Interference Minimization in Cooperative Relay Beamforming with Multiple Communicating Pairs

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***Abstract*—We consider a cellular network where each cell contains multiple source-destination pairs communicating through multiple amplify-and-forward relays using orthogonal channels. We propose an optimal relay beamforming design that minimizes the maximum interference at the neighboring cells subject to per-relay power limits and minimum received signal-to-noise ratio (SNR) requirements. Even though the problem is non-convex, we show that it has zero Lagrange duality gap, and we convert its dual problem to a semi-definite programming problem. Depending on the values of the optimal dual variables, we study three cases to obtain the optimal beam vectors accordingly. This results in an iterative algorithm that provides a semi-closed-form optimal solution. We extend our algorithm to the problem of maximizing the minimum SNR subject to some pre-determined maximum interference constraints at neighboring cells, by the solution to the min-max interference problem along with a bisection search. The solution to this max-min SNR problem gives insight into the worst-case signal-to-interference-and-noise ratio (SINR) given some maximum interference target. The performance of the proposed algorithm is studied numerically, both for when the knowledge of interference channel is perfect and for when it is imperfect due to either limited feedback or channel estimation error.**

***Index Terms*—Relay beamforming, multiple users, interference minimization, multi-channel system.**

I. Introduction

Next-generation wireless networks are characterized by heterogeneous infrastructure consisting of base stations (BS) and relays in cooperative communication. Furthermore, the new Internet-of-Things paradigm will foster a large population of diverse user equipment (UE), which may engage in complex communication patterns that include both traditional BS-UE transmission and UE-UE transmission, *e.g.,* in a device-to-device (D2D) mode. In these systems, radio interference is a crucial yet challenging issue, due to the many randomly located transmitters and receivers. Interference management in cellular networks is a challenging problem, and the difficulty will only be exacerbated in future systems that utilize a large number of small cells, relays, and D2D nodes. One approach for interference management is to control the maximum inter-cell interference (ICI) as a worst-performance guarantee.

We consider a cellular network where multiple source-destination (S-D) pairs communicating through the assistance of multiple amplify-and-forward (AF) relays within a cell. These S-D pairs are generally defined. For example, they may represent D2D communication between UEs, or the multiple communication links between one BS and multiple UEs in the cell. Using a multi-channel system, such as orthogonal frequency division multiple access (OFDMA) as in LTE-Advanced [2], [3], each communicating pair is assigned an orthogonal subchannel to avoid intra-cell interference. The relays assist each S-D pair's transmission by forming cooperative relay beamforming over the pair's assigned subchannel.

Although free of intra-cell interference due to orthogonal subchannel allocation, these communicating pairs still cause ICI to neighboring cells, which needs to be carefully controlled to provide satisfactory performance guarantee. In this work, we aim at designing optimal relay beamforming for multiple S-D pairs within each cell to minimize the maximum interference caused at the neighboring cells, while satisfying the minimum signal-to-noise ratio (SNR) requirements and per-relay power constraints. This formulation is suitable for certain types of applications or traffic patterns where some fixed rate is expected, *e.g.,* VoIP. We also intend to find a pre-determined maximum interference threshold in each neighboring cell under which the worst-case signal-to-interference-and-noise ratio (SINR) at the destinations of the desired cell is maximized.

The design of relay beamforming in order to minimize ICI is challenging. Most ICI mitigation techniques for relay networks in the literature focus on the scheduling problem, *i.e.,* resource block allocation [4]–[8]. These techniques could not precisely control the amount of interference at the neighboring cells. To the best of our knowledge, the problem of minimizing the maximum interference at neighboring cells by relay beamforming has not been studied in the literature. Furthermore, most of the existing results in beamforming consider only a total power constraint across the antennas, which increases analytical tractability. However, in practical scenarios, we often need to consider an individual power limit for each relay [9]–[12].

In the following, we first summarize the main results of this work and then explain their relation to prior work in the literature.

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A. Summary of Contributions

We first formulate the relay beamforming problem in order to minimize the maximum interference in multiple neighboring cells under minimum SNR requirements and per-relay power constraints. Although the problem is non-convex, we show that it has zero duality gap and hence can be solved in the Lagrange dual domain. We then transform the dual problem into a semi-definite programming (SDP) problem with a much fewer number of variables and constraints compared with the original optimization problem and as such can be solved efficiently using interior-point methods.

Depending on the values of the optimal dual variables, we identify three cases to obtain the optimal beam vectors accordingly. These cases represent whether the minimum SNR requirement and per-relay power constraint are met with equality, and whether at optimality the interference at a destination in a neighboring cell is the maximum among destinations in all neighboring cells. The first case corresponds to the infeasibility of the min-max interference problem. For the other two cases, we propose an iterative algorithm to obtain optimal relay beam vectors with a semi-closed-form structure.

We also consider the problem of maximizing the minimum received SNR subject to maximum interference at each neighboring cell and per-relay power constraints. We show that the max-min SNR is the inverse problem of minimizing the maximum interference subject to a pre-determined SNR constraint. We propose an algorithm to solve the max-min SNR problem iteratively using the solution to the problem of maximum interference minimization and bisection search. Furthermore, by limiting the interference from each neighboring cell, we propose a solution to the problem of maximizing the worst-case received SINR. To this end, we solve the max-min SNR problem under an appropriate maximum interference target.

In order to gain insight into designing this system in practice, we study the received worst-case SINR versus the maximum interference target numerically. Interestingly, a maximum worst-case SINR is identified for different system setups. Using the obtained optimal relay beamforming solution, we investigate the effect of the number of relays, S-D pairs, and neighboring cells on the maximum interference and worst-case SINR. We further study the performance of the proposed algorithm when the knowledge of interference CSI is imperfect due to either limited feedback or channel estimation error.

B. Relation to Prior Work

For single-channel systems, joint encoding and decoding across the base stations has been proposed to mitigate the ICI [13], [14]. In [15], joint optimization of source power allocation and relay beamforming to maximize the minimum SINR has been studied for a single-carrier FDMA system. Further base station cooperation or coordination, in the form of “virtual” or “network” multiple-input-multiple-output (MIMO) systems, have been extensively studied in the literature [16]–[18]. These base station coordination techniques demand a huge amount of back-haul communication to share the data streams among the cells. In this work, we do not consider data sharing between the base stations or relays.

For multi-channel systems, such as those based on OFDMA, ICI coordination techniques have been studied in [19]–[29]. The proposed approaches in the literature include power control, network MIMO, opportunistic spectrum access, adaptive frequency reuse factor, sphere decoding, and dirty paper decoding. The problem formulation in this paper is different from all of those available in the literature. In order to mitigate ICI, we consider relay beamforming, which leads to a uniquely complicated optimization problem. Relay cooperative communication in interference limited environments has been considered under various criteria such as capacity, throughput, area spectral efficiency, and received SINR [30]–[34]. However, the objectives of these works do not include ICI reduction.

ICI mitigation techniques for relay networks in multi-channel systems have been studied in [4]–[8], which focus on scheduling and resource management. The authors of [4] have proposed a radio resource management strategy for relay-user association, resource allocation, and power control, along with four scheduling methods for power allocation in the ICI environment. In [5], the performance of different relay strategies, one-way, two-way, and shared relays, has been studied in interference-limited cellular systems. Assuming Gaussian signaling, the achievable rate for each strategy is derived. In [6], a joint subcarrier allocation, scheduling, and power control scheme has been proposed for ICI-limited networks. For relay-aided cellular OFDMA-based systems, the authors of [7] have proposed an interference coordination heuristic scheme consisting of two phases, each performing a resource allocation algorithm. In [8], a game theoretic framework called interference coordination game has been developed to mitigate interference in OFDMA-based relay networks, and a low complexity algorithm is proposed to reach its equilibrium in a distributed way.

The above works are the most related to our work. However, none of them considers relay beamforming, which leads to a complex optimization problem as shown in this paper. Furthermore, none of these works aims to directly minimize ICI, which could be significant if the interference channel is strong. Finally, it is important to find the maximum worst-case received SINR in a multi-channel system with multiple S-D pairs, especially for delay-sensitive applications requiring guaranteed worst bit-rate. This paper is the first to address the min-max interference and max-min SINR problems with relay beamforming.

For a single S-D pair and a single multi-antenna relay, the problem of relay beamforming to minimize per-antenna power has been considered in [10]. In this paper, we consider multiple single-antenna relays in a multi-channel system, with multiple S-D pairs. Furthermore, in this paper, the power used across subchannels has a sum limit and interference minimization is the objective. In [35], we have studied the problem of relay beamforming to minimize per-relay power usage in a multi-user peer-to-peer network. Different from [35], in this paper we consider interference to multiple neighboring cells in a cellular system under relay power constraints. The new formulation and constraints add more difficulty to solving the problem. Although both problems use the dual method and involve dual variable case discussions, the case discussions involved in finding the optimal solutions in this paper are much more complicated and challenging than those in [35]. In addition, as shown through simulation in this paper, the min-max interference approach significantly outperforms the per-relay power approach in terms of the maximum interference to neighboring cells.

C. Organization and Notation

The rest of this paper is organized as follows: In Section II, the system model is described and the min-max interference problem is formulated. The min-max interference problem is solved in Section III. In Section IV, we study the problems of maximizing the minimum received SNR and SINR subject to some fixed maximum interference threshold at the neighboring cells. Numerical results are presented in Section V, and conclusions are drawn in Section VI.

Notation: We use $A \triangleq B$ to denote that A by definition is equivalent to B. We use $\|\cdot\|$ to denote the Euclidean norm of a vector. $\odot$ stands for the element wise multiplication. We use $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^\dagger$ to denote transpose, Hermitian, and matrix pseudo-inverse, respectively. The conjugate is represented by $(\cdot)^*$. The notation $\text{diag}(\mathbf{a})$ denotes a diagonal matrix consisting of the elements of a vector $\mathbf{a}$. We use $\mathbb{E}[\cdot]$ to denote the expectation and $\text{tr}(\mathbf{B})$ to represent the trace of $\mathbf{B}$. $\mathbf{I}$ denotes an $N \times N$ identity matrix. We use $\mathbf{Y} \succeq \mathbf{Z}$ to indicate that $\mathbf{Y} - \mathbf{Z}$ is a positive semi-definite matrix.

