on incoming flows [44]–[47]. To improve the performance of online flow classification, Jin et al. [47] proposed a light-weight modular architecture with several linear binary classifiers. Hullar et al. [44] used only the first few bytes of the first few packets to recognize P2P applications. Yan et al. [46] proposed a co-training algorithm for online traffic classification, which takes the packet sizes and inter-packet times of the first packets of a traffic flow as the features. However, none of these online solutions consider malicious feature manipulation.

There are a few recent works on robust flow classification [3], [48]–[50]. Wang et al. [48] proposed a flow classifier that is robust to the unclean flow data, i.e., mislabelled training samples, which incorporates noise elimination and suspected noise reweighting. Zhang et al. [3] addressed the problem of zero-day applications in traffic classification, and proposed a robust binary classifier that can identify flows of zero-day applications and accurately discriminate predefined application classes. To identify flows from known or unknown applications, Erman et al. [49] integrated a set of supervised training data with unsupervised learning. Wang et al. [50] proposed to combine flow clustering based on application signatures. These works studied robust flow classification from angles different from our work. None of these work takes into account malicious feature manipulation. Closer to our work, Li et al. [7] considered an offline setting and studied the problem of robust flow classification under malicious feature manipulation and proposed a solution framework based on Stackelberg games. To the best of our knowledge, no existing work studies the problem of robust online network flow classification with delayed feedback.

III. System Model and Problem Formulation

In this section, we present the system model and formulate the problem of robust online learning against malicious feature manipulation with delayed feedback.

A. Online Classification Model

We consider a learning model where the data samples each has M features, represented by $\mathbf{x} \in \mathbb{R}^M$, and each sample has a truthful class $k \in \mathcal{K} = \{1, 2, \cdots, K\}$. For example, in the application of network flow classification presented in Sec. VI, the features may include packet lengths, packet inter-arrival times, etc., and k may represent the level of delay sensitivity. Let $\mathbf{y} \in \mathbb{R}^K$ be a one-hot vector denoting the data sample's true class. We denote the k-th element of $\mathbf{y}$ by $\mathbf{y}^{(k)}$. Suppose the true label is k^*, then $\mathbf{y}^{(k^*)} = 1$ and $\mathbf{y}^{(k)} = 0, \forall k \neq k^*$. Thus, each data sample is associated with a tuple $(\mathbf{x}, \mathbf{y})$.

Time is slotted by rounds. In each round $t \in \{0, 1, ..., T\}$, the classifier decides its classification model, and then some data sample $(\mathbf{x}_t, \mathbf{y}_t)$ arrives. The classifier observes $\mathbf{x}_t$ and needs to estimate $\mathbf{y}_t$. We consider a linear classifier with a decision function $h_c(\mathbf{x}_t) = \arg\max_k \hat{\mathbf{y}}_t^{(k)}$, where $\hat{\mathbf{y}}_t = \mathbf{A}_t^T \mathbf{x}_t$, and matrix $\mathbf{A}_t \in \mathcal{H} \subseteq \mathbb{R}^{M \times K}$. We assume the feasible set $\mathcal{H}$ is convex. We further assume that $\mathbf{A}_t \in \mathcal{H}$ has Frobenius norm bounded by $Z_{\mathcal{H}}$. In the optimization literature, in particular the literature of online convex optimization (e.g., [13], [16]), researchers have used general assumptions in the system model in order to enable tractable analysis. In practice,

the bounded Frobenius norm assumption implies that, given limited hardware capacity, the values of the parameters in our classifier cannot be too large.

In cases where data are not linearly separable, non-linear feature transformation can be applied, so that the resulting transformed data features become linearly separable [51], [52]. That is, in case $\{(\mathbf{x}_t, \mathbf{y}_t)\}$ is not linearly separable we can use some non-linear function $\phi : \mathbb{V}^M \to \Phi$ to transform $\mathbf{x}_t \in \mathbb{R}^M$ to some $\phi(\mathbf{x}_t) \in \Phi$ so that the data $\{(\phi(\mathbf{x}_t), \mathbf{y}_t)\}$ is linearly separable. Thus, our consideration of linear classification can be generalized. For more complex non-linear classifiers, e.g., neural networks [35]. Due to the challenge of non-convex optimization, to achieve some performance guarantee, different solutions rather than the ones proposed in Secs. IV and V may be required. This is out of the scope of this paper and is left for future work. Furthermore, linear classifiers are beneficial, especially for online classification, as they can be executed efficiently. They are commonly used in practice for network traffic classification [47], [53], since they can quickly process a large volume of traffic. Our evaluation results of network flow classification based on real-world data, presented in Sec. VI, further demonstrate that a linear classifier can achieve high classification accuracy.

Let $L_c(\hat{\mathbf{y}}_t, \mathbf{y}_t)$ be the loss function of the classifier with respect to the data sample $(\mathbf{x}_t, \mathbf{y}_t)$. The loss function in this work is general and may include, for example, the commonly used *Hinge Loss* (HL) and *Categorical Cross-Entropy Loss* (CEL) [51]. We refer the readers to Sec. VI for more details. To lessen the chance of overfitting, we use a regularizer $R_c(\mathbf{A}_t) = \sum_i \sum_j A_{i,j}^2$, which is a standard technique when training a classification model. Therefore, the classifier's cost function is $\mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{y}}_t) = L_c(\hat{\mathbf{y}}_t, \mathbf{y}_t) + \gamma_c R_c(\mathbf{A}_t)$, where $\gamma_c \geq 0$ controls the trade-off of the regularizer against the loss. Since $\hat{\mathbf{y}}_t$ is a function of $\mathbf{x}_t$, we can write $\mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{y}}_t) = \mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \mathbf{x}_t)$ when the context is clear.

B. Feature Manipulation

The generator of data sample $(\mathbf{x}_t, \mathbf{y}_t)$ receives some utility based on the classifier's decision $h_c(\mathbf{x}_t)$. For example, in network traffic classification, a flow may be allocated network resource based on the classifier's estimation of its delay sensitivity. Without loss of generality, we assume that the data generator has some preference for the output of the classifier, e.g., to be classified as high-priority traffic, as expressed by the general loss function $L_g(\hat{\mathbf{y}}) = \mathbf{b}^T \hat{\mathbf{y}}$ where $\mathbf{b} \in \mathbb{R}^K$. A malicious data generator has the incentive to manipulate the features of its data, in order to game the classifier and receive higher utility. In this work, we allow the simultaneous existence of both normal and malicious data generators.

Feature manipulation incurs some cost to the malicious data generator. For example, in Sec. VI-A, we discuss several options for feature manipulation and their associated cost in network flow classification. Here, we consider a general model where the malicious data generator suffers a manipulation cost $C_m(\hat{\mathbf{x}}_t, \mathbf{x}_t) = \|\alpha \circ (\hat{\mathbf{x}}_t - \mathbf{x}_t)\|^2$, when the feature is changed from $\mathbf{x}_t$ to $\hat{\mathbf{x}}_t$, where α is a vector representing the weights of manipulating each feature.[1] Without loss of generality, in our analysis of the proposed algorithms, we normalize the feature values such that α is an all-ones vector. Thus, the cost function corresponding to the data sample $(\mathbf{x}_t, \mathbf{y}_t)$ is $\mathcal{C}_g(\mathbf{A}_t, \mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{x}}_t, \hat{\mathbf{y}}_t) = L_g(\hat{\mathbf{y}}_t) + \gamma_g C_m(\hat{\mathbf{x}}_t, \mathbf{x}_t)$, where g denotes the data generator, while $\gamma_g \geq 0$ controls the trade-off of the manipulation cost against the loss.

C. Clairvoyant and Non-Clairvoyant Data Generators

We assume that a malicious data generator can gain knowledge about the classification model. For example, the generator may observe the classification result from the past and use it to detect the classification model through training a local model on its own side, i.e., via reverse engineering attacks [19], [24]. Since detecting the classification model takes time, we assume that, at time t, the generator t can only observe the model $\mathbf{A}_{t'}$ for $t' \leq t - \xi$, for some non-negative integer ξ. This setting is general. Note that it does not rule out the possibility of $\xi = 0$, which represents the case where the data generator at any time t can accurately predict the classification model $\mathbf{A}_t$. In this case, we say that the data generator is *clairvoyant*. In the case $\xi > 0$, the data generator is called *non-clairvoyant*. Note that $\mathbf{A}_{t-\xi}$ is the most up-to-date classification model that the data generator in round t can observe. A malicious data generator can then manipulate its features from $\mathbf{x}_t$ to $\hat{\mathbf{x}}_t = \arg\min_{\mathbf{x}} \mathcal{C}_g(\mathbf{A}_{t-\xi}, \mathbf{x}_t, \mathbf{y}_t, \mathbf{x}, \hat{\mathbf{y}}_t)$ which best responds to $\mathbf{A}_{t-\xi}$. For a normal data generator, we have $\hat{\mathbf{x}}_t = \mathbf{x}_t$.

D. Model Updating Based on Delayed Feedback

At time t, after receiving a data sample with observed features $\hat{\mathbf{x}}_t$, which may be different from the true features $\mathbf{x}_t$, the online classifier computes the predicted scores $\hat{\mathbf{y}}_t = \mathbf{A}_t^T \hat{\mathbf{x}}_t$. Under a general online learning setup, the classifier receives delayed feedback of the true label $\mathbf{y}_t$, which it can use to estimate the accuracy of $\hat{\mathbf{y}}_t$ and adaptively update the classification model over time [13]. Detecting the true label for data samples takes time. For example, in Sec. VI-A, we discuss how various techniques can be used to obtain the true labels in online network flow classification. Initially, for clarity of presentation, we assume the classifier can observe the true label only after τ rounds, for some $\tau \geq 0$ that is fixed over time. That is, $\mathbf{y}_t$ is known to the classifier after $t' \geq t + \tau$. Later, in Sec. V, we extend this to the case of dynamic feedback delay.

E. Robust Online Learning Problem Formulation

The classifier aims at minimizing its *cumulative* cost $\sum_{t=1}^{T} \mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{y}}_t)$ to learn an accurate classification model while making prediction on the fly. During this process, the classifier has to be *robust* to feature manipulation by the malicious data generators. Such a learning process typically incurs an increase in the obtained cumulative cost compared with an

[1] Here, $\|\cdot\|$ is the Euclidean norm and "$\circ$"denotes element-by-element multiplication. The manipulation cost in practical systems can be more complex and dependent on how the features are extracted. In that case, our formula serves only as an approximation.

offline oracle, defined as $\arg\min_{\mathbf{A}} \sum_{t=1}^{T} \mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{y}}_t)$. Note that the offline oracle chooses the best static classifier with full knowledge of the whole sequence of data, including their features $\hat{\mathbf{x}}_t$ and their true labels $\mathbf{y}_t$. The *regret* at time T is thus defined as [16]

$$\text{Reg}(T) = \sum_{t=1}^{T} \mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{y}}_t) - \min_{\mathbf{A}} \sum_{t=1}^{T} \mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{y}}_t). \quad (1)$$

Our objective is to design an online algorithm to choose the classification parameters $\mathbf{A}_t, \forall t \in \{0, 1, ..., T\}$, such that the regret is minimized. Note that minimizing the regret is equivalent to minimizing the cumulative cost. We call the regret *sub-linear* if it grows sub-linearly with the number of rounds T, which implies that the difference between the *average* cost $\frac{1}{T}\sum_{t=1}^{T} \mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{x}}_t)$ of the classifier and the average cost of the offline oracle vanishes as T goes to infinity.

IV. Robust Online Classification with Delayed Feedback

We present algorithms ROLC−NC and ROLC−C for the cases where the malicious data generators are non-clairvoyant (Sec. IV-A) and clairvoyant (Sec. IV-B) respectively. We show that both of them have provable performance guarantee in terms of a sub-linear regret bound.

