Optimal Power Allocation in Device-to-Device Communication with SIMO Uplink Beamforming

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***Abstract*—Cellular and device-to-device (D2D) communication may cause significant inter-cell interference (ICI) at a neighboring base station (BS). In this work, we aim to maximize the sum rate of a cellular user (CU) and a D2D pair, with receive beamforming at the BS equipped with multiple antennas, subject to per-node power, maximum ICI, and minimum SINR constraints. We propose an efficient algorithm to maximize the sum rate in two steps. We first consider the D2D admissibility problem to determine whether the D2D pair can share the spectrum with the CU while satisfying all the constraints and SINR requirements. We then obtain the optimal beam vector and the optimal power levels of the CU and D2D transmitters in closed form. The performance of the proposed algorithm is studied numerically. It is shown the proposed optimal solution substantially outperforms a CU-priority heuristic approach that selects the maximum CU power with minimum D2D interference.**

I. INTRODUCTION

Local service requirements have led to the development of device-to-device (D2D) communication, where nearby users can transmit data directly to each other with reused cellular resource blocks [1], [2]. D2D communication has been shown to increase the overall network spectral efficiency and improve radio resource utilization because of resource reuse by both cellular users (CUs) and D2D pairs.

For a D2D underlaying cellular network, interference needs to be carefully controlled because cellular users and D2D users share the spectrum. In order to manage the interference to CUs in the same cell, several approaches have been proposed in the literature [1]–[9]. In [3], D2D users scale the transmission power according to their pathloss to the base station (BS). The authors of [4] have proposed a power control approach subject to constraints on the SINR degradation of the cellular link. In [5], the maximum D2D transmit power is limited using cellular power control information as a reference. A mixed integer nonlinear programming has been formulated in [6], and a greedy heuristic algorithm has been proposed to reduce the interference to the cellular network. In [7], a sum rate maximization has been studied for a cellular network with one CU and one D2D pair with rate constraints and a minimum quality-of-service (QoS) requirement for the CU. In [8], an interference limited area has been proposed, where a D2D pair cannot reuse the CU resources. In [9], a three-step algorithm has been proposed to maximize the overall network throughput subject to SINR requirements for both D2D users and CUs.

Despite the many studies summarized above, inter-cell interference (ICI) is a challenge that has not been addressed in the existing literature. It is important to set the cellular user and D2D transmit powers such that the ICI in the neighboring cell does not exceed some upper limit. Furthermore, receive beamforming at the BS is an efficient technique to take advantage of the spatial diversity provided by multiple antennas to improve the received SINR. However, the only existing work that studies joint CU and D2D power optimization under CU and D2D SINR requirements [9] considers neither ICI nor receive beamforming.

In this paper, we jointly design the receive beam vector at the BS and powers of CU and D2D transmitters to maximize the uplink sum rate. We focus our attention to one CU and one D2D pair that share the same spectrum in a cell, where the BS has multiple receive antennas. Our objective is to maximize the uplink sum rate of the CU and D2D pair under minimum CU and D2D SINR requirements, as well as per-node power and maximum ICI constraints. The proposed approach consists of two steps. We first consider the D2D admissibility problem, to determine whether all constraints and SINR requirements can be satisfied, if the D2D pair is admitted to share the spectrum with the CU. We then obtain the optimal beam vector and the optimal power levels of the CU and D2D transmitters in closed form. We show that the power optimization subproblem can be solved in sixteen unique scenarios. The necessary and sufficient conditions for those scenarios are discussed and the corresponding optimal solutions are given. Simulation results demonstrate that the optimal solution can substantially increase the sum rate over a heuristic where the CU is given higher priority over the D2D pair.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We study the underlaying D2D communication in a cellular system, where D2D devices reuse the spectrum resource already assigned to the CUs for uplink communication. We assume orthogonal spectrum resource allocation among CUs in a cell. Thus, these CUs do not interfere with each other. When a D2D pair reuses the channel of a CU, the D2D pair and the CU generate intra-cell interference to each other. In this work, we focus on the transmission design between a single CU and a single D2D pair attempting to reuse the CU's assigned channel. We assume the BS centrally coordinates the scheduling of the CU and D2D pair and their respective transmission power. The BS is equipped with N antennas, and the users are each equipped with a single antenna. In order to mitigate the intra-cell interference between the CU and the D2D pair, the transmit powers of the D2D transmitter and the CU should be optimally chosen. We assume that both the D2D pair and the CU have their respective minimum SINR requirements.