II. System Model and Problem Formulation

A. System Model

As illustrated in Fig. 1, we consider a cellular system where each cell contains M S-D pairs, N relays, and b neighboring cells, and all nodes are equipped with a single antenna. A multichannel communication system (*e.g.,* OFDMA) consisting of M orthogonal subchannels is used in each cell. Each source transmits data to its destination through the relays using a specific subchannel, and each subchannel is assigned to one S-D pair, so that the S-D pairs within the same cell do not interfere with each other. In this work, we study the interference caused by the relays in one cell (*desired cell*) to the destinations in its neighboring cells.

We consider the half-duplex AF protocol for relaying, where the direct path is ignored. Assume that S-D pair m communicates through N relays over subchannel m. The S-D communication is established in two phases. In phase one, each source transmits its signal to all the relays. In the desired cell, the received signal at relay i over subchannel m is given by

$$z_{m,i} = \sqrt{P_m} h_{m,i} s_m + n_{r,m,i} \tag{1}$$

where s_m is the transmitted symbol with unit power, *i.e.,* $\mathbb{E}[|s_m|^2] = 1$, P_m is the transmission power, and $n_{r,m,i}$ denotes the additive white Gaussian noise (AWGN) at relay i on subchannel m with zero mean and variance σ_r^2, which is i.i.d. across subchannels and relays. The vector of received signals at all relays over subchannel m is given by

$$\mathbf{z}_m = \sqrt{P_m} \mathbf{h}_m s_m + \mathbf{n}_{r,m} \tag{2}$$

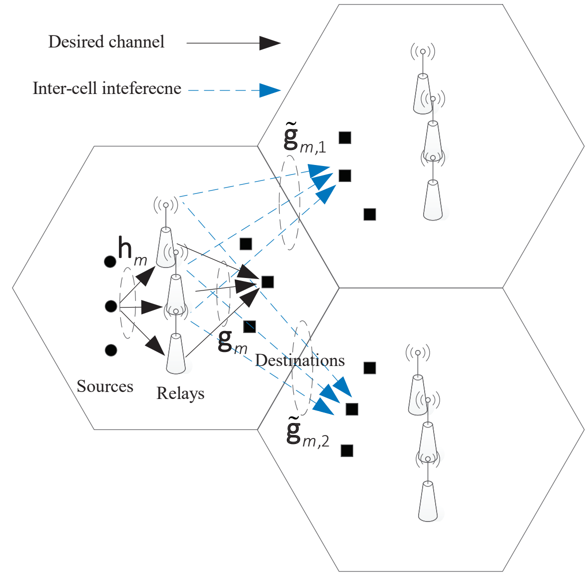

Fig. 1. The system model for multiple communication pairs. The solid and dashed lines show the desired and interference channels, respectively.

where $\mathbf{h}_m \triangleq [h_{m,1}, \cdots, h_{m,N}]^T$ and $\mathbf{n}_{r,m} \triangleq [n_{r,m,1}, \cdots, n_{r,m,N}]^T$ are the first-hop channel vector and the relay noise vector for S-D pair m, respectively.

In phase two, each relay i multiplies the received signal over subchannel m by a complex coefficient $w_{m,i}$ and forwards it to destination m, for $1 \le m \le M$.[1] The received signal at destination m from all relays over subchannel m is given by

$$\begin{aligned} r_m &= \mathbf{g}_m^T \mathbf{W}_m \mathbf{z}_m + n_{d,m} && (3) \\ &= \sqrt{P_m} \mathbf{g}_m^T \mathbf{W}_m \mathbf{h}_m s_m + \mathbf{g}_m^T \mathbf{W}_m \mathbf{n}_{r,m} + n_{d,m} && (4) \end{aligned}$$

where $\mathbf{g}_m \triangleq [g_{m,1}, \cdots, g_{m,N}]^T$ is the second-hop channel vector for S-D pair m, with $g_{m,i}$ denoting the channel coefficient over subchannel m from relay i to destination m, $\mathbf{W}_m \triangleq \text{diag}(\mathbf{w}_m)$, with $\mathbf{w}_m \triangleq [w_{m,1}, \cdots, w_{m,N}]^T$ denoting the relay beam vector for S-D pair m, and $n_{d,m}$ is the AWGN at destination m with zero mean and variance σ_d^2.

The power usage of relay i is expressed as

$$P_i = \sum_{m=1}^{M} \mathbb{E}[|w_{m,i} z_{m,i}|^2] = \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m \tag{5}$$

where $\mathbf{R}_m \triangleq \text{diag}([\mathbf{R}_{y,m}]_{1,1}, \cdots, [\mathbf{R}_{y,m}]_{N,N})$, with $\mathbf{R}_{y,m} \triangleq P_m \mathbf{h}_m \mathbf{h}_m^H + \sigma_r^2 \mathbf{I}$, for $m = 1, \cdots, M$, and $\mathbf{D}_i$ denotes the $N \times N$ diagonal matrix with 1 in the i-th diagonal and zero otherwise. We assume that the total power available at each relay, P_r, can be allocated across different subchannels.

The received signal power at destination m is given by

$$P_{S,m} = P_m [\mathbf{g}_m^T \mathbf{W}_m \mathbf{h}_m \mathbf{h}_m^H \mathbf{W}_m^H \mathbf{g}_m^*] = P_m \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m \tag{6}$$

[1]Note that multiple transmissions in a cooperative relay system may not be synchronized at a destination. An asynchronous transmission scheme is proposed in [36], [37].

where $\mathbf{F}_m \triangleq (\mathbf{f}_m\mathbf{f}_m^H)^*$, with $\mathbf{f}_m = \mathbf{g}_m \odot \mathbf{h}_m \triangleq [h_{m,1}g_{m,1}, \cdots, h_{m,N}g_{m,N}]^T$. The total noise power at destination m, including both the receiver noise and the relay amplified noise, is obtained as

$$\begin{aligned} P_{N,m} &= \mathbb{E}[\mathbf{n}_{r,m}^H \mathbf{W}_m^H \mathbf{g}_m^* \mathbf{g}_m^T \mathbf{W}_m \mathbf{n}_{r,m}] + \sigma_d^2 \\ &= \mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma_d^2 \end{aligned} \tag{7}$$

where $\mathbf{G}_m \triangleq \sigma_r^2 \operatorname{diag}((\mathbf{g}_m\mathbf{g}_m^H)^*)$. Hence, the SNR at destination m is given by

$$\mathrm{SNR}_m = \frac{P_m \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma_d^2}. \tag{8}$$

Each relay causes interference to the M destinations in each of neighboring cells. Let $\tilde{\mathbf{g}}_{m,j}$ denote the interference channel vector over subchannel m from the N relays of the desired cell to destination m in neighboring cell j. The received interference at destination m in neighboring cell j is given by

$$\tilde{r}_{m,j} = \tilde{\mathbf{g}}_{m,j}^T \mathbf{W}_m(\sqrt{P_m}\mathbf{h}_m s_m + \mathbf{n}_{r,m}). \tag{9}$$

The received interference power at destination m in neighboring cell j, including both the forwarded signal and the relay amplified noise, is given by

$$\mathcal{I}_{m,j} = P_m \mathbf{w}_m^H \tilde{\mathbf{F}}_{m,j} \mathbf{w}_m + \mathbf{w}_m^H \tilde{\mathbf{G}}_{m,j} \mathbf{w}_m \tag{10}$$

where $\tilde{\mathbf{F}}_{m,j} \triangleq (\tilde{\mathbf{f}}_{m,j}\tilde{\mathbf{f}}_{m,j}^H)^*$, $\tilde{\mathbf{f}}_{m,j} \triangleq \tilde{\mathbf{g}}_{m,j} \odot \mathbf{h}_m$, and $\tilde{\mathbf{G}}_{m,j} \triangleq \sigma_r^2 \operatorname{diag}((\tilde{\mathbf{g}}_{m,j}\tilde{\mathbf{g}}_{m,j}^H)^*)$ for $j = 1, \cdots, b$.

We assume the perfect knowledge of CSI, *i.e.,* $\{\mathbf{h}_m, \mathbf{g}_m, \tilde{\mathbf{g}}_{m,j}\}_{m=1}^M$, in designing the relay beam vectors, where a central controller in each cell may collect all intra- and inter-cell CSI for computing relay beam weights. In Section V-D, we further study the case where the interference CSI is imperfect through simulation.

B. Problem Formulation

Our focus is on designing the relay beam weights of the desired cell to minimize the maximum interference at the neighboring cells under per-relay power constraint and the received SNR requirement at each destination. This is expressed as the following optimization problem:

$$\begin{aligned} \text{P0:} \ & \min_{\{\mathbf{w}_m\}} \max_{m\in\mathcal{M}, j\in\mathcal{B}} \mathcal{I}_{m,j} \\ & \text{subject to} \ \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{R}_m \mathbf{D}_i \mathbf{w}_m \le P_r, \ i \in \mathcal{N}, \quad \text{(11a)} \\ & \qquad \frac{P_m \mathbf{w}_m^H \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{G}_m \mathbf{w}_m + \sigma_d^2} \ge \gamma_m, \ m \in \mathcal{M} \quad \text{(11b)} \end{aligned}$$

where $\mathcal{I}_{m,j}$ is as defined in (10), $\mathcal{M} \triangleq \{1, \cdots, M\}$, $\mathcal{N} \triangleq \{1, \cdots, N\}$, and $\mathcal{B} \triangleq \{1, \cdots, b\}$. To remove the inner maximization in P0, we note that the min-max optimization problem P0 is equivalent to the following:

$$\begin{aligned} \text{P1:} \ & \min_{\{\mathbf{w}_m\}, \mathcal{I}_{\max}} \mathcal{I}_{\max} \\ & \text{subject to} \ \mathbf{w}_m^H \tilde{\mathbf{B}}_{m,j} \mathbf{w}_m \le \mathcal{I}_{\max}, \ m \in \mathcal{M}, j \in \mathcal{B}, \quad \text{(12a)} \\ & \qquad \text{(11a), and (11b)} \end{aligned}$$

where $\mathcal{I}_{\max} = \max_{m\in\mathcal{M}, j\in\mathcal{B}} \mathcal{I}_{m,j}$ and $\tilde{\mathbf{B}}_{m,j} \triangleq P_m \tilde{\mathbf{F}}_{m,j} + \tilde{\mathbf{G}}_{m,j}$.

