Per-Relay Power Minimization for Multi-user Multi-channel Cooperative Relay Beamforming

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***Abstract*—We investigate the optimal relay beamforming problem for multi-user peer-to-peer communication with amplify-and-forward relaying in a multichannel system. Assuming each source-destination (S-D) pair is assigned an orthogonal channel, we formulate the problem as a min-max per-relay power minimization problem with minimum signal-to-noise (SNR) guarantees. After showing that strong Lagrange duality holds for this non-convex problem, we transform its Lagrange dual problem to a semi-definite programming problem and obtain the optimal relay beamforming vectors. We identify that the optimal solution can be obtained in three cases, depending on the values of the optimal dual variables. These cases correspond to whether the minimum SNR requirement at each S-D pair is met with equality, and whether the power consumption at a relay is the maximum among relays at optimality. We obtain a semi-closed form solution structure of relay beam vectors, and propose an iterative approach to determine relay beam vector for each S-D pair. We further show that the reverse problem of maximizing the minimum SNR with per-relay power budgets can be solved using our proposed algorithm with an iterative bisection search. Through simulation, we analyze the effect of various system parameters on the performance of the optimal solution. Furthermore, we investigated the effect of imperfect channel side information of the second hop on the performance and quantify the performance loss due to either channel estimation error or limited feedback.**

***Index Terms*—Multiple users, peer-to-peer, per-relay power, power minimization, relay beamforming.**

I. Introduction

Cooperative relaying is one of the key techniques to improve quality of service and efficient resource usage in our wireless systems. It has been adopted in current and future multi-channel based broadband access systems, such as the 4th generation (4G) orthogonal frequency division multiple access (OFDMA) systems with LTE and LTE-Advanced standards [1], [2]. It is also the underlying technique for many potential features for 5G evolution [3]. In such a network, there are typically multiple communicating pairs as well as available relays. Efficient physical layer design of cooperative relaying to support such simultaneous transmissions is crucial.

We consider a multi-user peer-to-peer relay network in a multi-channel communication system, where multiple source-destination (S-D) pairs communicate through multiple single-antenna relays using the amplify-and-forward (AF) relaying strategy. Orthogonal subchannel allocation to each communicating pair is assumed to avoid multi-user interference. For each S-D pair, all relays assist the pair's transmission over the assigned subchannel through cooperative relay beamforming. We consider that each relay has its own power budget, *i.e.*, it cannot share power with another relay. This is a more practical scenario, especially for distributed relay systems. Our focus is on designing the optimal relay beamformers, aiming at minimizing per-relay power usage while meeting the minimum received signal-to-noise (SNR) guarantees.

The vast majority of the existing literature on cooperative relay beamforming design is focused on a single S-D pair, considering perfect or imperfect CSI [4]–[7], multi-antenna relay processing matrix design [8]–[11], and relay beamforming design for two-way relaying [12]–[15]. For multi-user peer-to-peer relay networks, relay beamforming design has been considered for single-carrier systems [16]–[25]. For multi-user transmission in a single-carrier system, each S-D pair suffers from the interference from other pairs, causing significant performance degradation and is the main challenge in relay beamforming design. Due to the complexity involved in such a problem, an optimal solution is difficult to obtain. Typically, approximate solutions through numerical approaches are proposed or suboptimal problem structures are considered for analytical tractability.

In contrast, cooperative relay beamforming in a multi-channel system can avoid multi-user interference through subchannel orthogonalization. However, it adds a new design challenge of creating additional dimensions of power sharing. For each relay, its power is shared among subchannels for relaying signals of all S-D pairs. For each S-D pair, all relays participate in beamforming the transmitted signal, affecting the power usage of all relays. Thus, the optimal design of relay beamformers for per-relay power minimization remains a challenging problem.

A. Contributions

- In this paper, we study the optimal relay beamforming problem for multi-user peer-to-peer communication in a multi-channel system. Assuming perfect CSI, we formulate the multi-channel relay beamforming problem as

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a min-max per-relay power minimization problem with minimum SNR guarantees. Showing that strong Lagrange duality holds for this non-convex problem, we solve it in the dual domain. Through transformations, we express the dual problem as a semi-definite programming (SDP) problem to determine the optimal dual variables, which has a much smaller problem size than that of the original problem and can be solved efficiently.

- We identify that the optimal relay beamforming solution of the original problem can be obtained in three cases depending on the values of the optimal dual variables. These cases reflect, at optimality, whether the minimum SNR requirement at each S-D pair is met with equality, and whether the power consumption at a relay is the maximum among relays. Among these three cases, the first one corresponds to the feasibility of the original problem. For the second and third cases, we obtain a semi-closed form solution structure of relay beam vectors, and design an iterative approach to determine the relay beam vector for each S-D pair.
- We further study the reverse problem of max-min SNR subject to per-relay power constraints. We show the inverse relation of the two problems and propose an iterative bisection algorithm to solve the max-min SNR problem.
- Through simulation, we analyze the effect of the number of relays, as well as the number of S-D pairs on the power and SNR performance under the optimal relay beam vector solution. Furthermore, we investigate the effect of imperfect CSI of the second hop. We quantify the performance loss due to either quantization error with limited feedback or channel estimation error. It is found that the loss due to imperfect CSI is mild. Furthermore, the loss due to quantization is less sensitive to the number of relays than that due to channel estimation error.

B. Related Work

The problem of optimal relay beamforming design for a single S-D pair has been extensively studied under total and per-relay power constraints [4]–[15]. For the multi-user downlink broadcast channel, MIMO relay beamforming has been considered in [26], [27]. For transmission of multiple S-D pairs, the design of relay beam vectors has been studied under different metrics, including sum rate, sum mean square error (MSE), relay power, and total source and relay power, for single-carrier systems [16]–[25] and for multi-channel systems [28], [29]. Most of these works consider only the total power across relays either as the constraint or objective of the optimization problem, which renders the optimization problems analytically more tractable [16]–[24], [28].

There has been much study on MIMO relay beamforming for multiple S-D pairs. For example, in [16], a robust design of MIMO relay processing matrix to minimize the worst-case relay power has been proposed for multiple S-D pairs, where the relays have only CSI estimates. With multiple MIMO relays, the MIMO relay processing design has been considered to minimize the total relay power subject to SINR guarantees in [21] for the perfect CSI case and in [18] when only second-order statistics of CSI are known at the relays. In [19], a robust MIMO relay processing design with CSI estimates is considered for sum MSE minimization and MSE balancing under a total relay power constraint. For a network with multiple MIMO S-D pairs, the total source and relay power minimization problem subject to minimum received SINR is considered in [23] and an iterative algorithm is proposed to jointly optimize the source, relay, and receive beam vectors and the source transmission power.

For single-antenna cooperative relay beamforming, the problem of total relay power minimization subject to minimal SINR guarantees has been considered for multiple S-D pairs in [17], where an approximate solution is proposed based on the semi-definite relaxation approach. Joint optimization of the source power and distributed relay beamforming is considered for the total power minimization in [22]. For a single-carrier relay beamforming system with multiple S-D pairs, the relay sum power minimization problem is studied in [24] using an interference zero-forcing approach. In contrast, we consider a multi-channel system and we solve the per-relay power with optimal beamforming, which is technically far more challenging.

To the best of our knowledge, the per-relay power minimization problem in multi-channel multi-relay systems has been studied only in [29]. However, the solution provided there is incomplete. In this work, we propose an algorithm to provide a complete solution in several possible cases. It can be shown that the solution in [29] is one special case of our solution (*i.e.*, Case 3 in Section III-B3). Our algorithm transforms the dual problem into an efficient SDP problem and uses an iterative approach to find the solution. In [29], however, the dual problem is directly solved using a subgradient method. Moreover, we have investigated the effect of imperfect CSI due to quantization error or channel estimation error, while only the true CSI is assumed in [29].

C. Organization and Notations

The rest of this paper is organized as follows. In Section II, the system model is described and the min-max per-relay power problem is formulated. In Section III, the min-max per-relay problem is solved. We discuss three different cases and propose an SDP-based algorithm to obtain the optimal relay beam vectors. In Section IV, we discuss the reverse problem of maximizing the minimum SNR subject to per-relay power constraints. Numerical results are presented in Section V, and conclusions are drawn in Section VI.

Notation: We use $\|\cdot\|$ to denote the Euclidean norm of a vector. $\odot$ stands for the element wise multiplication. We use $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^\dagger$ to denote transpose, Hermitian, and matrix pseudo-inverse, respectively. The conjugate is represented by $(\cdot)^*$. The notation $\mathrm{diag}(\mathbf{a})$ denotes the diagonal matrix with diagonal entries consisting the elements of vector $\mathbf{a}$. $\mathbf{I}$ denotes an $N \times N$ identity matrix. We use $\mathbf{Y} \succeq \mathbf{Z}$ to indicate that $\mathbf{Y} - \mathbf{Z}$ is a positive semi-definite matrix.

