# Optimal Fixed Gain Linear Processing for Amplify-and-Forward Multichannel Relaying

Min Dong, *Senior Member, IEEE*, Mahdi Hajiaghayi, *Student Member, IEEE*, and Ben Liang, *Senior Member, IEEE*

## Abstract

In this paper, we consider the problem of linear processing design at the relay for amplified-and-forward relaying in a multichannel system. Assuming a fixed-gain power amplification at the relay, we study the linear processing structure to maximize the end-to-end achievable rate. For both the cases of relaying with or without direct path, we show that the optimal unitary processing matrix is of permutation structure, *i.e.,* channel pairing is optimal. Furthermore, in each case, the explicit optimal channel pairing strategy is obtained based on sorting certain function of received SNR over the incoming and outgoing subchannels. This result is especially noticeable for the case with direct path, where the optimal linear processing was not known before under any power allocation. Specifically, we show that the pairing is according to the ordering of the relative SNR ratio on a subchannel over first hop to its direct path, and that of SNR strengths on subchannels over the second hop. Simulation results are presented to demonstrate the achievable gain of optimal channel pairing over non-optimal linear processing or no-pairing cases. It is also shown that the performance of channel pairing under the simple fixed-gain power allocation outperforms that under the traditional uniform power allocation.

## Index Terms

Linear processing, channel pairing, multichannel relaying, fixed-gain power amplification

## I. INTRODUCTION

Multichannel-based transmission is an essential component of the physical-layer technology for next-generation wireless systems [1]. Correspondingly, future system designs are evolving toward the adoption of a multi-channel relaying architecture to allow broadband access and coverage improvement [2]. As opposed to a narrow-band single-channel relay, a multi-channel relay has access to multiple channels, e.g., the subcarriers (or subchannels) in an Orthogonal Frequency Division Multiplexing (OFDM) relaying system. Such multichannel system creates an additional frequency dimension for the relay to exploit, where it can process the incoming signals adaptively based on the strength of each channels for forwarding purpose. This capability is unique to relaying systems, and such exploitation can potentially improve the overall relay performance.

To maintain low-complexity processing, linear operation of incoming signals at the relay is often desired. In this paper, we aim to address how to optimally perform channel-aware linear processing of the incoming signals at the relay to maximize the relaying performance in a multichannel system.

Channel pairing, which maps incoming and outgoing channels at the relay, along with power allocation, can be viewed as a special case of linear processing. It was first proposed independently in [3] and [4] for an amplified-and-forward (AF) dual-hop OFDM relay system. This has sparked interests in finding optimal channel pairing schemes [5]–[10]. Existing work can be categorized as whether assuming the power allocation is given, or jointly optimized with channel pairing, as well as whether nor not the direct path exists. Without the *direct path*, for given power allocation, a paring scheme based on sorted SNR is shown optimal for dual-hop AF relaying [5]. In [7], the sorted SNR pairing scheme is also shown to be optimal for both AF and DF in general multi-hop relaying. In addition, in the multi-hop setting, [7] shows that joint channel pairing and power optimization can be treated separately, where the optimal pairing is obtained by sorting based on channel gain. Unlike the scenario without the direct path, when the *direct path* is present, given power allocation, currently no explicit optimal pairing scheme is available, and only suboptimal schemes exist [8]. Joint optimization of channel pairing and power allocation were studied in [9] for single-user relaying, and the efficient numerical method for jointly optimal channel-user assignment, channel pairing, and power allocation was proposed in [10] for multi-user relaying. In both cases, algorithms are designed to find a jointly optimal solution, although no explicit channel pairing strategy can be found. Except [9], [10], all of the above works focus on the relay path only, without direct-path transmission, perhaps partially due to the difficulty in finding optimal (explicit) channel pairing in the case with direct path. Apart from one-way relaying described above, channel pairing for two-way

relaying in a multichannel system (in the absence of direct path) is also considered recently in [11]. Channel pairing aside, there is a need to study the structure of general linear processing of incoming signals at the relay and its impact on the end-to-end performance in such multichannel systems. In particular, with much attention on finding the optimal channel pairing scheme for relaying, the natural questions arise on how good the performance of channel pairing is compared to other linear processing schemes, and whether there exist conditions such that pairing is optimal.

There is a similar problem of optimal linear processing design in the context of MIMO AF relaying, where an optimal processing matrix at the relay is sought for multi-antenna processing [12]–[14]. Multi-channel relaying model can be viewed as a special case of MIMO relaying model. For the case without direct path, the optimal linear processing design obtained in [12] can be adapted to provide the solution for multichannel relaying. It reveals that channel pairing is optimal under the optimal power allocation. Linear processing is studied in [15] for multi-relay in asynchronous frequency-selective fading channels, where the structure proposed there converts the system into an OFDM-like multichannel system. Similar to the result in [12], under the optimal power allocation, the optimal linear processing structure consisting of power amplification and channel pairing is shown. For the case with direct path, again no known results on the optimal linear processing design for MIMO AF relaying, and only some suboptimal solution exists [14]. Even though these existing results provide certain answer to the linear processing design for multichannel relaying, some important issues still remain to be investigated. Specifically, for the case without direct link, would the optimality of channel pairing structure still hold when power allocation is suboptimal, and how to do the pairing in that case? What should the optimal linear processing be when the direct path is available? The first question arises for systems implementing suboptimal power allocation due to the high overhead cost or implementation complexity associated with optimal power allocation. The second question remains to be the central design problem for the case with direct path.

In this paper, we consider the problem of linear processing design at the relay for a multichannel dual-hop AF relaying and aim to address the above mentioned problems. We study the linear processing design to maximize the end-to-end achievable rate under a fixed gain power amplification at the relay. The power amplification method is simple that incurs minimum complexity or overhead. We separate the processing structure into two components: a fixed gain power amplification and linear combining using a unitary linear processing matrix. We show that the optimal unitary processing matrix is of permutation structure, *i.e.,* channel pairing is optimal for both the cases with or without direct path. Furthermore, we are able to obtain the explicit optimal channel pairing strategy based on sorting certain function of received SNR over the incoming and outgoing subchannels. This result is especially important and

interesting for the case with direct path available, which was not known before. Specifically, we show that the pairing is according to the ordering of the relative SNR ratio on a subchannel over first hop to its direct path, and that of SNR strengths on subchannels over the second hop.

The rest of this paper is organized as follows. In Section II, we present the system model and problem formulation. Section III establish the optimality of channel pairing among unitary linear processing schemes, and determine the optimal channel pairing strategy. We present simulation results to demonstrate the performance gain achieved through optimal channel pairing in Section IV and finally conclude in Section V.

