Energy-Aware Power Allocation for Lifetime Maximization in Single-Source Relay Cooperation

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***Abstract*—We study the problem of optimal power allocation (PA) among relays for lifetime maximization in a dual-hop cooperative network operated by amplify-and-forward relays with battery limitation. We first formulate the optimization problem for power allocation in a static channel case and present a closed-form solution with the objective of lifetime maximization. This solution simply requires equally distributing energy over time for each participating relay. Based on this, we then develop a perceived lifetime (PLT) PA strategy that can be used in time-varying channel scenarios. We also present a minimum weighted total power (MWTP) strategy that depends only on the current channel condition and residual energy information. PLT and MWTP are compared through analysis and simulations. It is demonstrated that both result in significant lifetime improvement compared to the conventional strategy of minimizing the total power per transmission, especially when the link conditions or initial energy levels are nonuniform among relays.**

I. Introduction

Extensive studies have been devoted in the past to understanding the behaviors and benefits of cooperative relaying. Two types of relaying strategies are most commonly considered, amplify-and-forward (AF) and decode-and-forward (DF), with the former being simpler to implement [1].

In some battery-operated applications of cooperative relaying, extending the network *lifetime* is pivotal to maintain uninterrupted data exchange and to reduce the need for replenishing the batteries. However, existing works on relay power allocation mainly focus on per transmission power usage without energy limitation. Some of these studies concern optimal power splitting between the source and the relay in single-relay cooperation subject to a required data rate [2], outage probability [3], or bit error rate [4][5]. For multiple relays, optimal power allocation (PA) and relay selection for data-rate maximization have been considered in [6], and distributed relay selection schemes are studied in [7]. However, when the relays have limited energy, the above results do not necessarily indicate the relay network's lifetime. In particular, for lifetime consideration, power allocation should be chosen with residual energy as input as well, in addition to the link condition.

Studies on lifetime maximization for cooperative relaying have so far been scarce. In [8], the authors studied relay placement and power assignment for DF cooperative relaying in a multi-node network. Their goal was to maximize the minimum node lifetime, under bit-error-rate constraints in an uncoded M-PSK transmission system. Their power allocation was static, based on the channel statistics only. In [9], PA schemes are devised to prolong the lifetime of a single-source AF cooperative network. The authors focused on single-relay selection given some channel statistics. The network lifetime was defined by the required SNR at the destination to maintain a certain outage probability. We focus on relay cooperation instead of relay selection. Our network lifetime is defined as the duration when a certain data rate is achievable, and we consider a continues range of power levels.

In this paper, we investigate the lifetime of a dual-hop cooperative network operated by battery-limited relays using AF. For such network with one source, we initially formulate the problem of maximizing network lifetime for the static channel case and provide a closed-form and easy-to-implement optimal PA solution. Inspired by this solution, we then develop a PA algorithm for a more practical case where channels are slowly varying over time. This scheme is termed the *perceived lifetime* (PLT) algorithm. We further present a PA strategy based on the *minimum weighted total power* (MWTP), which incorporates both channel state information (CSI) and residual energy. Various numerical simulations associated with different network scenarios are provided to compare the performance of PLT, MWTP and the conventional minimum total power (MTP) schemes in various relaying setups. We conclude that PLT is more suitable when asymmetric initial energy levels and link conditions are present in the network.

II. Network Model

We consider a dual-hop AF cooperative network where a source node s transmits data to a destination node d with the assistance of N relays. We constrain ourselves to half-duplex transmission, where a relay node is either in transmission or reception but not simultaneously. A cooperative transmission takes place in two phases. In the first phase, the source node broadcasts its data to the relays and the destination. In the second phase, the relays forward an amplified version of the received signal from the source to the destination node with power $P_k(t)$, $k = 1, \cdots, N$. We assume each relay transmits the data using an orthogonal channel (e.g. frequency or time). Such arrangement arises in the case where coherent transmissions among relays are not possible, either in an asynchronous network, or no instantaneous channel side information available at the relays.