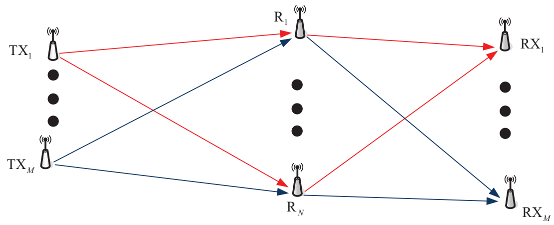

Fig. 1. The system model for multi-pair multi-channel relay communications.

II. System Model and Problem Formulation

A. System Model

We consider a two-hop wireless AF relaying system where M S-D pairs transmit data through N relays in a multi-channel communication system. All the nodes in the network are equipped with a single antenna. We assume that a direct link is not available between each S-D pair (*e.g.*, due to long distances). The multi-channel system is assumed to contain at least M frequency subchannels. Each S-D pair is pre-assigned a subchannel for its data transmission which is orthogonal to all other S-D pairs. Each relay can transmit received signals from all sources over their assigned respective subchannels. The system model is illustrated in Fig.1.

Since each S-D pair is pre-assigned a subchannel, without loss of generality, we assume S-D pair m communicates through N relays over subchannel m. The S-D transmission is established in two phases. In phase one, each source transmits its signal to all the relays. The received signal at relay i over subchannel m is given by

$$y_{m,i} = \sqrt{P_0} h_{m,i} s_m + n_{r,m,i} \tag{1}$$

where $h_{m,i}$ is the channel coefficient on subchannel m between source m and relay i, s_m is the transmitted symbol from source m with unit power, *i.e.*, $\mathbb{E}[|s_m|^2] = 1$, P_0 is the transmission power[1], and $n_{r,m,i}$ is the additive white Gaussian noise (AWGN) at relay i on subchannel m with zero mean and variance σ_r^2, which is i.i.d. across subchannels and relays. The received signal vector at all relays over subchannel m is given by

$$\mathbf{y}_m = \sqrt{P_0} \mathbf{h}_m s_m + \mathbf{n}_{r,m} \tag{2}$$

where $\mathbf{h}_m \triangleq [h_{m,1}, \cdots, h_{m,N}]^T$ and $\mathbf{n}_{r,m} \triangleq [n_{r,m,1}, \cdots, n_{r,m,N}]^T$ are the first-hop channel vector and the relay noise vector for S-D pair m, respectively.

In phase two, each relay i multiplies its received signal over subchannel m with a beamweight $w_{m,i}$ and forwards it to destination m. The received signal at destination m from all relays over subchannel m is given by

$$\begin{aligned} r_m &= \mathbf{g}_m^T \mathbf{W}_m \mathbf{y}_m + n_{d,m} \\ &= \sqrt{P_0} \mathbf{g}_m^T \mathbf{W}_m \mathbf{h}_m s_m + \mathbf{g}_m^T \mathbf{W}_m \mathbf{n}_{r,m} + n_{d,m} \end{aligned} \tag{3}$$

[1]Note that for simplicity, we assume the transmit power P_0 is the same for all sources. It is straightforward to extend our results to the scenario with different transmit power at different sources.

where $\mathbf{g}_m \triangleq [g_{m,1}, \cdots, g_{m,N}]^T$ is the second-hop channel vector for S-D pair m, with $g_{m,i}$ being the channel coefficient on subchannel m from relay i to destination m, $\mathbf{W}_m \triangleq \text{diag}(\mathbf{w}_m)$, with $\mathbf{w}_m \triangleq [w_{m,1}, \cdots, w_{m,N}]^T$ being the relay beam vector for S-D pair m, and $n_{d,m}$ is the AWGN at destination m with zero mean and variance σ_d^2, respectively.

The power usage of relay i is given by

$$P_{r,i} = \sum_{m=1}^{M} \mathbb{E}[|w_{m,i} y_{m,i}|^2] = \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m \tag{4}$$

where $\mathbf{R}_m \triangleq \text{diag}([\mathbf{R}_{y,m}]_{1,1}, \cdots, [\mathbf{R}_{y,m}]_{N,N})$, with $\mathbf{R}_{y,m} \triangleq P_0 \mathbf{h}_m \mathbf{h}_m^H + \sigma_r^2 \mathbf{I}$, for $m = 1, \cdots, M$, and $\mathbf{D}_i$ denotes the $N \times N$ diagonal matrix with 1 in the i-th diagonal entry and 0 otherwise.

Define $\mathbf{f}_m \triangleq \mathbf{g}_m \odot \mathbf{h}_m = [h_{m,1} g_{m,1}, \cdots, h_{m,N} g_{m,N}]^T$. The received signal power at destination m is obtained by

$$P_{S,m} = P_0 [\mathbf{g}_m^T \mathbf{W}_m \mathbf{h}_m \mathbf{h}_m^H \mathbf{W}_m^H \mathbf{g}_m^*] = P_0 \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m \tag{5}$$

where $\mathbf{F}_m \triangleq (\mathbf{f}_m \mathbf{f}_m^H)^*$. The total noise power at destination m including both the receiver noise and the relay amplified noise is given by

$$\begin{aligned} P_{N,m} &= \mathbb{E}[\mathbf{n}_{r,m}^H \mathbf{W}_m^H \mathbf{g}_m^* \mathbf{g}_m^T \mathbf{W}_m \mathbf{n}_{r,m}] + \sigma_d^2 \\ &= \mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma_d^2 \end{aligned} \tag{6}$$

where $\mathbf{G}_m \triangleq \sigma_r^2 \, \text{diag}\left((\mathbf{g}_m \mathbf{g}_m^H)^*\right)$. Thus, the SNR at destination m is given by

$$\text{SNR}_m = \frac{P_0 \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma_d^2}. \tag{7}$$

We use SNR as the quality-of-service (QoS) metric. Many other QoS metrics, such as BER and data rate, are monotonic functions of SNR. We assume perfect knowledge of CSI, *i.e.*, $\{\mathbf{h}_m, \mathbf{g}_m\}_{m=1}^M$, in designing the relay beam vectors.

B. Problem Formulation

We focus on a power efficient design of relay beamforming for multi-pair communications. Our goal is to minimize the maximum per-relay power usage by optimizing the relay beam vectors, while meeting the received SNR requirement at each destination. This min-max relay power optimization problem is given by

$$\min_{\{\mathbf{w}_m\}} \max_{1 \le i \le N} P_{r,i} \tag{8}$$

$$\text{subject to } \frac{P_0 \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma_d^2} \ge \gamma_m, \; m = 1, \cdots, M. \tag{9}$$

Denoting $P_{r,\max} = \max_i P_{r,i}$, the min-max optimization problem (8) is equivalent to the following problem

$$\min_{\{\mathbf{w}_m\}, P_{r,\max}} P_{r,\max} \tag{10}$$

$$\text{subject to } \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m \le P_{r,\max}, \; i = 1, \cdots, N, \tag{11}$$

and (9).

III. Minimizing Maximum Per-Relay Power Usage

The per-relay power minimization problem (10) is non-convex due to the SNR constraint (9). To solve it, we first examine the feasibility of the problem. Then we show that the solution can be obtained in the dual domain. The dual problem is further converted into an SDP with polynomial worst-case complexity. We obtain a semi-closed form structure of the beam vectors $\{\mathbf{w}_m\}$ and propose our algorithm to obtain the optimal dual variables in determining $\{\mathbf{w}_m\}$.

We first give the necessary condition for which the optimization problem (10) is feasible.

Proposition 1: A necessary condition for the feasibility of the relay power minimization problem (10) is

$$\min_{1\leq m\leq M} \frac{P_0}{\gamma_m}\mathbf{f}_m^H\mathbf{G}_m^{\dagger}\mathbf{f}_m > 1. \tag{12}$$

Proof: See Appendix A.

Note that the condition in (12) directly reflects the feasibility of the SNR constraint in (9), as shown in Appendix A. In other words, if the condition in (12) is not satisfied, the SNR constraint (9) cannot be satisfied for all m no matter what $\{\mathbf{w}_m\}$ is used.

A. The Dual Approach

Although the optimization problem (10) is non-convex, we show that the strong duality holds and hence the problem (10) can be solved in the Lagrange dual domain. The result is given below.

Proposition 2: The per-relay power minimization problem (10) has zero duality gap.

Proof: See Appendix B.

