Multi-channel Resource Allocation towards Ergodic Rate Maximization for Underlay Device-to-Device Communications

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***Abstract*—In underlay device-to-device (D2D) communications, a D2D pair reuses the cellular spectrum causing interference to regular cellular users. Maximizing the performance of underlay D2D communications requires joint consideration for the achieved D2D rate and the interference to cellular users. In this work, we consider the D2D power allocation optimization over multiple resource blocks (RBs), aiming at maximizing the either the ergodic D2D rate or the ergodic sum rate of D2D and cellular users, under the long-term sum-power constraint of the D2D users and per-RB probabilistic signal-to-interference-and-noise (SINR) requirements for all cellular users. We formulate stochastic optimization problems for D2D power allocation over time. The proposed optimization framework is applicable to both uplink and downlink cellular spectrum sharing. To solve the proposed stochastic optimization problems, we first convexify the problems by introducing a family of convex constraints as a replacement for the non-convex probabilistic SINR constraints. We then present two dynamic power allocation algorithms: a Lagrange dual based algorithm that is optimal but with a high computational complexity, and a low-complexity heuristic algorithm based on dynamic time averaging. Through simulation, we show that the performance gap between the optimal and heuristic algorithms is small, and effective long-term stochastic D2D power optimization over the shared RBs can lead to substantial gains in the ergodic D2D rate and ergodic sum rate.**

***Index Terms*—Device-to-Device communications, ergodic resource allocation, power allocation.**

I. INTRODUCTION

In D2D communications, two user equipments (UEs) directly communicate with each other without having the payload traversed through the backhaul network. Due to its local communications nature, D2D communication can be provided with a lower cost than cellular communications. Furthermore, D2D communications provides many benefits unavailable to uncoordinated communications [2]–[4]. There are many current and prospective applications for D2D communications. For example, D2D has been proposed for use in LTE-based public safety networks for its security and reliability [5]. Additionally, D2D communications is necessary for the scenarios where cellular transmission is not accessible [4].

To facilitate D2D communication, there are different challenges which should be addressed carefully. A survey on the challenges and proposed solutions for D2D communications can be found in [6]. In particular, sharing cellular recourses between D2D and regular cellular users may cause intra-cell and inter-cell interference. One possible option is to allocate different resources for cellular and D2D communications, i.e. overlay D2D communications. However, to achieve the highest possible spectral efficiency, underlay D2D communications has attracted more attention in the literature, where D2D and cellular users within a cell share the same spectrum resource and hence interfere with each other. In this paper we mainly focus on underlay D2D communications.

Underlaying requires effective interference management and resource sharing among all users. Many methods have been presented in the literature to address these problems. For example, Graph-based [7], [8] and game theoretic frameworks [9]–[14] were considered. Power back-off approaches were investigated in [15]–[17], and an interference cancelation method was proposed in [18]. These works do not directly address the optimization of spectrum resource and power allocation in D2D communications.

Closer to our interest, resource and power optimization methods have been proposed in [19]–[26] to maximize the D2D rate, D2D-cellular sum rate, or power-rate efficiency. An optimal power allocation solution for D2D users underlaying cellular users in downlink transmission was given in [19]. The solutions in [19] were achieved without imposing any constraint on the D2D power. In [20], a solution to encompass mode selection, resource allocation, and power control within a single framework was proposed. An energy efficient power control design for resource sharing between cellular and D2D users was proposed in [21]. The authors of [22] investigated a weighted sum-rate maximization with multi-carrier modulation for asynchronous D2D communications. Performance bounds in the maximization of power efficiency under signal-to-noise ratio (SNR) constraints were provided in [23]. The authors of [24] and [25] proposed sub-optimal power allocation solutions for D2D users in uplink transmission,

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which divide the original problem into several easier sub-problems. In [26], an optimal power allocation method based on maximizing application-dependent weighted cell utility was proposed.

However, the studies in [19]–[26] are incomplete and motivate further study in the following two aspects of D2D communications. First, the methods proposed in [19]–[21], [23]–[26] were designed for the simplified scenario where each D2D node accesses only a single channel at a time. They cannot be directly applied to the multi-channel scenario that is prevalent in most practical systems, such as supporting multiple RBs in an LTE network. Second, [20]–[26] considers only short-term power constraints. Yet, D2D nodes are often powered by batteries with limited energy storage capacity, which directly corresponds to long-term D2D power constraints. Furthermore, long-term D2D power allocation on individual RBs can give probabilistic guarantees on the interference from D2D transmitters to cellular users over the shared RBs. These are important characteristics of D2D communications that require further investigation beyond [19]–[26]. In [27] and [28], we solved the D2D-cellular sum-rate maximization problem over multiple RBs. However, the solution was short-term with regard to power and SINR constraints.

In this work, in a multi-channel communication environment, we aim to either maximize the ergodic D2D rate or the ergodic D2D-cellular sum rate by optimizing the power allocation of the D2D users, under the long-term power constraint on the D2D users and per-RB probabilistic SINR constraints for all cellular users. The combination of long-term power and SINR constraints with multi-channel communications leads to a complicated non-convex stochastic optimization problem. Building on our preliminary results presented in [1], the main contributions of this paper are as follows:

- We present a study on ergodic rate maximization with long-term power constraints and per-RB probabilistic SINR constraints in D2D communications. To address the non-convexity in our optimization problem, we propose a family of convex constraints that provides upper and lower bounds for the non-convex probabilistic SINR constraints. In particular, using the Chernoff bound, we further propose a method to reduce the gap between the probabilistic constraint and its convex replacement.
- Subsequently, to further convexify the D2D-cellular sum-rate maximization problem, we replace the objective by a function which, depending on the values of parameters, is either convex and decreasing, or concave and increasing. For the convex decreasing case, we show that optimal allocated power is zero, while for the concave decreasing case, we obtain a convex optimization problem.
- To solve the resulting convex optimization problem, we propose two dynamic algorithms for power allocation over time. The first algorithm is based on the Lagrange duality which provides the optimal power levels over all RBs at each time slot. However, the computational complexity of this algorithm can be prohibitive when the channel state space is large. Therefore, we propose an alternative heuristic algorithm based on dynamic time averaging, which drastically reduces the computational complexity.
- To show the tightness of the power allocation solutions by the proposed algorithms, we propose a method to reformulate the original problems to derive an upper bound of the original problems for comparison.
- Finally, we show that the proposed algorithms are easily scalable and can be applied to more general cases with multiple cells and additional power constraints.

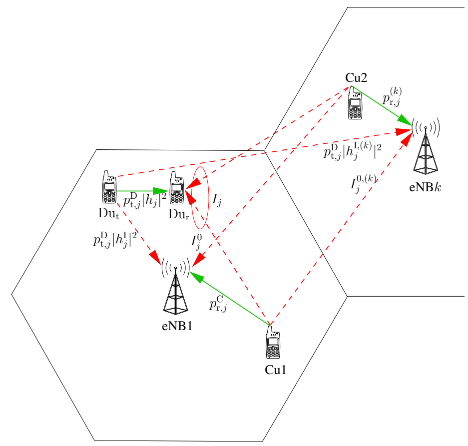


Fig. 1: A cellular network with underlaying D2D users in uplink resource sharing. $\mathrm{Du_t}$ and $\mathrm{Du_r}$: transmit and receive nodes of a D2D pair, respectively. Cu: cellular users. Solid and dashed lines: desired and interfering signals, respectively.

The rest of the paper is organized as follows. Section II presents the system model of the cellular network used in this paper and the resource allocation problem is defined in this section. The proposed methods for solving the D2D-rate maximization problem and the sum-rate maximization problem are presented in Section III and Section IV. In Section V, we discuss extensions of the proposed method to multi-cell scenarios and to accommodate additional power constraints. Section VI presents the simulation results. Section VII concludes the paper.

Notations: We use italic fonts and boldface small letters to represent scalar variables and vectors respectively. The notation $\mathbf{a} \succcurlyeq 0$ means all entries of vector $\mathbf{a}$ are nonnegative. We define $[x]_a^b \triangleq \max\{a, \min\{x, b\}\}$ and $[x]_+ \triangleq \max\{x, 0\}$. For a random process y, $y[n]$ indicates its outcome at time-slot n. We use $x \sim \mathcal{N}(m, \sigma^2)$ to denote a Gaussian random variable with mean m and variance σ^2.

II. System Model and Problem Definition

A. System Model

We consider a cellular system consisting of multiple cellular users and D2D users underlaying the cellular users. We assume that an idle D2D pair arrives at the cell of interest requesting access to spectrum for D2D communications. Due to the localized and low-power transmission of D2D users, we assume the resource planning (e.g., spectrum allocation and power control) of existing cellular users in the network is not modified. As a practical representation of cellular communications, e.g., LTE networks, we assume multiple RBs

TABLE I: Notation Definition

N	number of active cellular users in each cell
$\mathcal{C}$	set of all available RBs in the cell
$\mathcal{C}_l$	set of allocated RBs to the lth D2D pair
$\mathcal{S}_j$	set of neighboring cellular users using RB j
$p^{\mathrm{D}}_{\mathrm{t},j}$	D2D transmitted power over RB j
$p^{\mathrm{C}}_{\mathrm{r},j}$	cellular user received power over RB j
$p^{(k)}_{\mathrm{r},j}$	neighboring cellular user received power over the RB j (for $k \in \mathcal{S}_j$)
I_j	D2D received interference power over RB j
$p^{\mathrm{D}}_{\mathrm{t},j}\lvert h^{\mathrm{I}}_j\rvert^2$	cellular user received interference power over the RB j from the new D2D pair
$p^{\mathrm{D}}_{\mathrm{t},j}\lvert h^{\mathrm{I},(k)}_j\rvert^2$	neighboring cellular user received interference power over the RB j (for $k \in \mathcal{S}_j$) from the new D2D pair
h_j	D2D channel coefficient over RB j
I^0_j	cellular user received interference over RB j before entering the new D2D user
$I^{0,(k)}_j$	neighboring cellular user received interference over RB j (for $k \in \mathcal{S}_j$) before entering the new D2D user
$P^{\mathrm{D}}_{\max}$	maximum available power for a D2D pair
$\zeta^{\mathrm{intra}}_{j,\min}$	cellular user minimum required SINR over RB j
$\zeta^{(k)}_{j,\min}$	neighboring cellular user minimum required SINR over RB j (for $k \in \mathcal{S}_j$)
σ^2	noise power over each RB

are allocated to each user in the network. Since the D2D devices use licensed cellular spectrum, we assume that resource allocation is centrally controlled by the cellular operator. In particular, the RBs are allocated to the cellular and D2D users by the Evolved Node B (eNB). Furthermore, we assume that changes to RB allocation occur at a time scale much large than power allocation, so that when considering the power allocation problem, the RB allocation is viewed as fixed. There is no intra-cell interference among cellular users in a cell because of orthogonal assignment of RBs to the cellular users. However, due to frequency reuse at neighboring cells, these cellular users suffer from inter-cell interference. Fig. 1 shows the interference scenarios for a cellular network with D2D users in uplink resource sharing. The proposed algorithms can be similarly applied to the alternate case of downlink spectrum sharing.