Note that, in this work, we consider the interference minimization problem under per-relay total power constraint. For a fixed source transmit power, we can show that considering the direct ICI link from sources to destinations in neighboring cells does not change our analysis.[2] To see this, note that the total received interference at destination m in neighboring cell j contains interference from both relays and sources, given by

$$\tilde{\mathcal{I}}_{m,j} = \mathcal{I}_{m,j} + P_m|\tilde{h}_{m,j}|^2$$

where $\tilde{h}_{m,j}$ denotes the direct channel from source m to destination m in neighboring cell j. Assume $|\tilde{h}_{m,j}|^2$ is known in the desired cell. Given the fact that $P_m|\tilde{h}_{m,j}|^2$ does not depend on $\mathbf{w}_m$, we can treat it as a constant term when designing the beam vectors. To include the inference coming from the sources, we can replace the left-hand side of constraint (12a) with $\mathbf{w}_m^H \tilde{\mathbf{B}}_{m,j} \mathbf{w}_m + P_m|\tilde{h}_{m,j}|^2$. Then, a similar procedure as in our proposed algorithm can be followed to obtain the optimal beam vectors.

III. Minimizing Maximum Interference

The solution of P1 is provided in this section. Since the SNR constraint (11b) is not convex w.r.t. $\mathbf{w}_m$, P1 is non-convex. In order to solve this problem, we first provide a necessary condition for its feasibility. Then we show that P1 can be reformulated as a second-order-conic programming (SOCP) problem, and more importantly, the SOCP's conic dual and Lagrange dual are equivalent, so that P1 has zero Lagrange duality gap. In order to obtain the optimal dual variables, an SDP-based algorithm is proposed with polynomial worst-case complexity. We then propose an iterative algorithm to obtain the optimal beam vectors $\{\mathbf{w}_m\}$ with a semi-closed-form structure. Through complexity analysis, we show that our proposed algorithm is computationally more efficient in finding an optimal solution than directly solving the SOCP problem.[3]

A. Necessary Condition for Feasibility

We first introduce necessary condition for feasibility following the similar arguments in [35], which can be used to stop execution of the proposed algorithm if there exists $m \in \mathcal{M}$ such that SNR constraint (11b) cannot be satisfied.

A necessary condition for the feasibility of the min-max interference problem P1 is

$$\min_{m\in\mathcal{M}} \frac{P_m}{\gamma_m} \mathbf{f}_m^H \mathbf{G}_m^\dagger \mathbf{f}_m > 1. \tag{13}$$

[2] If the source transmission power $\mathbf{p} \triangleq [P_1, \cdots, P_M]^T$ is also an optimization variable, we have a joint optimization problem with $\{\mathbf{p}, \mathbf{w}\}$ as variables. This joint optimization problem becomes much more difficult to solve, as it is jointly non-convex. Whether it can be solved needs to be carefully investigated and is an open problem left for future research.

[3] We can show that solving SOCP directly increases complexity as compared with the proposed algorithm for the typical scenario of large number of relays and S-D pairs. In addition to complexity reduction, one can gain insights on the optimal solution structure using our proposed algorithm. However, the SOCP-based method does not provide any insight on the structure of the solution for $\mathbf{w}^o$.

Note that not satisfying (13) means that regardless of the values of $\{\mathbf{w}_m\}$ there always exists $m \in \mathcal{M}$ such that SNR constraint (13) cannot be satisfied, and thus P1 is infeasible. On the other hand, even if (13) holds, that does not guarantee that P1 is feasible. In that case, we will see later that Case 1 in Section III-C1 will identify the infeasibility of P1.

B. The Lagrange Dual Approach

In the following, we show that, despite P1 being non-convex, it has zero duality gap and can be solved in the Lagrange dual domain.

Proposition 1: Strong duality holds for the min-max interference problem P1.

Proof: We first show that P1 can be reformulated as an SOCP problem. It is known that the SOCP has zero *conic* duality gap [38]. Then we show that the Lagrange dual of P1 and *conic* dual of the SOCP are equivalent. For further details, see Appendix A. ∎

Using Proposition 1, we can obtain the optimum solution of P1 through the Lagrange dual approach. Let $\boldsymbol{\mu} \triangleq [\boldsymbol{\mu}_1, \cdots, \boldsymbol{\mu}_M]^T$ with $\boldsymbol{\mu}_m \triangleq [\mu_{m,1}, \cdots, \mu_{m,b}]^T$, $\boldsymbol{\lambda} \triangleq [\lambda_1, \cdots, \lambda_N]^T$, and $\boldsymbol{\alpha} \triangleq [\alpha_1, \cdots, \alpha_M]^T$ denote the Lagrange multipliers associated with the interference constraint (12a), per relay power constraint (11a), and SNR constraint (11b), respectively. The Lagrangian of P1 is given by

$$L(\{\mathbf{w}_m\}, \mathcal{I}_{\max}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}) = \sum_{m=1}^{M} \mathbf{w}_m^H (\mathbf{K}_m - \frac{\alpha_m P_m}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H) \mathbf{w}_m + \sum_{m=1}^{M} \alpha_m \sigma_d^2 + \mathcal{I}_{\max}(1 - \sum_{m=1}^{M} \sum_{j=1}^{b} \mu_{m,j}) - P_r(\sum_{i=1}^{N} \lambda_i) \quad (14)$$

where

$$\mathbf{K}_m \triangleq \mathbf{R}_m \mathbf{D}_{\boldsymbol{\lambda}} + \sum_{j=1}^{b} \mu_{m,j} \tilde{\mathbf{B}}_{m,j} + \alpha_m \mathbf{G}_m \quad (15)$$

and $\mathbf{D}_{\boldsymbol{\lambda}} \triangleq \mathrm{diag}(\lambda_1, \cdots, \lambda_N)$.

The dual problem of P1 is obtained by

$$\text{D0:} \max_{\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}} \min_{\{\mathbf{w}_m\}, \mathcal{I}_{\max}} L(\{\mathbf{w}_m\}, \mathcal{I}_{\max}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha})$$
$$\text{subject to } \boldsymbol{\lambda} \succeq \mathbf{0}, \boldsymbol{\mu} \succeq \mathbf{0}, \boldsymbol{\alpha} \succeq \mathbf{0}. \quad (16a)$$

Furthermore, the dual problem D0 can be reformulated as the following problem:

$$\text{D1:} \max_{\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}} \sum_{m=1}^{M} \alpha_m \sigma_d^2 - P_r(\sum_{i=1}^{N} \lambda_i)$$
$$\text{subject to } \mathbf{K}_m \succeq \frac{\alpha_m P_m}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H, \; m \in \mathcal{M}, \quad (17a)$$
$$\sum_{m=1}^{M} \sum_{j=1}^{b} \mu_{m,j} \leq 1, \quad (17b)$$
$$\text{and (16a).}$$

The equivalence of D0 and D1 can be shown by showing that constraints (17a) and (17b) are satisfied at optimality of D0. Suppose one of the constraints (17a) or (17b) is not satisfied. Then there is some $\{\mathbf{w}_m, \mathcal{I}_{\max}\}$ such that the inner minimization of D0 leads to $L(\{\mathbf{w}_m\}, \mathcal{I}_{\max}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}) = -\infty$, but clearly this cannot be the optimal objective of the dual problem. Hence, the optimal solution of D0 satisfies constraints (17a) and (17b). In this case, after the inner minimization of the Lagrangian in D0, we have the objective of D1. Thus, both D0 and D1 lead to the same optimal $\{\boldsymbol{\lambda}^o, \boldsymbol{\mu}^o, \boldsymbol{\alpha}^o\}$.

To solve the dual problem D1 we show that it can be reformulated as an SDP problem to determine the optimal $\{\boldsymbol{\alpha}^o, \boldsymbol{\lambda}^o, \boldsymbol{\mu}^o\}$.

Proposition 2: The dual problem D1 can be expressed as

$$\text{D2:} \min_{\mathbf{x}} \mathbf{a}^T \mathbf{x}$$
$$\text{subject to } \sum_{i=1}^{M(b+1)+N} x_i \boldsymbol{\Psi}_{m,i} \preceq \mathbf{0}, \; m \in \mathcal{M}, \quad (18a)$$
$$\mathbf{x} \succeq \mathbf{0}, \; \mathbf{b}^T \mathbf{x} \leq 1 \quad (18b)$$

where x_i is the i-th entry of the vector $\mathbf{x} \triangleq [\boldsymbol{\alpha}^T, \boldsymbol{\lambda}^T, \boldsymbol{\mu}^T]^T$, $\mathbf{a} \triangleq [-\sigma_d^2 \mathbf{1}_{M\times 1}^T, P_r \mathbf{1}_{N\times 1}^T, \mathbf{0}_{Mb\times 1}^T]^T$, $\mathbf{b} \triangleq [\mathbf{0}_{(M+N)\times 1}^T, \mathbf{1}_{Mb\times 1}^T]^T$, $\boldsymbol{\Psi}_{m,m} = \frac{P_m}{\gamma_m} \mathbf{f}_m \mathbf{f}_m^H - \mathbf{G}_m$, $\boldsymbol{\Psi}_{m,M+i} = -\mathbf{R}_m \mathbf{D}_i$ for $i \in \mathcal{N}$, $\boldsymbol{\Psi}_{m,M+N+(m-1)b+j} = -\tilde{\mathbf{B}}_{m,j}$ for $m \in \mathcal{M}$, $j \in \mathcal{B}$, and all other $\boldsymbol{\Psi}$ are zeros.