Assuming $x_s(t)$ is the source data to be sent at time t, the signals received at the kth relay and the destination in the first phase are given by

$$y_{rk}(t) = \sqrt{P_s} h_{sk}(t) x_s(t) + n_{rk}(t),$$
$$y_d(t) = \sqrt{P_s} h_{sd}(t) x_s(t) + n_d(t), \quad (1)$$

where $h_{sd}(t)$ and $h_{sk}(t)$ denote the channel gains between source and destination, and source and relay k, respectively. They capture the path loss with exponent α, shadowing, and flat fading. The source transmit power is denoted as P_s. The noise terms $n_{rk}(t),\ k=1,\cdots,N$, and $n_d(t)$ are the additive white Gaussian noises at time t. The forwarded signal at the destination by the kth relay in the second phase is given by

$$y_{dk}(t)=\sqrt{\frac{P_k(t)}{P_s|h_{sk}(t)|^2+\sigma^2}}h_{dk}(t)y_{rk}(t)+n_{dk}(t), \quad (2)$$

where $h_{kd}(t)$ denotes the channel gain between relay k and the destination, and $n_{dk}(t)$ is the corresponding additive Gaussian noise. Without loss of generality, we assume the noises on all links are i.i.d., zero mean complex Gaussian random variable with variance σ^2, i.e $n_{rk}(t), n_{dk}(t), n_d(t)\sim\mathcal{CN}(0,\sigma^2)$. From (2), the received SNR from the kth relay can be derived as

$$\gamma_k(P_k(t))=\frac{P_sP_k(t)b_k(t)c_k(t)}{1+P_sb_k(t)+P_k(t)c_k(t)}, \quad (3)$$

where $b_k(t)=|h_{sk}(t)|^2/\sigma^2$ and $c_k(t)=|h_{kd}(t)|^2/\sigma^2$ are the nominal received signal-to-noise ratio (SNR) with unit transmit power at relay k and the destination (from relay k), respectively. Through the maximum ratio combining (MRC) technique, the observations from direct path and relay nodes can be coherently added. Hence, the effective end-to-end data rate is given by

$$C(t)=\frac{1}{N+1}\log\left(1+P_sa(t)+\sum_{k=1}^{N}\gamma_k(P_k(t))\right), \quad (4)$$

where $a_k(t)=|h_{sd}(t)|^2/\sigma^2$ is the nominal received SNR from the direct path, and $1/(N+1)$ is the bandwidth efficiency factor, reflecting the orthogonal transmissions.

We assume that relays are battery powered. Let $\underline{\mathcal{E}}(t)=[\mathcal{E}_1(t),\cdots,\mathcal{E}_N(t)]^T$ be the relay residual energy vector with $\mathcal{E}_k(t)$ being the residual energy of relay k at time t. The initial energy is then given by $\mathcal{E}_k(0)$ for relay k. A relay gradually depletes its energy as it participates in forwarding the source message. We define relay network lifetime as the time interval during which the end-to-end data rate is maintained above a minimum required rate R, i.e

$$T=\max\{t: C(t')\geq R, 0<t'<t\}. \quad (5)$$

The transmission power allocated in a relay at time t should satisfy the energy constraint: $P_k(t)\Delta_t\leq\mathcal{E}_k(t)$, where Δ_t denotes the transmission duration. A network is called *functional* at time t, if there exists a feasible relay PA vector $\underline{P}(t)=[P_1(t),\cdots,P_N(t)]^T$ that satisfies both energy and rate requirement. A network should be functional during its entire lifetime. We denote the matrix $\mathbf{P}(T)=[\underline{P}(0),\underline{P}(1),\cdots,\underline{P}(T)]$ as the $N\times T$ PA matrix during the network lifetime, with columns and rows corresponding to time and relays, respectively.