By Proposition 2, since the zero duality gap holds for the problem (10), the optimal beam vectors $\{\mathbf{w}_m^o\}_{m=1}^M$ can be obtained through the Lagrange dual domain. Let $\boldsymbol{\lambda} \triangleq [\lambda_1,\cdots,\lambda_N]^T$ and $\boldsymbol{\alpha} \triangleq [\alpha_1,\cdots,\alpha_M]^T$ denote the Lagrange multipliers associated with the per-relay power constraint (11) and SNR constraint (9), respectively. The dual problem of the problem (10) is given by

$$\max_{\boldsymbol{\lambda},\boldsymbol{\alpha}} \min_{\{\mathbf{w}_m\},P_{r,\max}} L(\{\mathbf{w}_m\},P_{r,\max},\boldsymbol{\lambda},\boldsymbol{\alpha}) \tag{13}$$

$$\text{subject to } \boldsymbol{\lambda}\succeq 0, \boldsymbol{\alpha}\succeq 0. \tag{14}$$

The Lagrangian $L(\{\mathbf{w}_m\},P_{r,\max},\boldsymbol{\lambda},\boldsymbol{\alpha})$ in (13) is given by

$$L(\{\mathbf{w}_m\},P_{r,\max},\boldsymbol{\lambda},\boldsymbol{\alpha}) = \sum_{m=1}^M \alpha_m\sigma_d^2 + P_{r,\max}(1-\sum_{i=1}^N\lambda_i) + \sum_{m=1}^M \mathbf{w}_m^H\Big(\mathbf{K}_m - \frac{\alpha_m P_0}{\gamma_m}\mathbf{f}_m\mathbf{f}_m^H\Big)\mathbf{w}_m \tag{15}$$

where

$$\mathbf{K}_m \triangleq \mathbf{R}_m\mathbf{D}_{\boldsymbol{\lambda}} + \alpha_m\mathbf{G}_m \tag{16}$$

and $\mathbf{D}_{\boldsymbol{\lambda}} \triangleq \mathrm{diag}(\lambda_1,\cdots,\lambda_N)$.

The dual problem (13) can be shown to be equivalent to the following problem:

$$\max_{\boldsymbol{\lambda},\boldsymbol{\alpha}} \sum_{m=1}^M \alpha_m\sigma_d^2 \tag{17}$$

$$\text{subject to } \mathbf{K}_m \succeq \frac{\alpha_m P_0}{\gamma_m}\mathbf{f}_m\mathbf{f}_m^H,\ m=1,\cdots,M, \tag{18}$$

$$\sum_{i=1}^N \lambda_i \leq 1, \tag{19}$$

and (14).

To see the equivalence, note that if either (18) or (19) is not satisfied, there exists some $\{\mathbf{w}_m, P_{r,\max}\}$ resulting in $L(\{\mathbf{w}_m\},P_{r,\max},\boldsymbol{\lambda},\boldsymbol{\alpha}) = -\infty$, which cannot be an optimal solution of the dual problem (13). Therefore, the constraints (18) and (19) are met at the optimality of the problem (13). After the inner minimization with respect to (w.r.t.) $\{\mathbf{w}_m\}$ and $P_{r,\max}$, the objective of the dual problem (13) is equivalent to that in (17).

To solve the problem (17) for the optimal dual variables $\{\boldsymbol{\lambda}^o,\boldsymbol{\alpha}^o\}$, we now show that it can be reformulated into an SDP given below to obtain the solution.

$$\min_{\mathbf{y}} \mathbf{a}^T\mathbf{y} \tag{20}$$

$$\text{subject to } \mathbf{b}^T\mathbf{y}\leq 1,\ \mathbf{y}\succeq 0$$

$$\sum_{j=1}^{M+N} y_j\boldsymbol{\Psi}_{m,j} \preceq 0,\ m=1,\cdots,M$$

where $\mathbf{y} \triangleq [\boldsymbol{\alpha}^T,\boldsymbol{\lambda}^T]^T$, $\mathbf{a} \triangleq [-\sigma_d^2\mathbf{1}_{M\times1}^T, \mathbf{0}_{N\times1}^T]^T$, $\mathbf{b} \triangleq [\mathbf{0}_{M\times1}^T,\mathbf{1}_{N\times1}^T]^T$, $\boldsymbol{\Psi}_{m,m} \triangleq \frac{P_0}{\gamma_m}\mathbf{f}_m\mathbf{f}_m^H - \mathbf{G}_m$, $\boldsymbol{\Psi}_{m,M+i} \triangleq -\mathbf{R}_m\mathbf{D}_i$ for $m=1,\cdots,M$, $i=1,\cdots,N$, and all other $\boldsymbol{\Psi}_{m,j}$ are zeros.

The above SDP can be solved efficiently using a standard SDP solver [30]. Obtaining the optimal beam vector solution $\{\mathbf{w}_m^o\}_{m=1}^M$ of the problem (13) depends on the values of the optimal dual variables $\{\boldsymbol{\lambda}^o,\boldsymbol{\alpha}^o\}$. In the following, we partition the values of $\{\boldsymbol{\lambda}^o,\boldsymbol{\alpha}^o\}$ into three cases and derive $\{\mathbf{w}_m^o\}_{m=1}^M$ in each case. We first present the following lemma showing a certain condition on the value of $\boldsymbol{\alpha}^o$.

Lemma 1: If $\boldsymbol{\lambda}^o \succ 0$, then $\boldsymbol{\alpha}^o \succ 0$.

Proof: See Appendix C.

Note that $\boldsymbol{\lambda}^o$ and $\boldsymbol{\alpha}^o$ are the optimal dual variables associated with the per-relay power constraint (11) and SNR constraint (9), respectively. The Karush-Kuhn-Tucker (KKT) conditions require complementary slackness. Thus, Lemma 1 indicates that if the per-relay power constraint is active (*i.e.*, attained with equality) at optimality, then the SNR constraint is also active at optimality. However, note that α_m^o could be zero for some m, if λ_i^o is zero for some i.

B. The Optimal Beam Vector $\{\mathbf{w}_m^o\}$

Using Lemma 1, in the following, we partition the values of $\{\boldsymbol{\lambda}^o,\boldsymbol{\alpha}^o\}$ into three cases to derive $\{\mathbf{w}_m^o\}_{m=1}^M$.

1) Case 1: $\boldsymbol{\lambda}^o = 0$. In this case, $\mathbf{K}_m$ in (16) reduces to $\alpha_m \mathbf{G}_m$. For the constraint (18) to hold, we have $\boldsymbol{\alpha}^o = 0$ (also see Appendix C). As a result, the objective in (17) becomes zero. If the SNR constraint (9) could be satisfied for all m, *i.e.,* the original problem (10) is feasible, the optimal objective has to be strictly greater than zero which is a contradiction. This implies the per-relay power minimization problem (10) is infeasible. In other words, if the optimization problem (10) is feasible, there should be at least one i such that (11) is active at optimality, *i.e.,* $\lambda_i^o > 0$.

2) Case 2: $\boldsymbol{\lambda}^o \not\succ 0$ and $\boldsymbol{\lambda}^o \neq 0$. In this case, we have $\lambda_i^o = 0$ for some i's and $\alpha_m^o = 0$ for some m's. In the following, we first consider the case in which at optimality, only one entry in $\boldsymbol{\lambda}^o$ and $\boldsymbol{\alpha}^o$ is strictly positive. In other words, only one S-D pair and one relay meet the SNR constraint and power constraint with equality, respectively. Then, we explain how to extend our solution to the case in which $\lambda_i^o > 0$, $\alpha_m^o > 0$ for arbitrary i's and m's. Denote $\tilde{m}$ and $\tilde{i}$ such that $\alpha_{\tilde{m}}^o > 0$ and $\lambda_{\tilde{i}} > 0$, respectively, and $\alpha_m^o = 0$ for $m \neq \tilde{m}$ and $\lambda_i^o = 0$ for $i \neq \tilde{i}$. In this case, we have $\lambda_{\tilde{i}} = 1$ from the maximization problem (17), since its optimal objective is increasing w.r.t. $\lambda_{\tilde{i}}$.

In the following, we first obtain the optimal beam vector $\mathbf{w}_{\tilde{m}}^o$. For $m \neq \tilde{m}$, the optimal beam vector $\mathbf{w}_m^o$ cannot be derived in a similar way as that for $\mathbf{w}_{\tilde{m}}^o$. Instead, we formulate a new optimization problem to obtain $\mathbf{w}_m^o$.

Proposition 3: Assume $\alpha_{\tilde{m}}^o > 0$. The optimal beam vector $\mathbf{w}_{\tilde{m}}^o$ for the per-relay power minimization problem (10) is given by

$$\mathbf{w}_{\tilde{m}}^o = \zeta_{\tilde{m}} {\mathbf{K}_{\tilde{m}}^o}^{\dagger} \mathbf{f}_{\tilde{m}} \tag{21}$$

where

$$\zeta_{\tilde{m}} \triangleq \sigma_d \left[\frac{P_0}{\gamma_{\tilde{m}}} |\mathbf{f}_{\tilde{m}}^H {\mathbf{K}_{\tilde{m}}^o}^{\dagger} \mathbf{f}_{\tilde{m}}|^2 - \mathbf{f}_{\tilde{m}}^H {\mathbf{K}_{\tilde{m}}^o}^{\dagger} \mathbf{G}_{\tilde{m}} {\mathbf{K}_{\tilde{m}}^o}^{\dagger} \mathbf{f}_{\tilde{m}} \right]^{-\frac{1}{2}} \tag{22}$$

with $\mathbf{K}_{\tilde{m}}^o$ obtained by substituting the optimal dual variables $\alpha_{\tilde{m}}^o$ and $\boldsymbol{\lambda}^o$ into (16).

Proof: See Appendix D.