Proof: It is not difficult to show that D1 is equivalent to D2, and (16a) and (17b) are equivalent to (18b). Then, substituting (15) into (17a) and after some manipulation, (18a) is obtained. ∎

Note that standard interior point-based solvers, *e.g.,* CVX, could be used to solve D2 efficiently [38]. Then, depending on the values of the optimal dual variables $\{\boldsymbol{\alpha}^o, \boldsymbol{\lambda}^o, \boldsymbol{\mu}^o\}$, we identify three cases to obtain the optimal beam vectors $\{\mathbf{w}_m^o\}$. We first investigate a useful property of the constraint (17a) in the following lemma.

Lemma 1: If either $\mu_{m,j}^o > 0$ for some $\{m, j\}$ or $\boldsymbol{\lambda}^o \succ 0$, then $\alpha_m^o > 0$, *i.e.,* the Lagrange dual variable associated with the SNR requirement at destination m is strictly positive.

Proof: See Appendix B.

Recall that $\mu_{m,j}^o$, $\boldsymbol{\lambda}^o$, and α_m^o are the optimal dual variables corresponding to the interference constraint (12a), per-relay power constraint (11a), and SNR constraint (11b), respectively. Due to Proposition 1, the Karush-Kuhn-Tucker (KKT) conditions for P1 are satisfied. Hence, the complementary slackness condition holds. According to Lemma 1, if the interference constraint over subchannel m is active at optimality, *i.e.,* attained with equality, or the per-relay power constraint is active for each relay, then the SNR constraint for S-D pair m is also active at optimality.

C. The Optimal Beam Vector $\{\mathbf{w}_m^o\}$

Using Lemma 1, we classify the optimal dual variables $\{\boldsymbol{\lambda}^o, \boldsymbol{\mu}^o, \boldsymbol{\alpha}^o\}$ into three cases to obtain $\{\mathbf{w}_m\}$.

1) Case 1: $\boldsymbol{\mu}^o = \mathbf{0}$. In this case, we show that the min-max interference problem P1 is infeasible. Suppose the per-relay power constraint (11a) and minimum SNR requirement (11b) could be satisfied for every S-D pair m and relay i, *i.e.,* the original problem P1 is feasible. Since the objective of P1 clearly is sensitive to changes in the RHS of

(12a), $\boldsymbol{\mu}^o = \mathbf{0}$ implies that (12a) is inactive for all m and j. Then the optimal objective $\mathcal{I}^o_{\max}$ is strictly greater than $\hat{\mathcal{I}} \triangleq \max_{m\in\mathcal{M},j\in\mathcal{B}} \mathbf{w}^o_m{}^H \tilde{\mathbf{B}}_{m,j}\mathbf{w}^o_m$ at optimality. However, $\mathcal{I}^o_{\max}$ can be replaced by $\hat{\mathcal{I}}$ resulting in a smaller objective while satisfying all the constraints which is a contradiction. In this case, the only possible conclusion is that the min-max interference problem P1 is infeasible. Hence, if P1 is feasible, there should be at least one $\{m, j\}$ such that (12a) is active at optimality, *i.e.*, $\mu^o_{m,j} > 0$.

2) Case 2: $\boldsymbol{\mu}^o_m \neq \mathbf{0}$ for all m or $\boldsymbol{\lambda}^o \succ \mathbf{0}$.[4] According to Lemma 1, we have $\boldsymbol{\alpha}^o \succ \mathbf{0}$ in D1, *i.e.*, if $\mathbf{K}^o_m - \alpha^o_m\mathbf{G}_m \succ \mathbf{0}$, then $\alpha^o_m > 0$ for all $m \in \mathcal{M}$, and the solution is given by the following proposition.

Proposition 3: Suppose $\boldsymbol{\alpha}^o \succ \mathbf{0}$. The optimum beam vector $\mathbf{w}^o_m$ of the min-max interference problem P1 for $m \in \mathcal{M}$ is given by

$$\mathbf{w}^o_m = \zeta_m {\mathbf{K}^o_m}^\dagger \mathbf{f}_m \tag{19}$$

where

$$\zeta_m \triangleq \sigma_d \Big[\frac{P_m}{\gamma_m}|\mathbf{f}^H_m {\mathbf{K}^o_m}^\dagger \mathbf{f}_m|^2 - \mathbf{f}^H_m {\mathbf{K}^o_m}^\dagger \mathbf{G}_m {\mathbf{K}^o_m}^\dagger \mathbf{f}_m\Big]^{\frac{-1}{2}}, \tag{20}$$

and $\mathbf{K}^o_m$ is obtained by substituting the optimum dual variables $\{\boldsymbol{\lambda}^o, \boldsymbol{\mu}^o_m, \alpha^o_m\}$ into (15).

Proof: See Appendix C.

The following corollary provides the structure for the optimal value of P1 as a function of the optimal dual variables.

Corollary 1: The maximum received interference of P1 is given by

$$\begin{aligned}\mathcal{I}^o_{\max} &= \sum_{m=1}^{M} \alpha^o_m \sigma^2_d - P_r(\sum_{i=1}^{N}\lambda^o_i)\\ &= \sigma^2_d \sum_{m=1}^{M} \frac{\gamma_m}{P_m \mathbf{f}^H_m {\mathbf{K}^o_m}^{-1}\mathbf{f}_m} - P_r(\sum_{i=1}^{N}\lambda^o_i).\end{aligned} \tag{21}$$

Proof: The first equality follows from Proposition 1 due to the zero duality gap. According to the proof in Appendix C, $\frac{\alpha^o_m P_m}{\gamma_m}\mathbf{f}^H_m {\mathbf{K}^o_m}^{-1}\mathbf{f}_m = 1$ in Case 2 for $m \in \mathcal{M}$. Substituting α^o_m into the objective of D1, the second equality in (21) is derived. ■

3) Case 3: $\boldsymbol{\mu}^o \neq \mathbf{0}$, $\boldsymbol{\mu}^o_m = \mathbf{0}$ for some m, and $\boldsymbol{\lambda}^o \not\succ \mathbf{0}$. Using Lemma 1, we have $\alpha^o_m = 0$ for some m. In the following, we first consider the case where only one entry in $\boldsymbol{\alpha}^o$ is strictly positive. In other words, only one S-D pair meets the SNR requirement with equality. Later, we will generalize the solution to the case where multiple entries in $\boldsymbol{\alpha}^o$ are positive. Denote $\tilde{m}$ such that $\alpha^o_{\tilde{m}} > 0$ and $\alpha^o_m = 0$ for $m \neq \tilde{m}$.

Following the proof in Appendix C, we can show that $\frac{\alpha^o_{\tilde{m}} P_{\tilde{m}}}{\gamma_{\tilde{m}}}\mathbf{f}^H_{\tilde{m}} {\mathbf{K}^o_{\tilde{m}}}^{-1}\mathbf{f}_{\tilde{m}} = 1$. Then the optimal beam vectors $\mathbf{w}^o_{\tilde{m}}$ can be obtained using the solution in (19) as $\alpha^o_{\tilde{m}} > 0$. However, we cannot obtain the optimal beam vector $\mathbf{w}^o_m$ for $m \neq \tilde{m}$ in a similar way. Next, we formulate a new optimization problem and obtain $\mathbf{w}^o_m$ for $m \neq \tilde{m}$.

Denote $\mathcal{M}_{\tilde{m}} \triangleq \mathcal{M} \setminus \{\tilde{m}\}$. Using the fact that $\alpha^o_m = 0$ for $m \in \mathcal{M}_{\tilde{m}}$ and Proposition 1, we see that the optimal objective of P1 is $\mathcal{I}^o_{\max} = \alpha^o_{\tilde{m}}\sigma^2_d - P_r(\sum_{i=1}^{N}\lambda^o_i)$. Further define $P_{\tilde{m},i} \triangleq {\mathbf{w}_{\tilde{m}}}^H \mathbf{R}_{\tilde{m}}\mathbf{D}_i\mathbf{w}_{\tilde{m}}$ as the power usage at relay i over subchannel $\tilde{m}$. Then, obtaining the optimal beam vectors $\{\mathbf{w}_m, m \in \mathcal{M}_{\tilde{m}}\}$ is equivalent to solving the following feasibility problem:

$$\begin{aligned} \text{P2: find } & \{\mathbf{w}_m, m \in \mathcal{M}_{\tilde{m}}\}\\ \text{subject to } & \mathbf{w}^H_m \tilde{\mathbf{B}}_{m,j}\mathbf{w}_m \le \mathcal{I}^o_{\max},\ m \in \mathcal{M}, j \in \mathcal{B}, && \text{(22a)}\\ & P_{\tilde{m},i} + \sum_{\mathcal{M}_{\tilde{m}}} \mathbf{w}^H_m \mathbf{R}_m\mathbf{D}_i\mathbf{w}_m \le P_r,\ i \in \mathcal{N}, && \text{(22b)}\\ & \frac{P_m \mathbf{w}^H_m \mathbf{F}_m \mathbf{w}_m}{\mathbf{w}^H_m \mathbf{G}_m \mathbf{w}_m + \sigma^2_d} \ge \gamma_m,\ m \in \mathcal{M}_{\tilde{m}}. && \text{(22c)}\end{aligned}$$

Note that the solution to P2 is not unique. This is because the SNR constraint (22c) may not be active at optimality for $m \in \mathcal{M}_{\tilde{m}}$ since $\alpha^o_m = 0$. However, we can always scale $\mathbf{w}_m$ such that (22c) meets with equality for $m \in \mathcal{M}_{\tilde{m}}$ while satisfying the max interference constraint (22a) and per-relay power constraint (22b). In what follows, we provide the details of using this approach to find a solution to P2.