Note that minimizing relay transmission power at each time does not necessarily prolong the network lifetime, as residual energy also needs to be taken into account on how power should be allocated. Our objective is then to seek effective PA strategies to maximize the lifetime T.

III. Energy-aware Power Allocation Strategies

A. Optimal Power Allocation in the Static Channel Case

We first consider a special case when the channel over each link is static. We assume that the power of relays remain constant within each transmission slot with duration Δ_t. We set $\Delta_t=1s$ without loss of generality. Since $a_k(t)$, $b_k(t)$ and $c_k(t)$ are time-independent, the time index t for static channel can be dropped. However, the optimum power $P_k(t)$ is still subject to change due to the variation of residual energy $\mathcal{E}_k(t)$ over time. Expressing $T=n\Delta_t$ as the lifetime, the optimization is then given by

$$\max_{\mathbf{P}(n)} \quad n \quad (6)$$

$$s.t.(i)\quad \sum_{t=1}^{n}P_k(t)\leq\mathcal{E}_k(0),\quad k=1,\cdots,N;$$

$$(ii)\quad P_sa+\sum_{k=1}^{N}\frac{P_sP_k(t)b_kc_k}{1+P_sb_k+P_k(t)c_k}\geq\gamma_{th},\quad t=1,\cdots,n;$$

$$(iii)\quad P_k(t)\geq 0,\quad t=1,\cdots,n;\quad k=1,\cdots,N,$$

where $\gamma_{th}\triangleq(2^{(N+1)R}-1)$ is the SNR threshold for the required rate R. The first constraint ensures that the total expended energy by relay k will not exceed the initial energy $\mathcal{E}_k(0)$, while the second constraint provides the rate requirement. Finally, the power variables are non-negative.

Proposition 1: For static channels, the following PA strategy is optimal for the problem (6)

$$P_k^*(t)=\frac{\mathcal{E}_k(0)}{n^*},\quad t=1,\ldots,n^*, \quad (7)$$

where the maximum lifetime n^* is given by

$$n^*=\max\left\{n: P_sa+\sum_{k=1}^{N}\frac{\mathcal{E}_k(0)P_sb_kc_k}{n(1+P_sb_k)+\mathcal{E}_k(0)c_k}\geq\gamma_{th}\right\}.$$

Proof: Proof by contradiction. Assume there exists $\hat{n}>n^*$ whose associated power matrix $\hat{\mathbf{P}}$ satisfies both sets of constraints. Since $P_k^*(t)$ in (7) is not optimal, we must have $\sum_{k=1}^{N}\gamma_k\left(\frac{\mathcal{E}_k(0)}{\hat{n}}\right)<\gamma_{th}-P_sa$, and subsequently,

$$\sum_{k=1}^{N}\sum_{t=1}^{\hat{n}}\gamma_k\left(\frac{\mathcal{E}_k(0)}{\hat{n}}\right)<\hat{n}\left(\gamma_{th}-P_sa\right). \quad (8)$$

On the other hand, the power matrix $\hat{\mathbf{P}}$ must satisfy

$$\sum_{k=1}^{N}\sum_{t=1}^{\hat{n}}\gamma_k(\hat{P}_k(t))\geq\hat{n}(\gamma_{th}-P_sa). \quad (9)$$

Due to the concavity of $\gamma_k(\cdot)$ respect to $\hat{P}_k(t)$ in (3), by using the Lagrangian method and KKT conditions one can show that the maximum of inner sum $\sum_{t=1}^{\hat{n}}\gamma_k(\hat{P}_k(t))$ occurs when $\hat{P}_k(t)=\mathcal{E}_k(0)/\hat{n}$ for all $t=1,\cdots,\hat{n}$. Hence, from (9), we yield

$$\hat{n}(\gamma_{th}-P_sa)\leq\sum_{k=1}^{N}\sum_{t=1}^{\hat{n}}\gamma_k(\hat{P}_k(t))\leq\sum_{k=1}^{N}\sum_{t=1}^{\hat{n}}\gamma_k\left(\frac{\mathcal{E}_k(0)}{\hat{n}}\right),$$

which contradicts the inequality (8). Therefore, $\hat{n}$ must be equal to n^*, and the power allocation solution in (7) is optimal. ■