Let P_D and P_C denote the transmission power of the D2D pair and the CU, respectively. The SINR at the D2D receiver is given by

$$\gamma_D = \frac{P_D|h_D|^2}{\sigma_D^2 + P_C|g_C|^2} \tag{1}$$

where $h_D \in \mathbb{C}$ is the channel between the D2D pair, $g_C \in \mathbb{C}$ is the interference channel between the CU and the D2D receiver, and σ_D^2 is the noise variance at the D2D receiver. The uplink received SINR at the BS for the CU is given by

$$\gamma_C = \frac{P_C|\mathbf{w}^H\mathbf{h}_C|^2}{\sigma^2 + P_D|\mathbf{w}^H\mathbf{g}_D|^2} \tag{2}$$

where $\mathbf{h}_C \in \mathbb{C}^{N\times 1}$ is the channel between the CU and the BS, $\mathbf{g}_D \in \mathbb{C}^{N\times 1}$ is the interference channel between the D2D transmitter and the BS, $\mathbf{w}$ is the receive beam vector at the BS with unit norm, *i.e.,* $\|\mathbf{w}\|^2 = 1$, and σ^2 is the noise variance at the BS.

Both D2D and CU transmissions cause ICI in a neighboring cell. In this work, we consider ICI for uplink transmission at the neighboring BS. However, our approach can be applied to consider ICI in the downlink scenario. Let $\mathbf{f}_C \in \mathbb{C}^{N\times 1}$ and $\mathbf{f}_D \in \mathbb{C}^{N\times 1}$ denote the ICI channel from the CU and the D2D transmitter to the neighboring BS, respectively. Since the beam vector of the neighboring BS is usually unknown to the CU and D2D pair, we consider the worst-case maximum ICI power given by

$$P_{\mathcal{I}} = P_C\|\mathbf{f}_C\|^2 + P_D\|\mathbf{f}_D\|^2. \tag{3}$$

Let $\tilde{\mathbf{w}}$ denote the beam vector used by the neighboring BS. Since $|\tilde{\mathbf{w}}^H\mathbf{f}| \le \|\mathbf{f}\|$ for all $\mathbf{f}$, the power in (3) is an upper bound of the effective ICI. If $\tilde{\mathbf{w}}$ is known, then it is easy to take it into account by replacing $\|\mathbf{f}\|$ by $|\tilde{\mathbf{w}}^H\mathbf{f}|$ in (3).

We assume perfect knowledge of the communication channels and intra-cell interfering channels, which is a common assumption in the research literature. For the ICI channels, we note that only channel powers (*i.e.,* $\|\mathbf{f}\|^2$) are required to compute the interference power, which can be estimated in the neighboring BS and shared with the BS scheduler in the desired cell through the wired backhaul.

B. Problem Formulation

We assume per-node power constraints, with $P_C^{\max}$ and $P_D^{\max}$ denoting the maximum transmit power at the CU and D2D transmitters, respectively. Our goal is to maximize the sum rate of the D2D pair and the CU uplink transmission by optimizing the set of powers $\{P_D, P_C\}$ and beam vector $\mathbf{w}$, under the per-node power and maximum ICI constraints, as well as the SINR requirements. The sum rate maximization problem is given by

$$\max_{P_D,P_C,\mathbf{w}} \Big(\log(1+\gamma_C) + \log(1+\gamma_D)\Big) \tag{4}$$

$$\text{subject to } \gamma_C \ge \tilde{\gamma}_C, \tag{5}$$

$$\gamma_D \ge \tilde{\gamma}_D, \tag{6}$$

$$P_C \le P_C^{\max},\ P_D \le P_D^{\max}, \tag{7}$$

$$P_{\mathcal{I}} \le \tilde{\mathcal{I}} \tag{8}$$

where $\tilde{\gamma}_C$ and $\tilde{\gamma}_D$ are the minimum SINR requirements for the CU and D2D pair, respectively, and $\tilde{\mathcal{I}}$ is the maximum ICI power in the neighboring cell.