Define $\mathcal{M} \triangleq \{1, \cdots, M\} \setminus \{\tilde{m}\}$, and define $P_{i,\tilde{m}} \triangleq {\mathbf{w}_{\tilde{m}}^o}^H \mathbf{R}_{\tilde{m}} \mathbf{D}_i \mathbf{w}_{\tilde{m}}^o$ as the power used at relay i for S-D pair $\tilde{m}$. The beamforming vectors $\{\mathbf{w}_m, m \in \mathcal{M}\}$ are determined through solving the following feasibility problem

$$\begin{aligned} &\text{find } \{\mathbf{w}_m, m \in \mathcal{M}\} && (23)\\ &\text{subject to } \max_{1 \le i \le N} P_{i,\tilde{m}} + \sum_{m \in \mathcal{M}} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m = P_{r,\max}^o, \\ &\qquad \frac{P_0 \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma_d^2} \ge \gamma_m, \ m \in \mathcal{M}. && (24) \end{aligned}$$

There is no unique solution for the feasibility problem (23). However, we can always scale $\mathbf{w}_m$ such that (24) meets with equality for $m \in \mathcal{M}$. Since we assume $\alpha_m^o = 0$ for $m \neq \tilde{m}$, the optimal objective of the original problem (10) is $P_{r,\max}^o = \alpha_{\tilde{m}}^o \sigma_d^2$. By Proposition 2, this means, at optimality, the Lagrangian in (15) is $\alpha_{\tilde{m}}^o \sigma_d^2$. It follows that, under the assumed $\boldsymbol{\alpha}^o$, $\boldsymbol{\lambda}^o$, we have $\sum_{m \in \mathcal{M}} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_{\tilde{i}} \mathbf{w}_m = 0$. Since $\lambda_{\tilde{i}} > 0$, the power constraint (11) for $\tilde{i}$ is met with equality, and we have $P_{\tilde{i},\tilde{m}} = P_{r,\max}^o$.

As analyzed above, at optimality, except S-D pair $\tilde{m}$, relay $\tilde{i}$ does not forward signal from any other source $m \in \mathcal{M}$. Thus, to obtain w_m^o for $m \in \mathcal{M}$, we now propose the following relay power minimization problem by excluding the consideration of S-D pair $\tilde{m}$ and restricting the power usage on relay $\tilde{i}$

$$\begin{aligned} &\min_{\{\mathbf{w}_m, m \in \mathcal{M}\}, \tilde{P}_r} \tilde{P}_r && (25)\\ &\text{subject to } \sum_{m \in \mathcal{M}} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_{\tilde{i}} \mathbf{w}_m \le 0, \\ &\qquad \sum_{m \in \mathcal{M}} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m \le \tilde{P}_r, \ \forall i \neq \tilde{i}, && (26)\\ &\qquad \text{and (24).} \end{aligned}$$

Following similar argument as Proposition 2, we can show that zero duality gap holds for the problem (25). This problem can be reformulated in the dual domain into an SDP, given by

$$\begin{aligned} &\min_{\mathbf{y}} \mathbf{c}^T \mathbf{y} && (27)\\ &\text{subject to } \sum_{j=1}^{M+N} y_j \boldsymbol{\Psi}_{m,j} \preceq 0, \ m \in \mathcal{M}, \\ &\qquad \mathbf{y} \succeq 0, \mathbf{d}^T \mathbf{y} \le 1 \end{aligned}$$

where $\mathbf{y}$ is defined the same way as in (20), $\mathbf{c}$ is defined the same way as $\mathbf{a}$ in (20) except for the $\tilde{m}$-th entry, $c_{\tilde{m}}$, being zero, and $\mathbf{d}$ is defined the same way as $\mathbf{b}$ in (20) except for the $(M + \tilde{i})$-th entry being zero.

The terms corresponding to S-D pair $\tilde{m}$ are eliminated in both the objective and constraints of (27), which is consistent with the problem formulation (25).

For the optimization problem (25), we repeat our procedure to evaluate the values of $\{\alpha_m^o, \ m \in \mathcal{M}\}$.[2] If $\alpha_m^o > 0$ for all $m \in \mathcal{M}$, then we can find $\{\mathbf{w}_m^o, \ m \in \mathcal{M}\}$ similarly to Case 3 as discussed in the following. Otherwise, we follow the steps to obtain the solution in Case 2. For example, suppose the per-relay power constraint (26) and SNR constraint (24) are active for some relay $\hat{i}$ and some S-D pair $\hat{m}$, *i.e.,* $\lambda_{\hat{i}}^o > 0$ and $\alpha_{\hat{m}}^o > 0$. Let $\tilde{P}_r^o$ denote the minimum value of (25). As the minimum objective of (10) is $P_{r,\max}^o$, we have $\tilde{P}_r^o + P_{\hat{i},\tilde{m}} \le P_{r,\max}^o$, and we can find $\mathbf{w}_{\hat{m}}^o$ with similar structure as in (21) by substituting the optimal dual variables obtained from (27) into (16).

So far, we have proposed our algorithm to obtain the optimal beam vector solution $\{\mathbf{w}_m^o\}$, assuming only one entry in $\boldsymbol{\alpha}^o$ and $\boldsymbol{\lambda}^o$ is strictly positive. The proposed procedure can be extended to the general case where multiple entries in $\boldsymbol{\alpha}^o$ and $\boldsymbol{\lambda}^o$ are positive. In this case, we define $\mathcal{I}_\alpha \triangleq \{m \mid \alpha_m^o > 0\}$ and $\mathcal{I}_\lambda \triangleq \{i \mid \lambda_i^o > 0\}$. According to Proposition 3, the optimal $\mathbf{w}_m^o$ for $m \in \mathcal{I}_\alpha$ has a similar expression as in (21). Then, we can solve a feasibility problem similar to (23) to find $\mathbf{w}_m^o$ for $m \in \{1, \cdots, M\} \setminus \mathcal{I}_\alpha$. The feasibility problem can be reformulated into an SDP similar to (27) with updated $\mathbf{c}$, $\mathbf{d}$, and $\boldsymbol{\Psi}_{m,j}$ according to $\mathcal{I}_\alpha$ and $\mathcal{I}_\lambda$.

[2]Note that the problem (25) is feasible. This is because we consider Case 2 for the original problem, which means the problem is feasible. Thus, only Cases 2 or 3 will happen in the subsequent iterative procedure.

Algorithm 1 Solving the per-relay power minimization problem (10)

1: Check the feasibility condition (12).
2: Solve the SDP problem (20) to obtain the optimal dual variables $\{\boldsymbol{\alpha}^o, \boldsymbol{\lambda}^o\}$.
3: Obtain $\mathcal{I}_\alpha = \{m \mid \alpha_m^o > 0\}$ and $\mathcal{I}_\lambda = \{i \mid \lambda_i^o > 0\}$.
4: Set $\Pi_\alpha = \mathcal{I}_\alpha$.
5: **while** $\mathcal{I}_\alpha \neq \{1, \cdots, M\}$ **do**
6: Compute $\mathbf{K}_m^o$ and find $\mathbf{w}_m^o$ in (21) for all $m \in \Pi_\alpha$.
7: Update $\mathbf{c}$ and $\mathbf{d}$ as defined below the problem (27).
8: Solve the SDP problem (27).
9: Find $\Pi_\alpha = \{l \in \{1, \cdots, M\} \setminus \mathcal{I}_\alpha | \alpha_l^o > 0\}$ and $\Pi_\lambda = \{q \in \{1, \cdots, N\} \setminus \mathcal{I}_\lambda | \lambda_q^o > 0\}$.
10: Update $\mathcal{I}_\alpha = \mathcal{I}_\alpha \cup \Pi_\alpha$ and $\mathcal{I}_\lambda = \mathcal{I}_\lambda \cup \Pi_\lambda$.
11: **end while**
12: Compute $\mathbf{K}_m^o$ and find $\mathbf{w}_m^o$ in (21) for all $m \in \Pi_\alpha$.

3) Case 3: $\boldsymbol{\lambda}^o \succ 0$. According to Lemma 1, we have $\boldsymbol{\alpha}^o \succ 0$. From $\mathbf{K}_m$ in (16), this means that if $\mathbf{K}_m^o - \alpha_m^o \mathbf{G}_m \succ 0$, then $\alpha_m^o > 0$ for all m, and the solution is given by Proposition 3. According to the proof in Appendix D, it can be shown that $\frac{\alpha_m^o P_0}{\gamma_m} \mathbf{f}_m^H \mathbf{K}_m^{o\,\dagger} \mathbf{f}_m = 1$, for $m = 1, \cdots, M$. In this case, since $\boldsymbol{\alpha}^o \succ 0$, we obtain the optimal beam vectors $\mathbf{w}_m^o$ directly by the semi-closed form solution given by (21).

