

Minimum Cost Configuration of Relay and Channel Infrastructure in Heterogeneous Wireless Mesh Networks

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Abstract. Fixed broadband wireless access is a promising technology allowing Internet service providers to expand their customer base in sparsely populated rural areas. Because the size of the target service area is humongous, relay infrastructure is essential. Installing and maintaining this relay infrastructure is the main cost associated with such networks. Thus, we develop an optimization framework which computes the minimum number of relay stations and their corresponding channel configurations such that a pre-specified subscribers' traffic demand can be satisfied. Since the problem is a mixed-integer program, we propose an efficient optimization algorithm to compute the optimal solution in a reasonable amount of time. Our numerical results show that by using a few relay stations in a rural community, broadband Internet access can be established in a cost effective manner.

Keywords: fixed wireless broadband Internet access, relay stations, optimal placement and channel assignment

1 Introduction

Since high wiring cost is one of the biggest factors inhibiting wired broadband Internet access in sparsely populated rural areas, broadband wireless has long held the promise of delivering a wide range of data and information services to business and residential customers quickly and cost-effectively. With the publication of a comprehensive industry standard, namely IEEE 802.16, broadband wireless is ready to unleash its full potential. The IEEE 802.16 standard requires two separate physical layer specifications because the propagation characteristics of radio waves are so different in the lower- and upper-microwave regions. The WirelessMAN-OFDM and WirelessMAN-SC specifications utilize the 2-11 GHz and 10-66 GHz spectrum respectively. Lower frequency signals can penetrate walls and deflect from obstacles, while higher frequency transmissions must meet strict line-of-sight requirements. However, the advantage of using high frequency bands is an abundance of bandwidth. This intrinsic property of IEEE 802.16 technology makes it ideal for a heterogeneous architecture.

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For the network under investigation, we assume that there is a base station wired to the ISP core network, and this base station is assigned to serve sedentary subscribers in a particular area. Because of the size of the coverage area, the base station usually cannot serve every subscriber by single-hop communication. As a result, several relay stations (RSs) are installed in the network, for example, on the subscribers' rooftops, to relay traffic from and to the base station. If line-of-sight communications can be established among some RSs and the base station, the bandwidth abundant high frequency spectrum is used to form a backbone network. The lower spectrum, on the other hand, is used by the base station and RSs to communicate with the subscribers and form the corresponding local network. For this architecture, the cost of the network is dominated by the installation and maintenance cost of the RSs. Under this hypothetical heterogeneous mesh networking architecture, the focus of this work is to minimize the number of RSs used in the mesh network while maintaining the prespecified uplink and downlink demands of the subscribers. Note that the above IEEE 802.16 specifications are used only as an example; the analytical framework presented in this paper is general and can be applied to mesh networks based on other types of wireless technologies.

To the best of our knowledge, this work is among the first solutions to address the problem of joint relay equipment placement and channel assignment in a heterogeneous wireless mesh network. In this work, we describe a heterogeneous wireless mesh network architecture with relay infrastructure and develop an analytical framework which determines whether a network with a particular relay station placement and channel assignment can satisfy the subscribers' demands and interference constraints. Furthermore, we propose an optimization framework which combines a heuristic with Bender's decomposition to calculate the minimum deployment and maintenance cost of a given heterogeneous wireless mesh network.

The rest of this paper is organized as follows. In Section 2, we review the related work in multihop wireless networks. In Section 3, we describe the network infrastructure and equipment capabilities. In Section 4, we define our relay station placement and channel assignment problem mathematically, and describe an optimization solution. In Section 5, we discuss the convergence time and performance of the proposed optimization algorithm. Finally, concluding remarks are given in Section 6.