III. Admissibility Test and Power Allocation

In this section, we solve the sum-rate maximization problem (4). This problem is non-convex, since the objective in (4) is not convex. Nonetheless, we will derive a closed-form solution. Two steps are involved in this problem. First we need to determine whether the D2D pair can be admitted to reuse the CU's assigned channel, then we obtain the optimal power for each transmission. Towards this, we obtain the closed-form expression of the optimal beam vector $\mathbf{w}$, leading to a simple feasibility test. Then, we obtain the optimal powers P_D^o and P_C^o in closed form.

A. The Admissibility Test

Given the power constraints, SINR requirements, and ICI threshold, the admissibility of the D2D pair can be determined by evaluating the feasibility of the problem (4). That is, the D2D pair is allowed to reuse the CU's channel if it passes the following feasibility test

$$\text{find } \{P_D, P_C, \mathbf{w}\} \tag{9}$$
$$\text{subject to } (5), (6), (7), (8).$$

We first obtain the optimal beam vector $\mathbf{w}$ in terms of $\{P_C, P_D\}$ that maximizes γ_C at the left hand side of constraint (5). For a given set of $\{P_C, P_D\}$, this receive beamforming problem is given by

$$\max_{\mathbf{w}} \frac{P_C\mathbf{w}^H\mathbf{H}_C\mathbf{w}}{\mathbf{w}^H\mathbf{\Lambda}_D\mathbf{w}} \tag{10}$$

where $\mathbf{H}_C \triangleq \mathbf{h}_C\mathbf{h}_C^H$ and $\mathbf{\Lambda}_D \triangleq \sigma^2\mathbf{I} + P_D\mathbf{g}_D\mathbf{g}_D^H$. The maximization problem (10) is a generalized eigenvalue problem, and the optimum beam vector is given by

$$\mathbf{w}^o = \frac{\mathbf{\Lambda}_D^\dagger\mathbf{h}_C}{\|\mathbf{\Lambda}_D^\dagger\mathbf{h}_C\|}. \tag{11}$$

We can show that $\mathbf{\Lambda}_D \succ 0$; hence, the pseudo inverse in (11) becomes a matrix inversion. Substituting $\mathbf{w}^o$ in (11) into (2), the maximum SINR of the CU is given by

$$\max_{\mathbf{w}} \gamma_C = P_C\mathbf{h}_C^H\mathbf{\Lambda}_D^{-1}\mathbf{h}_C. \tag{12}$$

Let $\rho \triangleq \frac{|\mathbf{h}_C^H\mathbf{g}_D|}{\|\mathbf{h}_C\|\|\mathbf{g}_D\|}$ denote the correlation coefficient of the channels $\mathbf{h}_C$ and $\mathbf{g}_D$, where $|\rho| \le 1$. Applying the matrix inversion lemma to $\mathbf{\Lambda}_D^{-1}$ and after some algebraic manipulation, the SINR constraint (5) can be re-expressed as

$$\frac{P_C\|\mathbf{h}_C\|^2}{\sigma^2}\Big(1-\frac{\rho^2}{1+\frac{\sigma^2}{P_D\|\mathbf{g}_D\|^2}}\Big)\geq\tilde{\gamma}_C. \tag{13}$$

For notation simplicity, in the following, the D2D and CU powers P_D and P_C are denoted by x and y, respectively. Considering the x-y power plane, we study the constraints (6) and (13) in the following proposition to solve the feasibility problem (9).

Proposition 1: Consider constraints (6) and (13) with equality. The solution $\{x_{\mathcal{I}}, y_{\mathcal{I}}\}$ to the two equations is unique and is given by

$$x_{\mathcal{I}}=\frac{\xi}{2(1-K_1)},\quad y_{\mathcal{I}}=\frac{\xi}{2(1-K_1)\beta K_3}-\frac{\sigma_D^2}{K_3} \tag{14}$$

where $\xi = \beta\big(\alpha K_3 + \sigma_D^2(1-K_1)\big) - K_2 + \sqrt{K_4^2+4(1-K_1)\beta K_2(\alpha K_3+\sigma_D^2)}$, $K_4 \triangleq \beta(\alpha K_3 + \sigma_D^2(1-K_1)) - K_2$, $\alpha \triangleq \frac{\sigma^2\tilde{\gamma}_C}{\|\mathbf{h}_C\|^2}$, $\beta \triangleq \frac{\tilde{\gamma}_D}{|h_D|^2}$, $K_1 \triangleq \rho^2$, $K_2 \triangleq \frac{\sigma^2}{\|\mathbf{g}_D\|^2}$, and $K_3 \triangleq |g_C|^2$.