Since the optimal beam vector $\mathbf{w}_{\tilde{m}}$ is already obtained, we can reduce $P_{\tilde{m},i}$ from the maximum per-relay power target P_r to find the maximum available power that can be used over other subchannels. This motivates the following interference minimization problem by excluding S-D pair $\tilde{m}$ from consideration and limiting the power usage on each relay based on the new maximum power target, *i.e.*,

$$\begin{aligned} \text{P3: } \min_{\{\mathbf{w}_m, m\in\mathcal{M}_{\tilde{m}}\},\tilde{\mathcal{I}}} \ & \tilde{\mathcal{I}}\\ \text{subject to } & \mathbf{w}^H_m \tilde{\mathbf{B}}_{m,j}\mathbf{w}_m \le \tilde{\mathcal{I}},\ m \in \mathcal{M}_{\tilde{m}}, j \in \mathcal{B}, && \text{(23a)}\\ & \sum_{m\in\mathcal{M}_{\tilde{m}}} \mathbf{w}^H_m \mathbf{R}_m\mathbf{D}_i\mathbf{w}_m \le P_r - P_{\tilde{m},i},\ i \in \mathcal{N}, && \text{(23b)}\\ & \text{and (22c).}\end{aligned}$$

Similar to Proposition 1, we can show that zero duality gap holds for P3. We can reformulate the Lagrange dual problem of P3 into an SDP as follows:

$$\begin{aligned} \text{D3: } \min_{\mathbf{x}} \ & \mathbf{c}^T\mathbf{x}\\ \text{subject to } & \sum_{i=1}^{M(b+1)+N} x_i \boldsymbol{\Psi}_{m,i} \preceq \mathbf{0},\ m \in \mathcal{M}_{\tilde{m}}, && \text{(24a)}\\ & \mathbf{x} \succeq \mathbf{0},\ \mathbf{d}^T\mathbf{x} \le 1 && \text{(24b)}\end{aligned}$$

where $\mathbf{x}$ is as defined in D2; $\mathbf{c}$ is defined similarly to $\mathbf{a}$ in D2 except that the entries $\mathbf{a}_{\tilde{m}}$, $\mathbf{a}_{(M+1):(M+N)}$, and $\mathbf{a}_{(M+N+(\tilde{m}-1)b+1):(M+N+\tilde{m}b)}$ are zero, $[P_r - P_{\tilde{m},1}, \cdots, P_r - P_{\tilde{m},N}]^T$, and zero, respectively; and $\mathbf{d}$ is defined similarly to $\mathbf{b}$ in D2 except that the entries $\mathbf{b}_{(M+N+(\tilde{m}-1)b+1):(M+N+\tilde{m}b)}$ are zero. Thus, the essence of the proposed algorithm is to eliminate the terms associated with S-D pair $\tilde{m}$ in both the objective and the constraints of D3 such that the per-relay maximum power targets are updated.

In order to obtain $\{\mathbf{w}^o_m, m \in \mathcal{M}_{\tilde{m}}\}$, the above procedure is repeated to update the values of $\{\alpha^o_m, m \in \mathcal{M}_{\tilde{m}}\}$ through solving D3. If $\alpha^o_m > 0$ for all $m \in \mathcal{M}_{\tilde{m}}$, then we can obtain $\{\mathbf{w}^o_m, m \in \mathcal{M}_{\tilde{m}}\}$ similar to Case 2. Otherwise, the steps to find the solution in Case 3 are repeated. As an example, after solving SDP problem D3, suppose Case 3 happens, *i.e.*, $\alpha_{m'} >$

[4] Note that for Cases 2 and 3, it is implicitly assumed $\boldsymbol{\mu}^o \neq \mathbf{0}$.

Algorithm 1 Minimizing the maximum interference

Check the feasibility condition (13).
Solve the SDP problem D2 finding the optimal dual variables $\{\boldsymbol{\alpha}^o, \boldsymbol{\mu}^o, \boldsymbol{\lambda}^o\}$.
Obtain $\mathcal{P}_\alpha = \{m \mid \alpha_m^o > 0\}$.
Set $\Pi = \mathcal{P}_\alpha$.
while $\mathcal{P}_\alpha \neq \mathcal{M}$ **do**
    Compute $\mathbf{K}_m^o$ (15) and find $\mathbf{w}_m^o$ (19) for all $m \in \Pi$.
    Update available power at each relay, $\mathbf{c}$ and $\mathbf{d}$.
    Solve D3 finding $\Pi = \{l \in \mathcal{M} \setminus \mathcal{P}_\alpha | \alpha_l^o > 0\}$.
    Update $\mathcal{P}_\alpha = \mathcal{P}_\alpha \bigcup \Pi$.
end while
Compute $\mathbf{K}_m^o$ (15) and find $\mathbf{w}_m^o$ (19) for all $m \in \Pi$

0 for some $m' \in \mathcal{M}_{\tilde{m}}$ (the SNR constraint (22c) is active for S-D pair m'). Following the proof in Appendix C, we can find $\mathbf{w}_{m'}^o$ with a similar structure as in (19) through substituting the optimal dual variables given by D3 into (15). As long as P1 is feasible, this procedure can be repeated until $\mathbf{w}_m^o$ for all m are found.

So far, we have assumed only one entry in $\boldsymbol{\alpha}^o$ is strictly positive in the solution to the dual problem D1. We can extend our algorithm to the general case where the number of positive entries in $\boldsymbol{\alpha}^o$ is arbitrary. Define $\mathcal{P}_\alpha \triangleq \{m \mid \alpha_m^o > 0\}$. Using Proposition 3, we can obtain the optimal beam vector $\mathbf{w}_m^o$ for $m \in \mathcal{P}_\alpha$ with a similar expression as in (19). To obtain $\mathbf{w}_m^o$ for $m \in \mathcal{M} \setminus \mathcal{P}_\alpha$, we can solve a feasibility problem similar to P2. We can show zero duality gap holds and formulate the dual problem into an SDP similar to D3 through updating $\mathbf{c}$, $\mathbf{d}$, and $\boldsymbol{\Psi}_{m,i}$ according to $\mathcal{P}_\alpha$.

D. Summary of Algorithm

The steps proposed to solve the min-max interference problem P1 are summarized in Algorithm 1.[5]

We can further obtain a necessary and sufficient condition for the feasibility of P1 since both Cases 2 and 3 lead to a solution with the semi-closed-form structure in (19). Note that for ζ_m in (20) to be real, the expression in RHS of (20) should be strictly positive. Furthermore, substituting (19) into (11a), the per-relay power usage should not exceed the maximum target P_r. As a result, the necessary and sufficient conditions for feasibility of P1 is as follows.

Corollary 2: P1 is feasible if and only if there exists $\boldsymbol{\alpha} \succeq \mathbf{0}$, $\boldsymbol{\lambda} \succeq \mathbf{0}$, $\boldsymbol{\mu} \succeq \mathbf{0}$ with $\sum_{m\in\mathcal{M}} \sum_{j\in\mathcal{B}} \mu_{m,j} \leq 1$ such that

$$\min_{m\in\mathcal{M}} \frac{P_m}{\gamma_m} |\mathbf{f}_m^H \mathbf{K}_m^\dagger \mathbf{f}_m|^2 - \mathbf{f}_m^H \mathbf{K}_m^\dagger \mathbf{G}_m \mathbf{K}_m^\dagger \mathbf{f}_m > 0, \tag{25}$$

$$\max_{i\in\mathcal{N}} \sum_{m\in\mathcal{M}} \zeta_m^2 \mathbf{f}_m^H \mathbf{K}_m^\dagger \mathbf{R}_m \mathbf{D}_i \mathbf{K}_m^\dagger \mathbf{f}_m \leq P_r. \tag{26}$$

E. Complexity Analysis

To determine the complexity of our proposed algorithm, note that P0 has been converted to an SDP D2 with $M(b+1)+N$ variables and M linear matrix inequality constraints of size N. Typically there are only a few neighboring cells with dominant interference, so b is a small number. The SDP can be solved efficiently using interior-point methods. Based on the complexity analysis for the standard SDP form in [39, Section 5], the computation complexity per iteration to solve the SDP D2 is $\mathcal{O}\big((M+N)^2MN^2\big)$. The number of iterations to solve an SDP is typically between 5 to 50 regardless of problem size [39, Section 5]. Thus, the complexity to solve the SDP is $\mathcal{O}\big((M+N)^2MN^2\big)$.

The overall computation complexity to solve P0 depends on the values of the optimal dual variables. As shown in Section III-C, if Case 2 happens, only one SDP problem D2 is solved, *i.e.*, the complexity is given by $\mathcal{O}\big((M+N)^2MN^2\big)$. If Case 3 happens, at most M SDP problems formulated as D3 are solved, *i.e.*, the worst-case complexity is given by $\mathcal{O}\big((M+N)^2M^2N^2\big)$. In both cases, the algorithm has a polynomial worst-case complexity w.r.t. the number of relays and S-D pairs. Note that the above analysis is based on worst-case complexity estimates. In practice, the complexity is much lower than the worst-case estimate [39, Section 5].

As shown in Appendix B, we can also reformulate P1 into an SOCP problem (A.2) given in Appendix B. It has $MN+1$ variables and $M(b+1)+N$ constraints. This SOCP can be directly solved using interior-point methods with the complexity per iteration of $\mathcal{O}\big((M+N)M^3N^3\big)$. The number of iterations to solve an SOCP does not depend on the problem size [40]. Thus, the complexity of the SOCP compared with the worst-case complexity of our proposed algorithm (*i.e.*, the maximum number of iterations in Case 3 of Section III-C) is increased by a factor of $\mathcal{O}\left(MN/(M+N)\right)$. We note that MN is typically much larger than $M+N$. Therefore, our proposed algorithm to obtain the optimal solution offers much lower complexity than the SOCP method.

IV. SNR and SINR Maximization

We can used the method presented in Section III to maximize the minimum received SNR or SINR subject to per-relay power constraint and a maximum interference limit at the neighboring cells.

A. Maximizing the Minimum SNR

In the following, we first formulate the max-min SNR problem and show that this problem and P1 are inverse problems. Then using the bisection search, an iterative algorithm is proposed to solve the max-min SNR problem.

Typically the relays have the same front-end amplifiers and the destinations have the same minimum SNR requirements and received interference threshold. In the following, we assume identical per-relay power budget, minimum SNR requirements for destinations in the desired cell, and maximum interference limit for destinations in the neighboring cells. Extension to the case of non-uniform power, SNR, and interference requirement can follow a similar approach.