We assume that there are N active cellular users in each cell. A D2D pair attempts to reuse the assigned RBs of active cellular users in the cell and $\mathcal{C}$ is the set of all available RBs within the cell. Let $\mathcal{C}_l$ indicate the set of allocated RBs to the lth D2D pair. For $j \in \mathcal{C}_l$, let $p^{\mathrm{D}}_{\mathrm{t},j}$ denote the transmit power of the D2D pair over the jth RB and $p^{\mathrm{C}}_{\mathrm{r},j}$ denote the received power from the unique cellular user that is assigned to the jth RB. In addition, let $\mathcal{S}_j$ denote the set of all cellular users in the neighboring cells that are using the jth RB. Let $p^{(k)}_{\mathrm{r},j}$ denote the received power from the kth user in $\mathcal{S}_j$ over the jth RB.

The cellular users have both intra-cell interference from the D2D transmission and inter-cell interference from neighboring cells. For $j \in \mathcal{C}_l$, let I^0_j and $I^{0,(k)}_j$ denote the received interference power over the jth RB for the corresponding cellular user in the main cell and the kth neighboring user, respectively, excluding the interference from the D2D pair under consideration. For the uplink sharing, let $|h^{\mathrm{I}}_j|^2$ and $|h^{\mathrm{I},(k)}_j|^2$ denote the channel power gains over the jth RB between the D2D transmitter and the eNB and between the D2D transmitter and the kth neighboring cellular user's eNB, for $k \in \mathcal{S}_j$, respectively (for the downlink case, the same notation can be used, except that the eNBs are replaced by the corresponding cellular users.). Furthermore, let I_j denote the received interference power over the jth RB at the D2D receiver. And finally, let h_j denote the D2D channel coefficient over the jth RB. Under the fading environment, all channel power gains and interference power are random variables. The notation used throughout this paper is summarized in Table I.

B. *Ergodic Rate Optimization Problem*

For the uplink transmission, the received SINR of the cellular user over the jth RB at the eNB in the main cell, at the eNB of the kth neighboring cellular user in $\mathcal{S}_j$, and at the D2D receiver are respectively given by [1]

$$\mathrm{SINR}^{\mathrm{C}}_j = \frac{p^{\mathrm{C}}_{\mathrm{r},j}}{\sigma^2 + I^0_j + p^{\mathrm{D}}_{\mathrm{t},j}|h^{\mathrm{I}}_j|^2}, \tag{1}$$

$$\mathrm{SINR}^{\mathrm{C},(k)}_j = \frac{p^{(k)}_{\mathrm{r},j}}{\sigma^2 + I^{0,(k)}_j + p^{\mathrm{D}}_{\mathrm{t},j}|h^{\mathrm{I},(k)}_j|^2}, \quad k \in \mathcal{S}_j, \tag{2}$$

$$\mathrm{SINR}^{\mathrm{D}}_j = \frac{|h_j|^2 p^{\mathrm{D}}_{\mathrm{t},j}}{\sigma^2 + I_j}. \tag{3}$$

In order to maintain the quality of service for the cellular users at a specific level, it is important to control the interference from the D2D transmitter to the cellular users in the main cell and also in the neighboring cells. Therefore, the D2D power over each RB must be confined. We first consider the following constraints

$$\Pr\Big\{\mathrm{SINR}^{\mathrm{C}}_j \le \zeta^{\mathrm{intra}}_{j,\min}\Big\} \le \epsilon, \;\; j \in \mathcal{C}_l \tag{4}$$

$$\Pr\Big\{\mathrm{SINR}^{\mathrm{C},(k)}_j \le \zeta^{(k)}_{j,\min}\Big\} \le \epsilon, \;\; j \in \mathcal{C}_l, \;\; k \in \mathcal{S}_j \tag{5}$$

where $\zeta^{\mathrm{intra}}_{j,\min}$ and $\zeta^{(k)}_{j,\min}$ are minimum SINR targets for the cellular user in the main cell and the kth neighboring cellular user in $\mathcal{S}_j$, respectively. These constraints guarantee a specific long-term QoS for the cellular users in the main cell and

[1]For the downlink, SINR is defined by replacing the eNB with the cellular user.

neighboring cells. We define

$$\eta_j \triangleq \min\left\{\frac{p^{\mathrm{C}}_{\mathrm{r},j}/\zeta^{\mathrm{intra}}_{j,\min}-(\sigma^2+I^0_j)}{|h^{\mathrm{I}}_j|^2}\right.,\left.\left\{\frac{p^{(k)}_{\mathrm{r},j}/\zeta^{(k)}_{j,\min}-(\sigma^2+I^{0,(k)}_j)}{|h^{\mathrm{I},(k)}_j|^2}\right\}_{k\in\mathcal{S}_j}\right\}. \quad (6)$$

It is easy to show that (4) and (5) are equivalent to the following constraint:

$$\Pr\Big\{p^{\mathrm{D}}_{\mathrm{t},j}\geq\eta_j\Big\}\leq\epsilon. \quad (7)$$

Furthermore, in order to limit power usage for the D2D user, we additionally consider a long-term sum-power constraint for the D2D pair as follows:

$$\mathbb{E}\Big\{\sum_{j\in\mathcal{C}_l}p^{\mathrm{D}}_{\mathrm{t},j}\Big\}\leq P^{\mathrm{D}}_{\max}. \quad (8)$$

The statistical constraints on the D2D transmission power in (7) and (8) are more practical than the deterministic ones commonly assumed in the literature [19], [23]–[26]. Instead of imposing instantaneous, strict SINR and power constraints in each time slot, we allow their fluctuations over time. Constraint (7) models long-term QoS requirements, while the constraint (8) corresponds to the need to conserve energy especially for battery-powered D2D equipment. The resultant additional degree of freedom in dynamic adjustment of the D2D transmission power, tailored to the time-varying channel conditions, can lead to substantial gains in the ergodic D2D rate and D2D-cellular sum rate. This will be numerically demonstrated in Section III-E and IV-B, where we compare the cases where the D2D transmission power is properly designed over time under statistical constraints, and where it is deterministically bounded in each time slot. Furthermore, in Section V-B, we will discuss how the proposed solution can be easily extended to the case where there are both statistical and deterministic constraints on the D2D transmission power.

Thus, in this paper, we study the following two stochastic power allocation problems to find the optimal power in each time slot over each RB for the new D2D pair:

I) Ergodic D2D-Rate Maximization Problem

$$\mathcal{D}1:\quad \max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}\succcurlyeq\mathbf{0}}\ \mathbb{E}\Big\{\sum_{j\in\mathcal{C}_l}\log(1+\mathrm{SINR}^{\mathrm{D}}_j)\Big\}$$
$$\text{subject to}\quad (7)\text{ and }(8);$$

II) Ergodic Sum-Rate Maximization Problem

$$\mathcal{S}1:\max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}\succcurlyeq\mathbf{0}}\ \mathbb{E}\Big\{\sum_{j\in\mathcal{C}_l}\log(1+\mathrm{SINR}^{\mathrm{C}}_j)+\log(1+\mathrm{SINR}^{\mathrm{D}}_j)\Big\}$$
$$\text{subject to}\quad (7)\text{ and }(8),$$

where we define $\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}=[p^{\mathrm{D}}_{\mathrm{t},1},\cdots,p^{\mathrm{D}}_{\mathrm{t},|\mathcal{C}_l|}]^T$. Note that, in the above optimization problems, the transmission power $\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}$ is a mapping from the random channel state vector to a power allocation vector.

C. Feasibility Check

Consider the SINR constraints in (4)-(5). The feasible set is non-empty only if we have

$$\Pr\Big\{\frac{p^{\mathrm{C}}_{\mathrm{r},j}}{\sigma^2+I^0_j}\leq\zeta^{\mathrm{intra}}_{j,\min}\Big\}\leq\epsilon,\quad j\in\mathcal{C}_l, \quad (9)$$

$$\Pr\Big\{\frac{p^{(k)}_{\mathrm{r},j}}{\sigma^2+I^{0,(k)}_j}\leq\zeta^{(k)}_{j,\min}\Big\}\leq\epsilon,\quad j\in\mathcal{C}_l,\quad k\in\mathcal{S}_j. \quad (10)$$

For example, in the case that all signal and interface powers are exponentially distributed, the constraints in (4)-(5) are equivalent to

$$1-\frac{e^{-\lambda^{\mathrm{C}}_{\mathrm{p},j}\zeta^{\mathrm{intra}}_{j,\min}\sigma^2}}{1+\frac{\lambda^{\mathrm{C}}_{\mathrm{p},j}\zeta^{\mathrm{intra}}_{j,\min}}{\lambda_{\mathrm{I},j}}}\leq\epsilon,\quad j\in\mathcal{C}_l, \quad (11)$$

$$1-\frac{e^{-\lambda^{(k)}_{\mathrm{p},j}\zeta^{(k)}_{j,\min}\sigma^2}}{1+\frac{\lambda^{(k)}_{\mathrm{p},j}\zeta^{(k)}_{j,\min}}{\lambda^{(k)}_{\mathrm{I},j}}}\leq\epsilon,\quad j\in\mathcal{C}_l,\quad k\in\mathcal{S}_j, \quad (12)$$

where $\lambda^{\mathrm{C}}_{\mathrm{p},j}$, $\lambda^{(k)}_{\mathrm{p},j}$, $\lambda_{\mathrm{I},j}$ and $\lambda^{(k)}_{\mathrm{I},j}$ are the rates of exponentially distributed random variables $p^{\mathrm{C}}_{\mathrm{r},j}$, $p^{(k)}_{\mathrm{r},j}$, I^0_j and $I^{0,(k)}_j$, respectively.

III. D2D Rate Maximization

The stochastic optimization problem $\mathcal{D}1$ can be reformulated as an equivalent deterministic optimization problem, in which all expectations in the optimization problem $\mathcal{D}1$ can be written as probability-weighted sums of functions of realizations of the random channel state vector over all RBs. In this reformulation, the decision variable is $\mathbf{p}^{\mathrm{D}}_{\mathrm{t},j}$ corresponding to every realization of the channel state vector. However, the complexity of directly solving such an optimization problem would be prohibitive, due to the exponential size of the multi-dimensional channel state space. Instead, we propose to first convexify the optimization problem $\mathcal{D}1$. We will then show that using Lagrange multipliers, the problem of finding $p^{\mathrm{D}}_{\mathrm{t},j}$ can be solved *separately* over each observed realization of channel states.

A. Convexification of Problem $\mathcal{D}1$

The probabilistic individual power constraint in (7) is not convex. We consider instead stronger convex constraints using the following lemma.