Proof: We can rewrite (13) as

$$y=\eta(x)\triangleq\alpha\Big(1-\frac{K_1}{1+K_2/x}\Big)^{-1}. \tag{15}$$

Taking the first and second derivatives, we can show that $\eta(x)$ is a concave strictly increasing function. Note that constraint (6) is characterized by a line on the power plane, *i.e.,* $\frac{x}{\sigma_D^2+K_3y}=\beta$. Solving the intersection of this line and the curve (15), we obtain (14). ■

By Proposition 1, the necessary and sufficient condition for the D2D pair to be admissible is that the solution $\{x_{\mathcal{I}}, y_{\mathcal{I}}\}$ in (14) satisfies

$$0<x_{\mathcal{I}}\leq P_D^{\max}, \tag{16}$$

$$0<y_{\mathcal{I}}\leq P_C^{\max}, \tag{17}$$

$$c_1y_{\mathcal{I}}+c_2x_{\mathcal{I}}\leq 1 \tag{18}$$

where $c_1 \triangleq \|\mathbf{f}_C\|^2/\tilde{\mathcal{I}}$, and $c_2 \triangleq \|\mathbf{f}_D\|^2/\tilde{\mathcal{I}}$. Note that if either (16) or (17) does not hold, the maximum power of D2D or CU would not be enough to meet both SINR requirements. If (18) does not hold, the ICI constraint cannot be satisfied.

B. The Optimal Power Allocation Solution

We now solve the optimal power allocation problem to maximize the sum rate in (4). After substituting (11) into (2), the problem (4) can be rewritten as

$$\max_{(x,y)}\ \log\mathcal{R}(x,y) \tag{19}$$
$$\text{subject to } (6),(7),(8),(13)$$

where $\mathcal{R}(x,y) \triangleq (1+\frac{ax}{\sigma_D^2+K_3y})\big(1+by(1-\frac{K_1x}{K_2+x})\big)$, $a \triangleq |h_D|^2$, and $b \triangleq \|\mathbf{h}_C\|^2/\sigma^2$. We will solve the problem in the case when the ICI constraint (8) is inactive at optimality, as well as when it is active at optimality. Let $\mathcal{A}_{xy}$ denote the feasible solution region of the problem (19). A property of the objective function (19) is provided in the following lemma.

Lemma 1: Given any power pair (x,y) in the interior of $\mathcal{A}_{xy}$, there exists $\zeta>1$, such that $(\zeta x,\zeta y)\in\mathcal{A}_{xy}$. Furthermore, $\mathcal{R}(\zeta x,\zeta y)>\mathcal{R}(x,y)$.

Lemma 1 indicates that the optimal power solution pair (x^o,y^o) is at the boundary lines of $\mathcal{A}_{xy}$. In particular, when the constraint (8) is inactive at optimality, for (x^o,y^o), at least one of them equals the maximum power ($P_D^{\max}$ or $P_C^{\max}$). In other words, (x^o,y^o) is at either vertical or horizontal boundary of $\mathcal{A}_{xy}$. In the following, we analyze these boundary lines $h(x)\triangleq\mathcal{R}(x,P_C^{\max})$ and $g(y)\triangleq\mathcal{R}(P_D^{\max},y)$ to find the optimal power allocation.

Proposition 2: If the ICI constraint (8) is inactive, then $\{x^o,y^o\}$ is at one end of the vertical or horizontal boundary of $\mathcal{A}_{xy}$.

Proof: Taking the first and second derivatives, we can show that $h(x)$ and $g(y)$ are either strictly increasing or convex functions. We omit the details due to page limitation. ■

If the ICI constraint (8) is active at optimality, from (3) we have $c_1y^o+c_2x^o=1$, *i.e.,* (x^o,y^o) is on the tilted boundary line of $\mathcal{A}_{xy}$. The following lemma provides the solution in this case.