## II. SYSTEM MODEL AND PROBLEM STATEMENT

### A. System Model

We consider a dual-hop multi-channel AF relay system consisting of a source node, a destination node, and a relay node. A direct link may exist between the source and destination. The system consists of $N$ subchannels for data transmission. We consider half-duplex transmissions, where a relay node is either in transmission or reception but not simultaneously. The cooperative transmission takes place in two phases. In the first phase the source sends data through $N$ subchannels to the relay and destination simultaneously (if the direct path is available). The relay then linearly processes the received signals over $N$ subchannels, and forwards the amplified version of the processed signals to the destination.

We denote the channel gain over subchannel $k$ from source to relay, from relay to destination, and from source to destination by $h_{1k}$, $h_{2k}$, and $h_{0k}$, respectively. The symbol transmitted from the source on subchannel $k$ is denoted by $s_k$, with unit power $\mathrm{E}|s_k|^2 = 1$. The power coefficient to transmit $s_k$ is denoted by $d_{sk}$. The received signals at the relay and destination in the first phase are given by

$$\mathbf{y}_r = \mathbf{H}_1 \mathbf{D}_s \mathbf{s} + \mathbf{n}_r, \quad \mathbf{y}_d^{(1)} = \mathbf{H}_0 \mathbf{D}_s \mathbf{s} + \mathbf{n}_d^{(1)} \tag{1}$$

where $\mathbf{s} = [s_1, \cdots, s_N]^T$ is the transmitted symbol vector with i.i.d. entries; the vectors $\mathbf{y}_r = [y_{r1}, \cdots, y_{rN}]^T$ and $\mathbf{y}_d^{(1)} = [y_{d1}^{(1)}, \cdots, y_{dN}^{(1)}]^T$ are the received signal vectors at relay and destination, respectively, and the matrices $\mathbf{H}_1 = \mathrm{diag}(h_{11}, \cdots, h_{1N})$ and $\mathbf{H}_0 = \mathrm{diag}(h_{01}, \cdots, h_{0N})$ are the corresponding channel matrices; The diagonal matrix $\mathbf{D}_s = \mathrm{diag}(\mathbf{d}_s)$ is the power coefficient matrix, with $\mathbf{d}_s = [d_{s1}, \cdots, d_{sN}]^T$ being the power coefficient vector, reflecting the power allocation across $N$ subchannels at the source. Moreover, $\mathbf{n}_r = [n_{r1}, \cdots, n_{rN}]^T$ and $\mathbf{n}_d^{(1)} = [n_{d1}^{(1)}, \cdots, n_{dN}^{(1)}]^T$ are AWGN at the relay and the destination, with $\mathbf{n}_r \sim \mathcal{CN}(0, \sigma_r^2 \mathbf{I})$ and $\mathbf{n}_d^{(1)} \sim \mathcal{CN}(0, \sigma_d^2 \mathbf{I})$, respectively.

In the second phase, the received signal $\mathbf{y}_r$ at the relay is first linearly combined, and then the relay retransmits the amplified version of the processed signal to the destination. Denoting $\mathbf{W}$ as the linear processing matrix at the relay, we have the received signal vector at the destination by

$$\begin{aligned}
\mathbf{y}_d^{(2)} &= \mathbf{H}_2\mathbf{W}\mathbf{y}_r + \mathbf{n}_d^{(2)} \\
&= \mathbf{H}_2\mathbf{W}(\mathbf{H}_1\mathbf{D}_s\mathbf{s} + \mathbf{n}_r) + \mathbf{n}_d^{(2)}.
\end{aligned} \tag{2}$$

where $\mathbf{y}_d^{(2)} = [y_{d1}^{(2)}, \cdots, y_{dN}^{(2)}]^T$, $\mathbf{H}_2 = \mathrm{diag}(h_{21}, \cdots, h_{2N})^T$, and $\mathbf{n}_d^{(2)} = [n_{d1}^{(2)}, \cdots, n_{dN}^{(2)}]^T$ with $\mathbf{n}_d^{(2)} \sim \mathcal{CN}(0, \sigma_d^2\mathbf{I})$.

Let $P_r$ be the average power budget at the relay, the processing matrix $\mathbf{W}$ must satisfy the relay power constraint $\mathrm{E}\|\mathbf{W}\mathbf{y}_r\|^2 \leq P_r$, which leads to

$$\mathrm{tr}\{\mathbf{W}(\mathbf{H}_1\mathbf{D}_s^2\mathbf{H}^H + \sigma_r^2\mathbf{I})\mathbf{W}^H\} \leq \mathrm{P_r}. \tag{3}$$

Furthermore, let $P_s$ be the power budget at the source. Then, the transmit power matrix $\mathbf{D}_s$ must satisfy

$$\mathrm{E}\|\mathbf{D}_s\mathbf{s}\|^2 = \|\mathbf{d}_s\|^2 \leq P_s.$$

In this study, we focus on the effect of the processing matrix $\mathbf{W}$ on the relay performance, and assume a pre-determined power allocation at the source, *i.e.,* $\mathbf{D}_s$ is given.

We are interested in the achievable sum-rate obtained through the above described AF relaying in the multichannel system. With the sum-rate as the objective, our goal is to study the structure of the processing matrix $\mathbf{W}$, with or without the presence of the direct link.

### B. Fixed Gain Relay Processing Structure

To obtain the optimal $\mathbf{W}$ to maximize the achievable rate, the difficulty lies in the case when the direct link exists. Various attempts in the existing works, either for MIMO relaying or multi-channel systems, show that there is no analytical solution for this case, and $\mathbf{W}$ can only be obtained through numerical exhaustive search. As a result, most existing works studying $\mathbf{W}$ for various system setups neglect the direct link.