2 Related Works

Motivated by recent advances in ad hoc networking [1][2], wireless multihop mesh networking is now considered as the next evolutionary step for wireless data networks. To bring wireless mesh networks closer to reality, in [3], Draves *et al.* conducted a detailed empirical evaluation of several link-quality metrics on route computation performance in wireless mesh networks. The issue of interference management in wireless mesh networks has been discussed in several contexts. In [4], Jain *et al.* considered the fundamental question of how much throughput a given wireless mesh network can achieve under different interference conditions. To address operational issues, an interference-aware channel assignment algorithm for multi-radio wireless mesh networks was proposed in [5] by Ramachandran *et al.*

The problem of wireless network equipment placement has also been addressed in several works. In [6], So and Liang proposed a Lagrangian approach to compute the optimal placement of a fixed number of relay nodes, which relay traffic in a two-hop fashion, to improve throughput in a WLAN. In the context of community mesh networks, innovative integration techniques were developed by Begerano in [7] to minimize the number of *wired* access points in a mesh network to reduce wiring cost, while maintaining users' QoS constraints. To the best of our knowledge, there is no existing work that addresses the problem of joint relay equipment placement and channel assignment in community wireless mesh networks, which is what we investigate in this work.

3 Infrastructure and Equipment Capabilities

To establish a network in a rural area, an operator needs to establish a site for the initial base station and the central office, which should have high capacity backhaul connection with the ISP core network. One cost effective backhaul solution is to lease dark fiber from electrical utilities or railroad companies. However, by using this approach, the network point of access is already fixed. Thus, the ISP does not have the freedom to choose the location of the central office and initial base station. In this work, we consider the case where the location of the base station and central office is given.

Our goal is to place the minimum number of relay stations in the network such that the demands of the subscribers can be met. Since the subscriber locations are fixed and the high frequency spectrum is used, by using advanced antenna technologies and the three-dimensional space intelligently, we can effectively control interference in both the backbone and local networks. Adaptive array antenna technologies [8][9] have the ability to focus a beam very tightly toward a receiver, virtually eliminating the effects of interference. However, since such equipment is expensive, we assume that it is used only in the backbone network. For the local network, we assume a more affordable and common approach as follows [10]. Polarized directional antennas only dispose magnetic fields horizontally. When the antenna is tilted downward (or upward), beyond a certain distance, the radiation will simply be absorbed into the ground (or outer space). As shown in Fig. 1, two relay stations, e.g., Relay Station 2 and 3, can use the same channel for the local network and do not interfere with each other as long as they are placed far apart from each other. However, since subscribers who use the same RS are located in the same multipath environment, they have to share the channel in a time-multiplexed fashion. In the next section, we define the problem mathematically.

4 System Model and Optimization Framework

Suppose there are N subscribers and one base station in the system, and they are represented by the set $V = \{0, 1, \dots, N\}$, where the base station is represented by the index 0. Let $V_R \subseteq V$ be the set of nodes where the installation of relay stations are feasible.¹

¹ Whether a subscriber site is in V_R or not depends on the willingness of the subscriber and other physical conditions. Furthermore, the base station, which has index 0, is included in the set V_R .

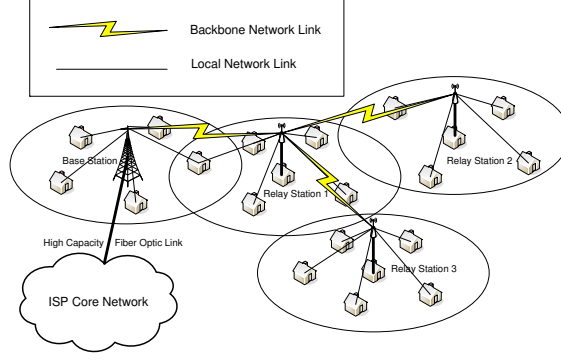


Fig. 1. Tilted polarized directional antenna systems.

We can use the set V_R to form a directed-complete graph representing the backbone network. The link weight from node i to node j , denoted C_{ij}^B , represents the capacity in terms of bit per second from node i to node j using the backbone technology. The capacity between two nodes is zero if line-of-sight communication cannot be established. Similar to the backbone network, we can use the set V to form a directed-complete graph representing the local network. Within the local technology, the capacity from node i to node j is denoted by C_{ij}^L . Since a link which handles local traffic has to be associated with the base station or a relay station, $C_{ij}^L = 0$ if both i and j are not in the set V_R .

As mentioned above, two relay stations using the same channel would interfere each other's local network operations if they are placed in each other's interfering zones. Let $N(i)$ be the set of nodes that interfere the operation of node i , where $i \in V_R$. Moreover, when there are N_C local channels available, let $\Lambda = \{1, 2, \dots, N_C\}$ be the local channel set. Furthermore, for each subscriber i , there is a pre-specified uplink demand, u_i , and downlink demand, d_i . Given the above as the input to our problem, we define the following decision variables.