Lemma 2: If the ICI constraint (8) is active at optimality, then the optimal D2D power x^o is one of the roots of the following quartic equation

$$e_4x^4+e_3x^3+e_2x^2+e_1x+e_0=0 \tag{20}$$

where $e_0 \triangleq aa_1K_2^2(b_1+1)-a_1^2b_1K_1K_2-a_1^2b_2K_2^2$, $e_1 \triangleq -2aa_1b_2K_2^2+aa_1K_2(b_1+1)+aa_1K_2-2aa_1K_1K_2b_1+aa_1b_1K_2+2a_1^2b_2K_2(K_1-1)+2a_1a_2b_1K_1K_2+2a_1a_2b_2K_2^2$, $e_2 \triangleq aa_1b_2K_2(3K_1-4)+aa_1\big(1+b_1(1-K_1)\big)-a_1^2b_2(1-K_1)+a_2b_2K_2^2(a-a_2)+a_2b_1K_1K_2(a-a_2)-4a_1a_2b_2K_2(K_1-1)$, $e_3 \triangleq -2aa_1b_2(1-K_1)+2a_1a_2b_2(1-K_1)-2a_2b_2K_2(K_1-1)(a-a_2)$, $e_4 \triangleq a_2(a-a_2)b_2(1-K_1)$, $a_1 \triangleq \sigma_D^2+K_3/c_1$, $a_2 \triangleq K_3c_2/c_1$, $b_1 \triangleq b/c_1$, and $b_2 \triangleq bc_2/c_1$. Furthermore, the optimal CU power is given by $y^o=(1-c_2x^o)/c_1$.

Proof: The optimal power on the line due to ICI is the solution of the following optimization problem

$$\max_{(x,y)}\ \big(1+\frac{ax}{\sigma_D^2+K_3y}\big)\Big(1+by(1-\frac{K_1x}{K_2+x})\Big) \tag{21}$$
$$\text{subject to } c_1y+c_2x=1.$$

Substituting $y=(1-c_2x)/c_1$ into (21), it becomes $\max_x\ \tilde{\mathcal{R}}(x)$, where $\tilde{\mathcal{R}}(x)\triangleq\big(1+\frac{ax}{a_1-K_4x}\big)\big(1+(b_1-b_2x)(1-\frac{K_1x}{K_2+x})\big)$. Since $\tilde{\mathcal{R}}(x)$ is continuous and has a first-order derivative, the optimum x^o is obtained by solving $\mathrm{d}\tilde{\mathcal{R}}(x)/\mathrm{d}x=0$, which results in a quartic equation in (20). ■

Not that quartic equations have closed-form solutions. Furthermore, there is no need to compute all the roots of (20), since not all of them are in $\mathcal{A}_{xy}$. In the following, we classify different scenarios of $\mathcal{A}_{xy}$ and the corresponding optimal power solutions (x^o,y^o), providing simple expressions to check the conditions under which these scenarios apply.

C. Weak ICI from CU and D2D

Now, we consider the case where both the CU and the D2D transmitter cause weak ICI. This happens if the boundary

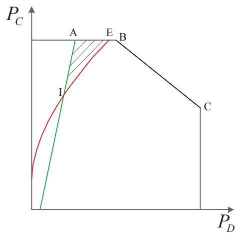

Fig. 1. Scenario 1

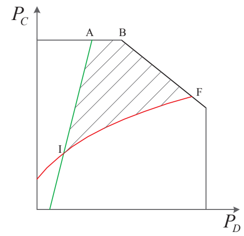

Fig. 2. Scenario 2

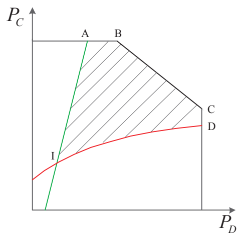

Fig. 3. Scenario 3

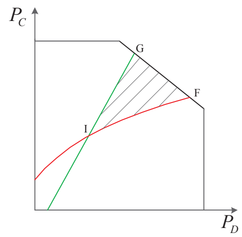

Fig. 4. Scenario 4

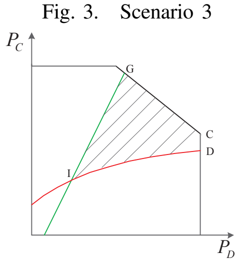

Fig. 5. Scenario 5

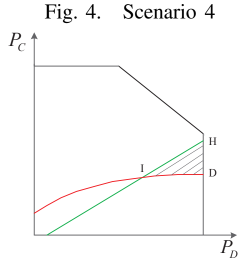

Fig. 6. Scenario 6

line $c_1x + c_2y = 1$ intersects both the horizontal and vertical boundary lines, as shown in Figs. 1-6. More precisely, we have

$$\frac{1 - c_2P_D^{\max}}{c_1} \leq P_C^{\max} \leq \frac{1}{c_1}, \ \frac{1 - c_1P_C^{\max}}{c_2} \leq P_D^{\max} \leq \frac{1}{c_2}.$$

Depending on the shape of $\mathcal{A}_{xy}$ (*i.e.,* shaded area in Figs. 1-6), we derive the optimal power allocation in the following six scenarios.