Lemma 1. *For any strictly increasing function $f(\cdot)$ such that $f(0)=1$, we have*

$$\Pr\Big\{p^{\mathrm{D}}_{\mathrm{t},j}\geq\eta_j\Big\}\leq\mathbb{E}\Big\{f(p^{\mathrm{D}}_{\mathrm{t},j}-\eta_j)\Big\}. \quad (13)$$

Proof: Since $f(\cdot)$ is a strictly increasing function, we have

$$\begin{aligned}\Pr\Big\{p^{\mathrm{D}}_{\mathrm{t},j}\geq\eta_j\Big\}&=\Pr\Big\{p^{\mathrm{D}}_{\mathrm{t},j}-\eta_j\geq0\Big\}\\&=\Pr\Big\{f(p^{\mathrm{D}}_{\mathrm{t},j}-\eta_j)\geq f(0)\Big\}\\&\leq\mathbb{E}\Big\{f(p^{\mathrm{D}}_{\mathrm{t},j}-\eta_j)\Big\},\end{aligned} \quad (14)$$

where the last inequality is achieved by applying Markov's inequality and the assumption that $f(0) = 1$. ■

Note that in Lemma 1, $f(\cdot)$ does not need to be convex. However, to obtain a convex optimization problem, we will use only convex increasing functions. We propose substituting the following constraint for the constraint in (7):

$$\mathbb{E}\Big\{ f_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j) \Big\} \le \epsilon, \tag{15}$$

where $f_j(\cdot)$'s are convex increasing functions for all $j \in \mathcal{C}_l$. By satisfying (15), the constraint in (7) will be guaranteed. Thus, we can find a lower-bound for $\mathcal{D}1$ by using (15) and solving the new convex optimization problem:

$$\begin{aligned} \mathcal{D}2: \quad & \max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t},j} \succcurlyeq \mathbf{0}} \ \mathbb{E}\Big\{ \sum_{j \in \mathcal{C}_l} \log\left(1 + \frac{|h_j|^2 p^{\mathrm{D}}_{\mathrm{t},j}}{\sigma^2 + I_j}\right) \Big\} \\ & \text{subject to} \ \ \mathbb{E}\Big\{ \sum_{j \in \mathcal{C}_l} p^{\mathrm{D}}_{\mathrm{t},j} \Big\} \le P^{\mathrm{D}}_{\max} \qquad (16) \\ & \qquad \mathbb{E}\Big\{ f_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j) \Big\} \le \epsilon, \quad j \in \mathcal{C}_l. \qquad (17) \end{aligned}$$

B. Solution via the Lagrange Method

The Lagrange function of $\mathcal{D}2$ can be written as

$$\begin{aligned} &\mathcal{L}_D(\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}, \boldsymbol{\lambda}, \mu) \\ &= \sum_{j \in \mathcal{C}_l} \mathbb{E}\Big\{ \log\left(1 + \frac{|h_j|^2 p^{\mathrm{D}}_{\mathrm{t},j}}{\sigma^2 + I_j}\right) - \mu p^{\mathrm{D}}_{\mathrm{t},j} - \lambda_j f_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j) \Big\} \\ &\quad + \mu P^{\mathrm{D}}_{\max} + \sum_{j \in \mathcal{C}_l} \lambda_j \epsilon \end{aligned} \tag{18}$$

where $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_{|\mathcal{C}_l|}]^T$ is a vector of Lagrange multipliers. The corresponding Lagrange dual function is

$$g_D(\boldsymbol{\lambda}, \mu) = \max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t}} \succcurlyeq \mathbf{0}} \quad \mathcal{L}_D(\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}, \boldsymbol{\lambda}, \mu). \tag{19}$$

To find the optimal $p^{\mathrm{D}}_{\mathrm{t},j}$, for fixed values of λ_j and μ, we need to solve the following optimization problem for each j and each channel realization of the jth RB:

$$\begin{aligned} p^{\mathrm{D}*}_{\mathrm{t},j}(\lambda_j, \mu) = \arg \max_{p^{\mathrm{D}}_{\mathrm{t},j} \ge 0} \log\left(1 + \frac{|h_j|^2 p^{\mathrm{D}}_{\mathrm{t},j}}{\sigma^2 + I_j}\right) \\ - \mu p^{\mathrm{D}}_{\mathrm{t},j} - \lambda_j f_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j) \end{aligned} \tag{20}$$

where $p^{\mathrm{D}*}_{\mathrm{t},j}(\lambda_j, \mu)$ is the optimal power allocation. Note that the problem in (20) is convex and the optimal value can be found efficiently.

The optimal values for $\boldsymbol{\lambda}$ and μ can be found through the dual optimization problem

$$\min_{\boldsymbol{\lambda} \succcurlyeq \mathbf{0}, \mu \ge 0} \quad g_D(\boldsymbol{\lambda}, \mu) \tag{21}$$

using the subgradient method [29]. To find subgradients, we note that

$$\begin{aligned} & g_D(\boldsymbol{\lambda}', \mu') \\ &= \max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t}} \succcurlyeq \mathbf{0}} \ \mathcal{L}_D(\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}, \boldsymbol{\lambda}', \mu') \\ &\ge \mathcal{L}_D(\mathbf{p}^{\mathrm{D}*}_{\mathrm{t}}(\boldsymbol{\lambda}, \mu), \boldsymbol{\lambda}', \mu') \\ &= g_D(\boldsymbol{\lambda}, \mu) + \sum_{j \in C_l} (\lambda'_j - \lambda_j)(\epsilon - \mathbb{E}\{f_j(p^{\mathrm{D}*}_{\mathrm{t},j}(\lambda_j, \mu) - \eta_j)\}) \\ &\quad + (\mu' - \mu)(P^{\mathrm{D}}_{\max} - \mathbb{E}\{\sum_{j \in C_l} p^{\mathrm{D}*}_{\mathrm{t},j}(\lambda_j, \mu)\}). \end{aligned} \tag{22}$$

Hence, the following are subgradients of $g(\boldsymbol{\lambda}, \mu)$:

$$\partial \mu = P^{\mathrm{D}}_{\max} - \sum_{j \in C_l} \mathbb{E}\{p^{\mathrm{D}*}_{\mathrm{t},j}(\lambda_j, \mu)\} \tag{23}$$

$$\partial \lambda_j = \epsilon - \mathbb{E}\{f_j(p^{\mathrm{D}*}_{\mathrm{t},j}(\lambda_j, \mu) - \eta_j)\} \quad j \in C_l. \tag{24}$$

Following the subgradient method, after a sufficient number of iterations, we can find the value of optimal Lagrange multipliers, i.e., μ^* and λ^*_j for all $j \in \mathcal{C}_l$. Then, for each channel realization, we need to solve the following optimization problem to find an optimal power allocation:

$$\begin{aligned} p^{\mathrm{D}*}_{\mathrm{t},j}(H) = \arg \max_{p^{\mathrm{D}}_{\mathrm{t},j} \ge 0} \ \log\left(1 + \frac{|h_j|^2 p^{\mathrm{D}}_{\mathrm{t},j}}{\sigma^2 + I_j}\right) \\ - \mu^* p^{\mathrm{D}}_{\mathrm{t},j} - \lambda^*_j f_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j(H)) \end{aligned} \tag{25}$$

where H is the vector of channel state.

Note that considering (23) and (24), the above formulation can only be applied over a time interval where we can assume the channel for D2D users is stationary.

C. Special Case for Function $f_j(\cdot)$: Chernoff Bound

Using the Chernoff bound, (15) becomes

$$\mathbb{E}\Big\{ e^{\omega_j (p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)} \Big\} \le \epsilon. \tag{26}$$

If we use the Chernoff bound as a substitute of the probabilistic SINR constraint, i.e., (7), it is important to properly choose the value of ω_j to achieve the minimal gap between the two constraint. In fact, we have:

$$\Pr\Big\{ p^{\mathrm{D}}_{\mathrm{t},j} \ge \eta_j \Big\} \le \min_{\omega_j} \mathbb{E}\Big\{ e^{\omega_j (p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)} \Big\} \le \mathbb{E}\Big\{ e^{\omega_j (p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)} \Big\}. \tag{27}$$

To find an optimal ω_j we have

$$\frac{\partial}{\partial \omega_j} \mathbb{E}\Big\{ e^{\omega_j (p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)} \Big\} = \mathbb{E}\Big\{ (p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j) e^{\omega_j (p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)} \Big\} = 0. \tag{28}$$

We define the random variable $x_j = p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j$. Unfortunately, the distribution of x_j is not known before solving the optimization problem. However, we observe that x_j is a complicated mixture of multiple random quantities, and our numerical results indicate it has roughly bell-shape distribution. Thus, we assume that $x_j \sim \mathcal{N}(m_j, \upsilon_j)$ and

$$\begin{aligned} & \mathbb{E}\Big\{ x_j e^{\omega_j x_j} \Big\} \\ &= \int_{-\infty}^{+\infty} x e^{\omega_j x} \frac{1}{\upsilon_j \sqrt{2\pi}} e^{-\frac{(x - m_j)^2}{2\upsilon_j^2}} dx \\ &= e^{\frac{(\omega_j \upsilon_j^2 + m_j)^2 - m_j^2}{2\upsilon_j^2}} \int_{-\infty}^{+\infty} y \frac{1}{\upsilon_j \sqrt{2\pi}} e^{-\frac{(y - (\omega_j \upsilon_j^2 + m_j))^2}{2\upsilon_j^2}} dy. \end{aligned} \tag{29}$$

Note that the integral in (29) is the mean value for the random variable $y_j \sim \mathcal{N}(\omega_j \upsilon_j^2 + m_j, \upsilon_j)$. Therefore, from (28) and (29), a suitable value for ω_j can be found as $-\frac{m_j}{\upsilon_j^2}$. Since

m_j and υ_j are not known, we use an iterative method where, starting from an initial point for ω_j, in each step we update the value of ω_j for the next step by estimating m_j and υ_j using the available information to compute the statistics of x_j in the current step.

D. Special Case for Function $f_j(\cdot)$: Polynomial Functions

It is easy to show that, for any $m \in \mathbb{N}$, the function $f(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j) = [1 + \frac{1}{\eta_j^{\max}}(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)]^m$, where $\eta_j^{\max}$ is the maximum possible value for η_j, is convex and increasing with $f(0) = 1$. In this case, we have the following constraint:

$$\mathbb{E}\Big\{[1 + \omega_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)]^m\Big\} \leq \epsilon, \tag{30}$$

where $\omega_j = \frac{1}{\eta_j^{\max}}$.

In the special case where (30) is linear, i.e., $m = 1$, we can provide a closed-form solution to (20) as follows. Setting the derivative of (20) equal to zero, for $f_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j) = 1 + \omega_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)$, we have

$$\frac{|h_j|^2}{|h_j|^2 p^{\mathrm{D}}_{\mathrm{t},j} + \sigma^2 + I_j} - \mu - \lambda_j\omega_j = 0. \tag{31}$$

Thus,

$$p^{\mathrm{D}*}_{\mathrm{t},j}(\lambda_j, \mu) = \Big[\frac{1}{\mu + \lambda_j\omega_j} - \frac{\sigma^2 + I_j}{|h_j|^2}\Big]_+. \tag{32}$$

E. Time-Averaging Based Heuristic Solutions

In the optimal solutions above, the calculation of subgradients has high complexity and needs full information about the statistics of the channels between D2D users and all interference channels. In this section, we present a new method to directly find the Lagrange multipliers by approximating the primal domain problem.