For discrete decision variables, let us set $X_i^\lambda = 1$ if an RS which uses channel λ is installed in node i ; otherwise we set $X_i^\lambda = 0$. We term X_i^λ the *location-channel variables*. Let us define the following continuous decision variables. Let f_{ij}^d and f_{ij}^u be the amount of downlink and uplink traffic flow from node i to node j by using the backbone technology respectively. Let h_{ij}^d and h_{ij}^u be the amount of downlink and uplink traffic flow from node i to node j by using the local technology respectively. All input and decision variables are non-negative. Moreover, we define $X_0^1 = 1$ since the base station is always present, and without loss of generality, we can also let the base station use channel 1 for its local network operation. Next, we formulate our problem as a mixed integer program.

4.1 Optimization Formulation

Our goal is to find the minimum number of RSs in the system which satisfies all the demand and interference constraints. The optimization formulation is as follows:

$$\min_{\mathbf{X}} : \sum_{i \in V_R, \lambda \in \Lambda} X_i^\lambda \quad (1)$$

$$s.t. \quad \sum_{i \in V_R \setminus \{0\}} f_{i0}^u + \sum_{i \in V \setminus \{0\}} h_{i0}^u = \sum_{i \in V \setminus \{0\}} u_i \quad (2)$$

$$\sum_{i \in V_R \setminus \{0\}} f_{0i}^d + \sum_{i \in V \setminus \{0\}} h_{0i}^d = \sum_{i \in V \setminus \{0\}} d_i \quad (3)$$

$$\sum_{j \in V_R, i \neq j} f_{ji}^u + \sum_{j \in V \setminus \{0\}} h_{ji}^u = \sum_{j \in V_R, i \neq j} f_{ij}^u \quad \forall i \in V_R \setminus \{0\} \quad (4)$$

$$\sum_{j \in V_R, i \neq j} f_{ji}^d - \sum_{j \in V \setminus \{0\}} h_{ji}^d = \sum_{j \in V_R, i \neq j} f_{ij}^d \quad \forall i \in V_R \setminus \{0\} \quad (5)$$

$$\sum_{j \in V_R} h_{ij}^u \geq u_i \quad \forall i \in V \setminus \{0\} \quad (6)$$

$$\sum_{j \in V_R} h_{ji}^d \geq d_i \quad \forall i \in V \setminus \{0\} \quad (7)$$

$$\sum_{j \in V, j \neq i} \frac{h_{ij}^d + h_{ij}^u}{C_{ij}^L} + \frac{h_{ji}^d + h_{ji}^u}{C_{ji}^L} \leq (1 - X_i^\lambda)k + 1 \quad \forall i \in V_R, \lambda \in \Lambda \quad (8)$$

$$\sum_{j \in V} h_{ij}^d + h_{ji}^u \leq k \sum_{\lambda \in \Lambda} X_i^\lambda \quad \forall i \in V_R \quad (9)$$

$$\sum_{\lambda \in \Lambda} X_i^\lambda \leq 1 \quad \forall i \in V_R \quad (10)$$

$$f_{ij}^u + f_{ij}^d \leq C_{ij}^B \sum_{\lambda \in \Lambda} X_i^\lambda \quad \forall i \in V_R, j \in V_R, i \neq j \quad (11)$$

$$f_{ij}^u + f_{ij}^d \leq C_{ij}^B \sum_{\lambda \in \Lambda} X_j^\lambda \quad \forall i \in V_R, j \in V_R, i \neq j \quad (12)$$

$$X_i^\lambda + \sum_{j \in N(i)} X_j^\lambda \leq 1 \quad \forall i \in V_R, \lambda \in \Lambda \quad (13)$$