1) Scenario 1: The feasible solution region $\mathcal{A}_{xy}$ is depicted in Fig. 1 as the shaded area. In this scenario, there is no intersection between the tilted boundary line B-C and the curves I-E and I-A (corresponding to SINR constraints (5) and (6), respectively). The condition under which this scenario happens is given by $K_2\big(\frac{K_1}{1-\frac{\bar{\gamma}_C}{bP_C^{\max}}} - 1\big)^{-1} \leq (1 - c_1P_C^{\max})/c_2$. By Lemma 1, it is sufficient to consider points A and E to find the optimal power allocation (x^o, y^o). Therefore, the set of candidate pairs is given by $\mathcal{P}^{(\mathrm{A}.1)} = \Big\{\Big(\beta(\sigma_D^2 + K_3P_C^{\max}), P_C^{\max}\Big), \Big(K_2\big(\frac{K_1}{1-\frac{\bar{\gamma}_C}{bP_C^{\max}}} - 1\big)^{-1}, P_C^{\max}\Big)\Big\}$.

2) Scenario 2: As shown in Fig. 2, in this scenario, the curve I-F intersects the tilted boundary line; However, line I-A does not intersect the tilted boundary line. The condition for this scenario is as follows:

$$\beta(\sigma_D^2 + K_3P_C^{\max}) \leq \frac{1 - c_1P_C^{\max}}{c_2}, \tag{22}$$

$$\frac{1 - c_1P_C^{\max}}{c_2} \leq \psi_1 \leq P_D^{\max} \tag{23}$$

where $\psi_1 \triangleq \frac{\mu + \sqrt{\mu^2 - 4c_2(1-K_1)K_2(\alpha c_1 - 1)}}{2c_2(1-K_1)}$ and $\mu \triangleq 1 - K_1 - c_2K_2 - \alpha c_1$. There are three candidate pairs for (x^o, y^o). If the optimal CU power is $P_C^{\max}$, then by Lemma 1, (x^o, y^o) is either point A or B. Using Lemma 2, we need to find the roots of (20) which are within the range of x-coordinate of tilted boundary line B-F. The set of candidate pairs is given by $\mathcal{P}^{(\mathrm{A}.2)} = \Big\{\Big(\beta(\sigma_D^2 + K_3P_C^{\max}), P_C^{\max}\Big), \Big((1 - c_1P_C^{\max})/c_2, P_C^{\max}\Big), \Big(\psi_1, (1 - c_2\psi_1)/c_1\Big), \mathcal{A}_2\Big\}$ where $\mathcal{A}_2 \triangleq \Big\{\Big(x_i^o, (1 - c_2x_i^o)/c_1\Big)\Big\}$ with x_i^o being a root of (20) satisfying $(1 - c_1P_C^{\max})/c_2 < x_i^o < \psi_1$.

3) Scenario 3: As illustrated in Fig. 3, in this scenario, the curves I-D and I-A do not intersect boundary line B-C. The entire tilted line is in the feasible region. The condition for this scenario is given by (22) and

$$\alpha\big(1 - \frac{K_1}{1 + K_2/P_D^{\max}}\big)^{-1} \leq \frac{1 - c_2P_D^{\max}}{c_1}. \tag{24}$$

Based on Lemmas 1 and 2, (x^o, y^o) could be at either of the end points A, B, C, D or any power given by the root of (20) within the range of x-coordinate of tilted boundary line B-C, *i.e.,* The set of candidate pairs is given by $\mathcal{P}^{(\mathrm{A}.3)} = \Big\{\Big(\beta(\sigma_D^2 + K_3P_C^{\max}), P_C^{\max}\Big), \Big((1 - c_1P_C^{\max})/c_2, P_C^{\max}\Big), \Big(P_D^{\max}, (1 - c_2P_D^{\max})/c_1\Big), \Big(P_D^{\max}, \alpha\big(1 - \frac{K_1}{1+K_2/P_D^{\max}}\big)^{-1}\Big), \mathcal{A}_3\Big\}$ where $\mathcal{A}_3 \triangleq \Big\{\Big(x_i^o, (1 - c_2x_i^o)/c_1\Big)\Big\}$ with x_i^o being a root of (20) satisfying $(1 - c_1P_C^{\max})/c_2 < x_i^o < P_D$.