Corollary 1: The maximum per-relay power for the original problem (10) is given by

$$P_{r,\max}^o = \sum_{m=1}^{M} \alpha_m^o \sigma_d^2 = \sigma_d^2 \sum_{m=1}^{M} \frac{\gamma_m}{P_0 \mathbf{f}_m^H \mathbf{K}_m^{o\,\dagger} \mathbf{f}_m}. \tag{28}$$

Proof: The first equality in (28) is due to the zero duality gap by Proposition 2. As shown in Appendix D for Case 3, we have $\frac{\alpha_m^o P_0}{\gamma_m} \mathbf{f}_m^H \mathbf{K}_m^{o\,\dagger} \mathbf{f}_m = 1$ at optimality. Substituting α_m^o into the objective of (17), we arrive at the expression at the right-hand side of (28). ∎

Combining Cases 2 and 3, we summarize our algorithm for solving per-relay power minimization problem (10) in Algorithm 1.

Note that for both Cases 2 and 3, the beam vector solution has the semi-closed form structure given in (21). Hence, we can provide the necessary and sufficient condition for the feasibility of (10). Note that for $\zeta_{\tilde{m}}$ in (22) to be real, the term in the bracket at the right-hand side of (22) should be positive. Therefore, the problem (10) is feasible if and only if there exists $\boldsymbol{\alpha} \succeq 0$, $\boldsymbol{\lambda} \succeq 0$, with $\sum_{i=1}^{N} \lambda_i \leq 1$ such that

$$\min_{1 \leq m \leq M} \frac{P_0}{\gamma_m} |\mathbf{f}_m^H \mathbf{K}_m^\dagger \mathbf{f}_m|^2 - \mathbf{f}_m^H \mathbf{K}_m^\dagger \mathbf{G}_m \mathbf{K}_m^\dagger \mathbf{f}_m > 0. \tag{29}$$

C. Complexity Analysis

Now we analyze the complexity of Algorithm 1. Note that the optimization problem (10) has been converted to an SDP problem in (20) with $M+N$ variables and M linear matrix inequality constraints. The SDP can be solved efficiently using interior-point methods with standard SDP solvers such as SeDuMi [31], [32]. In the following, we analyze the complexity based on the standard SDP form in [31]. Based on the complexity analysis of the standard SDP form, for the SDP with $M+N$ variables, and M linear matrix inequality constraints of the size given, the computation complexity per iteration to solve (20) is $\mathcal{O}\big((M+N)^2 MN^2\big)$. The number of iterations to solve SDP is typically between 5 to 50 regardless of problem size [31]. Thus, the complexity to solve the SDP is $\mathcal{O}\big((M+N)^2 MN^2\big)$.

Note that the overall computation complexity to solve the optimization problem (10) depends on the values of the optimal dual variables. As shown in Section III-B, if Case 3 happens, only one SDP problem (20) is solved, *i.e.*, the complexity is given by $\mathcal{O}\big((M+N)^2 MN^2\big)$. If Case 2 happens, at most M SDP problems formulated as (27) are solved, *i.e.*, the worst-case complexity is given by $\mathcal{O}\big((M+N)^2 M^2 N^2\big)$. In both cases, the algorithm has a polynomial worst-case complexity w.r.t. the number of relays and S-D pairs. Note that the above analysis is based on worst-case complexity estimates. In practice, the complexity is much lower than the worst-case estimate [31].

IV. Maximizing Minimum SNR

The ultimate end-to-end performance measures of the network such as the data rate or bit-error-rate (BER) are direct functions of the received SNR. It is often desirable to maximize the worst received SNR at the destinations under power constraints. In this section, we formulate the max-min SNR problem subject to per-relay power constraints, and show that it is the inverse problem of the min-max per-relay power subject to SNR constraints. Thus, we propose an iterative algorithm through bisection search to solve the max-min SNR problem.

In a typical system, the relays have the same front-end amplifiers and the destinations have the same minimum SNR requirements. In the following, we assume identical per-relay power budgets and minimum SNR requirements for the relays and destinations, respectively. Extension to the non-uniform power and/or SNR requirement scenarios can follow a similar approach, and is omitted for simplicity.

The problem of maximizing the minimum received SNR under a maximum per-relay power budget can be formulated as

$$\begin{aligned} &\max_{\{\mathbf{w}_m\},\gamma} \ \gamma \\ &\text{subject to} \ \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m \leq P_{r,0}, \ i = 1, \cdots, N, \\ &\qquad\qquad \text{SNR}_m \geq \gamma, \ m = 1, \cdots, M \end{aligned} \tag{30}$$

where $P_{r,0}$ denotes the relay power budget. The min-max relay power optimization problem (10) with a common SNR target γ_0 is given by

$$\begin{aligned} &\min_{\{\mathbf{w}_m\},P_r} \ P_r \\ &\text{subject to} \ \text{SNR}_m \geq \gamma_0, \ m = 1, \cdots, M, \\ &\qquad\qquad \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m \leq P_r, \ i = 1, \cdots, N. \end{aligned} \tag{31}$$

Algorithm 2 Solving the min SNR maximization problem (30)

1: Set $\gamma_{0,\min}$ such that $P_r^o(\gamma_{0,\min}) < P_{r,0}$ and $\gamma_{0,\max}$ such that $P_r^o(\gamma_{0,\max}) > P_{r,0}$. Set ε.
2: Set $\gamma_0 = \frac{\gamma_{0,\min}+\gamma_{0,\max}}{2}$.
3: Solve the optimization problem (31) under γ_0.
4: **if** $P_r^o(\gamma_0) > P_{r,0}$ **then**
5:     Set $\gamma_{0,\max} = \gamma_0$ and $P_r = 0$ (or $P_r < P_{r,0} - \varepsilon$).
6: **else**
7:     Set $\gamma_{0,\min} = \gamma_0$ and $P_r = P_r^o(\gamma_0)$.
8: **end if**
9: **if** $P_r < P_{r,0} - \varepsilon$ **then**
10:     Repeat (3)–(9); otherwise, return γ_0.
11: **end if**

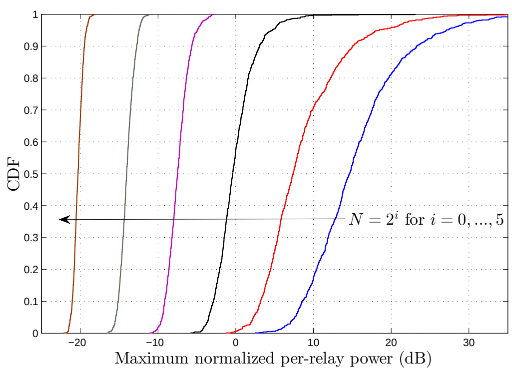

Fig. 2. CDF of maximum normalized relay power with $M = 2$.

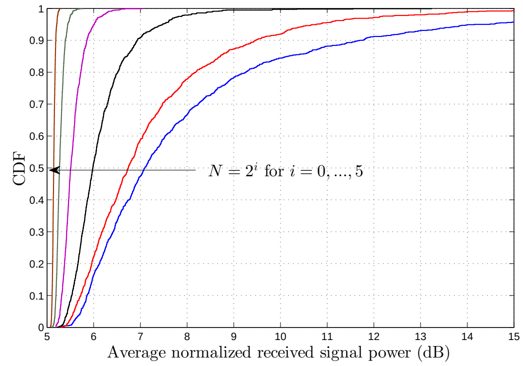

Fig. 3. CDF of average normalized received signal power with $M = 2$.

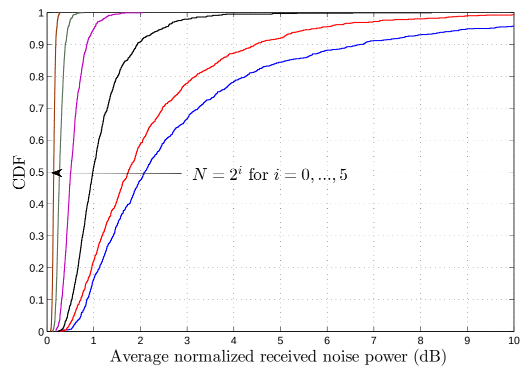

Fig. 4. CDF of average normalized received noise power with $M = 2$.

We use the notations $\gamma^o(P_{r,0})$ and $P_r^o(\gamma_0)$ to denote the optimal objectives in problems (30) and (31), to emphasize their dependency on $P_{r,0}$ and γ_0, respectively. The following proposition shows the property of $\gamma^o(P_{r,0})$ as a function of $P_{r,0}$.

Proposition 4: The optimal max received SNR $\gamma^o(P_{r,0})$ is a continuous and strictly monotonically increasing function of $P_{r,0}$, and any $\gamma < \gamma^o(P_{r,0})$ is achievable.

Proof: See Appendix E.