The PA scheme in Proposition 1 essentially suggests that equally distributing the energy of each relay over time (i.e. constant power) maximizes the network lifetime. Note that although this optimal solution turns out to be simple, it is a nontrivial solution. Considering the relays with different initial energy can be positioned anywhere and thus exhibit very different channel gains (i.e. b_k and c_k), it is not immediately obvious that the constant power solution for all relays would give the maximum lifetime. At the same time, the PA approach that minimizes the total power used by the relays at each time is suboptimal for lifetime maximization, as verified in the later simulation results.

From a practical implementation point of view, the allocation scheme is very simple and easy to implement. It requires the same fraction of the remaining energy to be allocated for each relay. All the information required for each relay is n^*, which can be broadcasted to each relay.

B. Time-Varying channel: Perceived Lifetime Strategy

Now we consider a more practical case when channels among source, relays, and destination are slowly varying over time. For each relay, the power to be used at each transmission can only be based on the current CSI and residual energy. Inspired by the optimal PA strategy in the static channel case, we propose the following *perceived lifetime* (PLT) PA approach.

At each time t, for given channel gains and remaining energy $(\{a_k(t), b_k(t), c_k(t)\}, \mathcal{E}_k(t))$, assuming the future CSI is the same as the current CSI, we compute the maximum *perceived lifetime* and the corresponding power allocation:

$$n^*(t) = \max\left\{n : P_s a(t) + \sum_{k=1}^{N} \frac{\mathcal{E}_k(t) P_s b_k(t) c_k(t)}{n(1 + P_s b_k(t)) + \mathcal{E}_k(t) c_k(t)} \geq \gamma_{th}\right\}, \quad (10)$$

$$P_k(t) = \frac{\mathcal{E}_k(t)}{n^*(t)}, \quad \text{for } k = 1, \cdots, N. \quad (11)$$

Essentially, given the current channel gains, the PLT algorithm tries to maximize the network lifetime at each transmission stage assuming the current channel gains will not change in the future. Moreover, following the static channel PA solution provided in Proposition 1, the PLT strategy for power allocation also has the following characteristics:

P1) It maintains equal energy efficiency among participating relays after each transmission, where the energy efficiency is defined as $\eta_k(t) = \frac{\mathcal{E}_k(t)}{P_k(t)} = n^*(t)$.

P2) It minimizes the sum of wasted residual energy at the end of the network lifetime.

C. Time-Varying channel: Minimum Weighted Total Power Strategy

We introduce another strategy that directly targets at reducing the current transmission powers among the relays without making any assumption of future CSI, in contrast to the PLT scheme. As mentioned earlier, minimizing the total per transmission power at relays without residual energy consideration is not necessarily prolonging the lifetime. Instead, we propose to minimize a weighted total power per transmission, where the power allocation at time t is the solution of the following optimization problem

$$\min_{\underline{P}(t)} \quad \sum_{k=1}^{N} \frac{P_k(t)}{\mathcal{E}_k(t)} \quad (12)$$

$$s.t \quad 1) \quad C(t) \geq R, \qquad 2) \quad \underline{P}(t) \preceq \underline{\mathcal{E}}(t)\ ,$$

where $\preceq$ denotes the element-wise inequality. The weight for each relay is the inverse of residual energy at time t, i.e. more weight is given for the relay with smaller residual energy, inducing the relay to use less power for relay cooperation. From the the static-channel case, we observe that to maximize the network lifetime, all relays would completely deplete their energy at the same time. The strategy in (12) is trying to achieve this goal through residual energy weighting.