4) Scenario 4: As shown in Fig. 4, in this scenario, both curves I-F and I-G intersect the tilted boundary line. The condition for this scenario is $\frac{1-c_1P_C^{\max}}{c_2} \leq \psi_2 \leq \psi_1 \leq P_D^{\max}$ where $\psi_2 \triangleq \frac{\sigma_D^2\beta + \beta K_3/c_1}{1 + \beta K_3c_2/c_1}$ is the x-coordinate of point G. In order to find the optimal power, we need to consider the end points G and F. In addition, we obtain the roots of (20) that are within the range of x-coordinate of tilted boundary line G-F. The set of candidate pairs is $\mathcal{P}^{(\mathrm{A}.4)} = \Big\{\Big(\psi_2, (1 - c_2\psi_2)/c_1\Big), \Big(\psi_1, (1 - c_2\psi_1)/c_1\Big), \mathcal{A}_4\Big\}$ where $\mathcal{A}_4 \triangleq \Big\{\Big(x_i^o, (1 - c_2x_i^o)/c_1\Big)\Big\}$ with x_i^o being a root of (20) satisfying $\psi_2 < x_i^o < \psi_1$.

5) Scenario 5: As shown in Fig. 5, in this scenario, the line I-G intersects tilted boundary line G-C, while the curve I-D does not. The condition for this scenario is given by (24) and $\frac{1-c_1P_C^{\max}}{c_2} \leq \psi_2 \leq P_D^{\max}$. Based on Lemmas 1 and 2, (x^o, y^o) could be either of the end points G, C, D or any power given by the root of (20) within the interval of x-coordinates of tilted boundary line G-C, *i.e.,* the set of candidate pairs is $\mathcal{P}^{(\mathrm{A}.5)} = \Big\{\Big(\psi_2, \frac{1-c_2\psi_2}{c_1}\Big), \Big(P_D^{\max}, (1 - c_2P_D^{\max})/c_1\Big), \Big(P_D^{\max}, \alpha\big(1 - \frac{K_1}{1+K_2/P_D^{\max}}\big)^{-1}\Big), \mathcal{A}_5\Big\}$ where $\mathcal{A}_5 \triangleq \Big\{\Big(x_i^o, (1 - c_2x_i^o)/c_1\Big)\Big\}$ with x_i^o being a root of (20) satisfying $\psi_2 < x_i^o < P_D^{\max}$.

6) Scenario 6: As depicted in Fig. 6, this scenario happens when there is no intersection between the tilted line and the curves I-H and I-D. The condition for this scenario is $(P_D^{\max} - \beta\sigma_D^2)/\beta K_3 \leq (1 - c_2P_D^{\max})/c_1$. By Lemma 1, it is sufficient to consider only points H and D to find the optimal power allocation. The set of candidates for the optimal powers is given by $\mathcal{P}^{(\mathrm{A}.6)} = \Big\{\Big(P_D^{\max}, \alpha\big(1 -$

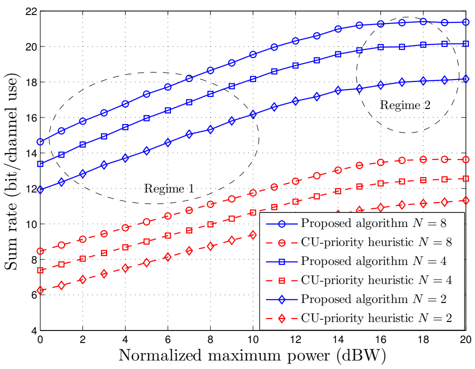

Fig. 7. The sum rate with $d_D/d_0 = 0.2$ for $N = 2, 4, 8$.