In this work, we are interested in studying the problem with the presence of direct link, when certain structure of $\mathbf{W}$ is imposed. Note that, the processing matrix $\mathbf{W}$ essentially determines two processes: 1) how to linear combine the $N$ incoming signals; and 2) what is the power amplification for the outgoing signals. In order to separate the two effects, we break the process into two steps: the linear combining and the power amplification. Specifically, the processing matrix is in the form of $\mathbf{W} = \mathbf{D}_r\mathbf{W}'$, where

$\mathbf{W}'$ is the linear processing matrix, and $\mathbf{D}_r = \text{diag}(d_{r1}, \cdots d_{rN})$ is the power amplification matrix with $d_{rk}$ being the power amplification factor for the processed signal at the $k$th outgoing subchannel. In this paper, we consider a fixed gain power amplification, *i.e.,* $\mathbf{D}_r = d_r\mathbf{I}$, where $d_r$ represents the fixed gain, in other words, the relay *equally* amplifies the processed signal over each subchannel. Since we separate the power amplification $d_r$ from the processing matrix, $\mathbf{W}'$ should not affect output signal power, *i.e.,* $\|\mathbf{W}'\mathbf{s}\| = \|\mathbf{s}\|$, thus in the following we assume the class of unitary processing matrices[*] for $\mathbf{W}'$, *i.e.,*

$$\mathbf{W} = d_r\mathbf{U} \tag{4}$$

where $\mathbf{U}$ is a unitary matrix with $\mathbf{U}^H\mathbf{U} = \mathbf{I}$. Thus, the received signal vector $\mathbf{y}_d^{(2)}$ in (2) is now rewritten as

$$\mathbf{y}_d^{(2)} = \mathbf{H}_2 d_r\mathbf{U}(\mathbf{H}_1\mathbf{D}_s\mathbf{s} + \mathbf{n}_r) + \mathbf{n}_d^{(2)}. \tag{5}$$

With the processing matrix structure in (4), the power constraint in (3) leads to[†]

$$d_r = \sqrt{\frac{P_r}{\sum_{k=1}^N d_{sk}^2|h_{1k}|^2 + N\sigma_r^2}}. \tag{6}$$

*C. Achievable Rate*

As mentioned earlier, we focus on the achievable end-to-end rate in such multi-channel system. Our objective is to find the optimal $\mathbf{U}^*$ to maximize the achievable rate. Regardless of whether the direct path is available or not, we can rewrite the end-to-end system equation in the following general form

$$\mathbf{y} = \widetilde{\mathbf{H}}(\mathbf{U})\mathbf{s} + \widetilde{\mathbf{n}}(\mathbf{U}) \tag{7}$$

where $\widetilde{\mathbf{H}}(\mathbf{U})$ and $\widetilde{\mathbf{n}}(\mathbf{U})$ are the equivalent channel matrix and the equivalent noise term, respectively. They are functions of the processing matrix $\mathbf{U}$. Given the system described earlier, the system achievable rate (or the capacity under the AF relaying) is given by [16]

$$R(\mathbf{U}) = \frac{1}{2}\log\det(\mathbf{I} + \mathbf{R_n}^{-1}\widetilde{\mathbf{H}}(\mathbf{U})\widetilde{\mathbf{H}}^H(\mathbf{U})) \tag{8}$$

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[*]Note that we are interested in finding what form of linear processing leads to the optimality of channel pairing. This helps bridge the relation of linear processing and channel pairing in multichannel environment. Within the class of unitary processing, we will show that channel pairing is optimal. For other classes of linear processing, we are not able to make such conclusion.

[†]The fixed gain power amplification should not be confused to uniform or fixed power allocation. The power allocation $P_k$ on subchannel $k$ is $P_k = \frac{P_r}{N} \cdot \frac{\sum_{j=1}^N w_{kj}'^2 d_{sj}^2|h_{1j}|^2 + \sigma_r^2}{\frac{1}{N}\sum_{k=1}^N d_{sk}^2|h_{1k}|^2 + \sigma_r^2}$, which is different across subchannels and channel gain dependent.

where $\mathbf{R_n} \triangleq \mathrm{E}[\widetilde{\mathbf{n}}(\mathbf{U})\widetilde{\mathbf{n}}^H(\mathbf{U})]$ is the covariance matrix of the equivalent noise term, and the factor $1/2$ reflects the half-duplex operation. Our goal is to find the optimal $\mathbf{U}^*$ to maximize the achievable rate

$$\mathbf{U}^* = \underset{\mathbf{U}:\mathbf{UU}^H=\mathbf{U}^H\mathbf{U}=\mathbf{I}}{\arg\max} \log\det(\mathbf{I} + \mathbf{R_n}^{-1}\widetilde{\mathbf{H}}(\mathbf{U})\widetilde{\mathbf{H}}^H(\mathbf{U})). \tag{9}$$

For the conventional multichannel relaying without linear processing, *i.e.,* $\mathbf{U} = \mathbf{I}$, the relay simply forwards the amplified signal to the destination over the same channel. However, such forwarding is in general not optimal in terms of maximizing the achievable rate. For improvement, channel pairing was proposed [3], [4], where a different subchannel may be used over the second hop for signal relaying. This technique was actively studied recently in a few specific relay models [5], [7], [8], [10] for improving the end-to-end data rate. Channel pairing can be essentially represented as a special class of $\mathbf{U}$. Indeed, when $\mathbf{U}$ is a permutation matrix $\mathbf{\Pi}$, linear processing reduces to channel pairing. Therefore, the question arises on how good is channel pairing among all possible linear combining, and under what condition it is optimal.

## III. Optimal Linear Processing Structure: Channel Pairing

To solve (9), we first give the following result needed for the subsequent development.

*Lemma 1:* Let $\mathbf{P} = \mathrm{diag}(p_1,\cdots,p_N)$ and $\mathbf{Q} = \mathrm{diag}(q_1,\cdots,q_N)$ be two diagonal matrices. Let $\{|p_{(i)}|\}$ and $\{|q_{(i)}|\}$ be the ordered sequences of $\{|p_i|\}$ and $\{|q_i|\}$ in descending order, respectively[‡]. For the following optimization

$$\max_{\mathbf{U}:\mathbf{UU}^H=\mathbf{U}^H\mathbf{U}=\mathbf{I}} \det(\mathbf{I} + (\mathbf{PUQ})^H(\mathbf{PUQ})), \tag{10}$$

the optimal $\mathbf{U}^*$ is give by $\mathbf{U}^* = \mathbf{\Pi}^*$, where $\mathbf{\Pi}^*$ is the optimal permutation matrix that matches $|p_{(i)}|$ and $|q_{(i)}|$ in the objective of (10), for $i = 1,\cdots,N$, *i.e.,*

$$\det(\mathbf{I} + (\mathbf{P\Pi}^*\mathbf{Q})^H(\mathbf{P\Pi}^*\mathbf{Q})) = \prod_{i=1}^{N}\left(1 + \left(|p_{(i)}||q_{(i)}|\right)^2\right). \tag{11}$$

*Proof:* See Appendix A.

Using Lemma 1, we now look at the relaying scenario without and with direct path separately.

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[‡]The ordered sequence $\{|p_{(i)}|\}$ satisfies $|p_{(i)}| \geq |p_{(i+1)}|$, for $i = 1,\cdots,N-1$.