A. Non-Clairvoyant Malicious Data Generators

Non-clairvoyant malicious data generators have $\xi > 0$. For this case, we propose the ROLC−NC algorithm, whose pseudo code is shown in Algorithm 1. The algorithm uses gradient descent, which is widely adopted in machine learning [51], [54], and combines it with delayed feedback information. A unique property of ROLC−NC is that it decides the classification model in each round, by leveraging *only* the data features presented in each round (regardless whether they are manipulated) and the delayed feedback of the true labels of some previously arrived data sample. Note that ROLC−NC does not require any additional information, e.g., whether the data sample is from a malicious or normal generator, the cost function of the malicious data generators, or the generator's delay ξ in observing the classification model.

1) Algorithm Design: ROLC−NC learns from the history and deploys a classification model parameterized by $\mathbf{A}_t$ in each round $t \in \{1, \cdots, T\}$ to predict the label for the corresponding data sample $(\mathbf{x}_t, \mathbf{y}_t)$.

As the feedback is delayed for τ rounds, the algorithm first initializes $\mathbf{A}_t$, for $t \in \{1, \cdots, \tau+1\}$, to some randomly picked $\mathbf{A}_0$ (Line 1). We recall that the parameters $\mathbf{A}_t$ are known to the data generators only after ξ rounds; that is, at time t, each data generator only knows the matrix $\mathbf{A}_{t'}$ for $t' \leq t - \xi$, for some positive integer ξ. Therefore, in rounds $t = 1, \cdots, \xi$, the malicious data generators manipulate the features arbitrarily. In each round t, ROLC−NC first commits to $\mathbf{A}_t$. Then a data sample $(\mathbf{x}_t, \mathbf{y}_t)$ arrives. If the data generator is malicious (but non-clairvoyant), it manipulates its data features to $\hat{\mathbf{x}}_t$ to obtain the best response to the latest model it observes $\mathbf{A}_{t-\xi}$; otherwise, $\hat{\mathbf{x}}_t = \mathbf{x}_t$. The classifier then observes $\hat{\mathbf{x}}_t$ and predicts a label $\hat{\mathbf{y}}_t$ with the model $\mathbf{A}_t$ (Line 3). In this round, the delayed true label $\mathbf{y}_{t-\tau}$ is known to the classifier, based on which the cost $\mathcal{C}_c(\mathbf{A}_{t-\tau}, \mathbf{y}_{t-\tau}, \hat{\mathbf{x}}_{t-\tau})$ from the prediction in that round is revealed (Line 4). This cost function is used to update the classification model in the next round. Then, ROLC−NC computes the (sub)gradient $\mathbf{g}_{t-\tau}$ of the cost function chosen at $\mathbf{A} = \mathbf{A}_{t-\tau}$ (Line 5). Finally, the algorithm updates the classification parameters $\mathbf{A}_{t+1} = \mathbf{A}_t - \eta \mathbf{g}_{t-\tau}$ for the next round, i.e., by taking a step proportional to the negative of the (sub)gradient, where η is the step size set by the algorithm (Line 6).

Algorithm 1: ROLC-NC: Robust OnLine Classification with Non-Clairvoyant Malicious Data Generators

Input: Data $\{(\mathbf{x}_t, \mathbf{y}_t)\}_{t=1}^{T}$, step size η, and delay τ for receiving feedback

Output: Models parameterized by $\{\mathbf{A}_t\}_{t=1}^{T}$

1 Initialization: Select $\mathbf{A}_0 \in \mathcal{H}$ randomly and set $\mathbf{A}_1, ..., \mathbf{A}_\tau, \mathbf{A}_{\tau+1} = \mathbf{A}_0$.
2 **for** $t = \tau + 1$ to T **do**
3 Commit to $\mathbf{A}_t$, observe $\hat{\mathbf{x}}_t$, predict $\hat{\mathbf{y}}_t$.
4 Observe $\mathbf{y}_{t-\tau}$ (and suffer the cost $\mathcal{C}_c(\mathbf{A}_{t-\tau}, \mathbf{y}_{t-\tau}, \hat{\mathbf{x}}_{t-\tau})$).
5 Set $\mathbf{g}_{t-\tau} \in \partial_{\mathbf{A}} \mathcal{C}_c(\mathbf{A}, \mathbf{y}_{t-\tau}, \hat{\mathbf{x}}_{t-\tau})$ at $\mathbf{A}_{t-\tau}$.
6 $\mathbf{A}_{t+1} = \arg\min_{\mathbf{A} \in \mathcal{H}} \|\mathbf{A} - (\mathbf{A}_t - \eta \mathbf{g}_{t-\tau})\|^2$.

2) Performance Analysis: In this part, we analyze the performance of ROLC−NC. Specifically, we show in Theorem 1 that ROLC−NC has a sub-linear regret bound.

We first note that a function $f(\mathbf{x}) : \mathcal{X} \to \mathcal{Y}$ is called Q-Lipschitz over $\mathbf{x}$ in $\mathcal{X}$ if there exists a real constant $Q \geq 0$ such that, for all $\mathbf{x}$ and $\mathbf{x}'$ in $\mathcal{X}$, $\|f(\mathbf{x}) - f(\mathbf{x}')\| \leq Q\|\mathbf{x} - \mathbf{x}'\|$.

Theorem 1. *Assume that, in any round t, the cost function $\mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$ is convex over $\mathbf{A}$ and Q-Lipschitz over $\mathbf{A}$ in $\mathcal{H}$. With step size $\eta = \frac{Z_{\mathcal{H}}}{Q} \frac{1}{\sqrt{(\frac{1}{4}+\tau)T}}$, ROLC−NC has regret bound $Reg_{\text{ROLC−NC}}(T) \leq \rho\sqrt{T}$, where $\rho = 2QZ_{\mathcal{H}}\sqrt{1+4\tau}$.*

Proof. A matrix can be converted to a column vector through vectorization.[2] For ease of presentation, we assume $\mathbf{A}$, $\mathbf{A}_t$ and $\mathbf{g}_t$, for all t, are vectors in this proof.

Let $\mathbf{A}^* \in \arg\min_{\mathbf{A}} \sum_{t=1}^{T} \mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$. Then, due to the convexity of $\mathcal{C}_c$, we have

$$\begin{aligned} \text{Reg}_{\text{ROLC−NC}}(T) &= \sum_{t=1}^{T} [\mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{x}}_t) - \mathcal{C}_c(\mathbf{A}^*, \mathbf{y}_t, \hat{\mathbf{x}}_t)] \\ &\leq \sum_{t=1}^{T} \mathbf{g}_t^T (\mathbf{A}_t - \mathbf{A}^*), \end{aligned}$$

where $\mathbf{g}_t \in \partial_{\mathbf{A}} \mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$ at $\mathbf{A}_t$. In this paper, we denote the transpose of any vector $\mathbf{p}$ as $\mathbf{p}^T$.

[2]The vectorization of a matrix is a linear transformation which converts the matrix into a column vector. Specifically, the vectorization of a $m \times n$ matrix $\mathbf{W}$ is the $mn \times 1$ column vector obtained by stacking the columns of the matrix $\mathbf{W}$ on top of each other.

For $t > \tau$, since $\mathbf{A}_{t+1}$ is the result of projecting $\mathbf{A}_t - \eta \mathbf{g}_{t-\tau}$ onto the convex set $\mathcal{H}$, we have

$$\begin{aligned}&\|\mathbf{A}_{t+1} - \mathbf{A}^*\|^2 - \|\mathbf{A}_t - \mathbf{A}^*\|^2 \\ \leq &\eta^2 \|\mathbf{g}_{t-\tau}\|^2 - 2\eta \mathbf{g}_{t-\tau}^T [(\mathbf{A}_{t-\tau} - \mathbf{A}^*) + (\mathbf{A}_t - \mathbf{A}_{t-\tau})]. \quad (2)\end{aligned}$$

Now we expand $\mathbf{g}_{t-\tau}^T (\mathbf{A}_t - \mathbf{A}_{t-\tau})$ as follows.

$$\begin{aligned}\mathbf{g}_{t-\tau}^T (\mathbf{A}_t - \mathbf{A}_{t-\tau}) &= \sum_{j=1}^{\tau} \mathbf{g}_{t-\tau}^T (\mathbf{A}_{t-(j-1)} - \mathbf{A}_{t-j}) \\ &= \sum_{j=1}^{\min(t-(\tau+1),\tau)} [-\eta \mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} - c_j], \quad (3)\end{aligned}$$

where $c_j = \mathbf{g}_{t-\tau}^T [(\mathbf{A}_{t-j} - \eta \mathbf{g}_{t-\tau-j}) - \mathbf{A}_{t-(j-1)}]$.

The proof will continue after the following lemma.

Lemma 1. $\|(\mathbf{A}_{t-j} - \eta \mathbf{g}_{t-\tau-j}) - \mathbf{A}_{t-(j-1)}\| \leq \|\eta \mathbf{g}_{t-\tau-j}\|$.

Proof. Since $\mathcal{H}$ is convex and $\mathbf{A}_{t-j}, \mathbf{A}_{t-(j-1)} \in \mathcal{H}$, we have $(1-\alpha)\mathbf{A}_{t-j} + \alpha \mathbf{A}_{t-(j-1)} \in \mathcal{H}$, for all $0 \leq \alpha \leq 1$. By the fact that $\mathbf{A}_{t-(j-1)}$ is the result of projecting $(\mathbf{A}_{t-j} - \eta \mathbf{g}_{t-\tau-j}^T)$ onto $\mathcal{H}$, we have, for all $0 \leq t \leq 1$,

$$\begin{aligned}&\|(\mathbf{A}_{t-j} - \eta \mathbf{g}_{t-\tau-j}) - \mathbf{A}_{t-(j-1)}\| \\ \leq &\|(\mathbf{A}_{t-j} - \eta \mathbf{g}_{t-\tau-j}) - [(1-\alpha)\mathbf{A}_{t-j} + \alpha \mathbf{A}_{t-(j-1)}]\|.\end{aligned}$$

Set $\alpha = 0$, and we have

$$\|(\mathbf{A}_{t-j} - \eta \mathbf{g}_{t-\tau-j}) - \mathbf{A}_{t-(j-1)}\| \leq \|\eta \mathbf{g}_{t-\tau-j}\|.$$

□

The Lipschitz property of $\mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$ gives $\|\mathbf{g}_t\| \leq Q, \forall t$. Thus, by Lemma 1, we have

$$c_j \leq \|(\mathbf{A}_{t-j} - \eta \mathbf{g}_{t-\tau-j}) - \mathbf{A}_{t-(j-1)}\| \cdot \|\mathbf{g}_{t-\tau}\| \leq \eta Q^2.$$

By (3), we have

$$\mathbf{g}_{t-\tau}^T (\mathbf{A}_t - \mathbf{A}_{t-\tau}) \geq \sum_{j=1}^{\min(t-(\tau+1),\tau)} [-\eta \mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} - \eta Q^2].$$

Substituting the above into (2), we have

$$\begin{aligned}&\mathbf{g}_{t-\tau}^T (\mathbf{A}_{t-\tau} - \mathbf{A}^*) \\ \leq &\frac{\eta}{2} \|\mathbf{g}_{t-\tau}\|^2 + \frac{\|\mathbf{A}_t - \mathbf{A}^*\|^2 - \|\mathbf{A}_{t+1} - \mathbf{A}^*\|^2}{2\eta} \\ &+ \sum_{j=1}^{\min(t-(\tau+1),\tau)} [\eta \mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} + \eta Q^2]. \quad (4)\end{aligned}$$

Due to the Lipschitz properties of $\mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$, we have $\|\mathbf{g}_t\| \leq Q$. Furthermore, by the Cauchy–Schwarz inequality, we have $\mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} \leq \|\mathbf{g}_{t-\tau-j}\| \cdot \|\mathbf{g}_{t-\tau}\| = Q^2$. Hence we have

$$\begin{aligned}&\sum_{t=\tau+1}^{T+\tau} \sum_{j=1}^{\min(t-(\tau+1),\tau)} \eta \mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} \\ \leq &\sum_{t=\tau+1}^{T+\tau} \min(t-(\tau+1),\tau) \eta Q^2 \\ = &[\sum_{t=\tau+1}^{2\tau} (t-(\tau+1)) + \sum_{t=2\tau+1}^{T+\tau} \tau] \eta Q^2 \\ = &[\frac{\tau(\tau-1)}{2} + (T-\tau)\tau] \eta Q^2 \\ = &(T\tau - \frac{\tau^2}{2} - \frac{1}{2}) \eta Q^2.\end{aligned}$$