The objective (1) minimizes the number of RSs to be installed in the network. Constraints (2) and (3) verify that the amount of traffic entering and exiting the base station equals the total uplink and downlink demands respectively. Constraints (4) and (5) verify that the amount of traffic entering each RS matches the amount of traffic exiting each RS (the conservation of flow at each RS). Constraints (6) and (7) verify that the uplink and downlink demands are met respectively. Constraints (8) and (9) work together with an arbitrary large number k . If an RS which uses channel λ is placed at node i , then $X_i^\lambda = 1$, and the right hand side of constraint (8) is 1. Since local uplink and downlink traffic share the channel in a time-multiplexed fashion, constraint (8) verifies that the local traffic enters and exits through the i^{th} RS does not exceed its capacity. If no RS is installed at node i , then $X_i^\lambda = 0 \forall \lambda \in \Lambda$ and the right hand side of constraint (8) is $k + 1$. Thus, constraint (8) does not impose any restriction on the traffic exiting and

entering node i . However, the right hand side of constraint (9) is 0. This ensures that no local uplink traffic enters node i and no local downlink traffic exits node i . Constraint (10) verifies that at most one channel can be assigned to an RS. Constraints (11) and (12) work together to ensure that a positive backbone traffic between node i and j exists only if an RS is placed at node i and an RS is placed at node j . Finally, constraint (13) ensures that no two RSs which use the same channel are placed in each other's interfering zone.

4.2 Problem Reformulation and Bender's Decomposition

Traditionally, any mixed integer problem can be solved by branch-and-bound. However, such approach is virtually intractable even for a small number of discrete variables because an exponential number of linear programs have to be solved. Given that we have a large number of continuous variables and a relatively small number of integer variables, Bender's decomposition breaks down the problem to a sequence of small 0-1 integer problems [11] which can be solved efficiently by commercial optimization softwares such as CPLEX. In the following, we first reformulate the above analytical framework so that it can be decomposed by Bender's method. Then, we describe the algorithm that we used to solve the RS placement and channel assignment problem.

To apply Bender's decomposition to a mixed integer program, the problem needs to be organized into the following form ²

$$\min_{x,y} c_1 y + c_2 x \quad (14)$$

$$s.t. \quad A_1 y + A_2 x \geq b, \quad (15)$$

where x is a vector represents the location-channel variables X_i^λ , y is a vector represents the set of continuous variables $f_{ij}^u, f_{ij}^d, h_{ij}^u, h_{ij}^d$, $c_1 = \mathbf{0}^t$, $c_2 = \mathbf{1}^t$, and $(\cdot)^t$ denotes vector transposition.

For a fixed value of the location-channel variables $x = \hat{x}$, problem (14) reduces to the following feasibility problem:

$$\min_y T(y|\hat{x}) \triangleq c_1 y \quad (16)$$

$$s.t. \quad A_1 y \geq b - A_2 \hat{x} \quad (17)$$

Obviously, given a particular RS placement and channel assignment, $\hat{x}$, the resulting problem, (16)(17), may or may not be feasible. To make all location-channel variables feasible, let us introduce one positive continuous variable, v , and a very large infeasibility constant, P . We can then modify (16)(17) by changing A_1 to $A_{1'} = [A_1 | \mathbf{1}]$, c_1 to $c_{1'} = [\mathbf{0}^t | P]$, and y to $y' = y \cup v$. Then, the modified feasibility problem is the following:

$$\min_{y'} T(y'|\hat{x}) = c_{1'} y' = Pv \quad (18)$$

$$s.t. \quad A_{1'} y' \geq b - A_2 \hat{x} \quad (19)$$

² $a = b$ is equivalent to $a \geq b$ and $b \geq a$.

For any $\hat{x}$ which makes (16)(17) infeasible, problem (18) (19) is still feasible, but it will suffer a very large infeasible penalty Pv .

Now, let us consider the dual of the modified feasibility problem (18)(19). Let u be the set of dual variables. The dual of the modified feasibility problem may now be formulated as follows:

$$\max_u D(u|\hat{x}) \triangleq (b - A_2\hat{x})^t u \quad (20)$$

$$s.t. \quad A_1^t u \leq c_1^t, \quad (21)$$

$$u \geq 0 \quad (22)$$

Denote the optimal solutions to the linear programs (18) and (20) by y'^* and u^* respectively. Then, by duality theory,