Considering the fact that μ is related to constraint (16), in each step, we can find $\mu[n+1]$ by associating it with the approximated constraint $\mathbb{E}_{n+1}\Big\{\sum_{j\in\mathcal{C}_l} p^{\mathrm{D}}_{\mathrm{t},j}\Big\} \leq P^{\mathrm{D}}_{\max}$, where n is the time slot index and $\mathbb{E}_n\{x\} \triangleq \frac{1}{n}\sum_{t=1}^{n} x[t]$. It is easy to show that we have $\mathbb{E}_{n+1}\{x\} = \frac{1}{n+1}(x[n+1] + n\mathbb{E}_n\{x\})$. Thus, the sum power constraint in (8) can be written as

$$\begin{aligned}\sum_{j\in\mathcal{C}_l} p^{\mathrm{D}}_{\mathrm{t},j}[n+1] &\leq (n+1)P^{\mathrm{D}}_{\max} - n\mathbb{E}_n\Big\{\sum_{j\in\mathcal{C}_l} p^{\mathrm{D}}_{\mathrm{t},j}\Big\}\\ &\triangleq P'^{\mathrm{D}}_{\max}[n+1].\end{aligned} \tag{33}$$

Similarly, the individual power constraints in (17) can be approximated as

$$\begin{aligned}&p^{\mathrm{D}}_{\mathrm{t},j}[n+1]\\ &\quad\leq \eta_j[n+1] + f_j^{-1}\Big((n+1)\epsilon - n\mathbb{E}_n\{f_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)\}\Big)\\ &\quad\triangleq \eta'_j[n+1],\end{aligned} \tag{34}$$

where $f_j^{-1}(\cdot)$ is the inverse function of $f_j(\cdot)$. Since, $f_j(\cdot)$ is a strictly increasing function, $f_j^{-1}(\cdot)$ is unique.

In time slot n, we may solve the following optimization problem

$$\begin{aligned}\mathcal{D}3: \quad &\max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t}} \succcurlyeq \mathbf{0}} \quad \sum_{j\in\mathcal{C}_l} \log\left(1 + \frac{|h_j[n]|^2 p^{\mathrm{D}}_{\mathrm{t},j}}{\sigma^2 + I_j[n]}\right)\\ &\text{subject to} \quad (33) \text{ and } (34).\end{aligned}$$

After applying the KKT optimality condition to the primal problem [29], we have two cases.
Case 1) $\sum_{j\in\mathcal{C}_l} \eta'_j[n] \leq P'^{\mathrm{D}}_{\max}$: $p^{\mathrm{D}*}_{\mathrm{t},j}[n] = \eta'_j[n]$, for all $j \in \mathcal{C}_l$.
Case 2) $\sum_{j\in\mathcal{C}_l} \eta'_j[n] > P'^{\mathrm{D}}_{\max}[n]$: for all $j \in \mathcal{C}_l$ we have

$$p^{\mathrm{D}*}_{\mathrm{t},j}[n] = \left[\frac{1}{\mu[n]} - \frac{\sigma^2 + I_j[n]}{|h_j[n]|^2}\right]_0^{\eta'_j[n]}. \tag{35}$$

Note that in Case 2, $\mu[n]$ must be found such that $\sum_{j\in\mathcal{C}_l} p^{\mathrm{D}}_{\mathrm{t},j}[n] = P'^{\mathrm{D}}_{\max}[n]$, which can be achieved using the bisection method.

Remark. It is worth mentioning that the sum-power constraint in (33) is equivalent to

$$\mathbb{E}_n\Big\{\sum_{j\in\mathcal{C}_l} p^{\mathrm{D}}_{\mathrm{t},j}\Big\} \leq P^{\mathrm{D}}_{\max}, \tag{36}$$

and also the individual power constraint in (34) is equivalent to

$$\mathbb{E}_n\Big\{f_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)\Big\} \leq \epsilon \tag{37}$$

which are valid for all n. For sufficiently large n, by assuming ergodicity for all channels in the network, satisfying the constraints in (33) and (34) guarantees satisfying the constraints in stochastic optimization problem in $\mathcal{D}2$. For small values of n, since there is no information on the future channel state, the feasible set of $\mathcal{D}3$ is a subset of the feasible set of $\mathcal{D}2$. By increasing the time-window size, i.e., for larger n, the feasible set of $\mathcal{D}3$ converges to the feasible set of $\mathcal{D}2$. In other words, the per-time-slot optimization problem $\mathcal{D}3$ provides a lower bound for the stochastic optimization problem $\mathcal{D}2$.

F. An Upper Bound for $\mathcal{D}1$

For a performance benchmark, we propose an upper bound for the optimization problem in $\mathcal{D}1$ and consider the gap between the lower and upper bound. In this section we try to reach a proper upper bound.

Theorem 1. *For any $\omega_j \geq 0$, $\Pr\Big\{p^{\mathrm{D}}_{\mathrm{t},j} \geq \eta_j\Big\} \geq 1 - \mathbb{E}\Big\{e^{-\omega_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)}\Big\}$.*

Proof: We have $\Pr\Big\{p^{\mathrm{D}}_{\mathrm{t},j} \geq \eta_j\Big\} = \Pr\Big\{-\omega_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j) \leq 0\Big\} = \Pr\Big\{e^{-\omega_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)} \leq 1\Big\}$. From the Markov inequality we have

$$\Pr\Big\{e^{-\omega(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)} \leq 1\Big\} \geq= 1 - \mathbb{E}\Big\{e^{-\omega_j(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j)}\Big\}. \tag{38}$$

■

The optimization problem for finding the upper bound can be written as follows

$$\mathcal{D}4: \quad \max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t}} \succcurlyeq \mathbf{0}} \quad \mathbb{E}\Big\{\sum_{j\in\mathcal{C}_l} \log\left(\frac{a_j + b_j p^{\mathrm{D}}_{\mathrm{t},j}}{a_j + c_j p^{\mathrm{D}}_{\mathrm{t},j}}\right)\Big\} \tag{39}$$

$$\text{subject to} \quad \mathbb{E}\Big\{ \sum_{j\in\mathcal{C}_l} p_{\mathrm{t},j}^{\mathrm{D}} \Big\} \le P_{\max}^{\mathrm{D}} \tag{40}$$

$$\mathbb{E}\Big\{ 1 - e^{-\omega_j (p_{\mathrm{t},j}^{\mathrm{D}} - \eta_j)} \Big\} \le \epsilon, \quad j \in \mathcal{C}_l. \tag{41}$$

Unfortunately, because the constraint in (41) is concave, $\mathcal{D}4$ is not a convex optimization problem. However, the dual problem is always convex and provides an upper bound for the primal problem. Therefore, we can use the dual problem to find an upper bound for $\mathcal{D}1$. The Lagrange function of (39) can be written as

$$\begin{aligned}
&\mathcal{L}_1(\mathbf{p}_{\mathrm{t}}^{\mathrm{D}}, \boldsymbol{\lambda}, \mu) \\
&\triangleq \sum_{j\in\mathcal{C}_l} \mathbb{E}\Big\{ \log\left(1 + \frac{|h_j|^2 p_{\mathrm{t},j}^{\mathrm{D}}}{\sigma^2 + I_j}\right) \Big\} + \mu (P_{\max}^{\mathrm{D}} - \sum_{j\in\mathcal{C}_l} \mathbb{E}\Big\{ p_{\mathrm{t},j}^{\mathrm{D}} \Big\}) \\
&\qquad + \sum_{j\in\mathcal{C}_l} \lambda_j (\epsilon - 1 + \mathbb{E}\Big\{ e^{-\omega_j (p_{\mathrm{t},j}^{\mathrm{D}} - \eta_j)} \Big\}) \\
&= \sum_{j\in\mathcal{C}_l} \mathbb{E}\Big\{ \log\left(\frac{a_j + b_j p_{\mathrm{t},j}^{\mathrm{D}}}{a_j + c_j p_{\mathrm{t},j}^{\mathrm{D}}}\right) - \mu p_{\mathrm{t},j}^{\mathrm{D}} + \lambda_j e^{-\omega_j (p_{\mathrm{t},j}^{\mathrm{D}} - \eta_j)} \Big\} \\
&\qquad + \mu P_{\max}^{\mathrm{D}} + \sum_{j\in\mathcal{C}_l} \lambda_j (\epsilon - 1).
\end{aligned} \tag{42}$$

The corresponding Lagrange dual function can be defined is

$$g_1(\boldsymbol{\lambda}, \mu) = \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}} \succcurlyeq \mathbf{0}} \quad \mathcal{L}_1(\mathbf{p}_{\mathrm{t}}^{\mathrm{D}}, \boldsymbol{\lambda}, \mu). \tag{43}$$

To find an optimal $p_{\mathrm{t},j}^{\mathrm{D}}$, for fixed values of λ_j and μ and for $j \in C_l$, in (43) we need to solve the following optimization problem for each j

$$\begin{aligned}
p_{\mathrm{t},j}^{\mathrm{D}*}(\lambda_j, \mu) = \arg \max_{p_{\mathrm{t},j}^{\mathrm{D}} \ge 0} \log\left(1 + \frac{|h_j|^2 p_{\mathrm{t},j}^{\mathrm{D}}}{\sigma^2 + I_j}\right) \\
- \mu p_{\mathrm{t},j}^{\mathrm{D}} + \lambda_j e^{-\omega_j (p_{\mathrm{t},j}^{\mathrm{D}} - \eta_j)}.
\end{aligned} \tag{44}$$

Note that (44) is not a convex optimization problem. Thus we need to apply an extremum-search method to find the global optimum point. Then, to solve (43), the subgradient method can be used.

Improving the value of ω_j: Similarly to Section III-C, to improve ω_j for the upper bound, we have

$$\begin{aligned}
\frac{\partial}{\partial \omega_j}(1 - \mathbb{E}\Big\{ e^{-\omega_j (p_{\mathrm{t},j}^{\mathrm{D}} - \eta_j)} \Big\}) = \mathbb{E}\Big\{ (p_{\mathrm{t},j}^{\mathrm{D}} - \eta_j) e^{-\omega_j (p_{\mathrm{t},j}^{\mathrm{D}} - \eta_j)} \Big\} \\
= 0.
\end{aligned} \tag{45}$$

Assuming that $x_j = p_{\mathrm{t},j}^{\mathrm{D}} - \eta_j \sim \mathcal{N}(m_j, \upsilon_j)$, we have

$$\begin{aligned}
&\mathbb{E}\Big\{ x_j e^{\omega_j x_j} \Big\} \\
&= \int_{-\infty}^{+\infty} x e^{-\omega_j x} \frac{1}{\upsilon_j \sqrt{2\pi}} e^{-\frac{(x - m_j)^2}{2\upsilon_j^2}} dx \\
&= e^{\frac{(m_j - \omega_j \upsilon_j^2)^2 - m_j^2}{2\upsilon_j^2}} \int_{-\infty}^{+\infty} y \frac{1}{\upsilon_j \sqrt{2\pi}} e^{-\frac{(y - (m_j - \omega_j \upsilon_j^2))^2}{2\upsilon_j^2}} dy = 0.
\end{aligned}$$

From the above equation, a suitable value for ω_j can be found as $\frac{m_j}{\upsilon_j^2}$. Hence, we can update the value of ω_j using an iterative method as discussed in Section III-C.