*A. Relay without Direct Path*

In this case, the destination is out of the transmission zone of the source. From (5), the equivalent channel matrix, noise vector, and its covariance matrix in (7) are given by

$$
\begin{aligned}
\widetilde{\mathbf{H}}(\mathbf{U}) &= d_r \mathbf{H}_2 \mathbf{U} \mathbf{H}_1 \mathbf{D}_s, \\
\widetilde{\mathbf{n}}(\mathbf{U}) &= d_r \mathbf{H}_2 \mathbf{U} \mathbf{n}_r + \mathbf{n}_d^{(2)}, \\
\mathbf{R_n} &= \sigma_r^2 d_r^2 \mathbf{H}_2 \mathbf{H}_2^H + \sigma_d^2 \mathbf{I}
\end{aligned}
\tag{12}
$$

Note that, $\mathbf{R_n}$ is not a function of $\mathbf{U}$, and all matrices in RHS of equations in (12) are diagonal, except $\mathbf{U}$. Using the property of the determinant, $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$, we can write the end-to-end achievable rate in (8) as

$$
R(\mathbf{U}) = \frac{1}{2} \log \det(\mathbf{I} + \widetilde{\mathbf{H}}(\mathbf{U})^H \mathbf{R_n}^{-1} \widetilde{\mathbf{H}}(\mathbf{U})).
\tag{13}
$$

Substituting $\mathbf{R}_n$ in (12) into (13), we have

$$
\begin{aligned}
R(\mathbf{U}) &= \frac{1}{2} \log \det(\mathbf{I} + (\mathbf{R_n}^{-\frac{1}{2}} \widetilde{\mathbf{H}}(\mathbf{U}))^H \mathbf{R_n}^{-\frac{1}{2}} \widetilde{\mathbf{H}}(\mathbf{U})) \\
&= \frac{1}{2} \log \det(\mathbf{I} + (\underbrace{d_r \mathbf{R_n}^{-\frac{1}{2}} \mathbf{H}_2}_{\mathbf{P}} \, \mathbf{U} \underbrace{\mathbf{H}_1 \mathbf{D}_s}_{\mathbf{Q}})^H \underbrace{d_r \mathbf{R_n}^{-\frac{1}{2}} \mathbf{H}_2}_{\mathbf{P}} \, \mathbf{U} \underbrace{\mathbf{H}_1 \mathbf{D}_s}_{\mathbf{Q}})
\end{aligned}
\tag{14}
$$

where we group matrices to be the equivalent $\mathbf{P}$ and $\mathbf{Q}$ as shown in (14). The corresponding $i$-th diagonal entries $p_i$ and $q_i$ of $\mathbf{P}$ and $\mathbf{Q}$ are respectively given by

$$
p_i = \frac{h_{2i} d_r}{\sqrt{\sigma_d^2 + \sigma_r^2 |h_{2i} d_r|^2}}, \quad q_i = h_{1i} d_{si}.
$$

Following Lemma 1, it is clear that the optimal $\mathbf{U}^*$ is $\mathbf{\Pi}^*$ that optimally pairs channels with $|p_{(i)}|$ and $|q_{(i)}|$, for $i = 1, \cdots, N$. Note that

$$
|p_i|^2 = \frac{\text{SNR}_{\text{rd},i}}{1 + \sigma_r^2 \text{SNR}_{\text{rd},i}}, \quad |q_i|^2 = \sigma_r^2 \text{SNR}_{\text{sr},i},
$$

where

$$
\text{SNR}_{\text{sr},i} \triangleq \frac{|h_{1i}|^2 d_{si}^2}{\sigma_r^2}, \quad \text{SNR}_{\text{rd},i} \triangleq \frac{|h_{2i}|^2 d_r^2}{\sigma_d^2}
\tag{15}
$$

are the received SNR from source to relay, and from relay to destination, over the $i$th subchannel, respectively. Since $|p_i|^2$ is a monotonically increasing function of $\text{SNR}_{\text{rd},i}$ and $\{|p_i|\}$ and $\{|p_i|^2\}$ have the same sorting order, the optimal pairing reduces to the pairing based on the sorted received SNRs. Let $\{\text{SNR}_{\text{sr},(i)}\}$ and $\{\text{SNR}_{\text{rd},(i)}\}$ be the ordered sequences of $\{\text{SNR}_{\text{sr},i}\}$ and $\{\text{SNR}_{\text{rd},i}\}$, we summarize the result as follow.

*Proposition 1:* For multi-channel fixed-gain AF relaying without direct path, optimizing $\mathbf{U}$ in (9) leads to the optimal channel pairing that pairs incoming and outgoing subchannels with $\mathrm{SNR}_{\mathrm{sr},(i)}$ and $\mathrm{SNR}_{\mathrm{rd},(i)}$, respectively, for $i = 1, \cdots, N$.

The result shows that the optimal unitary processing is given by channel pairing with the optimal pairing strategy based on sorted received SNRs. This shows that the optimality of channel pairing structure for linear processing holds under the fixed gain power allocation, besides the optimal power allocation. In addition, under given power allocation[§], the pairing based on sorted received SNRs has been shown to be optimal pairing under both noise-free relaying ($\sigma_r^2 = 0$) [4] and noisy relaying [5] in multi-channel systems. Here we show that the same optimality holds for the fixed-gain power amplification as well. Moreover, as we will see in the following, the approach we use allows us to find the optimal pairing strategy when direct path is available. This is the case where no explicit optimal pairing was known before.

### B. Relay with Direct Path

We now consider the case when the direct path is available. The received signals at the destination from both time slots can be written as $\mathbf{y} = [\mathbf{y}_d^{(1)} \ \mathbf{y}_d^{(2)}]^T$. Combining (1) and (5), we have the equivalent channel and noise terms in (7) as

$$\tilde{\mathbf{H}}(\mathbf{U}) = \begin{bmatrix} \mathbf{H}_o \mathbf{D}_s \\ \mathbf{H}_2 d_r \mathbf{U} \mathbf{H}_1 \mathbf{D}_s \end{bmatrix}, \tag{16}$$

$$\tilde{\mathbf{n}}(\mathbf{U}) = \begin{bmatrix} \mathbf{n}_d^{(1)} \\ \mathbf{H}_2 d_r \mathbf{U} \mathbf{n}_r + \mathbf{n}_d^{(2)} \end{bmatrix}, \tag{17}$$

$$\mathbf{R_n} = \begin{bmatrix} \sigma_d^2 \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\mathbf{n},r} \end{bmatrix} \tag{18}$$

[§]Note the difference between a given power allocation from a given power gain. For a given power allocation, the outgoing signal power is fixed, while for a given power gain, the amplification of the incoming signal power is fixed, and the outgoing signal power depends on both incoming signal power and the amplification gain.