Summing up (4) over $t = \tau + 1$ to $T + \tau$ yields

$$\begin{aligned}&\sum_{t=\tau+1}^{T+\tau} \mathbf{g}_{t-\tau}^T (\mathbf{A}_{t-\tau} - \mathbf{A}^*) \\ \leq &\frac{\eta}{2} \sum_{t=\tau+1}^{T+\tau} Q^2 + \frac{\|\mathbf{A}_{\tau+1} - \mathbf{A}^*\|^2}{2\eta} \\ &+ \sum_{t=\tau+1}^{T+\tau} \sum_{j=1}^{\min(t-(\tau+1),\tau)} \eta \mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} + \sum_{t=\tau+1}^{T+\tau} \tau \eta Q^2.\end{aligned}$$

Therefore,

$$\begin{aligned}\mathrm{Reg}_{\texttt{ROLC-NC}}(T) \leq &\frac{\eta}{2} T Q^2 + \frac{1}{2\eta} 4 Z_{\mathcal{H}}^2 + (T\tau - \frac{\tau^2}{2} - \frac{1}{2}) \eta Q^2 + \tau T Q^2 \eta \\ \leq &(\frac{1}{2} + \tau) \eta T Q^2 + \frac{2}{\eta} Z_{\mathcal{H}}^2 + \tau T Q^2 \eta \\ = &(\frac{1}{2} + 2\tau) T Q^2 \eta + \frac{2}{\eta} Z_{\mathcal{H}}^2.\end{aligned}$$

We pick the step size $\eta = \frac{Z_{\mathcal{H}}}{Q} \frac{1}{\sqrt{(\frac{1}{4}+\tau)T}}$ so that $\mathrm{Reg}_{\texttt{ROLC-NC}}(T) \leq 2QZ_{\mathcal{H}} \sqrt{1+4\tau} \sqrt{T}$. □

B. Clairvoyant Malicious Data Generators

In this section, we consider the case where data generators are clairvoyant (i.e., $\xi = 0$). This represents the worst-case challenge to the classifier. Note that the proposed ROLC−NC can be applied to classify the data when $\xi = 0$. However, to obtain guaranteed sub-linear regret in this case, the classifier requires more information. Thus, we extend ROLC−NC to propose ROLC−C. The pseudo code of ROLC−C is shown in Algorithm 2.

1) Malicious and Normal Rounds: We recall here that normal and malicious data co-exist. Let o_t indicate whether the data sample $(\mathbf{x}_t, \mathbf{y}_t)$ is normal (i.e., if normal, $o_t = 1$; otherwise, $o_t = 0$). We call a round *malicious round* (resp. *normal round*) if $o_t = 0$ (resp. $o_t = 1$).

We first analyze how a clairvoyant malicious data generator affects the classifier. Different from the non-clairvoyant data generators, a clairvoyant data generator can predict the classification model adopted by the classifier in the current round accurately. Therefore, after observing $\mathbf{A}_t$ committed by the classifier, the clairvoyant malicious generator manipulates its data features from $\mathbf{x}_t$ to $\hat{\mathbf{x}}_t$ that gives the best response to $\mathbf{A}_t$. Thus, $\hat{\mathbf{x}}_t$ is a solution of $\min_{\mathbf{x}} \mathcal{C}_g(\mathbf{A}_t, \mathbf{x}_t, \mathbf{y}_t, \mathbf{x}, \hat{\mathbf{y}}_t)$. That is, $\min_{\mathbf{x}} L_g(\hat{\mathbf{y}}_t(\mathbf{x})) + \gamma_g C_m(\mathbf{x}, \mathbf{x}_t)$, where $\mathbf{x}_t$ and $\mathbf{y}_t$ are the data features and true label of the data sample respectively. We observe that, for any fixed $\mathbf{A}_t$ and $\mathbf{x}_t$, the cost function $\mathcal{C}_g(\mathbf{A}_t, \mathbf{x}_t, \mathbf{y}_t, \mathbf{x}, \hat{\mathbf{y}}_t)$ is convex in $\mathbf{x}$. According to the Karush–Kuhn–Tucker (KKT) conditions, we

Algorithm 2: ROLC-C: Robust OnLine Classification with Clairvoyant Malicious Data Generators

Input: Data $\{(\mathbf{x}_t, \mathbf{y}_t)\}_{t=1}^T$, step size η and delay τ
Output: Models parameterized by $\{\mathbf{A}_t\}_{t=1}^T$

1 Initialization: Select $\mathbf{A}_0 \in \mathcal{H}$ randomly and set $\mathbf{A}_1, ..., \mathbf{A}_\tau, \mathbf{A}_{\tau+1} = \mathbf{A}_0$.
2 **for** $t = \tau + 1$ to T **do**
3 Commit to $\mathbf{A}_t$, observe $\hat{\mathbf{x}}_t$, predict $\hat{\mathbf{y}}_t$.
4 Observe $\mathbf{y}_{t-\tau}$, $o_{t-\tau}$ and the cost $\mathcal{C}_c(\mathbf{A}_{t-\tau}, \mathbf{y}_{t-\tau}, \hat{\mathbf{x}}_{t-\tau})$.
5 **if** $o_{t-\tau} = 1$ **then**
6 Set $\mathbf{x}_{t-\tau} = \hat{\mathbf{x}}_{t-\tau}$ and
7 $\mathbf{g}_{t-\tau} \in \partial_{\mathbf{A}} \mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}, \mathbf{y}_{t-\tau}, \mathbf{x}_{t-\tau})$ at $\mathbf{A}_{t-\tau}$.
8 **else**
9 Set $\mathbf{x}_{t-\tau} = \hat{\mathbf{x}}_{t-\tau} + \frac{1}{2\gamma_g}\mathbf{A}_{t-\tau}\mathbf{b}$ and
10 $\mathbf{g}_{t-\tau} \in \partial_{\mathbf{A}} \mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}, \mathbf{y}_{t-\tau}, \mathbf{x}_{t-\tau})$ at $\mathbf{A}_{t-\tau}$.
11 $\mathbf{A}_{t+1} = \arg\min_{\mathbf{A}\in\mathcal{H}} \|\mathbf{A} - (\mathbf{A}_t - \eta \mathbf{g}_{t-\tau})\|^2$.

have $\nabla_{\hat{\mathbf{x}}_t} \mathcal{C}_g(\mathbf{A}_t, \mathbf{x}_t, \mathbf{y}_t, \hat{\mathbf{x}}_t, \hat{\mathbf{y}}_t) = \mathbf{A}_t\mathbf{b} + 2\gamma_g(\hat{\mathbf{x}}_t - \mathbf{x}_t) = 0$. Therefore, $\hat{\mathbf{x}}_t$ is uniquely defined as

$$\hat{\mathbf{x}}_t = \mathbf{x}_t - \frac{1}{2\gamma_g}\mathbf{A}_t\mathbf{b}. \tag{5}$$

In the same round, the classifier aims to minimize its cost function $\mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{x}}_t)$ by choosing an $\mathbf{A}_t$.

By substituting $\hat{\mathbf{x}}_t = \mathbf{x}_t - \frac{1}{2\gamma_g}\mathbf{A}_t\mathbf{b}$ into the classifier's cost function, we can identify the actual cost function that the classifier should optimize in the malicious round, which we denote as $\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}_t, \mathbf{y}_t, \mathbf{x}_t)$. That is, $\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}_t, \mathbf{y}_t, \mathbf{x}_t) = L_c(\hat{\mathbf{y}}_t(\hat{\mathbf{x}}_t), \mathbf{y}_t) + \gamma_c R_c(\mathbf{A}_t) = L_c(\hat{\mathbf{y}}_t(\mathbf{x}_t - \frac{1}{2\gamma_g}\mathbf{A}_t\mathbf{b}), \mathbf{y}_t) + \gamma_c R_c(\mathbf{A}_t) = L_c(\hat{\mathbf{y}}_t(\mathbf{x}_t) - \frac{1}{2\gamma_g}\mathbf{A}_t^T\mathbf{A}_t\mathbf{b}, \mathbf{y}_t) + \gamma_c R_c(\mathbf{A}_t)$. In contrast, in the normal round, the classifier's cost function is $\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}_t, \mathbf{y}_t, \mathbf{x}_t) = L_c(\hat{\mathbf{y}}_t(\hat{\mathbf{x}}_t), \mathbf{y}_t) + \gamma_c R_c(\mathbf{A}_t) = L_c(\hat{\mathbf{y}}_t(\mathbf{x}_t), \mathbf{y}_t) + \gamma_c R_c(\mathbf{A}_t)$.

2) Algorithm Design: As discussed above, to compute the actual cost function of the classifier requires additional information on the data generator, including the cost function of the classifier corresponding to normal and malicious data generators, i.e., $\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}_t, \mathbf{y}_t, \mathbf{x}_t)$ and $\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}_t, \mathbf{y}_t, \mathbf{x}_t)$, and o_t. However, we recall that, because of feedback delay, the true label $\mathbf{y}_t$, and hence also o_t, are known to the classifier only in round $t' \geq t + \tau$. Therefore, ROLC−C is designed to utilize such delayed information.

ROLC−C first initializes $\mathbf{A}_t, \forall t \in \{1, \cdots, \tau + 1\}$, to some randomly picked $\mathbf{A}_0$ (Line 1). In each round t, ROLC−C first commits to $\mathbf{A}_t$. Then a data sample $(\mathbf{x}_t, \mathbf{y}_t)$ arrives, and the features presented to the classifier is $\hat{\mathbf{x}}_t$. If it is a malicious data sample, $\hat{\mathbf{x}}_t$ is the best response to $\mathbf{A}_t$; otherwise, $\hat{\mathbf{x}}_t = \mathbf{x}_t$. The classifier observes $\hat{\mathbf{x}}_t$ and predicts a label $\hat{\mathbf{y}}_t$ with the model $\mathbf{A}_t$ (Line 3). In the same round t, the true label $\mathbf{y}_{t-\tau}$ and the indicator $o_{t-\tau}$ become known to the classifier, based on which the cost $\mathcal{C}_c(\mathbf{A}_{t-\tau}, \mathbf{y}_{t-\tau}, \hat{\mathbf{x}}_{t-\tau})$ from the prediction in that round is revealed (Line 4). Then the classifier chooses the cost function and computes its (sub)gradient to update the classification model in the next round (Line 5 to 10) according to $o_{t-\tau}$. If round $t - \tau$ is a normal round (Line 5), the algorithm sets the feature $\mathbf{x}_{t-\tau} = \hat{\mathbf{x}}_{t-\tau}$ as observed at time $t - \tau$ (Line 6) and computes the gradient $\mathbf{g}_{t-\tau}$ of the cost function $\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}, \mathbf{y}_{t-\tau}, \mathbf{x}_{t-\tau})$ at $\mathbf{A} = \mathbf{A}_{t-\tau}$ (Line 7); otherwise, it sets $\mathbf{x}_{t-\tau} = \hat{\mathbf{x}}_{t-\tau} + \frac{1}{2\gamma_g}\mathbf{A}_{t-\tau}\mathbf{b}$ according to (5) (Line 9), and computes the gradient $\mathbf{g}_{t-\tau}$ of the cost function $\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}, \mathbf{y}_{t-\tau}, \mathbf{x}_{t-\tau})$ at $\mathbf{A} = \mathbf{A}_{t-\tau}$ (Line 10). Finally, the algorithm computes the classification parameters $\mathbf{A}_{t+1} = \mathbf{A}_t - \eta\mathbf{g}_{t-\tau}$ for next round (Line 11).

3) Performance Analysis: In this part, we analyze the performance of ROLC−C. Specifically, we show in Theorem 2 that ROLC−C has a sub-linear upper bound on regret.