$$c_1 y'^* = (b - A_2\hat{x})^t u^* \quad (23)$$

We now consider all the extreme points of the dual problem (20). Note that the extreme points are defined by the feasible region described by (21) and (22) which are independent of the location-channel variables x . Thus, the extreme points can be generated without any knowledge of the RS locations and channel assignments. Let us denote the i^{th} extreme point by u^i and total number of extreme points by p . We know from the theory of linear programming that at least one optimal solution to any linear problem occurs at an extreme point of the feasible region. Thus, the original problem can be reformulate as the following pure 0-1 problem:

$$\min_x c_2 x + D \quad (24)$$

$$s.t. \quad D \geq (b - A_2 x)^t u^i \quad \forall i \in [1, p], \quad (25)$$

or equivalently,

$$\min_x D' \quad (26)$$

$$s.t. \quad D' \geq c_2 x + (b - A_2 x)^t u^i \quad \forall i \in [1, p]. \quad (27)$$

The difficulty with problem (26) is that the number of extreme points of the dual problem is potentially very large. Thus, we do not want to enumerate all of the constraints in (27) explicitly. Also, at the optimal solution to (26), only a small subset of the constraints (27) are likely to be tight. Thus, even if we could enumerate all of them, many of them would prove to be unnecessary. On the other hand, if we solve (26) with only a subset of the constraints in (27), we will obtain a valid lower bound on the optimal value of the original objective function. Furthermore, if all of the constraints that are tight in the optimal solution to (26) happen to be in the subset of constraints that we include, then the value of the objective function (26) will exactly equal the optimal value.

To generate the desired subset of extreme points, Bender's method adds constraints to the constraint set (27) one by one [11]. When a new constraint is added, the optimal solution of (26) returns either a better (larger) lower bound value or the optimal solution to the original problem (14) if a feasible RS placement and channel assignment exists.³

³ If no feasible solution exists, the algorithm will return a very large number.

4.3 Modified Bender's Method

By using the original Bender's method, at each iteration, one needs to solve a small pure 0-1 minimization problem. Even though this approach makes the problem manageable, it could be time consuming and potentially require a large amount of time to compute. The purpose of finding the solution of (26) at each iteration is to find an appropriate extreme point to add to the constraint set (27). Instead of performing the minimization (26) at each iteration, we propose to use a heuristic to find a decent extreme point at each iteration, and only perform minimization (26) when the extreme point generated by the heuristic is invalid.

To further reduce the run time of the Bender's method, we propose to use the results generated by each iteration to reduce the solution space. If a feasible RS placement and channel assignment exists, the optimal solution must be an integer which equals the minimum number of required relay stations. To take advantage of this observation, we add the constraint, $L_l \leq \sum_{i \in V_R, \lambda \in \Lambda} X_i^\lambda \leq L_u$, to the problem, where the minimum number of required relay stations must be greater than or equal to L_l and smaller than or equal to L_u . We initialize $L_l = -\infty$ and $L_u = \infty$. We update L_u whenever a new (smaller) upper bound is found, and we update L_l whenever the lower bound, which is computed by the minimization of (26), increases by more than 1. Fig. 2 presents a flow chart of the modified Bender's decomposition approach to solve the RS placement and channel assignment problem.

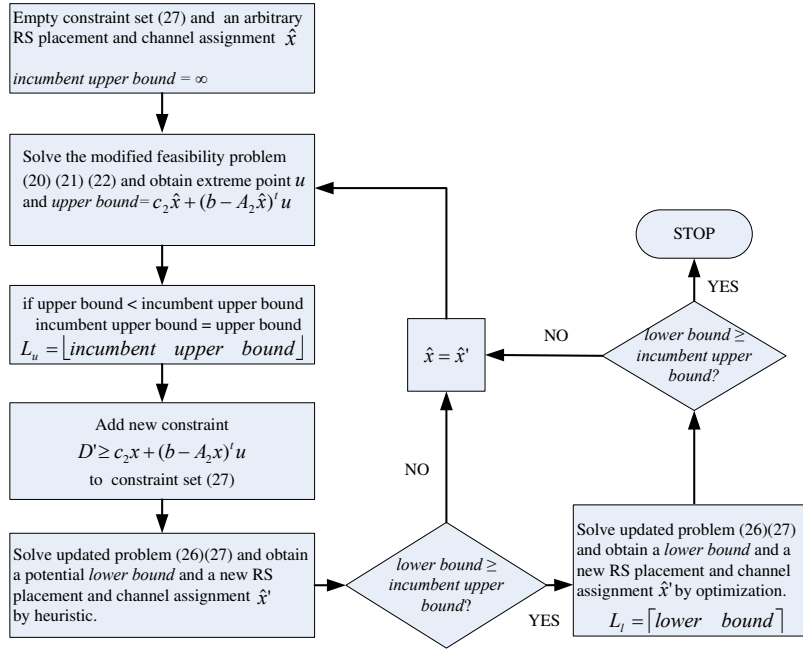


Fig. 2. Flowchart of the modified Bender's decomposition approach for solving the RS placement and channel assignment problem.