IV. Ergodic Sum-Rate Maximization

Similarly to the previous section, we first convexify the non-convex optimization problem in $\mathcal{S}1$ and then use the Lagrange duality to find the D2D power allocation $p_{\mathrm{t},j}^{\mathrm{D}}$ *separately* over each observed outcome (or "realization") of the channel state vector. However, the more complex non-convex form of the D2D-cellular sum-rate objective presents further challenges. In this section, we detail the additional procedures to solve $\mathcal{S}1$. We use the same definition and special cases of $f_j(\cdot)$ as in the previous section.

Note that the sum-rate of cellular users over RBs in $\mathcal{C}_l$, prior to the D2D pair entering the system, is given by $\sum_{j\in\mathcal{C}_l} \log(1 + \frac{p_{\mathrm{r},j}^{\mathrm{C}}}{\sigma^2 + I_j^{C}})$. It is independent of D2D transmitter power allocation. Thus, the sum-rate maximization problem $\mathcal{S}1$ is equivalent to the problem of maximizing the ergodic sum-rate *improvement* due to the addition of the new D2D pair, given by

$$\begin{aligned}
\mathcal{S}1' : \quad \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}} \succcurlyeq \mathbf{0}} \; \mathbb{E}\Big\{ \sum_{j\in\mathcal{C}_l} \log(1 + \mathrm{SINR}_j^{\mathrm{C}}) + \log(1 + \mathrm{SINR}_j^{\mathrm{D}}) \\
- \log\left(1 + \frac{p_{\mathrm{r},j}^{\mathrm{C}}}{I_j^0 + \sigma_j^2}\right) \Big\} \\
\text{subject to} \quad (7) \text{ and } (8).
\end{aligned}$$

A. Convexification of $\mathcal{S}1$

Typically, only those RBs over which the cellular users have a sufficiently high SINR condition are allocated to the D2D user. After D2D reuse, the SINR of the cellular user over such an RB is still relatively high. Therefore, we assume the minimum SINR requirement $\zeta_{j,\min}^{\mathrm{intra}} \gg 1$, for all $j \in \mathcal{C}_l$. With this assumption, we can use following approximation to approximate the objective of $\mathcal{S}1'$ as

$$\begin{aligned}
&\sum_{j\in\mathcal{C}_l} \log(1 + \mathrm{SINR}_j^{\mathrm{C}}) - \log\left(1 + \frac{p_{\mathrm{r},j}^{\mathrm{C}}}{I_j^0 + \sigma_j^2}\right) + \log(1 + \mathrm{SINR}_j^{\mathrm{D}}) \\
&\approx \sum_{j\in\mathcal{C}_l} \Bigg[\log\left(\frac{p_{\mathrm{r},j}^{\mathrm{C}}}{p_{\mathrm{t},j}^{\mathrm{D}} |h_j^{\mathrm{I}}|^2 + I_j^0 + \sigma^2}\right) - \log\left(\frac{p_{\mathrm{r},j}^{\mathrm{C}}}{I_j^0 + \sigma^2}\right) \\
&\qquad + \log\left(1 + \frac{|h_j|^2 p_{\mathrm{t},j}^{\mathrm{D}}}{I_j + \sigma^2}\right) \Bigg] \\
&= \sum_{j\in\mathcal{C}_l} \log\left(\frac{a_j + b_j p_{\mathrm{t},j}^{\mathrm{D}}}{a_j + c_j p_{\mathrm{t},j}^{\mathrm{D}}}\right),
\end{aligned} \tag{46}$$

where $a_j \triangleq (\sigma^2 + I_j^0)(\sigma^2 + I_j)$, $b_j \triangleq (\sigma^2 + I_j^0)|h_j|^2$, and $c_j \triangleq (\sigma^2 + I_j)|h_j^{\mathrm{I}}|^2$. Thus, we can approximate $\mathcal{S}1$ as

$$\begin{aligned}
\mathcal{S}2 : \quad \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}} \succcurlyeq \mathbf{0}} \quad \mathbb{E}\Big\{ \sum_{j\in\mathcal{C}_l} \log(\frac{a_j + b_j p_{\mathrm{t},j}^{\mathrm{D}}}{a_j + c_j p_{\mathrm{t},j}^{\mathrm{D}}}) \Big\} \\
\text{subject to} \quad (7) \text{ and } (8).
\end{aligned}$$

As discussed in section III, for the D2D-rate maximization problem, (7) is not a convex constraint, so we substitute it with (16). Therefore, we propose the following optimization problem:

$$\mathcal{S}3 : \quad \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}} \succcurlyeq \mathbf{0}} \quad \mathbb{E}\Big\{ \sum_{j\in\mathcal{C}_l} \log(\frac{a_j + b_j p_{\mathrm{t},j}^{\mathrm{D}}}{a_j + c_j p_{\mathrm{t},j}^{\mathrm{D}}}) \Big\}$$

$$\text{subject to} \quad (16) \text{ and } (17).$$

Furthermore, the objective function of $\mathcal{S}3$, for $\mathbf{p}_{\rm t}^{\rm D} \succcurlyeq \mathbf{0}$, can be upper bounded as follows:

$$\begin{aligned}
&\mathbb{E}\Big\{\sum_{j\in\mathcal{C}_l}\log(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}})\Big\}\\
&\quad=\sum_{j\in\mathcal{C}_l}\Pr\{b_j\geq c_j\}\mathbb{E}\Big\{\log(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}}\Big|b_j\geq c_j)\\
&\qquad\quad+\Pr\{b_j<c_j\}\mathbb{E}\Big\{\log(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}})\Big|b_j<c_j\Big\}\\
&\quad\leq\sum_{j\in\mathcal{C}_l}\Pr\{b_j\geq c_j\}\mathbb{E}\Big\{\log(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}})\Big|b_j\geq c_j\Big\}. \quad (47)
\end{aligned}$$

The last inequality comes from the fact that the function $\log(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}})$, for $b_j<c_j$ and $p_{{\rm t},j}^{\rm D}\geq 0$, is a decreasing (and convex) function. The upper bound is achievable if and only if for $b_j<c_j$ we have $p_{{\rm t},j}^{\rm D}=0$. In other words, $p_{{\rm t},j}^{\rm D}=0$ is an optimal solution when we have $b_j<c_j$ if and only if all the constraint in $\mathcal{S}3$ are satisfied.

The sum-power constraint is satisfied if we have

$$\begin{aligned}
\mathbb{E}\Big\{\sum_{j\in\mathcal{C}_l}p_{{\rm t},j}^{\rm D}\Big\}&=\sum_{j\in\mathcal{C}_l}(\Pr\{b_j\geq c_j\}\mathbb{E}\Big\{p_{{\rm t},j}^{\rm D}\Big|b_j\geq c_j\Big\}\\
&\qquad+\Pr\{b_j<c_j\}\mathbb{E}\Big\{p_{{\rm t},j}^{\rm D}\Big|b_j<c_j\Big\})\\
&=\sum_{j\in\mathcal{C}_l}\Pr\{b_j\geq c_j\}\mathbb{E}\Big\{p_{{\rm t},j}^{\rm D}\Big|b_j\geq c_j\Big\} \quad (48)\\
&\leq P_{\max}^{\rm D}.
\end{aligned}$$

The individual power constraints are satisfied if we have

$$\begin{aligned}
\mathbb{E}\Big\{f_j(p_{{\rm t},j}^{\rm D}-\eta_j)\Big\}&=\Pr\{b_j\geq c_j\}\mathbb{E}\Big\{f_j(p_{{\rm t},j}^{\rm D}-\eta_j)\Big|b_j\geq c_j\Big\}\\
&\quad+\Pr\{b_j<c_j\}\mathbb{E}\Big\{f_j(p_{{\rm t},j}^{\rm D}-\eta_j)\Big|b_j<c_j\Big\}\\
&=\Pr\{b_j\geq c_j\}\mathbb{E}\Big\{f_j(p_{{\rm t},j}^{\rm D}-\eta_j)\Big|b_j\geq c_j\Big\}\\
&\quad+\Pr\{b_j<c_j\}\mathbb{E}\Big\{f_j(-\eta_j)\Big|b_j<c_j\Big\}\\
&\leq\epsilon. \quad (49)
\end{aligned}$$

Therefore, $\mathcal{S}3$ is equivalent to the following optimization problem

$$\begin{aligned}
\mathcal{S}4:\ \max_{\mathbf{p}_{\rm t}^{\rm D}\succcurlyeq\mathbf{0}}\ &\sum_{j\in\mathcal{C}_l}\mathcal{P}_j\mathbb{E}\Big\{\log(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}})\Big|b_j\geq c_j\Big\}\\
\text{subject to}\ &\sum_{j\in\mathcal{C}_l}\mathcal{P}_j\mathbb{E}\Big\{p_{{\rm t},j}^{\rm D}\Big|b_j\geq c_j\Big\}\leq P_{\max}^{\rm D} \quad (50)\\
&\mathcal{P}_j\mathbb{E}\Big\{f_j(p_{{\rm t},j}^{\rm D}-\eta_j)\Big|b_j\geq c_j\Big\}\leq\epsilon_j' \quad j\in\mathcal{C}_l, \quad (51)
\end{aligned}$$

where we define $\mathcal{P}_j\triangleq\Pr\{b_j\geq c_j\}$ for all $j\in\mathcal{C}_l$ and $\epsilon_j'\triangleq\epsilon-(1-\mathcal{P}_j)\mathbb{E}\{f_j(-\eta_j)\Big|b_j<c_j\}$. We will later show that there is no need to calculate ϵ_j'.

It is easy to show that any solution for $\mathcal{S}4$, by applying $p_{{\rm t},j}^{\rm D}=0$ when we have $b_j<c_j$, is feasible for the problem $\mathcal{S}3$. Also, again by applying $p_{{\rm t},j}^{\rm D}=0$ when we have $b_j<c_j$, the objective function of the two problem are the same. Thus, the two problems $\mathcal{S}3$ and $\mathcal{S}4$ are equivalent.