Following Proposition 4, the min-max per-relay power $P_{r,0}$ is achieved when $\gamma^o(P_{r,0}) = \gamma_0$, for any γ_0, *i.e.*, $P_r^o\big(\gamma^o(P_{r,0})\big) = P_{r,0}$. Hence the optimization problem (30) is the inverse problem of (31), *i.e.*,

$$P_r^o\big(\gamma^o(P_{r,0})\big) = P_{r,0}, \;\; \gamma^o\big(P_r^o(\gamma_0)\big) = \gamma_0.$$

As a result, the SNR maximization problem (30) can be solved iteratively by solving the per-relay power minimization problem (31) with bisection search on the maximum per-relay power target P_r such that $P_r \to P_{r,0}$. The steps to solve the max-min SNR problem (30) using bisection search are summarized in Algorithm 2. It is shown in [31] that SDP problems have nearly linear convergence regardless of the problem size. Furthermore, it is well-known that the bisection algorithm used in Algorithm 2 converges in $\log(\gamma_{0,\max} - \gamma_{0,\min}) - \log\varepsilon$ iterations.

V. Numerical Results

In this section, we provide numerical results to evaluate the performance of the proposed min-max relay power algorithm. In simulation, the noise powers at the relay and destination are set to $\sigma_r^2 = \sigma_d^2 = 1$. The first and second hop channels $\mathbf{h}_m$ and $\mathbf{g}_m$ are assumed i.i.d. zero-mean Gaussian with variance 1. The normalized source transmit power (against destination noise power) is set to $P_0/\sigma_d^2 = 10$ dB. A total of 1000 feasible realizations are used. Unless otherwise specified, the default minimum SNR guarantees are set to $\gamma_m = \gamma_0 = 5$ dB for $m = 1, \cdots, M$.[3]

A. Effect of the Number of Relays

In order to study the effect of the number of relays, N, on the maximum relay power, we plot the CDF of $P_{r,\max}/\sigma_d^2$ obtained in problem (10) under different channel realizations, as shown in Fig. 2. We set $M = 2$. The number of relays are chosen as $N = 2^i$ for $i \in \{0, \cdots, 5\}$. It can be noticed that as N increases, the CDF is shifted to the left, and it also becomes more concentrated. In addition, the CDF curves do not converge as N becomes very large. In fact, those curves are uniformly shifted to the left. The uniform shift is because of the power gain achieved by relay beamforming. The tightening

[3]Note that because of the differences between [29] and ours as discussed in Section I-B, we do not perform any comparison of our solution with that of [29] in simulation.

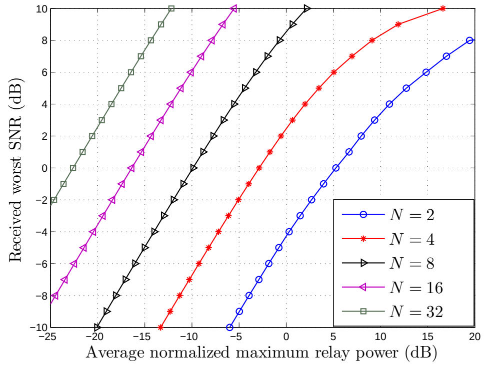

Fig. 5. Average $\min_m \mathrm{SNR}_m$ versus average $P_{r,\max}/\sigma_d^2$ with $M = 4$.

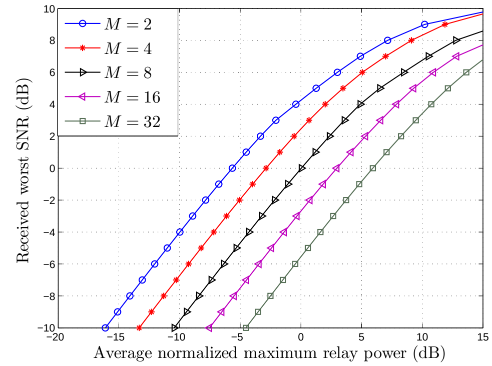

Fig. 7. Average $\min_m \mathrm{SNR}_m$ versus average $P_{r,\max}/\sigma_d^2$ with $N = 4$.

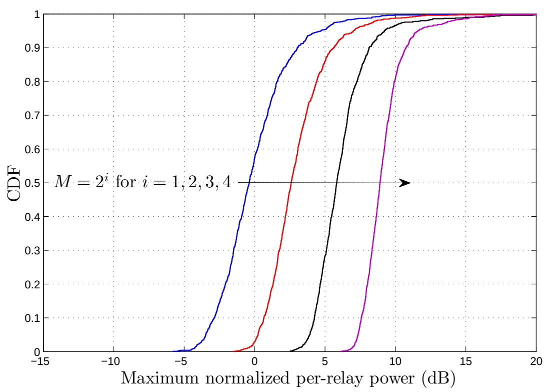

Fig. 6. CDF of maximum normalized relay power with $N = 4$.

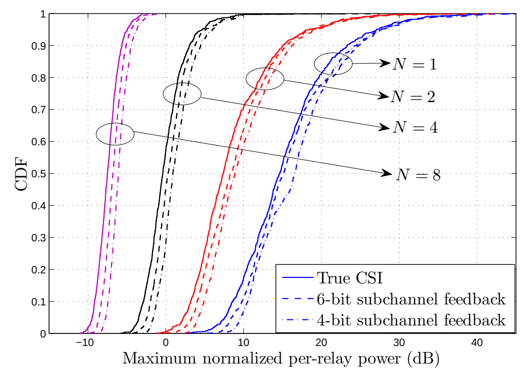

Fig. 8. Empirical CDF of $P_{r,\max}/\sigma_d^2$ for true CSI and limited feedback (Scenario 1) with $M = 2$.

of CDF curves reflects the "hardening" of the effective channel due to beamforming, in the sense that the distribution of the effective channel becomes tighter.

The CDFs of the average received signal in (5) and noise power in (6), each normalized against σ_d^2, with $N = 2^i$ for $i \in \{0, \cdots, 5\}$ and $M = 2$ are shown in Fig. 3 and Fig. 4, respectively. In both figures, we observe that, as N increases, the CDF is shifted to the left. Furthermore, the amount of shift decreases, and the CDF shape becomes tighter. In Fig. 4, as N increases, the amplified noise is reduced to zero, and the overall noise converges to the receiver noise, which is 0 dB. This happens because the beam vector norm $\|\mathbf{w}_m\|$ decreases as N increases. For Fig. 3, as N increases, the normalized received signal power converges to 5 dB which is the minimum SNR requirement.

To demonstrate the result of the max-min SNR problem (30), in Fig. 5, the average minimum received SNR, *i.e.*, $\min_m \mathrm{SNR}_m$ versus average $P_{r,\max}/\sigma_d^2$ is plotted with $M = 4$, and $N = 2^i$ for $i \in \{1, \cdots, 5\}$. To generate each curve, we set the minimum SNR requirement γ_0 from -10 dB to 10 dB. For each γ_0 value, 1000 realizations are generated and the average $P_{r,\max}/\sigma_d^2$ and $\min_m \mathrm{SNR}_m$ are computed for each realization. We see from Fig. 5 that, $\min_m \mathrm{SNR}_m$ is a monotonically increasing function of $P_{r,\max}/\sigma_d^2$. Also, for fixed $P_{r,\max}/\sigma_d^2$, the minimum received SNR $\min_m \mathrm{SNR}_m$ increases by more than 5 dB as N doubles.

B. Effect of the Number of S-D Pairs

For fixed $N = 4$, the CDF of maximum relay power $P_{r,\max}$ from the problem (10), normalized against σ_d^2, under various channel realizations is shown in Fig. 6, with $M = 2^i$ for $i \in \{1, \cdots, 4\}$. As expected, as M increases, more relay power is needed, *i.e.*, the CDF is shifted to the right.

In Fig. 7, the average minimum received SNR ($\min_m \mathrm{SNR}_m$) versus average $P_{r,\max}/\sigma_d^2$ is presented with $N = 4$, and $M = 2^i$ for $i \in \{1, \cdots, 5\}$. We see that, as expected, the average $\min_m \mathrm{SNR}_m$ increases with average $P_{r,\max}/\sigma_d^2$, while it decreases as M increases because the number of SNR constraints increases. Consequently, the relays increase transmission power in order to satisfy the SNR requirement γ_0 for all destinations.

C. Effect of Imperfect CSI

So far, true CSI is assumed. To observe the robustness of the proposed algorithm w.r.t. the limited number of CSI feedback bits and channel estimation error, we consider the following two scenarios when second-hop perfect CSI is not available.

In Scenario 1, there is no error in estimating the second-hop CSI. However, there is a limited number of feedback bits in order to send data to the relays. We consider equiprobable quantization of channel coefficients [33]. Let B denote the number of available feedback bits. In the equiprobable quantization, every real and imaginary part of the channel coefficient

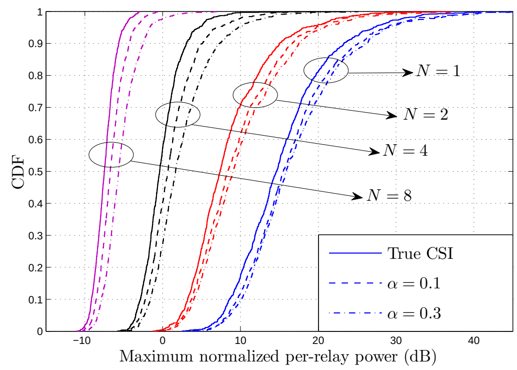

Fig. 9. Empirical CDF of $P_{r,\max}/\sigma_d^2$ for true CSI and Gaussian channel estimation error (Scenario 2) with $M = 2$.

on a subchannel is quantized with equal probability according to the CSI distribution, which is complex Gaussian.