When the context is clear, we write $\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}_t, \mathbf{y}_t, \mathbf{x}_t)$ (resp. $\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}_t, \mathbf{y}_t, \mathbf{x}_t)$) as $\mathcal{C}_c^{(\mathrm{n})}$ (resp. $\mathcal{C}_c^{(\mathrm{m})}$) for simplicity.

Theorem 2. *Assume that, in any round t, the cost functions of $\mathbf{A}$, $\mathcal{C}_c^{(n)}(\mathbf{A}, \mathbf{y}_t, \mathbf{x}_t)$ and $\mathcal{C}_c^{(m)}(\mathbf{A}, \mathbf{y}_t, \mathbf{x}_t)$, are convex over $\mathbf{A}$ in $\mathcal{H}$, and Q_n-Lipschitz and Q_m-Lipschitz over $\mathbf{A}$ in $\mathcal{H}$ respectively. With step size $\eta = \frac{2Z_{\mathcal{H}}}{\sqrt{T\kappa Q_n^2 + T(1-\kappa)Q_m^2 + 4T\tau B}}$, ROLC−C has regret bound $\mathrm{Reg}_{\text{ROLC−C}}(T) \leq \rho\sqrt{T}$, where $\rho = 2Z_{\mathcal{H}}\sqrt{\kappa Q_n^2 + (1-\kappa)Q_m^2 + 4\tau B}$, κ is the fraction of normal data over the time interval from 1 to T, and $B = \max(Q_n^2, Q_n Q_m, Q_m^2)$.*

Proof. Similar to the proof of Theorem 1, for ease of presentation, we assume $\mathbf{A}$, $\mathbf{A}_t$ and $\mathbf{g}_t$, for all t, are vectors in this proof.

For a fixed $\mathbf{A}$, we have

$$\sum_{t=1}^{T}[\mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{x}}_t) - \mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)]$$
$$= \sum_{t:o_t=1}[\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}_t) - \mathcal{C}_c^{(\mathrm{n})}(\mathbf{A})] + \sum_{t:o_t=0}[\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}_t) - \mathcal{C}_c^{(\mathrm{m})}(\mathbf{A})].$$

By convexity, for $\mathbf{g}_t{}' \in \partial\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}, \mathbf{y}_t, \mathbf{x}_t)$ at $\mathbf{A}_t$ in round t, we have

$$\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}_t) - \mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}) \leq \mathbf{g}_t{}'^T(\mathbf{A}_t - \mathbf{A}),$$

and, for $\mathbf{g}_t{}'' \in \partial\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}, \mathbf{y}_t, \mathbf{x}_t)$ at $\mathbf{A}_t$ in round t, we have

$$\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}_t) - \mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}) \leq \mathbf{g}_t{}''^T(\mathbf{A}_t - \mathbf{A}).$$

Let $\mathbf{A}^* \in \arg\min_{\mathbf{A}} \sum_{t=1}^T \mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$. Then, by (1), we have $\mathrm{Reg}_{\text{ROLC−C}}(T) \leq \sum_{t=1}^T \mathbf{g}_t^T(\mathbf{A}_t - \mathbf{A}^*)$, where $\mathbf{g}_t \in \partial_{\mathbf{A}}\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$ at $\mathbf{A}_t$ if $o_t = 1$; and $\mathbf{g}_t \in \partial_{\mathbf{A}}\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$ at $\mathbf{A}_t$ if $o_t = 0$. For $t > \tau$, since $\mathbf{A}_{t+1}$ is the result of projecting $\mathbf{A}_t - \eta\mathbf{g}_{t-\tau}$ onto the convex set $\mathcal{H}$, we have

$$\|\mathbf{A}_{t+1} - \mathbf{A}^*\|^2 - \|\mathbf{A}_t - \mathbf{A}^*\|^2$$
$$\leq \eta^2\|\mathbf{g}_{t-\tau}\|^2 - 2\eta\mathbf{g}_{t-\tau}^T[(\mathbf{A}_{t-\tau} - \mathbf{A}^*) + (\mathbf{A}_t - \mathbf{A}_{t-\tau})]. \tag{6}$$

Now we expand $\mathbf{g}_{t-\tau}^T(\mathbf{A}_t - \mathbf{A}_{t-\tau})$ in (6) as follows.

$$\mathbf{g}_{t-\tau}^T(\mathbf{A}_t - \mathbf{A}_{t-\tau}) = \sum_{j=1}^{\min(t-(\tau+1),\tau)} \mathbf{g}_{t-\tau}^T(\mathbf{A}_{t-(j-1)} - \mathbf{A}_{t-j})$$
$$= \sum_{j=1}^{\min(t-(\tau+1),\tau)} -\eta\mathbf{g}_{t-\tau-j}^T\mathbf{g}_{t-\tau} - c_j. \tag{7}$$

where $c_j = [(\mathbf{A}_{t-j} - \eta \mathbf{g}_{t-\tau-j}^T) - \mathbf{A}_{t-(j-1)}]\mathbf{g}_{t-\tau}$.

Note that the Lipschitz property of $\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}, \mathbf{y}_t, \mathbf{x}_t)$ and $\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}, \mathbf{y}_t, \mathbf{x}_t)$ give $\|\mathbf{g}_t\| \leq Q_{\mathrm{n}}, \forall t: o_t = 1$, and $\|\mathbf{g}_t\| \leq Q_{\mathrm{m}}, \forall t: o_t = 0$. Thus, we have $\|\mathbf{g}_{t-\tau-j}^T\| \cdot \|\mathbf{g}_{t-\tau}\| \leq B$. Due to Lemma 1 and the Lipschitz property of $\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}, \mathbf{y}_t, \mathbf{x}_t)$ and $\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}, \mathbf{y}_t, \mathbf{x}_t)$, we have

$$c_j \leq \|(\mathbf{A}_{t-j} - \eta \mathbf{g}_{t-\tau-j}) - \mathbf{A}_{t-(j-1)}\| \cdot \|\mathbf{g}_{t-\tau}\| \leq \eta B.$$

This inequation and (7) give

$$\mathbf{g}_{t-\tau}^T(\mathbf{A}_t - \mathbf{A}_{t-\tau}) \geq \sum_{j=1}^{\min(t-(\tau+1),\tau)} [-\eta \mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} - \eta B].$$

Substituting the above into (6), we have

$$\begin{aligned} &\mathbf{g}_{t-\tau}^T(\mathbf{A}_{t-\tau} - \mathbf{A}^*) \\ \leq &\frac{\eta}{2}\|\mathbf{g}_{t-\tau}\|^2 + \frac{\|\mathbf{A}_t - \mathbf{A}^*\|^2 - \|\mathbf{A}_{t+1} - \mathbf{A}^*\|^2}{2\eta} \\ &+ \sum_{j=1}^{\min(t-(\tau+1),\tau)} [\eta \mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} + \eta B]. \end{aligned} \tag{8}$$

Summing up (8) over $t = \tau + 1$ to $T + \tau$ yields

$$\begin{aligned} &\sum_{t=\tau+1}^{T+\tau} \mathbf{g}_{t-\tau}^T(\mathbf{A}_{t-\tau} - \mathbf{A}^*) \\ \leq &\frac{\eta}{2} \sum_{t=\tau+1}^{T+\tau} \|\mathbf{g}_{t-\tau}\|^2 + \frac{\|\mathbf{A}_{\tau+1} - \mathbf{A}^*\|^2}{2\eta} \\ &+ \sum_{t=\tau+1}^{T+\tau} \sum_{j=1}^{\min(t-(\tau+1),\tau)} \eta \mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} + \sum_{t=\tau+1}^{T+\tau} \tau \eta B. \end{aligned}$$

Next we bound the second last term in the right hand side of the above inequation. Since $\mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} \leq \|\mathbf{g}_{t-\tau-j}^T\| \cdot \|\mathbf{g}_{t-\tau}\| \leq B$, we have

$$\begin{aligned} &\sum_{t=\tau+1}^{T+\tau} \sum_{j=1}^{\min(t-(\tau+1),\tau)} \eta \mathbf{g}_{t-\tau-j}^T \mathbf{g}_{t-\tau} \\ \leq &\sum_{t=\tau+1}^{T+\tau} \min(t - (\tau+1), \tau) \eta B \\ = &(T\tau - \frac{\tau^2}{2} - \frac{1}{2})\eta B. \end{aligned}$$

Therefore, we have

$$\begin{aligned} &\mathrm{Reg}_{\texttt{ROLC-C}}(T) \\ \leq &\frac{\eta}{2}[T\kappa Q_{\mathrm{n}}^2 + T(1-\kappa)Q_{\mathrm{m}}^2] + \frac{2}{\eta} Z_{\mathcal{H}}^2 + 2\tau T B \eta. \end{aligned}$$

We pick the step size $\eta = \frac{2Z_{\mathcal{H}}}{\sqrt{T\kappa Q_{\mathrm{n}}^2 + T(1-\kappa)Q_{\mathrm{m}}^2 + 4T\tau B}}$. Then, we have

$$\mathrm{Reg}_{\texttt{ROLC-C}}(T) \leq Z_{\mathcal{H}} \sqrt{\kappa Q_{\mathrm{n}}^2 + (1-\kappa)Q_{\mathrm{m}}^2 + 4\tau B} \sqrt{T}.$$

□

Algorithm 3: ROLC-NC-D: Robust OnLine Classification with Non-Clairvoyant Malicious Data Generators and Dynamic Delay

Input: Data $\{(\mathbf{x}_t, \mathbf{y}_t)\}_{t=1}^T$, step size η, delay τ_t for receiving feedback

Output: Models parameterized by $\{\mathbf{A}_t\}_{t=1}^T$

1 Initialization: Select $\mathbf{A}_1 \in \mathcal{H}$ randomly
2 **for** $t = 1$ *to* T **do**
3 Commit to $\mathbf{A}_t$, observe $\hat{\mathbf{x}}_t$, predict $\hat{\mathbf{y}}_t$.
4 **if** $\mathcal{D}_t = \{w \in \{1, \cdots, T\} : w + \tau_w = t\} \neq \emptyset$ **then**
5 **for** $u \in \mathcal{D}_t$ **do**
6 Observe $\mathbf{y}_u$ (and suffer the cost $\mathcal{C}_c(\mathbf{A}_u, \mathbf{y}_u, \hat{\mathbf{x}}_u)$).
7 Set $\mathbf{g}_u \in \partial_{\mathbf{A}} \mathcal{C}_c(\mathbf{A}, \mathbf{y}_u, \hat{\mathbf{x}}_u)$ at $\mathbf{A}_u$.
8 $\mathbf{A}_{t+1} = \arg\min_{\mathbf{A} \in \mathcal{H}} \|\mathbf{A} - (\mathbf{A}_t - \eta \sum_{u \in \mathcal{D}_t} \mathbf{g}_u)\|^2$.
9 **else**
10 $\mathbf{A}_{t+1} = \mathbf{A}_t$.

V. Extension to Dynamic Feedback Delay

In the previous section, we have proposed two online classification algorithms under the assumption that the feedback delay τ is static over time. However, in many practical systems, the feedback information may require a variable amount of processing or transmission time to be observed by the classifier. In this section, we extend our algorithms for both non-clairvoyant and clairvoyant data generators to the scenarios where the classifier receives feedback after *dynamic* delay, which varies over time and is unknown a priori. We assume that the feedback delay for information of the data sample that arrives in round t is $\tau_t, \forall t$. That is, the true label $\mathbf{y}_t$ in round t is known to the classifier only at time $t' \geq t + \tau_t$.

Similar to Sec. IV, we propose two algorithms, `ROLC-NC-D` and `ROLC-C-D`, for the cases when malicious data generators are non-clairvoyant (Sec. V-A) and clairvoyant (Sec. V-B), respectively. We show that both of them have provable performance guarantee in terms of a sub-linear regret bound.

A. Non-Clairvoyant Malicious Data Generators

Recall that this is the case when malicious data generators have $\xi > 0$. For this case, we propose the `ROLC-NC-D` algorithm, whose pseudo code is shown in Algorithm 3. Similar to `ROLC-NC` (Algorithm 1), it decides the classification model in each round by leveraging *only* the data features presented in each round (regardless whether they are manipulated) and the delayed feedback(s) of true label(s) of some previously arrived data sample(s). Note that `ROLC-NC-D` does not require additional information, e.g., whether the data sample is from a malicious or normal generator, the cost function of a malicious data generator, or the generator's delay ξ in observing the classification model.