where $\mathbf{R}_{\mathbf{n},r} \triangleq \sigma_r^2 d_r^2 \mathbf{H}_2 \mathbf{H}_2^H + \sigma_d^2 \mathbf{I}$; It is the noise covariance matrix without direct path as in (12). In this case, the achievable rate in (8) is derived as

$$R(\mathbf{U}) = \frac{1}{2} \log \det \left( \mathbf{I} + \frac{1}{\sigma_d^2} \mathbf{H}_o \mathbf{D}_s (\mathbf{H}_o \mathbf{D}_s)^H + \mathbf{R}_{\mathbf{n},r}^{-1} d_r \mathbf{H}_2 \mathbf{U} \mathbf{H}_1 \mathbf{D}_s (d_r \mathbf{H}_2 \mathbf{U} \mathbf{H}_1 \mathbf{D}_s)^H \right)$$

$$= \frac{1}{2} \log \det \left( \mathbf{I} + \frac{1}{\sigma_d^2} \mathbf{H}_o \mathbf{D}_s (\mathbf{H}_o \mathbf{D}_s)^H + (d_r \mathbf{R}_{\mathbf{n},r}^{-\frac{1}{2}} \mathbf{H}_2 \mathbf{U} \mathbf{H}_1 \mathbf{D}_s)^H (d_r \mathbf{R}_{\mathbf{n},r}^{-\frac{1}{2}} \mathbf{H}_2 \mathbf{U} \mathbf{H}_1 \mathbf{D}_s) \right)$$

$$= \frac{1}{2} \log \det \left( \mathbf{I} + \mathbf{\Upsilon}_0^H \mathbf{\Upsilon}_0 + (\mathbf{\Upsilon}_2 \mathbf{U} \mathbf{\Upsilon}_1)^H \mathbf{\Upsilon}_2 \mathbf{U} \mathbf{\Upsilon}_1 \right) \tag{19}$$

where $\mathbf{\Upsilon}_0 \triangleq \frac{1}{\sigma_d} \mathbf{H}_0 \mathbf{D}_s$, $\mathbf{\Upsilon}_1 \triangleq \mathbf{H}_1 \mathbf{D}_s$, and $\mathbf{\Upsilon}_2 \triangleq d_r \mathbf{R}_{\mathbf{n},r}^{-\frac{1}{2}} \mathbf{H}_2$. Note that the second term inside the determinant of (19) corresponds to the direct path, and the third term corresponds to the relay path. This shows that, when a permutation matrix (channel pairing) is applied at the relay, the rate in (19) is achieved by maximum ratio combining (MRC) of the received signals from the direct path and the paired relay path that are used to transmit the same symbol. In other words, $y_{d,i}^{(1)}$ from the direct path and $y_{d,j}^{(2)}$ from the relay path are combined using MRC, when the $j$th outgoing subchannel at the relay is paired with the $i$th incoming subchannel, for $i = 1, \cdots, N$.

To find the optimum $\mathbf{U}^*$ that maximizes $R(\mathbf{U})$, we re-arrange (19) as follows

$$R(\mathbf{U}) = \frac{1}{2} \log \det \left[ (\mathbf{I} + \mathbf{\Upsilon}_0^H \mathbf{\Upsilon}_0) \times \right.$$

$$\left. (\mathbf{I} + (\mathbf{I} + \mathbf{\Upsilon}_0^H \mathbf{\Upsilon}_0)^{-1} (\mathbf{\Upsilon}_2 \mathbf{U} \mathbf{\Upsilon}_1)^H \mathbf{\Upsilon}_2 \mathbf{U} \mathbf{\Upsilon}_1) \right]$$

$$= \frac{1}{2} \left[ \log \det (\mathbf{I} + \mathbf{\Upsilon}_0^H \mathbf{\Upsilon}_0) + \right.$$

$$\left. \log \det \left( \mathbf{I} + (\mathbf{\Upsilon}_2 \mathbf{U} \mathbf{\Upsilon}_1)(\mathbf{I} + \mathbf{\Upsilon}_0^H \mathbf{\Upsilon}_0)^{-1} (\mathbf{\Upsilon}_2 \mathbf{U} \mathbf{\Upsilon}_1)^H \right) \right], \tag{20}$$

where the second term in (20) follows from the property $\det(\mathbf{I} + \mathbf{A}\mathbf{B}) = \det(\mathbf{I} + \mathbf{B}\mathbf{A})$. The first term of (20) is the quantity corresponding to the direct path; it is independent of $\mathbf{\Pi}$. We are only interested to maximize the second term as a function of $\mathbf{\Pi}$, which can be written as

$$R_2(\mathbf{\Pi}) \triangleq \log \det \left( \mathbf{I} + (\mathbf{\Upsilon}_2 \mathbf{\Pi} \mathbf{\Upsilon}_1)(\mathbf{I} + \mathbf{\Upsilon}_0^H \mathbf{\Upsilon}_0)^{-1} (\mathbf{\Upsilon}_2 \mathbf{\Pi} \mathbf{\Upsilon}_1)^H \right)$$

$$= \log \det \left( \mathbf{I} + \mathbf{\Upsilon}_2 \mathbf{\Pi} \mathbf{\Upsilon}_1 (\mathbf{I} + \mathbf{\Upsilon}_0^H \mathbf{\Upsilon}_0)^{-\frac{1}{2}} (\mathbf{\Upsilon}_2 \mathbf{\Pi} \mathbf{\Upsilon}_1 (\mathbf{I} + \mathbf{\Upsilon}_0^H \mathbf{\Upsilon}_0)^{-\frac{1}{2}})^H \right). \tag{21}$$

Again using $\det(\mathbf{I} + \mathbf{A}\mathbf{B}) = \det(\mathbf{I} + \mathbf{B}\mathbf{A})$, and noticing that $\mathbf{\Upsilon}_0$, $\mathbf{\Upsilon}_1$, and $\mathbf{\Upsilon}_2$ are all diagonal matrices, we can apply the result in Lemma 1 to obtain the optimal $\mathbf{\Pi}^*$. Specifically, let $\mathbf{P} = \mathbf{\Upsilon}_2$ and $\mathbf{Q} = \mathbf{\Upsilon}_1 (\mathbf{I} + \mathbf{\Upsilon}_0^H \mathbf{\Upsilon}_0)^{-\frac{1}{2}}$. We then can express (21) as the form in (10). Based on this, we obtain the optimal channel pairing scheme for relaying with direct path available. It is essentially pairing the corresponding

entries in the ordered sequences $\{|p_{(i)}|\}$ and $\{|q_{(i)}|\}$, with $p_i$ and $q_i$ in $\mathbf{P}$ and $\mathbf{Q}$ respectively given by

$$p_i = \frac{d_r h_{2i}}{\sqrt{\sigma_r^2 d_r^2 |h_{2i}|^2 + \sigma_d^2}}, \quad q_i = \frac{d_{si} h_{1i}}{\sqrt{1 + \frac{d_{si}^2}{\sigma_d^2} |h_{0i}|^2}}. \tag{22}$$