In Scenario 2, the second-hop channels are estimated with estimation error; however, no feedback limit is imposed. Specifically, let us define $\hat{h} = h + \alpha\tilde{h}$, where h is the true subchannel, $\hat{h}$ is the estimated subchannel used in the optimization problem. The estimation error $\tilde{h}$ is assumed Gaussian, *i.e.*, $\tilde{h} \sim \mathcal{CN}(0,1)$. The weight α is set to adjust the variance of error w.r.t. the variance of true CSI.

In Fig. 8, the CDF of $P_{r,\max}/\sigma_d^2$ under true CSI is compared with that under imperfect CSI Scenario 1 with $2B$ bits (B bits for each real and imaginary parts), where $B = 2$ and 3. Note that the performance under limited feedback is close to the case of true CSI. The degradation is similar for all N values.

Finally, Fig. 9 shows the CDF of $P_{r,\max}/\sigma_d^2$ of true CSI as compared with that under imperfect CSI Scenario 2 with the channel estimation error being $\alpha = 0.1$ and 0.3. Again, we observe that the performance gap from the true CSI case is relatively small. Furthermore, we observe that, unlike Scenario 1, the performance is sensitive to N. In particular, the performance degradation increases as N increases.

VI. Conclusions

In this paper, we have investigated the problem of relay beamforming design in a multi-user peer-to-peer relay network in a multi-channel system. Assuming perfect CSI, the problem of minimizing the maximum per-relay power usage subject to minimum received SNR guarantees is formulated. It is shown that the non-convex problem satisfies strong duality. We have expressed its dual problem as an SDP with polynomial worst-case complexity. Based on the values of the optimal dual variables, we have studied the optimal relay beamforming vectors of the original problem in three cases. These cases have reflected at optimality whether the minimum SNR requirement at each S-D pair is met with equality, and whether the power consumption at a relay is the maximum among relays. Furthermore, we have shown that maximizing the minimum received SNR subject to a fixed maximum relay power constraint is the inverse problem of min-max relay power subject to a minimum SNR constraint. The max-min SNR problem is solved iteratively using a bisection search. We have numerically evaluated the proposed algorithm, and analyzed the effect of various system parameters on the performance of the optimal solution. Furthermore, we have investigated the effect of imperfect CSI over the second hop, and quantified the performance loss due to limited feedback or channel estimation error.

Appendix A
Proof of Proposition 1

Proof: The upper-bound of SNR_m is given by (7) by ignoring the receiver noise σ_d^2 in the denominator, *i.e.*,

$$\overline{\mathrm{SNR}}_m \triangleq \frac{P_0|\mathbf{f}_m^H\mathbf{w}_m|^2}{\mathbf{w}_m^H\mathbf{G}_m\mathbf{w}_m}. \tag{A.1}$$

Note that a feasible $\mathbf{w}_m$ is not in the null space of $\mathbf{G}_m$, *i.e.*, $\mathbf{w}_m \notin \mathrm{null}\{\mathbf{G}_m\}$. The upper-bound (A.1) is invariable w.r.t. the scale of $\mathbf{w}$. For a fixed SNR upper-bound, the per-relay power constraint (11) can be satisfied by scaling $\{\mathbf{w}\}$. Hence, a necessary feasibility condition of (10) is given by

$$\max_{\mathbf{w}_m\notin\mathrm{null}\{\mathbf{G}_m\}} \frac{P_0|\mathbf{f}_m^H\mathbf{w}_m|^2}{\mathbf{w}_m^H\mathbf{G}_m\mathbf{w}_m} > \gamma_m,\ m = 1,\cdots,M. \tag{A.2}$$

Using the solution of the generalized eigenvalue problem, the left-hand side of (A.2) is maximized by substituting $\mathbf{w}_m = \mathbf{G}_m^\dagger\mathbf{f}_m$ into (A.1). Noting that the maximum value of (A.1) is $P_0\mathbf{f}_m^H\mathbf{G}_m^\dagger\mathbf{f}$, (12) is obtained and the proof is complete. ■

Appendix B
Proof of Proposition 2

Proof: In order to prove the strong duality property, (10) is rewritten as an SOCP problem in conic form. The SOCP in conic form is convex and therefore has zero duality gap [30]. We need to show that the dual of (10) is equivalent to the dual of the SOCP.

The per-relay power constraint (11) is convex w.r.t. $\mathbf{w} \triangleq [\mathbf{w}_1^T,\cdots,\mathbf{w}_M^T]^T$. However, the minimum received SNR constraint (9) is non-convex. Reformulating the SNR constraint (9) in a conic form, we have

$$\sqrt{P_0}|\mathbf{w}_m^H\mathbf{f}_m| \geq \sqrt{\gamma_m}\left\|\begin{bmatrix}\mathbf{G}_m^{1/2}\mathbf{w}_m\\ \sigma_d\end{bmatrix}\right\|,\ m = 1,\cdots,M. \tag{B.1}$$

Note that $\mathbf{w}_m$ can have any arbitrary phase, *i.e.*, it is obtained uniquely up to a phase shift. The phase could be adjusted such that $\mathbf{w}_m^H\mathbf{f}_m$ becomes real-valued for $m = 1,\cdots,M$. Hence, the optimization problem (10) can be recast as

$$\min_{\{\mathbf{w}_m\},P_{r,\max}} \quad P_{r,\max} \tag{B.2}$$

$$\text{subject to} \ \sqrt{\frac{P_0}{\gamma_m}}\mathbf{w}_m^H\mathbf{f}_m \geq \left\|\begin{bmatrix}\mathbf{G}_m^{1/2}\mathbf{w}_m\\ \sigma_d\end{bmatrix}\right\|,\ m = 1,\cdots,M, \tag{B.3}$$

and (11)

which is an SOCP. The problem (B.2) is non-convex since the constraint (B.3) is not in conic form. It is known that strong duality holds for SOCP in the conic form, but it may not hold in general forms [30]. However, the primal-dual optimality conditions for the problems with constraints in the form of (B.3) are provided in [34, Proposition 3]. Following a similar proof, it can be shown that (B.2) has zero duality gap. In the following, we show that the Lagrangian of (10) is the same as the Lagrangian of (B.2) using a similar proof as in [35, Proposition 1].

The Lagrangian of (10) is given by

$$L_1 = P_{r,\max} + \sum_{i=1}^{N} \lambda_i \Big(\sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m - P_{r,\max} \Big) \quad \text{(B.4)}$$
$$+ \sum_{m=1}^{M} \alpha_m \big(\sigma_d^2 + \mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m - \frac{P_0}{\gamma_m} |\mathbf{w}_m^H \mathbf{f}_m|^2 \big).$$

The Lagrangian of (B.2) is obtained by

$$L_2 = P_{r,\max} + \sum_{i=1}^{N} \tilde{\lambda}_i \Big(\sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m - P_{r,\max} \Big) \quad \text{(B.5)}$$
$$+ \sum_{m=1}^{M} \tilde{\alpha}_m \left(\left\| \begin{bmatrix} \mathbf{G}_m^{1/2} \mathbf{w}_m \\ \sigma_d \end{bmatrix} \right\| - \sqrt{\frac{P_0}{\gamma_m}} |\mathbf{w}_m^H \mathbf{f}_m| \right).$$

Denoting $\varphi_m \triangleq \left\| \begin{bmatrix} \mathbf{G}_m^{1/2} \mathbf{w}_m \\ \sigma_d \end{bmatrix} \right\| + \sqrt{\frac{P_0}{\gamma_m}} |\mathbf{w}_m^H \mathbf{f}_m| \geq \sigma_d$ and converting the last term of the Lagrangian (B.5), it is equivalent to

$$L_2 = P_{r,\max} + \sum_{i=1}^{N} \tilde{\lambda}_i \Big(\sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m - P_{r,\max} \Big)$$
$$+ \sum_{m=1}^{M} \frac{\tilde{\alpha}_m}{\varphi_m} \big(\sigma_d^2 + \mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m - \frac{P_0}{\gamma_m} |\mathbf{w}_m^H \mathbf{f}_m|^2 \big).$$

Since $\varphi_m \geq \sigma_d$, by changing the variables $\alpha_m = \frac{\tilde{\alpha}_m}{\varphi_m}$, there exists $\alpha_m \geq 0$ for any $\tilde{\alpha}_m \geq 0$ and $m = 1, \cdots, M$ such that (B.4) and (B.5) become exactly the same. As a result, strong Lagrange duality holds for the non-convex problem (10). ■

APPENDIX C
PROOF OF LEMMA 1

Proof: Substituting (16) into (18), the constraint (18) is equivalent to

$$\mathbf{R}_m \mathbf{D}_{\boldsymbol{\lambda}} + \alpha_m \big(\mathbf{G}_m - \frac{P_0}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H \big) \succeq 0. \quad \text{(C.1)}$$