Note that the function $\log(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}})$, for $b_j\geq c_j$ and $p_{{\rm t},j}^{\rm D}\geq 0$, is a concave (and increasing) function, thus we can use the method of Lagrange multipliers to solve $\mathcal{S}4$. The Lagrange function of $\mathcal{S}4$ can be written as

$$\begin{aligned}
&\mathcal{L}_S(\mathbf{p}_{\rm t}^{\rm D},\boldsymbol{\lambda},\mu)\\
&\triangleq\sum_{j\in\mathcal{C}_l}\mathcal{P}_j\mathbb{E}\Big\{\log(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}})\Big|b_j\geq c_j\Big\}\\
&\qquad+\mu(P_{\max}^{\rm D}-\sum_{j\in\mathcal{C}_l}\mathcal{P}_j\mathbb{E}\Big\{p_{{\rm t},j}^{\rm D}\Big|b_j\geq c_j\Big\})\\
&\qquad+\sum_{j\in\mathcal{C}_l}\lambda_j(\epsilon'-\mathcal{P}_j\mathbb{E}\Big\{f_j(p_{{\rm t},j}^{\rm D}-\eta_j)\Big|b_j\geq c_j\Big\})\\
&=\sum_{j\in\mathcal{C}_l}\mathcal{P}_j\mathbb{E}\Big\{\log(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}})-\mu p_{{\rm t},j}^{\rm D}-\lambda_jf_j(p_{{\rm t},j}^{\rm D}-\eta_j)\Big|b_j\geq c_j\Big\}\\
&\qquad+\mu P_{\max}^{\rm D}+\sum_{j\in\mathcal{C}_l}\lambda_j\epsilon_j'. \quad (52)
\end{aligned}$$

By applying the fact that for $b_j<c_j$ we have $p_{{\rm t},j}^{\rm D}=0$, the Lagrange function also can be written as follows:

$$\begin{aligned}
&\mathcal{L}_S(\mathbf{p}_{\rm t}^{\rm D},\boldsymbol{\lambda},\mu)\\
&\quad=\sum_{j\in\mathcal{C}_l}\mathbb{E}\Big\{\log(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}})-\mu p_{{\rm t},j}^{\rm D}-\lambda_jf_j(p_{{\rm t},j}^{\rm D}-\eta_j)\Big\}\\
&\qquad+\mu P_{\max}^{\rm D}+\sum_{j\in\mathcal{C}_l}\lambda_j\epsilon. \quad (53)
\end{aligned}$$

The Lagrange dual function is

$$g_S(\boldsymbol{\lambda},\mu)=\max_{\mathbf{p}_{\rm t}^{\rm D}\succcurlyeq\mathbf{0}}\quad\mathcal{L}_S(\mathbf{p}_{\rm t}^{\rm D},\boldsymbol{\lambda},\mu). \quad (54)$$

To find optimal $p_{{\rm t},j}^{\rm D}$, for fixed values of λ_j and μ and for $j\in C_l$, in (54) we need to solve following optimization problem for each j

$$\begin{aligned}
p_{{\rm t},j}^{\rm D*}(\lambda_j,\mu)=\arg\max_{p_{{\rm t},j}^{\rm D}\geq 0}\log\left(\frac{a_j+b_jp_{{\rm t},j}^{\rm D}}{a_j+c_jp_{{\rm t},j}^{\rm D}}\right)\\
-\mu p_{{\rm t},j}^{\rm D}-\lambda_jf_j(p_{{\rm t},j}^{\rm D}-\eta_j). \quad (55)
\end{aligned}$$

Note that (55) is only valid for $b_j\geq c_j$, and for $b_j<c_j$ we have $p_{{\rm t},j}^{\rm D*}(\lambda_j,\mu)=0$. Hence, it is a convex optimization problem thus the optimal value can be found efficiently.

We next consider the following convex optimization problem to find the optimal $\boldsymbol{\lambda}$ and μ:

$$\min_{\boldsymbol{\lambda}\succcurlyeq\mathbf{0},\mu\geq 0}\quad g_S(\boldsymbol{\lambda},\mu). \quad (56)$$

To solve (56), the subgradient method can be exploited. It is easy to show that the subgradients are

$$\begin{aligned}
\partial\lambda_j&=\epsilon_j'-\mathcal{P}_j\mathbb{E}\{f_j(p_{{\rm t},j}^{\rm D*}(\lambda_j,\mu)-\eta_j)\Big|b_j\geq c_j\},\quad j\in C_l,\\
\partial\mu&=P_{\max}^{\rm D}-\sum_{j\in C_l}\mathcal{P}_j\mathbb{E}\{p_{{\rm t},j}^{\rm D*}(\lambda_j,\mu)\Big|b_j\geq c_j\}.
\end{aligned}$$

Using (48) and (49), the subgradients can be re-written as

$$\partial\lambda_j=\epsilon-\mathbb{E}\{f_j(p_{{\rm t},j}^{\rm D*}(\lambda_j,\mu)-\eta_j)\},\quad j\in C_l, \quad (57)$$

$$\partial\mu = P_{\max}^{\mathrm{D}} - \sum_{j\in C_l} \mathbb{E}\{p_{\mathrm{t},j}^{\mathrm{D}*}(\lambda_j,\mu)\}. \quad (58)$$

We note that, (57) and (58) are exactly the same as (23) and (24). After a sufficient number of iterations in the subgradient method, the value of optimal lagrange multipliers, i.e., λ_j^*, for all $j \in C_{l^*}$, and μ^*, are found. Then, for each channel state, we solve the following optimization problem to find the optimal power allocation:

$$p_{\mathrm{t},j}^{\mathrm{D}*}(H) = \arg\max_{p_{\mathrm{t},j}^{\mathrm{D}}\geq 0} \log\left(\frac{a_j(H)+b_j(H)p_{\mathrm{t},j}^{\mathrm{D}}}{a_j(H)+c_j(H)p_{\mathrm{t},j}^{\mathrm{D}}}\right) - \mu^* p_{\mathrm{t},j}^{\mathrm{D}} - \lambda_j^* e^{\omega_j(p_{\mathrm{t},j}^{\mathrm{D}}-\eta_j(H))}. \quad (59)$$

Note that considering (57) and (58), the above formulation can only be applied over a time-interval where we can assume the channel for D2D users is stationary.

Remark. It is worth mentioning that since we have the same constraints for both optimization problem $\mathcal{S}1$ and $\mathcal{D}1$, the same approach, as discussed in Section III-C, can be applied to sum-rate maximization to improve the Chernoff bound.

Remark. Using the family of linear constraints in (30), we can provide a closed-form solution for (55). Setting the derivative of (55) equal to zero, for $f_j(p_{\mathrm{t},j}^{\mathrm{D}}-\eta_j) = \omega_j(p_{\mathrm{t},j}^{\mathrm{D}}-\eta_j)+1$, we have

$$\frac{a_jb_j - a_jc_j}{(a_j+b_jp_{\mathrm{t},j}^{\mathrm{D}})(a_j+c_jp_{\mathrm{t},j}^{\mathrm{D}})} - \mu - \lambda_j\omega_j = 0, \quad (60)$$

i.e., we have $(\omega_j\lambda_j+\mu)b_jc_j(p_{\mathrm{t},j}^{\mathrm{D}})^2 + (\omega_j\lambda_j+\mu)a_j(b_j+c_j)p_{\mathrm{t},j}^{\mathrm{D}} + ((\omega_j\lambda_j+\mu)a_j^2 - a_jb_j + a_jc_j) = 0$ or equivalently $\kappa_j(p_{\mathrm{t},j}^{\mathrm{D}})^2 + \beta_jp_{\mathrm{t},j}^{\mathrm{D}} + (1-\frac{\gamma_j}{(\omega_j\lambda_j+\mu)}) = 0$ where $\kappa_j \triangleq \frac{b_jc_j}{a_j^2} > 0$, $\beta_j \triangleq \frac{b_j+c_j}{a_j} > 0$ and $\gamma_j \triangleq \frac{b_j-c_j}{a_j} > 0$. Summation of the roots of this equation is $-\frac{\beta_j}{\kappa_j}$, which is negative, so we have at least one negative root. Hence, only the greater root can be accepted or otherwise the solution for (55) is zero. Therefore, we have

$$p_{\mathrm{t},j}^{\mathrm{D}*}(\lambda_j,\mu) = \left[\frac{-\beta_j + \sqrt{\beta_j^2 - 4\kappa_j(1-\frac{\gamma_j}{(\mu+\omega_j\lambda_j)})}}{2\kappa_j}\right]_+. \quad (61)$$

B. Time-Averaging Based Heuristic Solutions

Using the same idea of time-averaging instead of statistical means, as we explained previously for $\mathcal{D}2$, we propose the following a heuristic solution for $\mathcal{S}3$. In time slot n, we solve the following optimization problem:

$$\mathcal{S}5: \quad \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}}\succcurlyeq\mathbf{0}} \quad \sum_{j\in\mathcal{C}_l} \log\left(\frac{a_j[n]+b_j[n]p_{\mathrm{t},j}^{\mathrm{D}}}{a_j[n]+c_j[n]p_{\mathrm{t},j}^{\mathrm{D}}}\right)$$
$$\text{subject to} \quad (33) \text{ and } (34).$$

After applying the KKT optimality condition to the primal problem [29], we have two cases:
Case 1) $\sum_{j\in\mathcal{C}_l}\eta_j'[n] \leq P_{\max}^{'\mathrm{D}}$: $p_{\mathrm{t},j}^{\mathrm{D}*}[n] = \eta_j'[n]$, for all $j\in\mathcal{C}_l$.
Case 2) $\sum_{j\in\mathcal{C}_l}\eta_j'[n] > P_{\max}^{'\mathrm{D}}[n]$: for all $j\in\mathcal{C}_l$ we have

$$p_{\mathrm{t},j}^{\mathrm{D}*}[n] = \left[\frac{-\beta_j[n] + \sqrt{\beta_j[n]^2 - 4\kappa_j[n](1-\frac{\gamma_j[n]}{\mu[n]})}}{2\kappa_j[n]}\right]_0^{\eta_j'[n]}. \quad (62)$$

In each time slot, $\mu[n]$ should be found such that $\sum_{j\in\mathcal{C}_l}p_{\mathrm{t},j}^{\mathrm{D}}[n] \leq P_{\max}^{'\mathrm{D}}[n]$. This can be achieved using the bisection method.

Since we have the same constraints for the optimization problems $\mathcal{D}3$ and $\mathcal{S}5$, all other discussions in Section III-E on solving $\mathcal{D}3$ are valid for $\mathcal{S}5$.