1) Algorithm Design: `ROLC-NC-D` first initializes $\mathbf{A}_1$ for $t = 1$ (Line 1). In each round t, `ROLC-NC` first commits to $\mathbf{A}_t$.

Then a data sample $(\mathbf{x}_t, \mathbf{y}_t)$ arrives. If the data generator is a (non-clairvoyant) malicious one, it manipulates its data feature to $\hat{\mathbf{x}}_t$ to obtain the best response to the latest model it observes $\mathbf{A}_{t-\xi}$; otherwise, $\hat{\mathbf{x}}_t = \mathbf{x}_t$. The classifier then observes $\hat{\mathbf{x}}_t$ and predicts a label $\hat{\mathbf{y}}_t$ with the model $\mathbf{A}_t$ (Line 3). Here, $\mathcal{D}_t = \{w \in \{1, \cdots, T\} : w + \tau_w = t\}$ is the set of rounds in which the feedbacks of the previously arrived data samples are observed in round t (Line 4). In round t, feedbacks may arrive, i.e., $\mathcal{D}_t \neq \emptyset$. Different from ROLC-NC and ROLC-C, more than one feedbacks (corresponding to multiple previously arrived data samples) may arrive in a round, i.e., $|\mathcal{D}_t|$ can be greater than one. Based on the arrived feedback(s), the cost $\mathcal{C}_c(\mathbf{A}_u, \mathbf{y}_u, \hat{\mathbf{x}}_u)\ \forall u \in \mathcal{D}_t$, from the prediction in corresponding rounds are revealed (Line 6). Then, the algorithm computes the (sub)gradient $\mathbf{g}_u$ of the cost function $\mathcal{C}_c(\mathbf{A}, \mathbf{y}_u, \hat{\mathbf{x}}_u)$ chosen at $\mathbf{A} = \mathbf{A}_u$ (Line 7). Finally, the algorithm updates the classification parameters $\mathbf{A}_{t+1} = \mathbf{A}_t - \eta \sum_{u \in \mathcal{D}_t} \mathbf{g}_u$ used in the next round, i.e., by taking a step proportional to the negative of the summation of all (sub)gradients computed in this round, where η is the step size set by the algorithm (Line 8). If $\mathcal{D}_t = \emptyset$, the algorithm sets $\mathbf{A}_{t+1} = \mathbf{A}_t$ for the next round (Line 10).

2) Performance Analysis: In this part, we analyze the performance of ROLC-NC-D. Specifically, we show in Theorem 3 that ROLC-NC-D has provable performance guarantee in terms of a sub-linear regret bound.

Let $\tau_{\max} = \max_{t \in \{1, \cdots, T\}} \tau_t$, i.e., the maximum delay among all rounds. Note that $|\mathcal{D}_t| \leq \tau_{\max}$ for all $t \in \{1, \cdots, T\}$. Let $D_T = \sum_{t=1}^{T} \tau_t$, i.e., the sum of delay over time T.

Theorem 3. *Assume that the cost function $\mathcal{C}_c$ is convex, and Q-Lipschitz over $\mathbf{A}$ in $\mathcal{H}$. Further assume that $T - |\cup_{t=1}^{T} \mathcal{D}_t| = O(1)$. With step size $\eta = \frac{Z_{\mathcal{H}}}{Q} \frac{1}{\sqrt{\frac{T\tau_{\max}}{4} + D_T}}$, we have regret bound $Reg_{\text{ROLC-NC-D}}(T) \leq 2Z_{\mathcal{H}} Q \sqrt{T\tau_{\max} + 4D_T} + O(1)$.*

Proof. For ease of presentation, we assume $\mathbf{A}$, $\mathbf{A}_t$ and $\mathbf{g}_t$, for all t, are vectors.

Let $\mathbf{A}^* \in \arg\min_{\mathbf{A}} \sum_{t=1}^{T} \mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{x}}_t)$. Then, we have

$$\text{Reg}_{\text{ROLC-NC-D}}(T) = \sum_{t=1}^{T} [\mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{x}}_t) - \mathcal{C}_c(\mathbf{A}^*, \mathbf{y}_t, \hat{\mathbf{x}}_t)].$$

We fix some $t \in \{1, \cdots, T\}$. Suppose $\mathcal{D}_t$ is nonempty. Let $\mathcal{D}_{t,u} = \{r \in \mathcal{D}_t : r < u\}, \forall u \in \mathcal{D}_t$, $\mathbf{A}_{t,u} = \mathbf{A}_t - \eta \sum_{r \in \mathcal{D}_{t,u}} \mathbf{g}_r$, and $u' = \max \mathcal{D}_t$ (i.e., the largest round index in $\mathcal{D}_t$). We recall that $\mathbf{g}_u \in \partial \mathcal{C}_c(\mathbf{A}, \mathbf{y}_u, \hat{\mathbf{x}}_u)$ at $\mathbf{A}_u$, for all $u = 1, \cdots, T$. Since $\mathbf{A}_{t+1}$ is the result of projecting $\mathbf{A}_t - \eta \sum_{u \in \mathcal{D}_t} \mathbf{g}_u$ onto the convex set $\mathcal{H}$, we have

$$\begin{aligned}
&\|\mathbf{A}_{t+1} - \mathbf{A}^*\|^2 - \|\mathbf{A}_{t,u'} - \mathbf{A}^*\|^2 \\
\leq &\|\mathbf{A}_{t,u'} - \eta \mathbf{g}_{u'} - \mathbf{A}^*\|^2 - \|\mathbf{A}_{t,u'} - \mathbf{A}^*\|^2 \\
= &\eta^2 \|\mathbf{g}_{u'}\|^2 - 2\eta \mathbf{g}_{u'}^T (\mathbf{A}_{t,u'} - \mathbf{A}^*).
\end{aligned}$$

Then, we have

$$\begin{aligned}
&\|\mathbf{A}_{t+1} - \mathbf{A}^*\|^2 - \|\mathbf{A}_t - \mathbf{A}^*\|^2 \\
= &\|\mathbf{A}_{t+1} - \mathbf{A}^*\|^2 - \|\mathbf{A}_{t,u'} - \mathbf{A}^*\|^2 \\
&+ \|\mathbf{A}_{t,u'} - \mathbf{A}^*\|^2 - \ldots - \|\mathbf{A}_t - \mathbf{A}^*\|^2 \\
\leq &\eta^2 \sum_{u \in \mathcal{D}_t} \|\mathbf{g}_u\|^2 - 2\eta \sum_{u \in \mathcal{D}_t} \mathbf{g}_u^T (\mathbf{A}_{t,u} - \mathbf{A}^*).
\end{aligned}$$

For each $u \in \mathcal{D}_t$, by convexity, we have

$$\begin{aligned}
&- \mathbf{g}_u^T (\mathbf{A}_{t,u} - \mathbf{A}^*) \\
= &\mathbf{g}_u^T (\mathbf{A}^* - \mathbf{A}_{t,u}) \\
= &\mathbf{g}_u^T (\mathbf{A}^* - \mathbf{A}_u + \mathbf{A}_u - \mathbf{A}_{t,u}) \\
\leq &\mathcal{C}_c(\mathbf{A}^*, \mathbf{y}_u, \hat{\mathbf{x}}_u) - \mathcal{C}_c(\mathbf{A}_u, \mathbf{y}_u, \hat{\mathbf{x}}_u) + \mathbf{g}_u^T (\mathbf{A}_u - \mathbf{A}_{t,u}).
\end{aligned}$$

Due to the Lipschitz properties of $\mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$, we have $\|\mathbf{g}_t\| \leq Q, \forall t$. Then, we have

$$\begin{aligned}
&\|\mathbf{A}_{t+1} - \mathbf{A}^*\|^2 - \|\mathbf{A}_t - \mathbf{A}^*\|^2 \\
\leq &\eta^2 |\mathcal{D}_t| Q^2 + 2\eta \sum_{u \in \mathcal{D}_t} [\mathcal{C}_c(\mathbf{A}^*, \mathbf{y}_u, \hat{\mathbf{x}}_u) - \mathcal{C}_c(\mathbf{A}_u, \mathbf{y}_u, \hat{\mathbf{x}}_u) \\
&+ \mathbf{g}_u^T (\mathbf{A}_u - \mathbf{A}_{t,u})].
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&\text{Reg}_{\text{ROLC-NC-D}}(T) \\
= &\sum_{t=1}^{T} \sum_{u \in \mathcal{D}_t} [\mathcal{C}_c(\mathbf{A}_u, \mathbf{y}_u, \hat{\mathbf{x}}_u) - \mathcal{C}_c(\mathbf{A}^*, \mathbf{y}_u, \hat{\mathbf{x}}_u)] \\
&+ \sum_{u'' \notin \cup_{t=1}^{T} \mathcal{D}_t} [\mathcal{C}_c(\mathbf{A}_{u''}, \mathbf{y}_{u''}, \hat{\mathbf{x}}_{u''}) - \mathcal{C}_c(\mathbf{A}^*, \mathbf{y}_{u''}, \hat{\mathbf{x}}_{u''})] \\
\leq &\frac{2}{\eta} Z_{\mathcal{H}}^2 + \frac{\eta}{2} T \tau_{\max} Q^2 + \sum_{t=1}^{T} \sum_{u \in \mathcal{D}_t} \mathbf{g}_u^T (\mathbf{A}_u - \mathbf{A}_{t,u}) \\
&+ \sum_{u'' \notin \cup_{t=1}^{T} \mathcal{D}_t} 2QZ_{\mathcal{H}}. \qquad (9)
\end{aligned}$$

Since $T - |\cup_{t=1}^{T} \mathcal{D}_t| = O(1)$, we have $\sum_{u'' \notin \cup_{t=1}^{T} \mathcal{D}_t} 2QZ_{\mathcal{H}} = O(1)$.

By the Cauchy-Schwartz inequality, the second last term in the above inequality is bounded by

$$\begin{aligned}
\sum_{t=1}^{T} \sum_{u \in \mathcal{D}_t} \mathbf{g}_u^T (\mathbf{A}_u - \mathbf{A}_{t,u}) &\leq \sum_{t=1}^{T} \sum_{u \in \mathcal{D}_t} \|\mathbf{g}_u^T\| \cdot \|\mathbf{A}_u - \mathbf{A}_{t,u}\| \\
&\leq Q \sum_{t=1}^{T} \sum_{u \in \mathcal{D}_t} \|\mathbf{A}_u - \mathbf{A}_{t,u}\|. \qquad (10)
\end{aligned}$$

Recall that, when $\mathcal{D}_t$ is nonempty, $\mathcal{D}_{t,u} = \{r \in \mathcal{D}_t : r < u\}, \forall u \in \mathcal{D}_t$. By the triangle inequality, we have

$$\begin{aligned}
\|\mathbf{A}_{t,u} - \mathbf{A}_u\| &\leq \|\mathbf{A}_{t,u} - \mathbf{A}_t\| + \|\mathbf{A}_t - \mathbf{A}_u\| \\
&\leq \eta \sum_{r \in \mathcal{D}_{t,u}} \|\mathbf{g}_r\| + \sum_{v=u}^{t-1} \|\mathbf{A}_{v+1} - \mathbf{A}_v\|. \qquad (11)
\end{aligned}$$

Since $\mathbf{A}_{v+1}$ is the result of projecting $\mathbf{A}_v - \eta \sum_{l \in \mathcal{D}_v} \mathbf{g}_l$ onto the convex set $\mathcal{H}$, we have

$$\|\mathbf{A}_{v+1} - \mathbf{A}_v\| \leq \|\sum_{l \in \mathcal{D}_v} \mathbf{g}_l\| \leq \sum_{l \in \mathcal{D}_v} \|\mathbf{g}_l\|. \qquad (12)$$