We begin by using an empty constraint set (27), and we add new constraints to it iteratively. By using the RS locations and channel assignments, $\hat{x}$, generated by the previous iteration⁴, we solve the modified feasibility problem (20). When this problem is solved, we obtain an extreme point u . From this, we obtain the minimum infeasibility penalty for the RS placement and channel assignment, $\hat{x}$, suggested by the previous iteration. The sum of the infeasibility penalty and the cost of the RSs constitute an upper bound of the problem. That is to say the *upper bound* = $c_2\hat{x} + (b - A_2\hat{x})^t u$. We keep the best (lowest) upper bound found so far and save it as the *incumbent upper bound*. The newly generated extreme point is then used to add a new constraint, $D' \geq c_2x + (b - A_2x)^t u$, to the constraint set (27). In the original Bender's method, one solves the updated problem (26) and obtain a lower bound value and a new RS placement and channel assignment $\hat{x}'$. In this work, we propose to use a simple heuristic⁵ to generate decent values of $\hat{x}'$, and we only perform minimization if the lower bound value generated by the heuristic is higher than or equal to the incumbent upper bound. Otherwise we set $\hat{x} = \hat{x}'$ and generate another extreme point.⁶ If the lower bound generated by the minimization and the incumbent upper bound are equal, we stop. Otherwise, we set $\hat{x} = \hat{x}'$ and go back to the beginning of the iterative phase.

5 Numerical Analysis

In this section, we present numerical results based on a hypothetical IEEE 802.16 network. A link capacity model, similar to that in [6], is used to determine the operational bit rate between any pair of nodes. The optimal RS placement and channel assignment in a typical rural environment will be derived by the proposed modified Bender's decomposition method.

By using the proposed optimization framework, we evaluate the cost of deploying a heterogeneous wireless mesh network with relay stations in a sparse rural area. The cost of the network is the minimum number of RSs required. We set the infeasibility constant P to 1000. For the backbone network, a 20MHz spectrum is occupied, and the IEEE 802.16 WirelessMAN-SC technology is used. For the local network, we use the IEEE 802.16 WirelessMAN-OFDM technology and a 20MHz spectrum as well. As shown in Fig. 4, the subscribers are distributed in a $12\text{km} \times 12\text{km}$ rural area, and the base station and central office are located at node 0, where they can be connected to the ISP core network via the fiber optic network of the railroad company, which is assumed to be underutilized.

According to the IEEE 802.16 WirelessMAN-OFDM specifications, channel bandwidth can be adjusted dynamically. However, the bandwidth occupied by each channel is vendor specific. In this work, as an example of illustration, we assume that the local network spectrum is divided into three channels.⁷ Each RS or base station has a 4km

⁴ For the first iteration, we use an arbitrary RS placement and channel assignment.

⁵ In our numerical result, we use a simple descent algorithm.

⁶ Problem (26) is a small pure 0-1 minimization problem. This problem can be solved to optimality in a reasonable amount of time by any commercial optimization softwares if the number of integer variables is not too large (e.g. less than 1000).

⁷ The link rate of one channel is one third of the original 20MHz channel.

interference zone. In other words, if a base station or RS using a particular channel is placed in one location, another RS which uses the same channel cannot be placed within a 4km radius of the former base station or RS. Among the 58 nodes, we assume the ISP has access to 50 of them for the installation of RSs. Furthermore, we set the uplink and downlink demand of each subscriber to 1Mbps, and 2Mbps respectively.