C. An Upper Bound for $\mathcal{S}1$

Similarly to Section III-F, the optimization problem for finding the upper bound can be written as follows

$$\mathcal{S}6: \quad \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}}\succcurlyeq\mathbf{0}} \quad \mathbb{E}\Big\{\sum_{j\in\mathcal{C}_l}\log(\frac{a_j+b_jp_{\mathrm{t},j}^{\mathrm{D}}}{a_j+c_jp_{\mathrm{t},j}^{\mathrm{D}}})\Big\}$$
$$\text{subject to} \quad (40) \text{ and } (41).$$

As discussed before, $\mathcal{S}6$ is not a convex problem and an upper bound cannot be achieved using straight-forward convex optimization methods. We instead use the dual problem to find an upper bound. The Lagrange function of $\mathcal{S}6$ can be written as

$$\begin{aligned}
&\mathcal{L}_2(\mathbf{p}_{\mathrm{t}}^{\mathrm{D}},\boldsymbol{\lambda},\mu) \\
&\triangleq \sum_{j\in\mathcal{C}_l}\mathbb{E}\Big\{\log(\frac{a_j+b_jp_{\mathrm{t},j}^{\mathrm{D}}}{a_j+c_jp_{\mathrm{t},j}^{\mathrm{D}}})\Big\} + \mu(P_{\max}^{\mathrm{D}} - \sum_{j\in\mathcal{C}_l}\mathbb{E}\Big\{p_{\mathrm{t},j}^{\mathrm{D}}\Big\}) \\
&\quad + \sum_{j\in\mathcal{C}_l}\lambda_j(\epsilon - 1 + \mathbb{E}\Big\{e^{-\omega_j(p_{\mathrm{t},j}^{\mathrm{D}}-\eta_j)}\Big\}) \\
&= \sum_{j\in\mathcal{C}_l}\mathbb{E}\Big\{\log(\frac{a_j+b_jp_{\mathrm{t},j}^{\mathrm{D}}}{a_j+c_jp_{\mathrm{t},j}^{\mathrm{D}}}) - \mu p_{\mathrm{t},j}^{\mathrm{D}} + \lambda_je^{-\omega_j(p_{\mathrm{t},j}^{\mathrm{D}}-\eta_j)}\Big\} \\
&\quad + \mu P_{\max}^{\mathrm{D}} + \sum_{j\in\mathcal{C}_l}\lambda_j(\epsilon-1). \quad (63)
\end{aligned}$$

The corresponding Lagrange dual function can be defined is

$$g_2(\boldsymbol{\lambda},\mu) = \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}}\succcurlyeq\mathbf{0}} \quad \mathcal{L}_2(\mathbf{p}_{\mathrm{t}}^{\mathrm{D}},\boldsymbol{\lambda},\mu). \quad (64)$$

To find an optimal $p_{\mathrm{t},j}^{\mathrm{D}}$, for fixed values of λ_j and μ and for $j\in C_l$, in (64) we need to solve the following optimization problem for each j:

$$p_{\mathrm{t},j}^{\mathrm{D}*}(\lambda_j,\mu) = \arg\max_{p_{\mathrm{t},j}^{\mathrm{D}}\geq 0}\log\left(\frac{a_j+b_jp_{\mathrm{t},j}^{\mathrm{D}}}{a_j+c_jp_{\mathrm{t},j}^{\mathrm{D}}}\right) - \mu p_{\mathrm{t},j}^{\mathrm{D}} + \lambda_je^{-\omega_j(p_{\mathrm{t},j}^{\mathrm{D}}-\eta_j)}. \quad (65)$$

Noting that (65) is not a convex optimization problem, we apply an extremum-search method to find the global optimum point. After finding the global maximum point, we can use the subgradient method to find the optimal Lagrange multipliers.

V. Discussion on Generalizations

A. Multi-D2D Multi-cell Scenario

The proposed sum-rate and D2D rate maximization framework can be applied to a realistic multi-cell network with multiple D2D pairs and cellular users in each cell. In general, each D2D user has its own arrival time and duration of stay in the network. Upon the arrival of any D2D user, a scheduler within each cell (which can reside within the eNB) can decide the resources to be allocated to the D2D user. To avoid large overhead and to decrease the computational complexity, the resource allocation decision for each D2D user can be made separately by the scheduler, which is in harmony with the fact that the nodes in a cellular system have different arrival time and duration of stay in the network. Furthermore, by implementing the same algorithms (i.e. sum-rate maximization or D2D rate maximization) in all neighboring cells, we guarantee all the constraints in (7) and (8), i.e., sum-power constraint for all D2D users and a pre-defined SINR level for all cellular users in the network, while optimizing the throughput performance separately for each cell.

However, in the multi-cell scenario, applying the subgradient method for sum-rate and D2D rate maximization may cause some difficulties. As we notice before, the subgradient method can be applied only under the assumption that the signal and interference channels to the D2D user under consideration is stationary. In the multi-cell scenario, the entry of a new D2D user into the network changes the interference distribution for all other D2D users in the neighboring cells which use the same RBs as allocated to the new user. In that case, the stationarity assumption is not valid anymore. Only in a very crowded network, or in a large cell with D2D users that are not very close to the edges of the cell, can we assume that the interference to D2D users is approximately stationary.

Note that for the proposed time-averaging heuristic solution does not require such a stationarity assumption. Therefore, it can be applied to any multi-cell network efficiently while guaranteeing all the constraints in (7) and (8) for all users in the network.

B. More Generalized Optimization Problem

So far, we have considered the optimization problems in $\mathcal{D}1$ and $\mathcal{S}1$ with long-term sum-power constraint (8) on the D2D users. The constraint in (8) addresses the battery usage of the D2D users. In reality, the instantaneous power may also be confined due to physical design of the mobile device such as power amplifier and antenna power limitations. We can add a short-term power constraint as follows:

$$\sum_{j\in\mathcal{C}_l} p^{\mathrm{D}}_{\mathrm{t},j} \leq P^{\mathrm{D}}_{\mathrm{max,s}}. \tag{66}$$

The same procedure as described in this paper can be used to solve the new optimization problems with this additional constraint. In particular, for the D2D-rate optimization problem, to find the optimal power allocation we need to solve the maximization problem in (20) subject to (66). To solve the sum-rate optimization problem, the maximization problem in (55) must be solved subject to constraint in (66). Since (66) is a linear constraint, solving (20) and (55) under this constraint still admits efficient convex optimization methods.

VI. Simulation Evaluation

We consider a cellular network with 10 cellular users in each cell and $N = 10$ distinct RBs are assigned to each cellular user. Cellular and D2D users are uniformly randomly distributed over each cell. We apply a power control mechanism which compensates for the free-space path loss effects for all cellular users. For the link between any two nodes, we use a simple path loss model $K_0 D^{-\alpha}$, where we set the path loss constant $K_o = 0.01$, and the pass loss exponent $\alpha = 3.5$. We assume Rayleigh fading for all interference, cellular and D2D links. The default values of different system parameters are presented in Table II.

TABLE II: Default values for simulation parameters

$N = 10$
Number of RBs per cellular user $= 10$
RB bandwidth $=$ 12x15 KHz
Cell radius $(R_c) = 100$ m
D2D distance $(d) = 20$ m
Number of D2D pairs in each cell $(N_d) = 7$
Ave. cellular SNR $= 30$ dB
Free space path-loss factor $= 3.5$
Standard deviation for shadowing $= 6$ dB

In each cell, when there are multiple active D2D pairs requesting to share RBs, they are queued based on a first-come-first-serve rule. We solve the proposed optimization problems for the D2D pairs one by one based on their order in the queue [2]. To allocate RBs to each D2D pair, first we need to find all available RBs that provide a non-empty feasible set for each optimization problem. Let us assume that $\mathcal{C}^*_l$ is the set of all available RBs such that we have (11)-(12) satisfied. We need to find $\mathcal{C}_l \subseteq \mathcal{C}^*_l$ such that $|\mathcal{C}_l| \leq N_l$. To find the optimal $\mathcal{C}_l$ we would need to solve $\binom{N^*_l}{N_l}$ optimization problems where $N^*_l = |\mathcal{C}^*_l|$ (assuming $N^*_l \geq N_l$). Instead, we use the following heuristic method for a more practical solution. Using the Markov's inequality instead of the probabilistic constraints in (7) yields

$$\mathbb{E}\Big\{p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j + 1\Big\} \leq \epsilon. \tag{67}$$

Equivalently, we have

$$\mathbb{E}\Big\{p^{\mathrm{D}}_{\mathrm{t},j}\Big\} \leq \mathbb{E}\Big\{\eta_j\Big\} + \epsilon - 1. \tag{68}$$

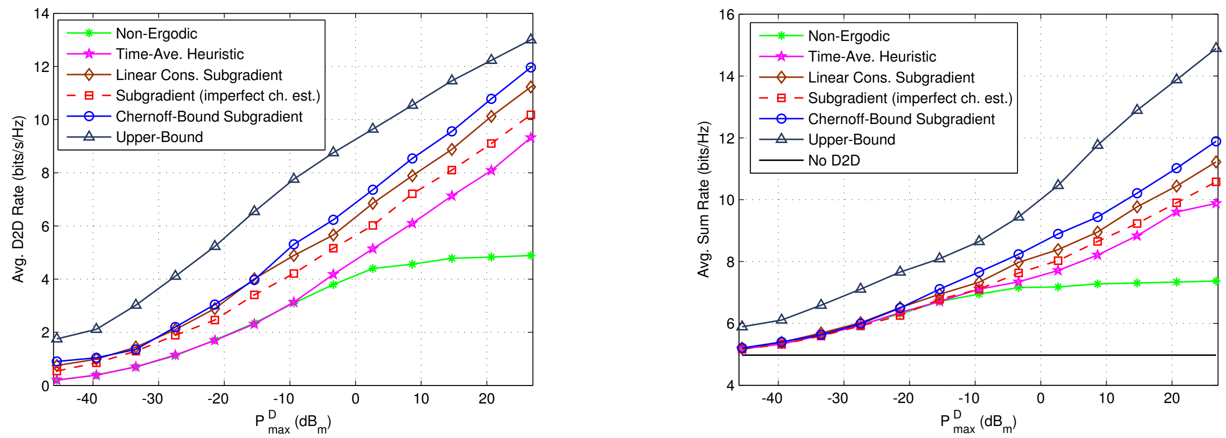
This means that the larger $\mathbb{E}\{\eta_j\}$ value, the larger the feasible set for the optimization problems $\mathcal{S}1$ and $\mathcal{D}1$. Therefore, by sorting the values of $\mathbb{E}\Big\{\eta_j\Big\}$ for all $j \in \mathcal{C}^*_l$, the lth D2D user be opportunistically assigned the RBs with the highest $\mathbb{E}\{\eta_j\}$ such that (11)-(12) are satisfied. It is worth mentioning that other methods with higher complexity can be adapted for RB allocation., e.g. the readers may consider [31]- [32].

For each data point, we average the results over a large number of random positions of cellular and D2D users and also random channel realizations. To avoid redundancy, we only present the results for the uplink case, in Figs. 2-5.

A. Efficacy of Convexification

In this section we analyze the efficacy of applying the convexification approach we proposed in Section III-A and IV-A to solve the ergodic sum-rate and D2D rate optimization problems. To focus on convexification and remove the effects

[2] For alternative approaches the readers may consider [30]



(a) Throughput of D2D users under D2D rate maximization,

(b) Throughput of the main cell under sum-rate maximization.

Fig. 2: The effect of convexification on throughput in a single cell scenario.

of inter-cell dynamics, in this subsection, we only consider the single-cell scenario. We compare the performance of the proposed solution methods with the upper-bounds developed in Sections III-F and IV-C, and also with a naive non-ergodic huristic method. For the non-ergodic approach, similar to [27] and [28], instead of long-term constraints in (7) and (8), we use short-term constraints, i.e., we set $\epsilon = 0$ and we use the constraint $\sum_{j\in\mathcal{C}_l} p^{\mathrm{D}}_{\mathrm{t},j} \leq P^{\mathrm{D}}_{\max}$ as the sum-power constraint. We set $\zeta^{\mathrm{intra}}_{j,\min} = 3\mathrm{dB}$, for all j. Also, for ergodic methods we set $\epsilon = 0.05$.