The problem of maximizing the minimum SNR under a predetermined maximum interference threshold, $\mathcal{I}_0$, is formally

[5] Algorithm 1 requires at most $M-1$ iterations to complete.

stated as follows:

$$\begin{aligned} \text{P4:}\ \max_{\{\mathbf{w}_m\},\gamma}\ & \gamma \\ \text{subject to}\ & \mathbf{w}_m^H \tilde{\mathbf{B}}_{m,j} \mathbf{w}_m \le \mathcal{I}_0,\ m \in \mathcal{M},\ j \in \mathcal{B}, & \text{(27a)} \\ & \text{SNR}_m \ge \gamma,\ m \in \mathcal{M}, & \text{(27b)} \\ & \text{and (11a)} \end{aligned}$$

where $\mathcal{I}_0$ denotes a pre-determined maximum interference threshold.

The min-max interference problem P1 with a common SNR target γ_0 is given by

$$\begin{aligned} \text{P5:}\ \min_{\{\mathbf{w}_m\},\mathcal{I}}\ & \mathcal{I} \\ \text{subject to}\ & \mathbf{w}_m^H \tilde{\mathbf{B}}_{m,j} \mathbf{w}_m \le \mathcal{I},\ m \in \mathcal{M},\ j \in \mathcal{B}, & \text{(28a)} \\ & \text{SNR}_m \ge \gamma_0,\ m \in \mathcal{M}, & \text{(28b)} \\ & \text{and (11a).} \end{aligned}$$

We denote the optimal objective of P4 and P5 as $\gamma^o(\mathcal{I}_0)$ and $\mathcal{I}^o(\gamma_0)$ to focus on their dependencies on $\mathcal{I}_0$ and γ_0, respectively. We first study the optimal maximum SNR $\gamma^o(\mathcal{I}_0)$ as a function of $\mathcal{I}_0$ following the similar arguments in [35].

The optimal objective $\gamma^o(\mathcal{I}_0)$ is continuous and strictly monotonically increasing function of $\mathcal{I}_0$; for given $\mathcal{I}_0$ any $\gamma < \gamma^o(\mathcal{I}_0)$ is achievable.

Hence, for any γ_0, the minimum interference $\mathcal{I}_0$ is obtained when $\gamma^o(\mathcal{I}_0) = \gamma_0$, *i.e.*, $\mathcal{I}^o\big(\gamma^o(\mathcal{I}_0)\big) = \mathcal{I}_0$. This indicates that P4 and P5 are inverse problems, *i.e.*,

$$\mathcal{I}^o\big(\gamma^o(\mathcal{I}_0)\big) = \mathcal{I}_0, \gamma^o\big(\mathcal{I}^o(\gamma_0)\big) = \gamma_0.$$

The solution for P4 can be obtained by iteratively solving the min-max interference problem P5 with bisection search on the max interference threshold $\mathcal{I}$. The stopping criterion is when $\mathcal{I} \to \mathcal{I}_0$.

In Algorithm 2, we provide the steps to solve P4 using P5 and bisection.[6]

Algorithm 2 Maximizing the minimum SNR

1: Set convergence threshold $\delta > 0$.
2: Find $\gamma_{0,\min}$ such that $\mathcal{I}^o(\gamma_{0,\min}) < \mathcal{I}_0$.
3: Find $\gamma_{0,\max}$ such that $\mathcal{I}^o(\gamma_{0,\max}) > \mathcal{I}_0$.
4: Set $\gamma_0 = \frac{\gamma_{0,\min}+\gamma_{0,\max}}{2}$.
5: Solve P5 under γ_0
6: **if** $\mathcal{I}^o(\gamma_0) > \mathcal{I}_0$ **then**
7:     Set $\gamma_{0,\max} = \gamma_0$ and $\mathcal{I} = 0$.
8: **else**
9:     Set $\gamma_{0,\min} = \gamma_0$ and $\mathcal{I} = \mathcal{I}^o(\gamma_0)$.
10: **end if**
11: **while** $\mathcal{I} < \mathcal{I}_0 - \delta$ **do**
12:     (4)–(10).
13: **end while**
14: Return γ_0.

B. Maximizing the Worst-Case Received SINR

Since the performance measures for cellular networks, *e.g.*, data rate or bit-error-rate (BER), are direct functions of the received SINR, our ultimate goal is to maximize the worst-case SINR at the destinations. For many practical scenarios, we make local decisions in each cell. Without jointly optimizing all cells, each cell optimizes its own resources in a distributed way based on some messages passed between cells over the backhaul.

The received SINR at destination m in the desired cell is given by $\text{SINR}_m = \frac{P_{S,m}}{\mathcal{I}_m + P_{N,m}} = \frac{\text{SNR}_m}{\mathcal{I}_m/P_{N,m}+1}$, where $\mathcal{I}_m$ denotes the total interference at destination m. It can be determined by solving the max-min SNR problem P4, if we constrain the interference from each neighboring cell to be below a given value $\mathcal{I}_0$ such that $\mathcal{I}_m \le b\mathcal{I}_0$, for all $m \in \mathcal{M}$. Thus, we propose solving P4 under different values of $\mathcal{I}_0$. Then, an optimal $\mathcal{I}_0$ value can be chosen to maximize the worst-case SINR among all destinations in a cell, given by $\min_m \text{SINR}_m$. Intuitively, when $\mathcal{I}_0$ is too low, SINR at the destination is noise dominant. Also, low $\mathcal{I}_0$ limits the relay power, and the received signal power at the intended user is low as well, resulting in low SINR. As $\mathcal{I}_0$ increases, SINR increases due to power (and beamforming) gain. As $\mathcal{I}_0$ continues increasing, the interference becomes dominant over the gain received by relay beamforming, and SINR decreases. The $\mathcal{I}_0$ values that maximize the worst-case SINR are illustrated in Section V.

[6]The values of $\gamma_{0,\min}$ and $\gamma_{0,\max}$ can be set based on the typical range of SNRs in a particular application. For simplicity, we can always set $\gamma_{0,\min} = 0$ and $\gamma_{0,\max} = \max_m P_m \mathbf{f}_m^H \mathbf{G}_m^H \mathbf{f}_m$.

V. Simulation Results

In this section, we applied the proposed min-max interference algorithm in various simulation settings. We are mainly interested in how the maximum interference and the worst-case SINR behave under different system parameter values, *i.e.*, different number of relays, S-D pairs, and neighboring cells. We set $\sigma_r^2 = \sigma_d^2 = 1$, $P_m = P_0$ for $m \in \mathcal{M}$ with $P_0/\sigma_r^2 = 10$ dB, and $P_r/\sigma_d^2 = 20$ dB. The minimum SNR target is set to $\gamma_m = \gamma_0 = 5$ dB for $m \in \mathcal{M}$. The first and second hop channels $\mathbf{h}_m$ and $\{\mathbf{g}_m, \tilde{\mathbf{g}}_{m,j}\}$ are assumed i.i.d. zero-mean Gaussian with variance 1. This model essentially captures the worst-case interference scenario, where the distance from relays to cell-edge users at the neighboring cells is similar to that between the relays and destinations, causing strong interference. We further study the effect of imperfect interference CSI in Section V-D, and consider the scenario of random user locations in Section V-E.

A. Effect of the Number of Relays

To study the behavior of maximum interference as the number of relays N increases, we plot the CDF of $\mathcal{I}_{\max}$ in the objective of P1, normalized against noise variance σ_d^2, with $M = 2$ and $b = 1$ in Fig. 2. Also shown in Fig. 2 is the maximum interference under an alternate optimization problem where the objective is to minimize the maximum transmission power over all relays while meeting the minimum SNR requirements [35]. This min-max relay power problem may be viewed as a simpler alternative to reduce interference, which is created by the relays. The number of relays are

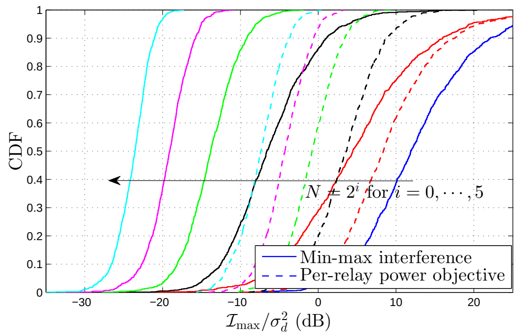

Fig. 2. CDF of normalized $\mathcal{I}_{\max}$ when $M = 2$ and $b = 1$.

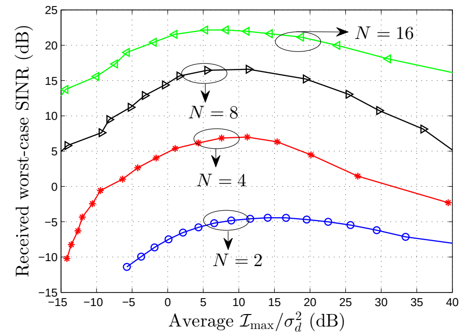

Fig. 4. Average $\min_m \text{SINR}_m$ versus average normalized $\mathcal{I}_{\max}$ for $M = 8$ and $b = 2$.

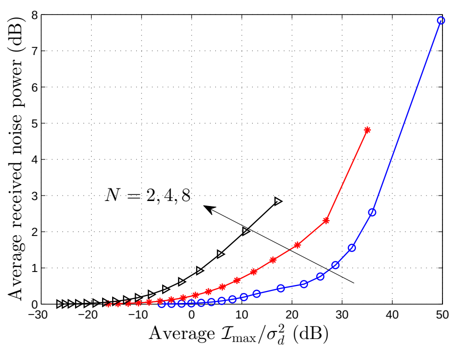

Fig. 3. Average received noise power versus average normalized $\mathcal{I}_{\max}$ for $M = 2$ and $b = 4$.

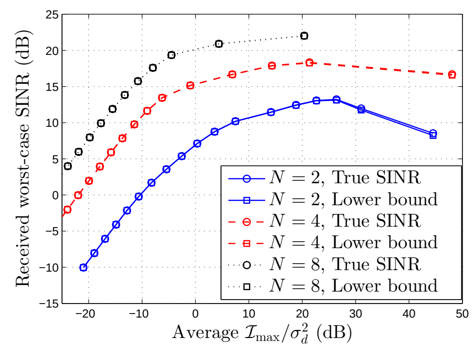

Fig. 5. Comparing the true $\min_m \text{SINR}_m$ with the SINR lower bound versus average normalized $\mathcal{I}_{\max}$ for $M = 2$ and $b = 1$.

chosen as $N = 2^i$ for $i \in \{0, \cdots, 5\}$. We see that as N increases, the interference CDF curves are shifted to the left for both optimization approaches. Note that the min-max interference approach significantly outperforms the per-relay power approach for each N, and the performance gap increases as N increases.