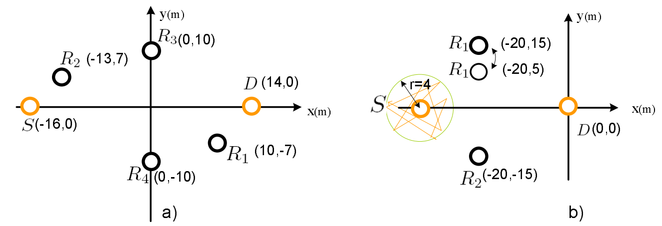

Fig. 1. Simulation configurations used for static channel and source-moving experiments

Since the optimization in (12) is convex, by writing the Lagrangian function and using KKT conditions, the optimal power $P_k(t)$ allocated to relay k is determined as

$$P_k(t) = \min\left\{\mathcal{E}_k(t), \left(\frac{\sqrt{\lambda \mathcal{E}_k(t)\beta_k(t)} - P_s b_k(t) - 1}{c_k(t)}\right)^+\right\}, \quad (13)$$

where $\beta_k(t) = P_s b_k(t) c_k(t)(1 + P_s b_k(t))$ and $(x)^+ = \max(x, 0)$. Parameter λ is chosen such that the minimum rate requirement $C(t) = R$ is met, where $C(t)$ is given in (4). Note that, without considering the residual energy, (12) would reduce to the conventional strategy to minimize the total power (MTP). The solution of MTP can be obtained by simply removing $\mathcal{E}_k(t)$ in (13).

IV. Simulation Results

In this section, we compare the performance of PLT, MWTP, and MTP in static and slow-varying channel conditions for different relay cooperation setups. Fig.1 shows two types of network setups we consider: a) Fixed node locations giving rise to a static channel environment; b) A moving source creating large-scale fading channels due to pathloss variation. Throughout the simulations, the pathloss exponent is assumed $\alpha = 2$, and the noise level $\sigma^2 = -40$dBW. We further assume that the source power and the transmission time slot are normalized to unity, i.e $P_s = 1$W and $\Delta_t = 1$s.

We first consider a static-channel case, and use Fig.1(a) for the network setup. A pair of source-destination is fixed at S and D and four relay nodes are placed at coordinates R_1, R_2, R_3, and R_4. Their initial energies are arbitrarily set to $\underline{\mathcal{E}}(0) = [10\text{KJ}, 2\text{KJ}, 20\text{KJ}, 2\text{KJ}]^T$. Fig.2 depicts the performance of achieved lifetime vs. the minimum required rate under the PLT, MWTP, and MTP schemes. The PLT scheme, which is the optimal allocation for the static channel case, clearly outperforms both MWTP and MTP. Moreover, the MWTP scheme achieves considerably longer lifetime than that of the MTP scheme. For example, at the rate requirement of $R = 1.3$bps/Hz, the lifetime of MWTP is almost doubled compared to that of MTP.

We now consider channel variation due to the moving source causing pathloss variation. The network setup is shown in Fig.1(b). Assume that the source moves with a constant speed

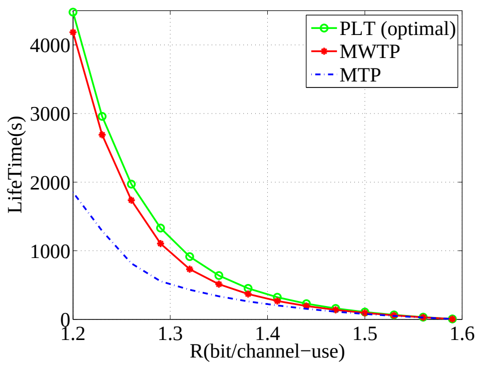

Fig. 2. Lifetime vs. rate requirement R, comparison among PLT, MWTP and MTP schemes for a static-channel case