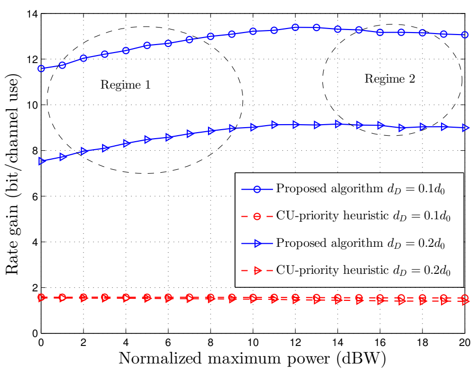

Fig. 8. The rate gain with $N = 4$ for $d_D/d_0 = 0.1, 0.2$.

$$\frac{K_1}{1+K_2/P_D^{\max}}\Big)^{-1}\Big), \Big(P_D^{\max}, \frac{P_D^{\max}-\beta\sigma_D^2}{\beta K_3}\Big)\Big\}.$$

D. Other Scenarios

Depending on the condition of ICI channels, four other cases are possible: i) strong CU ICI and weak D2D ICI, ii) weak CU ICI and strong D2D ICI, iii) strong CU ICI and strong D2D ICI, and iv) negligible ICI. There are ten scenarios in total for these four cases. Similar to our discussion in Section III-C, we can obtain the condition and the optimal power candidate pairs in each scenario. Details are omitted due to page limitation.

IV. Numerical Results

We provide numerical results to evaluate the performance of the proposed algorithm. We set $\sigma^2 = \sigma_D^2 = 1$, $\tilde{\gamma}_C = \tilde{\gamma}_D = 2$, $P_C^{\max} = P_D^{\max} = P^{\max}$, and $\tilde{\mathcal{I}} = 2N$. Let d_D, d_C, d_{gC}, d_{gD}, d_{fC}, d_{fD}, and d_0 denote the distances between the D2D transmitter and receiver, CU and BS, CU and D2D receiver, D2D transmitter and BS, CU and neighboring BS, D2D transmitter and neighboring BS, and the cell radius, respectively. We set $d_C = 0.5d_0$, $d_{gC} = 1.25d_0 - d_D/2$, $d_{gD} = 0.75d_0 + d_D/2$, $d_{fC} = 2.0616d_0$, and $d_{fD} = \sqrt{2^2 + (0.75 + 0.5d_D/d_0)^2}d_0$. The path loss exponent is set to 4. Thus, the channels are Gaussian with zero-mean and variance $(d/d_0)^{-4}$. We use 5000 realizations for each data point. For performance comparison, we consider a *CU-priority heuristic* algorithm: It selects the maximum feasible CU power with the minimum feasible D2D power, in order to maximize the SINR of the CU.

The sum rate versus the normalized maximum power, $P^{\max}/\sigma^2$, for various number of antennas, N, is shown in Fig. 7. We observe two regimes in this figure: Regime 1, where the ICI is low and the sum rate is limited by $P^{\max}$; and Regime 2, where $P^{\max}$ is high and the sum rate is controlled by the ICI threshold $\tilde{\mathcal{I}}$. It can be seen that the sum rate is an increasing function of the maximum power, in Regime 1, and then it converges in Regime 2 due to the ICI limit. We also see that the proposed algorithm significantly outperforms the CU-priority heuristic in both regimes for all values of N.

In order to evaluate the benefit of D2D communication, the difference between the sum rate under the proposed algorithm and the maximum sum rate when there is no D2D is defined as the rate gain. The rate gain for $d_D/d_0 = 0.1, 0.2$ and $N = 4$ is shown in Fig. 8. It can be seen that the rate gain is increasing in Regime 1 and decreasing at the beginning of Regime 2. This happens because we have two sources of ICI in the D2D mode as compared with one source when there is no D2D. Hence, in non-D2D mode, the CU can use a higher power for transmission. As expected, when the D2D channel is very strong, *i.e.,* the D2D distance is small, significant rate gain is achieved.

V. Conclusion

In this paper, we have formulated an uplink sum rate maximization problem of one CU and one D2D pair subject to minimum QoS requirements in terms of SINR, per-node maximum power, and maximum ICI constraints, with receive beamforming at the BS. Furthermore, we have developed a simple feasibility test to admit the D2D pair to share the spectrum with the CU. An algorithm has been proposed to obtain the optimal beam vector and powers of CU and D2D transmitters in closed form. Simulation results have shown significant rate gain obtained by the proposed algorithm.

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