The average received noise power (7) versus average normalized $\mathcal{I}_{\max}$ with $N = 2,\ 4,\ 8$, $M = 2$, and $b = 4$ is shown in Fig. 3. It can be seen that noise power increases as N increases. Note that received noise is the total amplified noise and AWGN at the destination. The amplified noise decreases to zero as N increases, and the overall noise converges to the destination noise, *i.e.*, 0 dB. This happens as the beam vector norm $\|\mathbf{w}_m\|$ decreases as N increases due the power (and beamforming) gain achieved by relay beamforming.

To evaluate the performance of the max-min SNR problem P4, in Fig. 4, the average received worst-case SINR, *i.e.*, $\min_m \text{SINR}_m$ versus average normalized $\mathcal{I}_{\max}$ is represented with $b = 2$, $M = 8$, and $N = 2^i$ for $i \in \{1, \cdots, 4\}$. To plot each curve, the minimum SNR requirement γ_0 is set to -10 dB to 24 dB. For each γ_0, 500 realizations are generated, and then $\mathcal{I}_{\max}$ and $\min_m \text{SINR}_m$ are computed for each realization. As discussed at the end of Section IV-B, we see that $\min_m \text{SINR}_m$ first increases and then decreases as a function of $\mathcal{I}_{\max}$. Hence, we can numerically identify the maximum $\min_m \text{SINR}_m$ for each N.

In Fig. 5, the true $\min_m \text{SINR}_m$ is compared with the SINR lower bound with $N = 2,\ 4,\ 8$ and $b = 1$. For each realization, an SINR lower bound is obtained by substituting $\mathcal{I}_{\max}$ at the denominator of SINR instead of the true received interference. The curves for true SINR and lower bound are very close for each N. Hence, $\mathcal{I}_{\max}$ can be used to gain insight into $\min_m \text{SINR}_m$. If $\mathcal{I}_{\max}$ is set and γ_0 is obtained accordingly, then it reflects the received $\min_m \text{SINR}_m$.

B. Effect of the Number of S-D pairs

In Fig. 6, the average $\min_m \text{SINR}_m$ versus average normalized $\mathcal{I}_{\max}$ is shown for $N = 4$, and $M = 2^i$ for $i \in \{1, \cdots, 4\}$, and $b = 2$. For each curve, a maximum $\min_m \text{SINR}_m$ is observed. We see that $\min_m \text{SINR}_m$ decreases as M increases because the number of SNR constraints in each cell increases. Hence, the relays increase transmission power in each cell and generate more interference at the neighboring cells.

C. Effect of the Number of Neighboring Cells

In Fig. 7, the average $\min_m \text{SINR}_m$ versus average normalized $\mathcal{I}_{\max}$ is shown for $N = 4$, $M = 8$, and $b \in \{1, 2, 4, 6\}$. A maximum $\min_m \text{SINR}_m$ for each curve is identified. For fixed average $\mathcal{I}_{\max}$, increasing b leads to degradation on $\min_m \text{SINR}_m$. As b increases, the number of interference sources corresponding to each subchannel increases. The total received interference increases and hence, $\min_m \text{SINR}_m$ decreases as b increases.

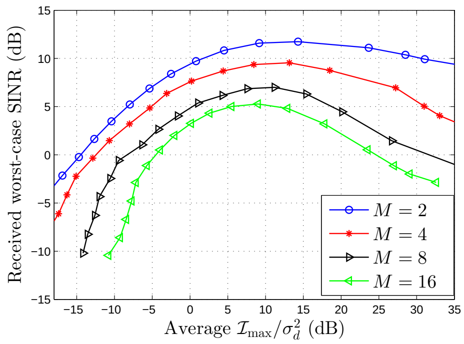

Fig. 6. Average $\min_m$ SINR$_m$ versus average normalized $\mathcal{I}_{\max}$ for $N = 4$ and $b = 2$.

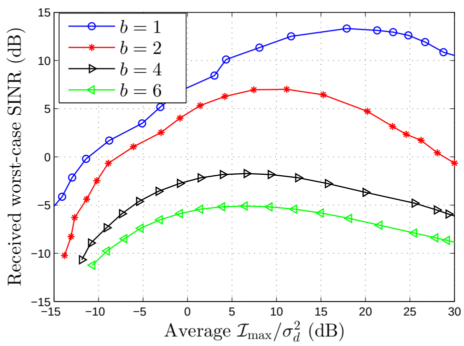

Fig. 7. Average $\min_m$ SINR$_m$ versus average normalized $\mathcal{I}_{\max}$ for $N = 4$ and $M = 8$.

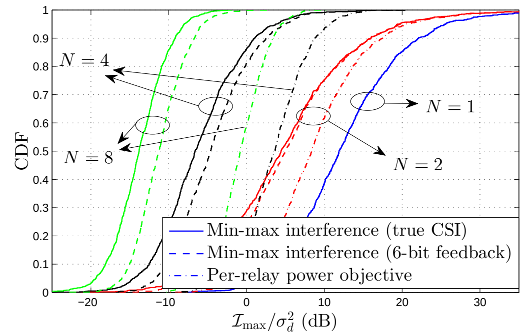

Fig. 8. Empirical CDF of normalized $\mathcal{I}_{\max}$ under limited feedback (Scenario 1) with $M = 2$ and $b = 1$.

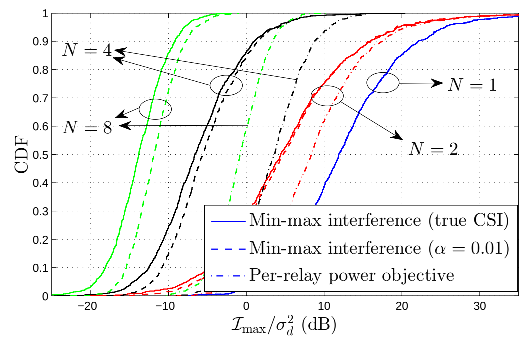

Fig. 9. Empirical CDF of normalized $\mathcal{I}_{\max}$ under Gaussian channel estimation error (Scenario 2) with $M = 2$ and $b = 1$.

D. Effect of Imperfect CSI

So far, true interference CSI is assumed to be known perfectly at the relays. In practice, obtaining such interference CSI may not be possible. In order to observe how robust the proposed algorithm is w.r.t. imperfect CSI, we consider the following scenarios with two types of imperfect CSI: limited number of CSI feedback bits and channel estimation error.

In Scenario 1, the receiver knows the interference CSI perfectly. However, the feedback bits to the relays are limited. We consider equiprobable quantization of channel values. Let B denote the number of available feedback bits. Every real and imaginary part of a channel is quantized with equal probability according to the CSI distribution, which is complex Gaussian.

In Scenario 2, the channels are estimated at the receiver with error and the estimated channel is fed back to the relays. In order to model the channel estimation error, let us define $\hat{h} = h + \alpha\tilde{h}$, where h is the true channel, $\hat{h}$ is the estimated channel used for optimization, $\tilde{h} \sim \mathcal{CN}(0, 1)$, and the weight α is set to adjust the variance of error w.r.t. the variance of true CSI.

In Fig. 8, the CDF of normalized $\mathcal{I}_{\max}$ under true interference CSI is compared with that of the imperfect CSI in Scenario 1 with 6 feedback bits (3 bits for each real and imaginary parts). It can be seen the interference in this limited feedback scenario is very close to the true CSI case even when the number of relays is large (*e.g.*, $N = 8$). As expected, the performance gap between the limited feedback scenario and true CSI case increases as N increases. In addition, the min-max interference approach under limited feedback still substantially outperforms the min-max per-relay power approach.

Fig. 9 shows the CDF of normalized $\mathcal{I}_{\max}$ under imperfect CSI in Scenario 2 with the channel estimation error being $\alpha = 0.01$. The interference in this case is close to that of true CSI. In addition, we see that the performance degradation increases as N increases. Again, the min-max interference approach under imperfect CSI in Scenario 2 outperforms the min-max per-relay approach.

The average $\min_m$ SINR$_m$ versus average normalized $\mathcal{I}_{\max}$ compared with that of the imperfect CSI in Scenario 1 with 4 feedback bits is shown in Fig. 10 for $N = \{2, 4, 8\}$, $M = 2$, and $b = 4$. The performance degradation due to limited feedback increases as N increases. Note that $\mathcal{I}_{max}$ corresponding to the maximum $\min_m$ SINR$_m$ decreases as N increases, reflecting higher diversity gain attained with more relays through achieving smaller $\mathcal{I}_{max}$.

E. Performance under Random Relay and User Locations

Previous simulation setup has captured the worse-case interference scenario, by assuming i.i.d. channel distribution for all relays and users. If some relays and destinations are far away from the cell edge, or users in neighboring cells are away from the cell edge, the ICI caused to these neighboring users is low and less critical.

We now study the pattern of the maximum interference in a scenario with random user and relay locations. We

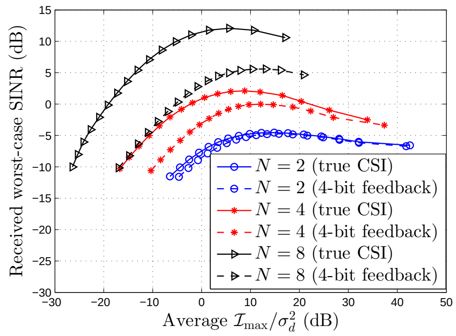

Fig. 10. Average $\min_m \text{SINR}_m$ under limited feedback (Scenario 1) with $M = 2$ and $b = 4$.