Fig. 2 investigates the effects of changing $P^{\mathrm{D}}_{\max}$ on the D2D and cell throughput. Fig. 2(a) shows the average D2D rate per RB for the cell under the D2D-rate maximization objective, and Fig. 2(b) shows the average D2D-cellular sum rate per RB for the cell under the sum-rate maximization objective. It can be seen that through applying the convexification method, we reach a small gap between the upper-bound and the Chernoff-bound and linear-constraint approximations. It is worth mentioning that by increasing $P^{\mathrm{D}}_{\max}$, the percentage of this gap, for specially D2D rate, is decreasing.

The Chernoff-bound method provides slightly higher throughput than the linear-constraint method. This is because we can improve the Chernoff-bound through the iterative method proposed in Section III-C. On the other hand, the linear-constraint method offers lower computational complexity, due to its semi-closed-form solution. For the heuristic time-averaging method, the Chernoff-bound constraints and the linear constraints lead to solutions with the same computational complexity and negligible performance gap, so only one curve is shown. We observe that the performance gap between the time-averaging heuristic and the subgradient methods is less than 17%, with drastically reduced computational complexity. Finally, Fig. 2 shows that for D2D power more than $-10\mathrm{dB_m}$, which is almost always true in practice, the performance gap between ergodic and non-ergodic power allocation is considerable, indicating the need for the proposed stochastic optimization solutions.

Besides the performance under perfect CSI, in Fig. 2, we also consider the performance of the convexification method with the Chernoff bound using imperfect CSI where channel estimation errors present. We model the channel estimation error as a complex Gaussian noise with zero mean and variance being 5% of the corresponding true channel variance. As can be seen from Figs. 2(a) and (b), the rate loss by using imperfect CSI for power allocation is less than 10%.

TABLE III: MATLAB Run-Time for Different Algorithms

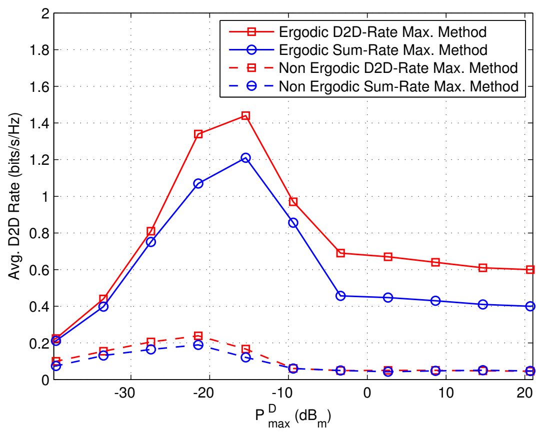
Non-Ergodic	5.91 s
Time-Ave. Heuristic	5.92 s
Linear Cons. Subgradient	1302.89 s
Chernoff-Bound Subgradient	1772.59 s

In Table III, we compare the computational complexity of the methods discussed in this paper based on the MATLAB run time of simulating 900 LTE frames (equivalent to 4.5 seconds) under all algorithms. It can be seen from Table III that the proposed time-averaging heuristic is nearly identical to the naive non-ergodic method in run time and it is around 300 times faster than the standard subgradient methods with Chernoff-bound or linear constraints.

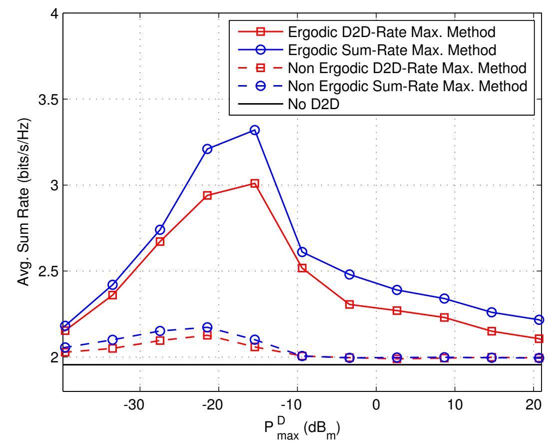
Note that all these methods have the same communication complexity. In general, beside a common overhead in LTE that is necessary to estimate CSI and the interference level, to calculate η_j, channel and interference feedback is required over each time slot. To avoid a large amount of information exchange, we can interpret power constraint (7) as per-RB power constraints set by the eNB in the main cell. In other words, η_j, for all $j \in C_l$, is set by the eNB such that we have SINR constraints (4)-(5) satisfied. In this case, each D2D transmitter directly receives the value of η_j, for $j \in C_l$, from the eNB of its own cell. Furthermore, after the initial setting of η_j by the eNB, any change in the value of η_j can be reported using limited feedback, e.g., using differential coding.

B. Multi-cell Performance Comparison

Under the multi-cell scenario, we compare the proposed heuristic solution with the non-ergodic approach. As default values, we set $P^{\mathrm{D}}_{\max} = -8.5\mathrm{dB_m}$, $\zeta^{\mathrm{intra}}_{j,\min} = \zeta^{(k)}_{j,\min} = -3\mathrm{dB}$,

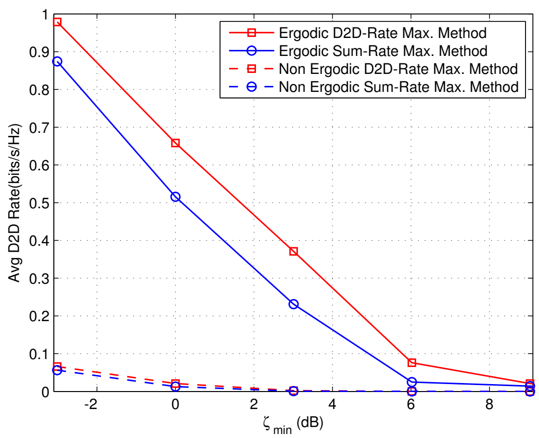


(a) Achieved throughput for D2D users,

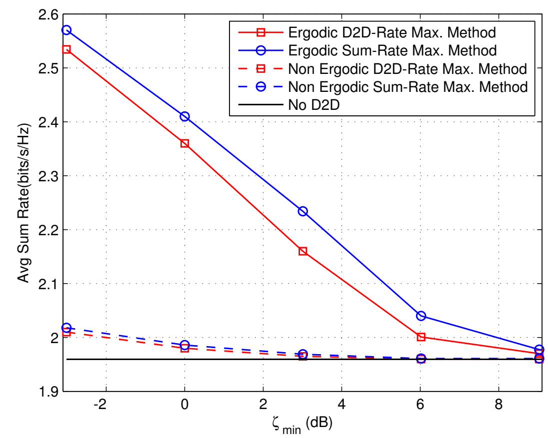


(b) Achieved throughput in the main cell.

Fig. 3: The achieved throughput vs. $P^{\mathrm{D}}_{\max}$.

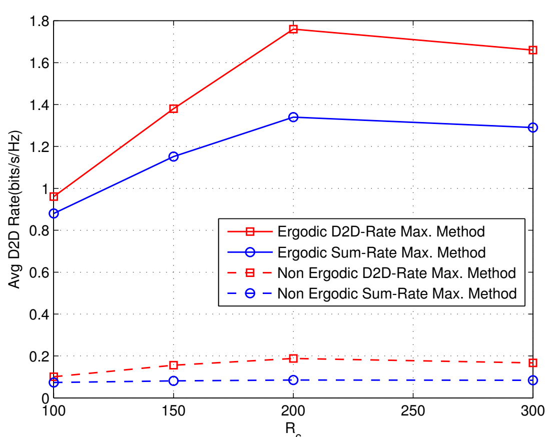


(a) Achieved throughput for D2D users,

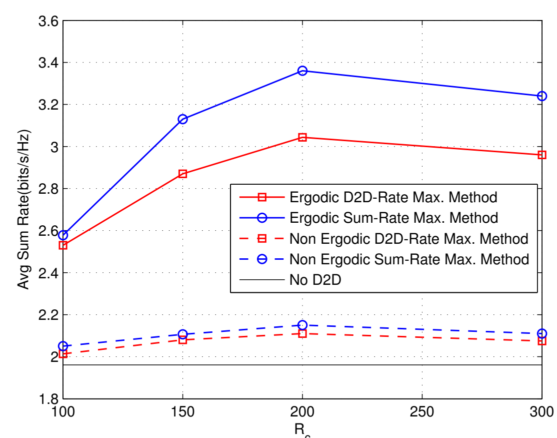


(b) Achieved throughput for the main cell.

Fig. 4: The achieved throughput vs. minimum required SINR.



(a) Achieved throughput for D2D users,



(b) Achieved throughput for the cell.

Fig. 5: The achieved throughput for different cell radius.

for all j and for all $k \in \mathcal{S}_j$, unless otherwise specified. For ergodic methods we set $\epsilon = 0.1$.

1) Maximum Power for D2D Users: Fig. 3 shows the effects of changing $P_{\max}^{\mathrm{D}}$ on the D2D and cell throughput. It can be seen that, by increasing $P_{\max}^{\mathrm{D}}$, at first the D2D-rate and sum-rate increase, but after some point they start to degrade. This happens because all neighboring cells use the same power allocation algorithm, and by increasing the D2D power, we increase the interference to and from the neighboring cells.

2) Minimum SINR Requirement for Cellular Users: Fig. 4 shows the effect of changing the the minimum required SINR on the D2D and cell throughput. It can be seen that, by increasing the minimum required SINR for the cellular users, there is less room for controlling the power of D2D users and this decreases the D2D throughput and the total cell throughput.

3) Cell Size: Fig. 5 shows the effect of changing the cell radius on the D2D and cell throughput. It can be seen from that, by increasing the cell radius, we see an increase and then a decrease in the throughput. In fact by increasing the cell radius, we have two different effects. The cellular and D2D users are spread over a larger area and thus the distance between interfering users is decreased, but on the other hand because of the uplink power control method for cellular users, their power increases, leading to more interference to D2D users.

VII. Conclusion

In this paper, we have considered optimal power allocation by the D2D users in a cellular network for underlay D2D communications in order to maximize the D2D rate and the sum rate between D2D and cellular users. The proposed optimization problems accommodates a long-term sum-power constraint and probabilistic individual power constraints over each accessible RB. This enables consideration for battery energy limits at D2D transmitters and the interference created by D2D communications. To solve the optimization problem, several approximate convex constraints are introduced, as replacement for the non-convex probabilistic individual power constraints. After such convexification, optimal solutions to the approximate D2D-rate and sum-rate maximization problems are developed, which are shown to give throughput performance that is close to an upper bound. We then propose a heuristic method by using time-averaging to approximate for long-term measures. The time-averaging heuristic method has low computational complexity, and it can be easily applied to the multi-cell scenario. Through simulation we observe that the performance gap between the standard subgradient solution and the proposed time-averaging heuristic method is less than 17%, with drastically reduced computational complexity for the heuristic method.

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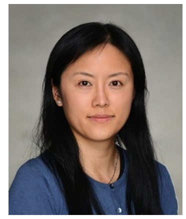

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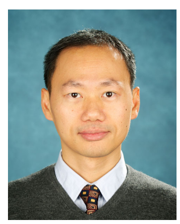

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