As mentioned in Section 4.2, the original version of Bender's decomposition method requires solving a small pure 0-1 optimization problem in each iteration, which potentially takes a very long time to perform. In the modified version as shown in Fig.2, we have integrated a classic descent algorithm [12] to reduce the runtime of the Bender's method. The convergence of the original and modified Bender's decomposition methods are shown in Fig. 3, and the resulting configuration of the network is shown in Fig. 4. It takes about 22 hours for the modified version of the Bender's decomposition method to converge to the optimal value, while in the same amount of time, the gap between the upper bound and the lower bound generated by the original method is still very large. Even though the extreme points generated by the heuristic at some iterations are not the desired extreme points, the heuristic can rapidly generate a set of useful extreme points which leads to faster convergence than the original approach.

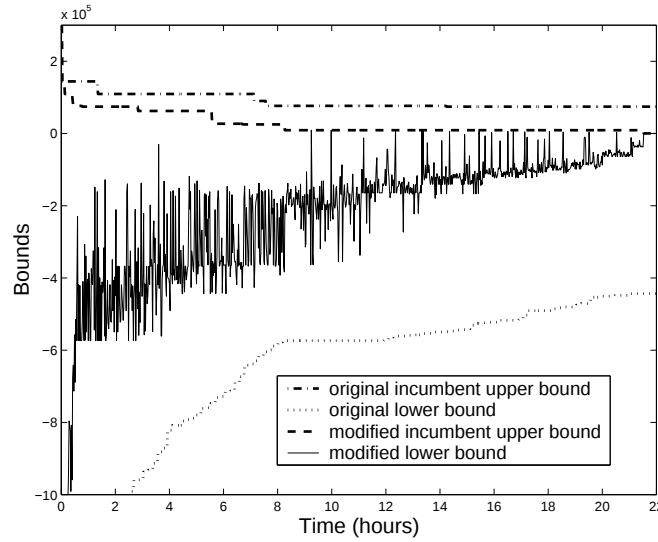


Fig. 3. Convergence of the original and modified Bender's decomposition method.

6 Conclusion

In this work, we investigate the optimal placement and channel assignment of wireless relay stations to minimize the operational cost of a wireless mesh network. We have pre-

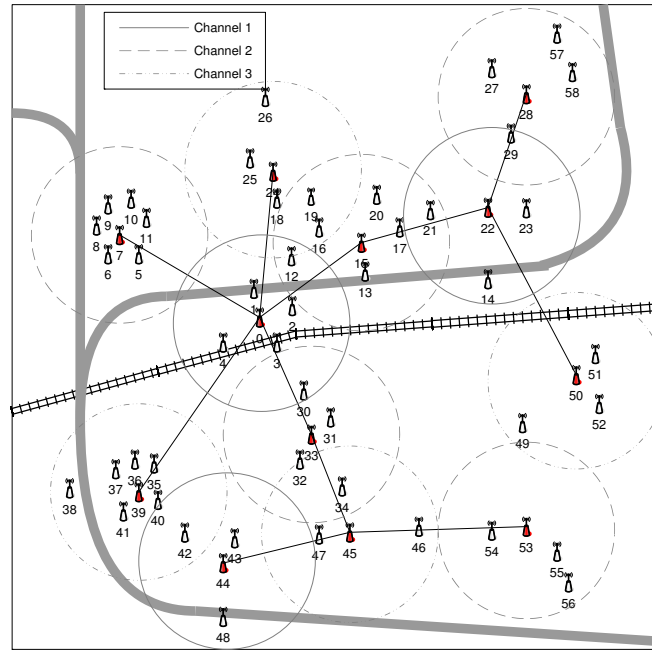


Fig. 4. Network configuration of a heterogeneous wireless mesh network.

sented a heterogeneous wireless mesh network architecture which uses relay stations to form a backbone and a local network. Furthermore, we have developed an analytical model to investigate whether a particular RS placement and channel assignment can satisfy the user demands and interference constraints. We use Bender's decomposition to compute the optimal number of RSs and their corresponding placement and channel assignment which minimize the operational cost of a heterogeneous wireless mesh network. Furthermore, we integrate heuristics in the algorithm to reduce the runtime of the Bender's decomposition method. Given a set of network parameters, the proposed framework and optimization technique can offer significant run time advantages, when used by network designers to compute the optimal placement and channel assignment of relay stations and to provide design guidelines on the network setup and maintenance cost estimations.

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