Using contradiction, we show that $\mathbf{G}_m - \frac{P_0}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H$ is an indefinite matrix. Suppose that $\mathbf{G}_m \succeq \frac{P_0}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H$. Since $\mathbf{G}_m$ is a positive-definite matrix, we have $P_0 \mathbf{f}_m^H \mathbf{G}_m^{-1} \mathbf{f}_m \leq \gamma_m$. ([35, Lemma 1]). This contradicts the necessary condition for the feasibility of (10) as shown in Proposition 1. If $\boldsymbol{\lambda}^o \succ 0$, there exists $\alpha_m^o > 0$ such that constraint (18) is satisfied. Note that the objective of the dual problem increases as α_m increases. If there exists $\lambda_i^o = 0$ for some i, then α_m^o can be zero for some m. ■

APPENDIX D
PROOF OF THEOREM 3

Proof: Suppose that $\boldsymbol{\lambda}^o$ satisfies the necessary condition in Lemma 1, *i.e.*, the optimal dual variables are in the set defined by Lemma 1. The constraint (18) can be rewritten as an equivalent inequality using [35, Lemma 1] as follows. The dual problem (17) is equivalent to

$$\max_{\boldsymbol{\lambda}} \max_{\boldsymbol{\alpha}} \sum_{m=1}^{M} \alpha_m \sigma_d^2 \quad \text{(D.1)}$$
$$\text{subject to } \frac{\alpha_m P_0}{\gamma_m} \mathbf{f}_m^H \mathbf{K}_m^{-1} \mathbf{f}_m \leq 1, \; m = 1, \cdots, M, \quad \text{(D.2)}$$
$$(19), \text{ and } (14).$$

In the following, we show the duality between (D.1) and SIMO beamforming problem similarly to [35]. Comparing (D.1) with the optimization problem

$$\max_{\boldsymbol{\lambda}} \min_{\boldsymbol{\alpha}} \sum_{m=1}^{M} \alpha_m \sigma_d^2 \quad \text{(D.3)}$$
$$\text{subject to } \frac{\alpha_m P_0}{\gamma_m} \mathbf{f}_m^H \mathbf{K}_m^{-1} \mathbf{f}_m \geq 1, \; m = 1, \cdots, M, \quad \text{(D.4)}$$
$$(19), \text{ and } (14),$$

we see that the inner maximization in (D.1) becomes minimization in (D.3) and the SNR inequality is reversed. Substituting (16) into the left-hand side of (D.2), we define

$$\Phi_m(\alpha_m) \triangleq \frac{\alpha_m P_0}{\gamma_m} \mathbf{f}_m^H \mathbf{K}_m^{-1} \mathbf{f}_m, \quad \text{(D.5)}$$

which is a monotonically increasing function of $\alpha_m > 0$ for $\boldsymbol{\lambda}^o$. Therefore, the constraints (D.2) and (D.4) are met with equality at optimality. The two problems (D.1) and (D.3) have the same optimal value α_m^o satisfying $\Phi_m(\alpha_m^o) = 1$ for $m = 1, \cdots, M$, *i.e.*, the optimization problems (D.1) and (D.3) are equivalent. The SIMO beamforming problem (D.3) is given by substituting $\tilde{\mathbf{w}}_m = \frac{\alpha_m P_0}{\sum_{m=1}^{M} \alpha_m \sigma_d^2} \mathbf{K}_m^{\dagger} \mathbf{f}_m$ into

$$\max_{\boldsymbol{\lambda}} \min_{\mathbf{w}_m, \boldsymbol{\alpha}} \sum_{m=1}^{M} \alpha_m \sigma_d^2 \quad \text{(D.6)}$$
$$\text{subject to } \frac{\alpha_m P_0 |\mathbf{w}_m^H \mathbf{f}_m|^2}{\|\mathbf{K}_m^{\frac{1}{2}} \mathbf{w}_m\|^2} \geq \gamma_m, \; m = 1, \cdots, M, \quad \text{(D.7)}$$
$$(19), \text{ and } (14).$$

For M destinations each equipped with N antennas, the inner minimization of (D.6) is the SIMO beamforming problem, where the transmit power and destination m noise covariance matrix are $\sum_{m=1}^{M} \alpha_m \sigma_d^2$ and $\tilde{\mathbf{K}}_m \triangleq \frac{\sum_{m=1}^{M} \alpha_m \sigma_d^2}{\alpha_m P_0} \mathbf{K}_m$, respectively. The solution of the inner minimization of the SIMO beamforming problem (D.6), is obtained by $\tilde{\mathbf{w}}_m^o = \tilde{\mathbf{K}}_m^{\dagger} \mathbf{f}_m$. Note that (D.3) is given by substituting $\tilde{\mathbf{w}}_m^o$ into (D.6). The solution $\tilde{\mathbf{w}}_m^o$ can be scaled by any non-zero coefficient $\tilde{\xi}$ such that the scaled $\tilde{\xi} \tilde{\mathbf{w}}_m^o$ is also an optimal solution. Hence, the optimization problems (D.1) and (D.6) are equivalent. Considering the condition for $\boldsymbol{\alpha}^o$ in Section III-B2, we have

$\Phi_{\tilde{m}}(\alpha^o_{\tilde{m}}) = 1$ since $\alpha^o_{\tilde{m}} > 0$. Hence, the solution of (D.6) can be used to obtain only $\mathbf{w}^o_{\tilde{m}}$ in (10). The optimal $\mathbf{w}^o_m$ for $m \neq \tilde{m}$ cannot be obtained using the solution of (D.6) because the constraints (D.2) and (D.4) are not met with equality if $\alpha^o_m = 0$. The optimal $\tilde{m}$-th beam vector in (10) is given by $\mathbf{w}^o_{\tilde{m}} = \zeta_{\tilde{m}} \mathbf{K}^o_{\tilde{m}} \mathbf{f}_{\tilde{m}}$ since the strong duality holds for (10) as shown in Proposition 2 and the solution $\tilde{\mathbf{w}}^o_{\tilde{m}}$ is unique only up to a scale factor. Due to KKT conditions and $\alpha^o_{\tilde{m}} > 0$, the SNR constraint (9) is met with equality. The coefficient (22) is obtained by substituting $\mathbf{w}^o_{\tilde{m}}$ into $\frac{P_0 \mathbf{w}^H_{\tilde{m}} \mathbf{F}_{\tilde{m}} \mathbf{w}_{\tilde{m}}}{\mathbf{w}^H_{\tilde{m}} \mathbf{G}_{\tilde{m}} \mathbf{w}_{\tilde{m}} + \sigma^2_d} = \gamma_{\tilde{m}}$, which completes the proof. ■

APPENDIX E
PROOF OF PROPOSITION 4

Proof: Using contradiction, it can be shown that the optimal $\gamma^o(P_{r,0})$ is strictly monotonically increasing function of $P_{r,0}$. Suppose that $\{\mathbf{w}_m\}_{m=1}^M$ is the optimal beam vector of the max-min problem (30) achieving $\gamma^o(P_{r,0})$. Let us assume $P_{r,1} > P_{r,0}$ and $\gamma^o(P_{r,1}) \leq \gamma^o(P_{r,0})$ for some $P_{r,1}$ and $P_{r,0}$. The beam vectors $\{\mathbf{w}_m\}_{m=1}^M$ can be scaled by a real-valued $0 < \chi < 1$ such that, under $\{\chi \mathbf{w}_m\}_{m=1}^M$, the SNR becomes $\gamma^o(P_{r,1})$ with the resulting maximum per-relay power usage $\chi^2 P_{r,0} < P_{r,1}$. This contradicts with the assumption that $P_{r,1}$ is optimal for $\gamma = \gamma^o(P_{r,1})$. It is not difficult to show that $\gamma^o(P_{r,0})$ is continuous w.r.t. $P_{r,0}$. In order to show that any $\gamma < \gamma^o(P_{r,0})$ is achievable, let us denote $\nu \triangleq \arg\min_{m=1,\cdots,M} \mathrm{SNR}_m$ and

$$\eta \triangleq \frac{\sigma_d}{\left(\frac{P_0}{\gamma} \mathbf{w}^H_\nu \mathbf{F}_\nu \mathbf{w}_\nu - \mathbf{w}^H_\nu \mathbf{G}_\nu \mathbf{w}_\nu\right)^{\frac{1}{2}}} > 0. \qquad \text{(E.1)}$$

Note that the denominator of η is positive since $\gamma < \gamma^o(P_{r,0})$. After some manipulation, it can be shown that $\{\eta \mathbf{w}_m\}_{m=1}^M$ achieves any arbitrary $\gamma < \gamma^o(P_{r,0})$. ■

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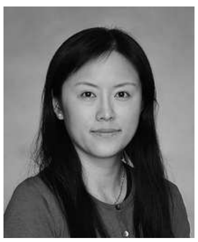

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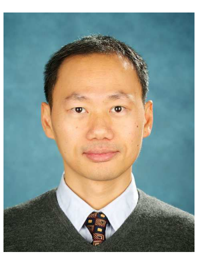

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