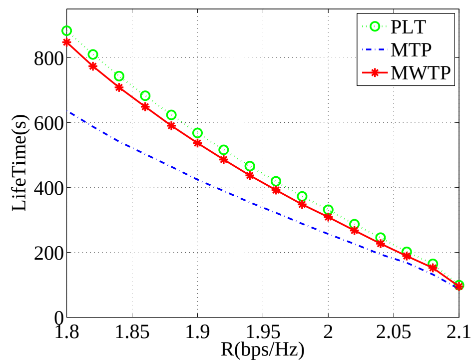

Fig. 3. Lifetime vs. minimum rate requirement R for a source-moving dual-relay case $N = 2$

$v = 0.3$m/s in a random direction confined in a circle with center $S(-30m, 0m)$, radius $r = 4m$, and two relays are placed at $R_1(-20m, 5m)$ and $R_2(-20m, -15m)$. In this asymmetric configuration, equal initial energy $\underline{\mathcal{E}}(0) = [1\text{KJ}, 1\text{KJ}]^T$ are considered for R_1 and R_2. Using the Monte Carlo simulation method, the average lifetime of the PA schemes vs. the required rate is plotted in Fig.3. Comparing the performance under each scheme, we see that PLT is slightly better than MWTP. Compared with MTP, both energy-aware PLT and MWTP schemes provide significant lifetime improvement. For example, at $R = 1.8$bps/Hz, the lifetime increment is approximately 30%. This gain demonstrates the importance of energy-aware PA in a energy-limited network environment.

In the next experiment, we study the effect of the initial energy levels on the lifetime of different power allocation schemes. We again use the network setup in Fig.1(b). To remove the effect from the asymmetrical relay links, we move R_1 to $(-20m, 15m)$, and fix the source at $S(-30m, 0m)$. We vary the ratio of initial energy, $\beta = \frac{\mathcal{E}_1(0)}{\mathcal{E}_2(0)}$, from 1 to 10, while keeping the sum initial energy unchanged, i.e., $\mathcal{E}_1(0) + \mathcal{E}_2(0) = 2$KJ. In Fig.4, we plot the network lifetime for two rate requirements: $R = 1.7$ or 1.9bits/s/Hz. For $\beta = 1$, the network setup is symmetrical in terms of relay link condition as well as initial energy. In this case, the performance of all three schemes coincide as shown in the figure, and this coincidence can be analytically verified through their formulation as well. As β increases, i.e. the asymmetry increases, the gap of the lifetime among the three schemes becomes larger, with PLT giving the best performance. Results

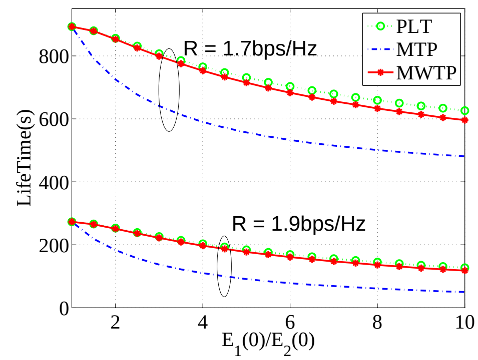

Fig. 4. Effect of initial energy ratio on lifetime for static channel with two relays

in Fig.3 and Fig.4 demonstrate that using the energy-aware power allocation schemes are particularly more effective in the asymmetric network setup.

V. Conclusion

Energy-aware power allocation for network lifetime maximization was considered in this paper for a dual-hop cooperative network operated by battery-limited AF relays. For a given minimum rate requirement, for the case of static channels, the problem was formulated as a convex optimization problem, and then a closed-form, easy-to-implement solution was provided. Inspired by this solution, we presented a causal implementation of this optimal solution for the time-varying scenarios that we termed the PLT scheme. Furthermore, we proposed an alternative scheme called MWTP, which relies on the instantaneous CSI and residual energy without predicting the future CSI. Both proposed schemes demonstrated a significant lifetime improvement over the MTP scheme, and they were particularly effective for asymmetric networks in terms of either relay link condition or the initial energy condition, which is common for relay networks.

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