Examining (22), we have

$$|p_i|^2 = \frac{\mathrm{SNR}_{\mathrm{rd},i}}{1 + \sigma_r^2 \mathrm{SNR}_{\mathrm{rd},i}}, \quad |q_i|^2 = \frac{\sigma_r^2 \mathrm{SNR}_{\mathrm{sr},i}}{1 + \mathrm{SNR}_{\mathrm{sd},i}} \tag{23}$$

where $\mathrm{SNR}_{\mathrm{sd},i} \triangleq \frac{|h_{0i}|^2 d_{si}^2}{\sigma_d^2}$ is the received SNR from source to destination over the $i$th channel, and $\mathrm{SNR}_{\mathrm{sr},i}$ and $\mathrm{SNR}_{\mathrm{rd},i}$ are the received SNR from source to relay, and from relay to destination, respectively, as given in (15). Let

$$\Gamma_i \triangleq \frac{\sigma_r^2 \mathrm{SNR}_{\mathrm{sr},i}}{1 + \mathrm{SNR}_{\mathrm{sd},i}}. \tag{24}$$

Similar as in the no direct path case, since $|p_i|^2$ is a monotonically increasing functions of $\mathrm{SNR}_{\mathrm{rd},i}$, we conclude that the optimal pairing is to pair the incoming and outgoing subchannels based on the ordered quantities $\{\Gamma_{(i)}\}$ and $\{\mathrm{SNR}_{\mathrm{rd},(i)}\}$. The result is summarized as follow.

*Proposition 2:* For multi-channel fixed-gain AF relaying with direct path available, optimizing $\mathbf{U}$ in (9) leads to the optimal channel pairing that pairs incoming and outgoing subchannels with $\Gamma_{(i)}$ and $\mathrm{SNR}_{\mathrm{rd},(i)}$, respectively, for $i = 1, \cdots, N$.

Proposition 2 again reveals the optimality of channel pairing among all possible unitary processing, even when the direct path is available. Furthermore, the optimal pairing is an *explicit* sorting strategy based on the received SNR on each path: It is to match the incoming and outgoing subchannels at the relay, according to the ordering of SNR strengths on the relay-destination subchannels, and that of the relative ratio of SNR strengths on the source-relay to source-destination subchannels. The benefit of such explicit sorting strategy for pairing is eminent: Various sorting algorithms can be employed with the computational complexity of $\mathcal{O}(N \log N)$ [17].

Based on Proposition 2, we comment on the following two cases:

- When $\mathrm{SNR}_{\mathrm{sd},i} \ll 0\mathrm{dB}$, for $i = 1, \cdots, N$: This case corresponds to the scenario where the direct path is weak, and $\Gamma_i \approx \sigma_r^2 \mathrm{SNR}_{\mathrm{sr},i}$. This means the ordering of $\{\Gamma_{(i)}\}$ is the same as $\{\mathrm{SNR}_{\mathrm{sr},(i)}\}$. Thus, the sorting metrics used in optimal pairing is naturally reduced to the one given in Proposition 1 for relaying without direct path. Note that, this is regardless of the link condition on the relay path. In other words, this case happens when the absolute link condition on the direct path is less than 0dB, instead of its relative strength to the relay path.

- When $\mathrm{SNR}_{\mathrm{sd},i} \gg 0\mathrm{dB}$, for $i = 1, \cdots, N$: This case corresponds to the scenario where the direct path is at least moderately strong. In this case, $\Gamma_i \approx \frac{\sigma_r^2 \mathrm{SNR}_{\mathrm{sr},i}}{\mathrm{SNR}_{\mathrm{sd},i}}$. Thus, the subchannel with a wider

difference on the quality of source-relay link and direct link will be paired with a stronger relay-destination subchannel.

*C. Overhead and Implementation Cost of Channel Pairing*

Out results show that to conduct channel pairing, the relay only needs to know $\mathrm{SNR}_{\mathrm{sr},i}$ and $\mathrm{SNR}_{\mathrm{rd},i}$ for the case without direct path, and additionally $\mathrm{SNR}_{\mathrm{sd},i}$ when direct path is available. The value of $\mathrm{SNR}_{\mathrm{sr},i}$ over the first hop is readily available at the relay. The values of $\mathrm{SNR}_{\mathrm{rd},i}$ and $\mathrm{SNR}_{\mathrm{sd},i}$ at the destination can be provided by the receiver through feedback. Note that regardless of channel pairing, SNR (or channel state) feedback to the relay is typically needed in the relay networks to gain channel information for various transmission designs. In practical implementation, usually only limited feedback is possible. The exact effect of channel feedback on the rate is different for the relay-destination path and the direct path, for which an exact quantification is challenging and outside the scope of this study. To implement channel pairing, the required sorting has the computational complexity in the order of $O(N \log N)$. Overall, implementing channel pairing incurs minimum additional overhead and implementation cost. In addition, fixed-gain power amplification greatly simplifies relay power allocation, and reduces the implementation complexity.

## IV. SIMULATION RESULTS

We compare the performance of the optimal channel pairing scheme with other non-optimal linear processing schemes through simulations. We use a 5MHz OFDM system with $N = 128$ subchannels as an example of multichannel system. A source-destination pair is placed at a distance $d_{sd}$ apart, and the distances between source and relay, and relay and destination are set at $d_{sr}$ and $d_{rd}$, respectively. The pathloss exponent of 2 is assumed. We denote $\mathrm{SNR} \triangleq \frac{P_s d_{sd}^{-2}}{N \sigma_d^2}$ as the average per subchannel received SNR over the direct path. In addition, we assume $P_s = P_r$. The achievable rate is normalized by $N$ subchannels and averaged over randomly generated multi-tap frequency selective channels.