Substituting (11) and (12) into (10), we have

$$\sum_{t=1}^{T} \sum_{u \in \mathcal{D}_t} \mathbf{g}_u^T (\mathbf{A}_u - \mathbf{A}_{t,u})$$

$$\leq Q\sum_{t=1}^{T}\sum_{u\in\mathcal{D}_t}(\eta\sum_{r\in\mathcal{D}_{t,u}}\|\mathbf{g}_r\|+\eta\sum_{v=u}^{t-1}\sum_{l\in\mathcal{D}_v}\|\mathbf{g}_l\|)$$
$$\leq \eta Q^2\sum_{t=1}^{T}\sum_{u\in\mathcal{D}_t}(|\mathcal{D}_{t,u}|+\sum_{v=u}^{t-1}|\mathcal{D}_v|). \quad (13)$$

For any t and $u\in\mathcal{D}_t$, either $\cup_{v=u}^{t-1}\mathcal{D}_v\cup\mathcal{D}_{t,u}\neq\emptyset$ or $\cup_{v=u}^{t-1}\mathcal{D}_v\cup\mathcal{D}_{t,u}=\emptyset$. If $\cup_{v=u}^{t-1}\mathcal{D}_v\cup\mathcal{D}_{t,u}=\emptyset$, $|\mathcal{D}_{t,u}|+\sum_{v=u}^{t-1}|\mathcal{D}_v|=0$. Next, we focus on the case in which $\cup_{v=u}^{t-1}\mathcal{D}_v\cup\mathcal{D}_{t,u}\neq\emptyset$. For any $k\in\cup_{v=u}^{t-1}\mathcal{D}_v\cup\mathcal{D}_{t,u}$, to capture the two cases, $k>u$ and $k\leq u$, over all rounds, we denote a round with different notations k' and u' in the analysis. For any round $r=1,\cdots,T$, we have

$$|\{k':k'>r \text{ and } k'\in\cup_{v=r}^{t-1}\mathcal{D}_v\cup\mathcal{D}_{t,r},\forall t\}|<\tau_r-1, \quad (14)$$

$$\text{and } |\{u':r\leq u' \text{ and } r\in\cup_{v=u'}^{t-1}\mathcal{D}_v\cup\mathcal{D}_{t,u'},\forall t\}|\leq\tau_r. \quad (15)$$

Summing up (14) and (15) over T rounds yields

$$\sum_{t=1}^{T}\sum_{u\in\mathcal{D}_t}(|\mathcal{D}_{t,u}|+\sum_{v=u}^{t-1}|\mathcal{D}_v|)\leq\sum_{t=1}^{T}2\tau_t.$$

Substituting the above inequality into (13), we have

$$\sum_{t=1}^{T}\sum_{u\in\mathcal{D}_t}\mathbf{g}_u^T(\mathbf{A}_u-\mathbf{A}_{t,u})\leq 2\eta Q^2\sum_{t=1}^{T}\tau_t.$$

Therefore, we have

$$\text{Reg}_{\texttt{ROLC-NC-D}}(T)\leq\frac{2}{\eta}Z_{\mathcal{H}}^2+\frac{\eta}{2}T\tau_{\max}Q^2+2\eta Q^2\sum_{t=1}^{T}\tau_t+O(1).$$

We then pick the step size $\eta=\frac{Z_{\mathcal{H}}}{Q}\frac{1}{\sqrt{\frac{T\tau_{\max}}{4}+D_T}}$, where $D_T=\sum_{t=1}^{T}\tau_t$, and we have

$$\text{Reg}_{\texttt{ROLC-NC-D}}(T)\leq 2Z_{\mathcal{H}}Q\sqrt{T\tau_{\max}+4D_T}+O(1).$$

□

B. Clairvoyant Malicious Data Generators

We now consider the case where data generators are clairvoyant (i.e., $\xi=0$). As we discussed in Sec. III, this represents the worst-case challenge to the classifier. Similar to how we extend ROLC-NC to ROLC-C, we propose another online algorithm called ROLC-C-D, whose pseudo code is shown in Algorithm 4.

Our analysis in Sec. IV-B shows that, to obtain guaranteed sub-linear regret, the classifier in the case when $\xi=0$ requires additional information including the cost function of classifiers corresponding to normal and malicious data generators, i.e., $\mathcal{C}_c^{(\text{n})}(\mathbf{A},\mathbf{y}_t,\mathbf{x}_t)$ and $\mathcal{C}_c^{(\text{m})}(\mathbf{A},\mathbf{y}_t,\mathbf{x}_t)$, and $o_t,\forall t$, indicating whether the data sample is from a malicious or normal generator. However, we recall that, because of feedback delay, the true label $\mathbf{y}_t$, and hence also o_t, are known to the classifier only at time $t'\geq t+\tau_t$. Therefore, ROLC-C-D is designed to utilize such delayed information. Same as ROLC-NC-D, ROLC-C-D does not require the information of the feedback delay before the corresponding feedback arrives.

Algorithm 4: ROLC-C-D: Robust OnLine Classification with Clairvoyant Malicious Data Generators and Dynamic Delay

Input: Data $\{(\mathbf{x}_t,\mathbf{y}_t)\}_{t=1}^T$, step size η and delay $\{\tau_t\}$
Output: Models parameterized by $\{\mathbf{A}_t\}_{t=1}^T$

1 Initialization: Select $\mathbf{A}_1\in\mathcal{H}$ randomly
2 **for** $t=1$ *to* T **do**
3 Commit to $\mathbf{A}_t$, observe $\hat{\mathbf{x}}_t$, predict $\hat{\mathbf{y}}_t$.
4 **if** $\mathcal{D}_t=\{w\in\{1,\cdots,T\}:w+\tau_w=t\}\neq\emptyset$ **then**
5 **for** $u\in\mathcal{D}_t$ **do**
6 Observe $\mathbf{y}_u$, o_u and the cost $\mathcal{C}_c(\mathbf{A}_u,\mathbf{y}_u,\hat{\mathbf{x}}_u)$.
7 **if** $o_u=1$ **then**
8 Set $\mathbf{x}_u=\hat{\mathbf{x}}_u$ and
9 $\mathbf{g}_u\in\partial_{\mathbf{A}}\mathcal{C}_c^{(\text{n})}(\mathbf{A},\mathbf{y}_u,\mathbf{x}_u)$.
10 **else**
11 Set $\mathbf{x}_u=\hat{\mathbf{x}}_u+\frac{1}{2\gamma_g}\mathbf{A}_u\mathbf{b}$ and
12 $\mathbf{g}_u\in\partial_{\mathbf{A}}\mathcal{C}_c^{(\text{m})}(\mathbf{A},\mathbf{y}_u,\mathbf{x}_u)$.
13 $\mathbf{A}_{t+1}=\arg\min_{\mathbf{A}\in\mathcal{H}}\|\mathbf{A}-(\mathbf{A}_t-\eta\sum_{u\in\mathcal{D}_t}\mathbf{g}_u)\|^2$.
14 **else**
15 $\mathbf{A}_{t+1}=\mathbf{A}_t$.

1) Algorithm Design: ROLC-C-D first initializes $\mathbf{A}_1$ for $t=1$ (Line 1). In each round t, ROLC-C-D first commits to $\mathbf{A}_t$. Then a data sample $(\mathbf{x}_t,\mathbf{y}_t)$ arrives. If it is from a malicious data generator, the data features are manipulated to $\hat{\mathbf{x}}_t$ to best respond to $\mathbf{A}_t$; otherwise, for the normal data sample, $\hat{\mathbf{x}}_t=\mathbf{x}_t$ (see the analysis in Sec. IV-B1). The classifier then observes $\hat{\mathbf{x}}_t$ and predicts a label $\hat{\mathbf{y}}_t$ with the model $\mathbf{A}_t$ (Line 3). Recall that $\mathcal{D}_t$ is the set of rounds in which the feedbacks of the previously arrived data samples are observed in round t (Line 4). In this round, feedbacks may arrive, i.e., $\mathcal{D}_t\neq\emptyset$ (Line 4). Note that, in one round, more than one feedbacks (corresponding to one or more previously arrived data samples) may arrive. The feedback from prediction in one previous round $u\in\mathcal{D}_t$ includes the true label $\mathbf{y}_u$ and the indicator o_u. Based on the arrived feedbacks, the cost $\mathcal{C}_c(\mathbf{A}_u,\mathbf{y}_u,\hat{\mathbf{x}}_u)\ \forall u\in\mathcal{D}_t$, from the prediction in corresponding rounds are revealed (Line 6). Next, the classifier chooses the cost function and computes its (sub)gradient to update the classification model in the next round (Line 7 to 12) according to o_u. Specifically, if round u is a normal round (Line 7); the algorithm sets $\mathbf{x}_u=\hat{\mathbf{x}}_u$ as it observed in round u (Line 8) and computes the (sub)gradient $\mathbf{g}_u$ of the cost function $\mathcal{C}_c^{(\text{n})}(\mathbf{A},\mathbf{y}_u,\mathbf{x}_u)$ chosen at $\mathbf{A}=\mathbf{A}_u$ (Line 9); otherwise, it sets $\mathbf{x}_u=\hat{\mathbf{x}}_u+\frac{1}{2\gamma_g}\mathbf{A}_u\mathbf{b}$ according to the closed form (5) of the manipulated feature for a malicious data generator (Line 11), and computes the (sub)gradient $\mathbf{g}_u$ of the cost function $\mathcal{C}_c^{(\text{m})}(\mathbf{A},\mathbf{y}_u,\mathbf{x}_u)$ at $\mathbf{A}=\mathbf{A}_u$ (Line 12). Finally, the algorithm computes the classification parameters $\mathbf{A}_{t+1}=\mathbf{A}_t-\eta\sum_{u\in\mathcal{D}_t}\mathbf{g}_u$ in the next round (Line 13). If $\mathcal{D}_t=\emptyset$, no feedback arrives in this round, and the algorithm sets $\mathbf{A}_{t+1}=\mathbf{A}_t$ for the next round (Line 15).

2) Performance Analysis: In this part, we analyze the performance of ROLC-C-D. Specifically, we show in Theorem 4 that ROLC-C-D has a sub-linear regret bound.

Theorem 4. *Assume that the cost functions of* $\mathbf{A}$, $\mathcal{C}_c^{(n)}$ *and* $\mathcal{C}_c^{(m)}$, *are convex over* $\mathbf{A}$ *in* $\mathcal{H}$, *and* Q_n*-Lipschitz and* Q_m*-Lipschitz over* $\mathbf{A}$ *in* $\mathcal{H}$ *respectively. Further assume that* $T - |\cup_{t=1}^{T}\mathcal{D}_t| = O(1)$. *With step size* $\eta = \frac{2Z_{\mathcal{H}}}{\sqrt{T\tau_{\max}\kappa Q_n^2 + T\tau_{\max}(1-\kappa)Q_m^2 + 4BD_T}}$, *we have regret bound* $Reg_{ROLC\text{-}C\text{-}D}(T) \leq 2\sqrt{2}Z_{\mathcal{H}}\sqrt{T\tau_{\max}\kappa Q_n^2 + T\tau_{\max}(1-\kappa)Q_m^2 + 4BD_T} + O(1)$, *where* κ *is the fraction of normal datas over the time horizon from* 1 *to* T, *and* $B = \max(Q_n^2, Q_nQ_m, Q_m^2)$.

Proof. Again, for ease of presentation, we assume $\mathbf{A}$, $\mathbf{A}_t$ and $\mathbf{g}_t$, for all t, are vectors.