We first compare different linear processing schemes and study when channel pairing is the most beneficial with respect to the relay location. With fixed $d_{sd} = 20$m, we vary the relay position between source and destination. We assume $P_s$ is equally allocated across subchannels, *i.e.,* $d_{si} = \sqrt{\frac{P_s}{N}}$. When the direct path is available, the change of relay position will affect the relative SNR strengths, between relay and direct paths. Fig.1 depicts the effect of such change on the achievable rate under different linear processing schemes. The average rate vs. the relative distance $d_{sr}/d_{rd}$ for $\mathrm{SNR} = 4$dB is plotted. We consider the following four processing schemes: 1) optimal channel pairing (CP) scheme $\mathbf{\Pi}^*$; 2) No

CP used, *i.e.,* $\mathbf{U} = \mathbf{I}$; 3) A random $\mathbf{U}$ used; 4) Using the pairing scheme $\mathbf{\Pi}^*$ that is obtained assuming no direct path (*i.e.,* pairing $\{\mathrm{SNR}_{\mathrm{sr},(i)}\}$ and $\{\mathrm{SNR}_{\mathrm{rd},(i)}\}$). The reason we consider the fourth scheme is that, in some cases, it may be easier for the relay to compute the optimal pairing only based on the SNRs obtained on the two relay paths, although the receiver may use signals from the direct path for combining to improve performance. We see that, when the relay moves closer to the source, the gain of using the optimal channel pairing over other schemes become more substantial. On the other hand, when the relay is very close to destination, the performance of all schemes coincide. The reason is that $\mathrm{SNR}_{rd}$ is relatively high when the relay is close to the destination. In this case, $|p_i|^2 \to 1/\sigma_r^2$ for all $i$ in (23). Thus, sorting becomes ineffective, and channel pairing provides little benefit.

Next, we compare the performance under different relay power allocation and pairing combinations. For the case without direct path, we plot the average rate vs. $d_{sr}/d_{rd}$ in Fig. 2 under four cases: 1) jointly optimal CP and power allocation (PA), which can be obtained using [12]; 2) Optimal CP under fixed gain power amplification obtained in this paper; 3) No CP but with optimal PA; 4) No CP and with fixed gain power amplification. We see that the optimal CP with fixed gain power amplification provides higher gain than the optimal PA alone without CP does. Jointly optimal CP and PA provides the overall best performance. For the case with direct path, we did similar comparison in Fig. 3. Since there is no jointly optimal CP and PA solution available, we only include a suboptimal scheme. Three cases are compared: 1) Optimal CP under fixed gain power amplification; 2) Suboptimal CP with equal power allocation: this is obtained using the CP result obtained in 1) and applying equal power allocation instead of fixed gain power amplification at the relay; 3) No CP and with fixed gain power amplification; 4) No CP and equal PA. We can clearly see the gain provided by optimal CP across different relay positions. In addition, we see that, although equal PA and fixed gain power amplification have the similar performance when no channel pairing is performed. The performance of pairing with fixed gain power amplification outperforms that with equal power allocation.

In Fig. 4, we compare different source power allocation schemes: equal power allocation vs. water-filling approach. The water-filling power allocation is determined based on the subchannels on the first hop only, *i.e.,* $\{|h_{1i}|^2\}$. As we see, for both cases with and without direct path, when the relay is closer to the source, the performance of the two power allocation schemes coincides. This is because the water-filling approach converges to the equal power allocation when the channel quality (at the first hop) is high. On the other hand, as the relay moves away from the source, the relative performance is different for the cases with and without direct path: compared to the equal power allocation, the water-filling approach improves the performance for the case without direct path, but degrades the performance for

the case with direct path which indicates its suboptimality in this case.

Finally, we study how the level of channel gain variation across subchannels affects the performance of various linear processing schemes. In Fig. 5, we plot the achievable rate vs. the number of taps of the frequency-selective channel for the case with direct path. We set $\mathrm{SNR} = 4\mathrm{dB}$, and increases the number of channel taps to increase the channel frequency selectivity. As we observe, the average rate increase with the number of channel taps under the optimal CP, demonstrating that the optimal CP benefits from an increased level of channel diversity, which is utilized effectively through channel pairing. On the other hand, the relative gain of random $\mathbf{U}$ or no CP is insensitive to such change and remains relatively constant.
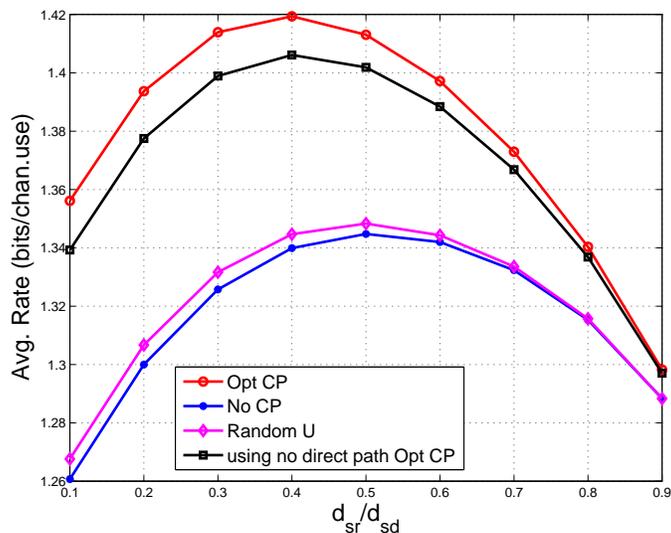


Fig. 1: With direct path: Rate vs. relative distance $\frac{d_{sr}}{d_{sd}}$ under different linear processing. (SNR=4dB)

## V. CONCLUSION

In this paper, AF relaying with linear processing capability at the relay for a multichannel system is considered. We have proposed a method to analyze how to select the linear processing matrix to maximize the end-to-end achievable rate, where fixed gain power amplification over channels at the relay is assumed. We have demonstrated the optimality of the optimal channel pairing among unitary processing for achievable rate maximization, for both with and without direct path. Our approach have allowed us to obtain the corresponding optimal explicit channel pairing strategy based on sorting certain
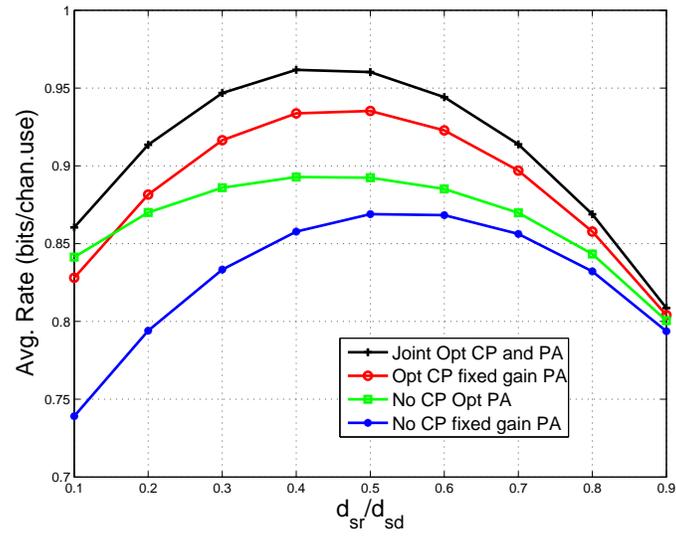
Fig. 2: Without direct path: Comparison of rate vs. $\frac{d_{sr}}{d_{sd}}$ under various relay power allocation. (SNR=4dB)
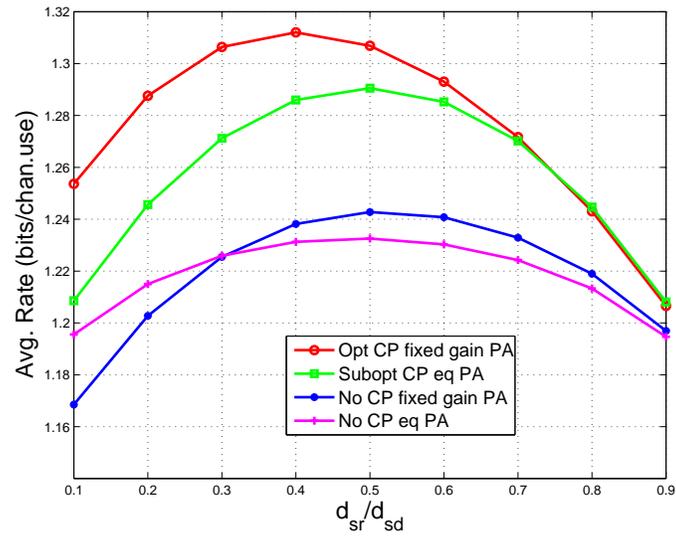