Let $\mathbf{A}^* \in \arg\min_{\mathbf{A}} \sum_{t=1}^{T} \mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{x}}_t)$. Then, we have

$$\mathrm{Reg}_{\text{ROLC-C-D}}(T) = \sum_{t=1}^{T}[\mathcal{C}_c(\mathbf{A}_t, \mathbf{y}_t, \hat{\mathbf{x}}_t) - \mathcal{C}_c(\mathbf{A}^*, \mathbf{y}_t, \hat{\mathbf{x}}_t)].$$

We fix a $t \in \{1, \cdots, T\}$. According to Line 9 and 12 in ROLC-C-D, we have $\mathbf{g}_t \in \partial_{\mathbf{A}}\mathcal{C}_c^{(\mathrm{n})}(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$ at $\mathbf{A}_t$ if $o_t = 1$; and $\mathbf{g}_t \in \partial_{\mathbf{A}}\mathcal{C}_c^{(\mathrm{m})}(\mathbf{A}, \mathbf{y}_t, \hat{\mathbf{x}}_t)$ at $\mathbf{A}_t$ if $o_t \neq 1$. Similar to the proof of Theorem 3, we have

$$\begin{aligned}&\|\mathbf{A}_{t+1} - \mathbf{A}^*\|^2 - \|\mathbf{A}_t - \mathbf{A}^*\|^2\\ \leq&\eta^2 \sum_{u\in\mathcal{D}_t} \|\mathbf{g}_u\|^2 - 2\eta \sum_{u\in\mathcal{D}_t} \mathbf{g}_u^T(\mathbf{A}_{t,u} - \mathbf{A}^*).\end{aligned}$$

For each $u \in \mathcal{D}_t$, by convexity, we have

$$\begin{aligned}&-\mathbf{g}_u^T(\mathbf{A}_{t,u} - \mathbf{A}^*)\\ \leq&\mathcal{C}_c(\mathbf{A}^*, \mathbf{y}_u, \hat{\mathbf{x}}_u) - \mathcal{C}_c(\mathbf{A}_u, \mathbf{y}_u, \hat{\mathbf{x}}_u) + \mathbf{g}_u^T(\mathbf{A}_u - \mathbf{A}_{t,u}).\end{aligned}$$

Then, we have

$$\begin{aligned}&\|\mathbf{A}_{t+1} - \mathbf{A}^*\|^2 - \|\mathbf{A}_t - \mathbf{A}^*\|^2\\ \leq&2\eta \sum_{u\in\mathcal{D}_t} (\mathcal{C}_c(\mathbf{A}^*, \mathbf{y}_u, \hat{\mathbf{x}}_u) - \mathcal{C}_c(\mathbf{A}_u, \mathbf{y}_u, \hat{\mathbf{x}}_u)\\ &+ \mathbf{g}_u^T(\mathbf{A}_u - \mathbf{A}_{t,u})) + \eta^2 \sum_{u\in\mathcal{D}_t} \|\mathbf{g}_u\|^2.\end{aligned}$$

Thus, we have

$$\begin{aligned}&\mathrm{Reg}_{\text{ROLC-C-D}}(T)\\ \leq&\frac{2}{\eta}Z_{\mathcal{H}}^2 + \frac{\eta}{2}[T\tau_{\max}\kappa Q_{\mathrm{n}}^2 + T\tau_{\max}(1-\kappa)Q_{\mathrm{m}}^2]\\ &+ \sum_{t=1}^{T}\sum_{u\in\mathcal{D}_t} \mathbf{g}_u^T(\mathbf{A}_u - \mathbf{A}_{t,u}) + \sum_{u''\notin\cup_{t=1}^{T}\mathcal{D}_t} 2Q_{\max}Z_{\mathcal{H}},\end{aligned}$$

where $Q_{\max} = \max\{Q_n, Q_m\}$. Since $T - |\cup_{t=1}^{T}\mathcal{D}_t| = O(1)$, we have $\sum_{u''\notin\cup_{t=1}^{T}\mathcal{D}_t} 2Q_{\max}Z_{\mathcal{H}} = O(1)$.

By the Cauchy-Schwartz inequality, the second last term in the above inequality is bounded by

$$\begin{aligned}&\sum_{t=1}^{T}\sum_{u\in\mathcal{D}_t} \mathbf{g}_u^T(\mathbf{A}_u - \mathbf{A}_{t,u})\\ \leq&\sum_{t=1}^{T}\sum_{u\in\mathcal{D}_t} \|\mathbf{g}_u\| \cdot (\eta \sum_{r\in\mathcal{D}_{t,u}} \|\mathbf{g}_r\| + \eta \sum_{v=u}^{t-1}\sum_{l\in\mathcal{D}_v} \|\mathbf{g}_l\|) \leq 2\eta B \sum_{t=1}^{T} \tau_t.\end{aligned}$$

Thus, we have

$$\begin{aligned}&\mathrm{Reg}_{\text{ROLC-C-D}}(T)\\ \leq&\frac{2}{\eta}Z_{\mathcal{H}}^2 + \frac{\eta}{2}[T\tau_{\max}\kappa Q_{\mathrm{n}}^2 + T\tau_{\max}(1-\kappa)Q_{\mathrm{m}}^2] + 2\eta BD_T + O(1).\end{aligned}$$

We pick the step size $\eta = \frac{2Z_{\mathcal{H}}}{\sqrt{T\tau_{\max}\kappa Q_{\mathrm{n}}^2 + T\tau_{\max}(1-\kappa)Q_{\mathrm{m}}^2 + 4BD_T}}$, and we have

$$\begin{aligned}&\mathrm{Reg}_{\text{ROLC-C-D}}(T)\\ \leq&2\sqrt{2}Z_{\mathcal{H}}\sqrt{T\tau_{\max}\kappa Q_{\mathrm{n}}^2 + T\tau_{\max}(1-\kappa)Q_{\mathrm{m}}^2 + 4BD_T} + O(1).\end{aligned}$$

□

VI. Application to Network Flow Classification

In this section, we present an application of the proposed robust online learning algorithms to network flow classification. We evaluate their performance by experimenting with real-world traffic data traces.

A. Robust Online Network Flow Classification

We consider the problem of network flow classification where *malicious flow generators* may exist, which can manipulate their flow features to best respond to the classification model to increase the likelihood of a certain outcome so as to increase their own utility, e.g., to be prioritized for network resource allocation.

There is a cost to manipulate flow features. In a high-speed network, it is generally expensive to build hardware for packet flow manipulation at line rate. On the other hand, software-based packet flow manipulation requires frequent interaction with the memory (e.g., read and write operations) [8]. Furthermore, some manipulation such as changing header fields requires more complex operation [10]. Therefore, software-based manipulation at line rate is non-trivial and often reduces the performance of the flow. Here we adopt the general cost model as explained in Sec. III-B. In addition, often it may take some time for a malicious flow generator to detect the classification model through reverse engineering attacks [19], [24]. We consider both the *non-clairvoyant* (i.e., $\xi > 0$) and *clairvoyant* (i.e., $\xi = 0$) cases as defined in Sec. III-C.

An online learning approach requires comparison between the true and predicted labels of a previously arrived and classified flow. For online network flow classification, the true label can be obtained, for example, through deep packet inspection (DPI), or by a sophisticated robust classifier residing in some powerful remote server, which is trained using sufficient data to give accurate prediction in spite of the altered features. Both options incur computation or communication delay. State-of-the-art DPI systems for encrypted traffic has high runtime overhead and cannot process packets in real-time [2], [55]–[57]. For example, BlindBox requires minutes for every new end-to-end connection [55]. Embark enables a cloud provider (e.g., Amazon EC2) to outsource DPI processing, which incurs communication delay [56].

Fortunately, as discussed in Secs. IV and V, the proposed algorithms ROLC-NC, ROLC-C, ROLC-NC-D, and ROLC-C-D

all can accommodate both malicious flow generators and feedback delay, while providing performance guarantee in terms of sub-linear regret bounds. Next, we present further experimental details and results when these algorithms are applied to online network flow classification.

B. Trace-Driven Experiments

1) Data Trace: To apply the proposed algorithms to online network flow classification, we use packet traces from [58] and [59]. All packets in the trace dataset are TCP packets, and each TCP connection corresponds to a flow. The dataset contains 377,526 flows in total. In each round of an experiment, a randomly selected flow from this set is sent to the classifier. The record of each flow contains a variety of characteristic features. We consider 100 standard features including the minimum, mean, maximum, and standard deviation of packet lengths and packet inter-arrival times, number of packets and bytes, and the duration of the network connection. They are features numbered 3-9, 195-208, 10-30, 153-194, 210-215, 31-40 in [36].[3]

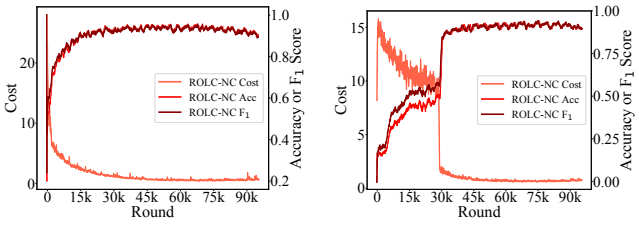
Each flow belongs to 12 application types: www, mail, ftp-control, ftp-pasv, attack, p2p, database, ftp-data, multimedia, services, interactive, and games. We map these application types into four different QoS labels roughly based on the application's delay requirement. The label assignment is as follows. $k = 1$: multimedia, interactive, games, and ftp-control; $k = 2$: attack, www, and p2p; $k = 3$: database, ftp-data, and services; $k = 4$: mail and ftp-pasv. Thus, the lower k is, the more sensitive to delay the corresponding flow is.

2) Performance Metrics and Benchmarks: We take the classification *accuracy* as our primary metric, which is more useful than the regret in practice. In addition, since the class distribution is imbalanced in the dataset, we also take the F_1 *score* as a second performance metric [51], [60].

We compare the proposed algorithms with two offline benchmarks as follows. We first train a static classification model by minimizing its cumulative cost $\sum_{t=1}^{T} \mathcal{C}_c(\mathbf{A}, \mathbf{y}_t, \mathbf{x}_t)$, assuming knowledge of the true features and true labels of all flows for $t = 1, 2, \cdots, T$. The optimizer we use is Sequential Least Squares Programming (SLSQP) [61]. Then, the two benchmarks are the performance metrics obtained by this classification model under two different test datasets as follows.

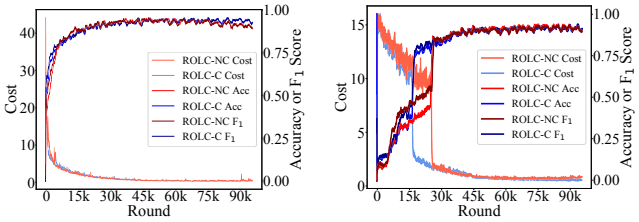
- **`Offline`**: Performance obtained with test dataset $(\hat{\mathbf{x}}'_t, \mathbf{y}_t), \forall t$, which contains the observed flow features (manipulated version if the flow comes from a malicious generator) of the same sequence of flows presented to the online algorithms, except that the manipulated flow features $\hat{\mathbf{x}}'_t$ best respond to the above offline classification model.
- **`Offline-Norm`**: Performance obtained with test dataset $(\mathbf{x}_t, \mathbf{y}_t), \forall t$, the normal flows containing unmanipulated original features.

[3]These features are chosen since they can be manipulated by the flow generators in practice.



(a) HL, $\xi = 10$ (b) CEL, $\xi = 10$

Fig. 1: Learning process of `ROLC-NC` with $\xi = 10$.



(a) HL, $\xi = 0$ (b) CEL, $\xi = 0$

Fig. 2: Learning process of `ROLC-NC` and `ROLC-C` with $\xi = 0$.

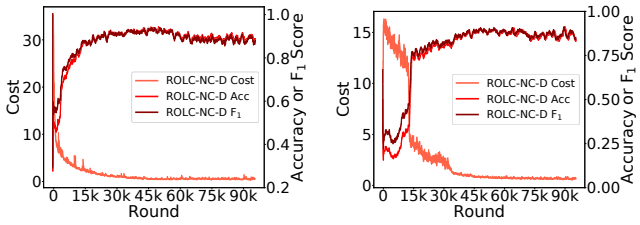
3) Experiment Setting: The loss function of the classifier is general in this work (see Sec. III). In our experiments, we use two common loss functions, Hinge Loss (HL) and Categorical Cross-Entropy Loss (CEL) defined as follows [51]:

- HL: $L_c(\hat{\mathbf{y}}, \mathbf{y}) = \sum_{k \neq k^*} \max(0, 1 + \hat{\mathbf{y}}^{(k)} - \hat{\mathbf{y}}^{(k^*)})$, where k^* is the true QoS level;
- CEL: $L_c(\hat{\mathbf{y}}, \mathbf{y}) = -\log \frac{e^{\hat{\mathbf{y}}^T \mathbf{y}}}{\sum_k e^{\hat{\mathbf{y}}^{(k)}}}$.