Fig. 3: With direct path: Comparison of rate vs. $\frac{d_{sr}}{d_{sd}}$ under various relay power allocation. (SNR=4dB)
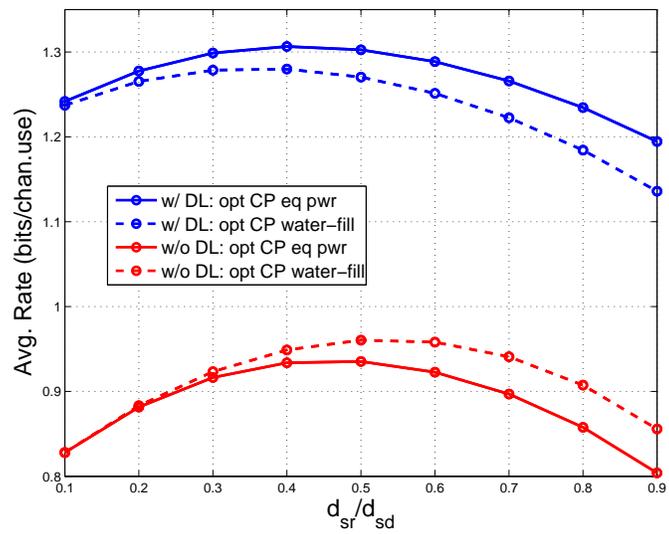
Fig. 4: Rate vs. $\frac{d_{sr}}{d_{sd}}$ under different source power allocation. (SNR=4dB)
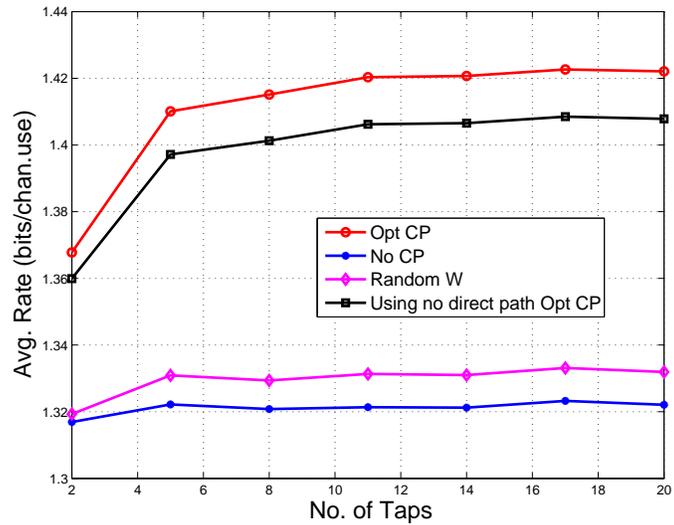


Fig. 5: With direct path: Rate vs. the number of channel taps. (SNR=4dB)

function of SNR at the first and second hops, for both cases with and without direct path. Simulation results also demonstrate the gain which can be achieved through optimal channel pairing as compared to the non-optimal linear processing and non-pairing cases.

## APPENDIX A

### PROOF OF LEMMA 1

Using the property of the determinant $\det(\mathbf{AB}) = \det(\mathbf{A}) \det(\mathbf{B})$, we see that the objective function in (10) can be rewritten as

$$\det(\mathbf{I} + (\mathbf{PUQ})^H(\mathbf{PUQ})) = \det(\mathbf{QQ}^H) \det((\mathbf{QQ}^H)^{-1} + \mathbf{U}^H\mathbf{P}^H\mathbf{PU}) \tag{25}$$

Since $\mathbf{QQ}^H$ is not a function of $\mathbf{U}$, we only need to optimize $\mathbf{U}$ to maximize the second determinant, *i.e.,*

$$\mathbf{U}^* = \arg\max_{\mathbf{U}} \det((\mathbf{QQ}^H)^{-1} + \mathbf{U}^H\mathbf{P}^H\mathbf{PU}). \tag{26}$$

By the property of determinant [18], we have $\det(\mathbf{A} + \mathbf{B}) \leq \prod_{n=1}^{N}(\lambda_n(\mathbf{A}) + \lambda_{N+1-n}(\mathbf{B}))$, where $\lambda_n(\mathbf{A})$ and $\lambda_n(\mathbf{B})$ are the eigenvalues of $\mathbf{A}$ and $\mathbf{B}$, respectively, sorted in ascending order. The equality is reached when $\mathbf{A}$ and $\mathbf{B}$ are both diagonal with the diagonal entries being inverse-order matched. Using this result, we have

$$\det((\mathbf{QQ}^H)^{-1} + \mathbf{U}^H\mathbf{P}^H\mathbf{PU}) \leq \prod_{n=1}^{N}\left(\frac{1}{|q_{(n)}|^2} + |p_{(N+1-n)}|^2\right) \tag{27}$$

Since $\mathbf{P}$ and $\mathbf{Q}$ are diagonal, and for any permutation matrix $\mathbf{\Pi}$, the matrix $\mathbf{\Pi}^H\mathbf{P}^H\mathbf{P}\mathbf{\Pi}$ is still diagonal, it immediately follows that $\mathbf{U}^* = \mathbf{\Pi}^*$, $\mathbf{\Pi}^*$ is the permutation matrix such that the entries of the ordered sequences $\{|p_{(i)}|\}$ and $\{|q_{(i)}|\}$ are one-to-one matched. $\blacksquare$

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