In each experiment, the proposed algorithms are run using either HL or CEL for $T = 100,000$ rounds, which is sufficient for them to go into the steady state. Over the T rounds, a fraction κ (resp. $1 - \kappa$) of rounds are randomly assigned as normal (resp. malicious) rounds. In each round, a flow is uniformly randomly chosen from the data trace. We evaluate the performance of the proposed algorithms in different settings by varying the value of key parameters around the default setting $(\kappa, \tau, \gamma_c, \gamma_g, \mathbf{b}) = (0.5, 10, 0.1, 0.8, [1, 2, 4, 8])$. The comparison benchmarks are calculated based on the same test data as those of the online algorithms. All experiments are run on a machine with two Intel(R) Xeon(R) CPU E5-2650 v4 2.20GHz with 32GB memory and 1.8TB hard drive.

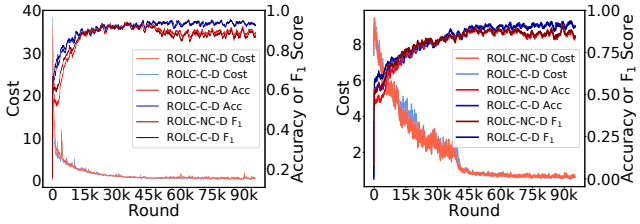
4) Accuracy and F_1 Score over Time: We first evaluate the performance of `ROLC-NC` and `ROLC-C` in terms of the cost of each round, the accuracy, and the F_1 score, over a sliding window of size 2000 rounds during the learning process. For the first 2000 rounds, the performance metrics are calculated over all arrived flows. Fig. 1 shows the learning process of `ROLC-NC` when the delay for a malicious flow generator to detect the classification model is $\xi = 10$. Fig. 2 shows the learning process of `ROLC-NC` and `ROLC-C` when $\xi = 0$.[4] We observe that, in all experiments, the cost value decreases quickly at the beginning and converges to its steady state after 15k (resp. 30k) rounds for `ROLC-NC` using HL (resp. CEL)

[4]Recall that `ROLC-NC` is designed for $\xi > 0$ but it can also be applied to the case when $\xi = 0$, while `ROLC-C` is used only when $\xi = 0$.



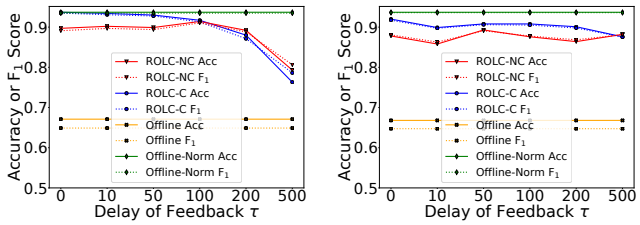
(a) HL, $\xi = 10$ (b) CEL, $\xi = 10$

Fig. 3: Learning process of ROLC−NC−D and ROLC−C−D with $\xi = 10$.



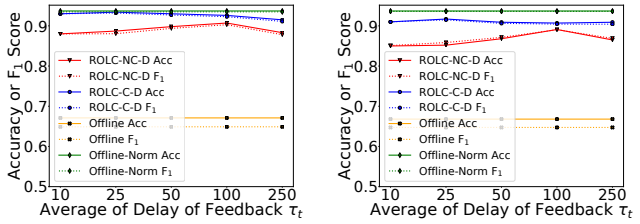
(a) HL, $\xi = 0$ (b) CEL, $\xi = 0$

Fig. 4: Learning process of ROLC−NC−D and ROLC−C−D with $\xi = 0$.



(a) HL (b) CEL

Fig. 5: Impact of static delay τ.



(a) HL (b) CEL

Fig. 6: Impact of dynamic delay τ_t.

and after 15k rounds for ROLC−C for both HL and CEL. The accuracy and F_1 score at convergence are both greater than 0.9. We also find that both algorithms learn faster and have higher accuracy and F_1 score using HL than using CEL. After 45k rounds, ROLC−NC using HL achieves 0.91 accuracy and 0.91 F_1 score when $\xi = 10$, and ROLC−C using HL achieves 0.93 accuracy and 0.93 F_1 score when $\xi = 0$.

When $\xi = 0$, we can further compare the performance of ROLC−NC and ROLC−C. Fig. 2 shows that, ROLC−C converges faster than ROLC−NC does, and performs slightly better than ROLC−NC in classification accuracy and F_1 score. This is because ROLC−C uses the real cost function $C_c^{(\mathrm{m})}$ in a malicious round to update the model, while ROLC−NC does not. This, as well as similar results presented in Figs. 5 and 7, suggests that ROLC−C can be a better choice in the case when the flow generators are clairvoyant.

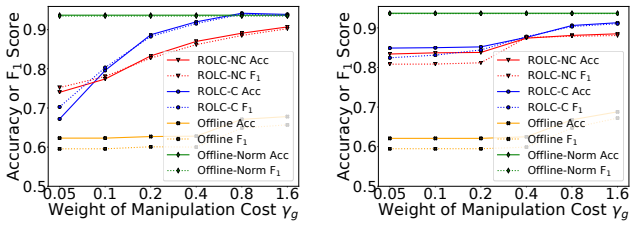
We then evaluate the performance of ROLC−NC−D and ROLC−C−D when the feedback delay is dynamic. We keep the default parameter setting described previously and choose the delay τ_t in each round t uniformly randomly from $\{0, 1, \cdots, 20\}$. Fig. 3 shows the learning process of ROLC−NC−D when $\xi = 10$. Fig. 4 shows the learning process of ROLC−NC−D and ROLC−C−D when $\xi = 0$. We observe that, both ROLC−NC−D and ROLC−C−D learn faster and have higher accuracy and F_1 score using HL than using CEL. ROLC−NC−D using HL (resp. CEL) achieves 0.88 (resp. 0.84) accuracy and F_1 score in all experiments. When $\xi = 0$, ROLC−C−D using HL (resp. CEL) achieves 0.93 (resp. 0.91) accuracy and F_1 score. ROLC−NC−D (resp. ROLC−C−D) take longer time to converge than ROLC−NC (resp. ROLC−C) does, which may be caused by the dynamic delay.

We also evaluate the convergence time for these algorithms to reach steady state in the learning process. We find that, with our machine, the computation time for each round is less than 10 milliseconds, and thus the average convergence time is between 75 and 150 seconds. To further reduce this time in practical implementation, one may use more powerful computing hardware and/or fewer features, e.g., only features from the initial packets of a flow [39].

5) Impact of Feedback Delay: We further evaluate the impact of the feedback delay τ on the performances of ROLC−NC and ROLC−C in steady state and present the results in Fig. 5. We calculate the accuracy and F_1 score by averaging over 2000 rounds in steady state. Keeping the default parameter setting described previously and ξ=0, we vary τ from 0 to 500. For each data point in the figure, we take the average of 10 repeated experiments, where we fix the sequences of the arriving flows $\{(\mathbf{x}_t, \mathbf{y}_t)\}_{t=1}^T$ and indicators $\{o_t\}_{t=1}^T$ while varying the random initialization of the algorithms (Line 1 of Algorithms 1 and 2).

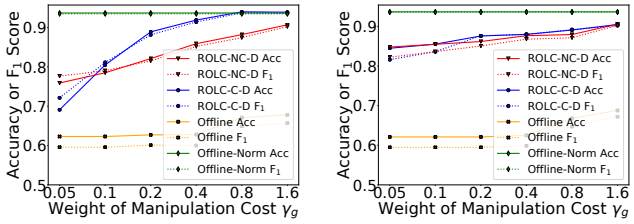
Fig. 5 presents the accuracy and F_1 score of ROLC−NC, ROLC−C, and the offline benchmarks. Note that the offline benchmarks are not affected by τ. When using HL (Fig. 5(a)), both ROLC−NC and ROLC−C have stable performance when $\tau \leq 100$, and ROLC−C achieves 0.92 accuracy and F_1 score on average, which is only 0.01 lower than those of Offline−Norm, while ROLC−NC achieves 0.90 accuracy and F_1 score on average. When the delay τ becomes longer, both ROLC−NC and ROLC−C yield lower accuracy and F_1 score. When $\tau \geq 200$, ROLC−NC performs slightly better than ROLC−C. When using CEL (Fig. 5(b)), ROLC−NC and ROLC−C both perform worse than when using HL. However, in this case, ROLC−NC and ROLC−C are less sensitive to the parameter τ, achieving above 0.88 accuracy and F_1 score even when τ is large. We note that, although not shown in this figure, a larger τ incurs a longer convergence time for both algorithms, which is expected for online learning.

We also evaluate the impact of the dynamic delay τ_t on the performances of ROLC−NC−D and ROLC−C−D in steady state. The results are presented in Fig. 6. Keeping



(a) HL (b) CEL

Fig. 7: Impact of weight of manipulation cost γ_g, with static feedback delay.



(a) HL (b) CEL

Fig. 8: Impact of weight of manipulation cost γ_g, with dynamic feedback delay.

the default parameter setting described previously and $\xi = 0$, we set τ_t to be chosen uniformly randomly from the sets $\{0, \cdots, 20\}, \{0, \cdots, 50\}, \{0, \cdots, 100\}, \{0, \cdots, 200\}$, and $\{0, \cdots, 500\}$. In Fig. 6, each x-axis label represents the mean of the corresponding set. In most cases, `ROLC-C-D` performs better than `ROLC-NC-D`. Similar to the results in Fig. 5, `ROLC-NC-D` and `ROLC-C-D` both perform better when using HL.

6) Impact of Manipulation Cost Weight γ_g: We vary the value of γ_g to study its impact on the performance of `ROLC-NC`, `ROLC-C`, `ROLC-NC-D`, and `ROLC-C-D`. We use the default parameter setting with $\xi = 0$. Figs. 7 and 8 present the accuracy and F_1 score of the proposed algorithms and the offline benchmarks. Note that `Offline-Norm` is not affected by γ_g as expected. With either HL or CEL, the proposed algorithms and `Offline` all yield higher accuracy and F_1 score when the manipulation cost γ_g increases. This is because a larger γ_g encourages the malicious flow generators to stay closer to the original features. However, when $\gamma_g \leq 0.4$, the accuracy and F_1 score of `Offline` becomes flat, because `Offline` performs so poorly that it can hardly classify any manipulated flow correctly.

VII. Conclusion

In this work, we address the problem of robust online learning against malicious manipulation. The data features may be manipulated by malicious generators to best respond to the classification models committed by the classifier. Practical issues such as delayed feedback (for the classifier) and delayed observation of the classification model (for malicious data generators) are captured in the problem. We propose four online classification algorithms `ROLC-NC`, `ROLC-C`, `ROLC-NC-D`, and `ROLC-C-D`, for different scenarios where the data generators are non-clairvoyant or clairvoyant, and the feedback delay is static or dynamic. Our theoretical analysis shows that all proposed algorithms have a sub-linear regret bound when the classifier's cost function is convex. We further evaluate the performance of the proposed algorithms in network flow classification via experiments using real-world data traces. We observe that the proposed algorithms are effective, in terms of steady-state accuracy and the F_1 score, and they compare favorably with an optimal static offline classification strategy under different testing scenarios.

For possible future research, one interesting question is whether classification accuracy can be improved by a more complex non-linear classifier (e.g., a neural network). Due to the challenge of non-convex online optimization, to achieve similar performance guarantee under malicious flow generators, different techniques rather than online convex optimization may be desired. Another avenue for future research in robust network flow classification is to account for how the flows features are extracted in the cost of feature manipulation. This may lead to an interesting new problem in jointly optimizing feature exaction and robust online classification.

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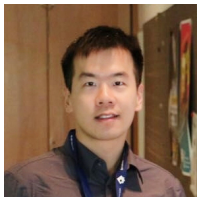

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