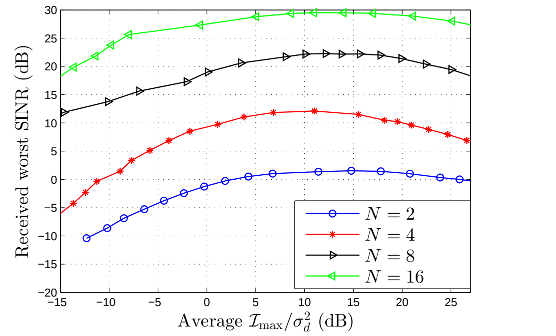

Fig. 12. Average $\min_m \text{SINR}_m$ versus average normalized $\mathcal{I}_{\max}$ for $M = 8$ and $b = 2$.

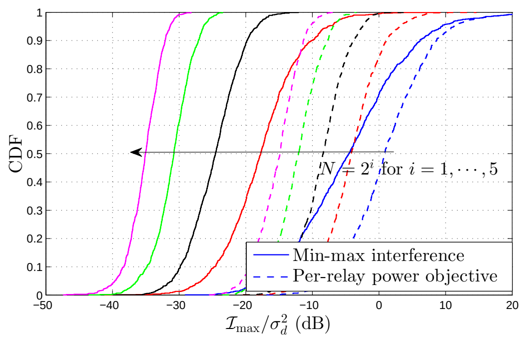

Fig. 11. CDF of normalized $\mathcal{I}_{\max}$ when $M = 2$ and $b = 1$.

set the distance between each source and relay, relay and destination in the desired cell, and relay and destination in the neighboring cells by κR, where R is the cell radius and κ is a random variable with uniform distribution in the range $[0.5\ 1]$, $[0.5\ 1]$, and $[1\ 1.5]$, respectively. The channel over each link is generated as zero-mean Gaussian with variance using the distance-based pathloss. We assume the path loss exponent is 3.

Similar to Fig. 2, we plot the CDF of normalized $\mathcal{I}_{\max}$ with $M = 2$, $b = 1$, and increasing N in Fig. 11, where we compared $\mathcal{I}_{\max}$ under our solution for P1 with that under the per-relay power objective. As expected, based on the above discussion, comparing Fig. 11 with Fig. 2, we observe that the interference CDF is shifted to the left, indicating a smaller $\mathcal{I}_{\max}$ as the the users are randomly located. However, the general trend remains the same.

Similar to Fig. 4, we also plot the average $\min_m \text{SINR}_m$ versus average normalized $\mathcal{I}_{\max}$ with $b = 2$, $M = 8$, and increasing N in Fig. 12, where we evaluate the performance of the max-min SNR problem P4. As expected, we see that $\min_m \text{SINR}_m$ first increases and then decreases as a function of $\mathcal{I}_{\max}$. Comparing Fig. 12 with Fig. 4, we observe that the average received worst-case SINR increases. It verifies that the $\mathcal{I}_{\max}$ decreases for random user locations, but again the general trend remains the same.

VI. Conclusions

In this paper, we have considered a multi-relay cellular network, where each cell has multiple S-D pairs communicating in orthogonal channels with assistance from the relays. In order to manage ICI, we have formulated the min-max interference problem under per-relay power constraints and minimum SNR requirements. We have shown that the strong duality property holds for this non-convex problem. Solving the Lagrange dual problem, three cases have been identified based on the optimal dual variables. We then propose an iterative algorithm to obtain the optimal beam vectors in semi-closed-form expressions. Numerical results have shown 10 dB reduction in the maximum interference with 4 relays for the min-max interference approach over the per-relay power approach, while the performance degradation when only 6 CSI feedback bits are used is within 3 dB. We have also solved the max-min SNR problem, under maximum interference and per-relay power constraints, using bisection search. Under different problem setups, we have evaluated the maximum interference and the corresponding worst-case received SINR. A maximum worst-case SINR has been observed as we vary the maximum interference target, which provides insight into designing relay beamforming in a multi-cell network.

Appendix A
Proof of Proposition 1

Proof: The interference constraint (12a) and per-relay power constraint (11a) are convex w.r.t. $\mathbf{w} \triangleq [\mathbf{w}_1^T, \cdots, \mathbf{w}_M^T]^T$. However, the minimum received SNR constraint (11b) is non-convex. Reformulating the SNR constraint (11b) in a conic form, we have

$$\sqrt{P_m}|\mathbf{w}_m^H \mathbf{f}_m| \geq \sqrt{\gamma_m} \left\| \begin{bmatrix} \mathbf{G}_m^{1/2}\mathbf{w}_m \\ \sigma_d \end{bmatrix} \right\|, \ m \in \mathcal{M}. \quad \text{(A.1)}$$

Note that $\mathbf{w}_m$ can have any arbitrary phase, *i.e.*, it is obtained uniquely up to a phase shift. The phase could be adjusted such that $\mathbf{w}_m^H \mathbf{f}_m$ becomes real-valued for $m \in \mathcal{M}$. The min-max interference problem P1 can be recast as

$$\min_{\mathbf{w}_1,\cdots,\mathbf{w}_M,\mathcal{I}_{\max}} \mathcal{I}_{\max} \quad \text{(A.2)}$$
$$\text{subject to (A.1), (11a), and (12a).}$$

The primal-dual optimality conditions for the problems with constraints in the form of (A.1) are provided in [41, Proposition 3]. Following a similar proof, it can be shown that (A.2) has zero duality gap with its Lagrangian dual. To prove

Proposition 1, we are left to show that the Lagrangian of P1 is the same as the Lagrangian of (A.2) by using a similar proof as in [9, Proposition 1].

The Lagrangian of P1 is given by

$$L = \mathcal{I}_{\max} + \sum_{m=1}^{M}\sum_{j=1}^{b}\mu_{m,j}\big(\mathbf{w}_m^H\tilde{\mathbf{B}}_{m,j}\mathbf{w}_m - \mathcal{I}_{\max}\big) \quad \text{(A.3)}$$
$$+ \sum_{i=1}^{N}\lambda_i\big(\sum_{m=1}^{M}\mathbf{w}_m^H\mathbf{R}_m\mathbf{D}_i\mathbf{w}_m - P_r\big)$$
$$+ \sum_{m=1}^{M}\alpha_m\big(\sigma_d^2 + \mathbf{w}_m^H\mathbf{G}_m\mathbf{w}_m - \frac{P_m}{\gamma_m}|\mathbf{w}_m^H\mathbf{f}_m|^2\big).$$

The Lagrangian of (A.2) is obtained by

$$\hat{L} = \mathcal{I}_{\max} + \sum_{m=1}^{M}\sum_{j=1}^{b}\hat{\mu}_{m,j}\big(\mathbf{w}_m^H\tilde{\mathbf{B}}_{m,j}\mathbf{w}_m - \mathcal{I}_{\max}\big) \quad \text{(A.4)}$$
$$+ \sum_{i=1}^{N}\hat{\lambda}_i\big(\sum_{m=1}^{M}\mathbf{w}_m^H\mathbf{R}_m\mathbf{D}_i\mathbf{w}_m - P_r\big)$$
$$+ \sum_{m=1}^{M}\hat{\alpha}_m\left(\left\|\begin{bmatrix}\mathbf{G}_m^{1/2}\mathbf{w}_m\\ \sigma_d\end{bmatrix}\right\| - \sqrt{\frac{P_m}{\gamma_m}}|\mathbf{w}_m^H\mathbf{f}_m|\right).$$

Denoting $u_m \triangleq \left\|\begin{bmatrix}\mathbf{G}_m^{1/2}\mathbf{w}_m\\ \sigma_d\end{bmatrix}\right\| + \sqrt{\frac{P_m}{\gamma_m}}|\mathbf{w}_m^H\mathbf{f}_m| \geq \sigma_d$ and converting the last term of the Lagrangian (A.4), it is equivalent to

$$\hat{L} = \mathcal{I}_{\max} + \sum_{m=1}^{M}\sum_{j=1}^{b}\hat{\mu}_{m,j}\big(\mathbf{w}_m^H\tilde{\mathbf{B}}_{m,j}\mathbf{w}_m - \mathcal{I}_{\max}\big) \quad \text{(A.5)}$$
$$+ \sum_{i=1}^{N}\hat{\lambda}_i\big(\sum_{m=1}^{M}\mathbf{w}_m^H\mathbf{R}_m\mathbf{D}_i\mathbf{w}_m - P_r\big)$$
$$+ \sum_{m=1}^{M}\frac{\hat{\alpha}_m}{u_m}\big(\sigma_d^2 + \mathbf{w}_m^H\mathbf{G}_m\mathbf{w}_m - \frac{P_m}{\gamma_m}|\mathbf{w}_m^H\mathbf{f}_m|^2\big).$$

Since $u_m \geq \sigma_d$, by changing the variables $\alpha_m = \frac{\hat{\alpha}_m}{u_m}$, there exists $\alpha_m \geq 0$ for any $\hat{\alpha}_m \geq 0$ and $m \in \mathcal{M}$ such that (A.3) and (A.4) become exactly the same. As a result, the strong Lagrange duality holds for the non-convex problem P1. ■

APPENDIX B
PROOF OF LEMMA 1

Proof: Substituting (15) into (17a), the constraint (17a) is equivalent to

$$\mathbf{R}_m\mathbf{D}_{\boldsymbol{\lambda}} + \sum_{j=1}^{b}\mu_{m,j}\tilde{\mathbf{B}}_{m,j} + \alpha_m\big(\mathbf{G}_m - \frac{P_m}{\gamma_m}\mathbf{f}_m\mathbf{f}_m^H\big) \succeq \mathbf{0}. \quad \text{(B.1)}$$

Using contradiction, we show that $\mathbf{G}_m - \frac{\alpha_m P_m}{\gamma_m}\mathbf{f}_m\mathbf{f}_m^H$ is an indefinite matrix. Suppose that $\mathbf{G}_m \succeq \frac{P_m}{\gamma_m}\mathbf{f}_m\mathbf{f}_m^H$. Since $\mathbf{G}_m$ is a positive-definite matrix, we have $P_m\mathbf{f}_m^H\mathbf{G}_m^{-1}\mathbf{f}_m \leq \gamma_m$ ([9, Lemma 1]). This contradicts the necessary condition for the feasibility of P1 as shown in Section III-A. If either $\mu_{m,j}^o > 0$ for some $\{m,j\}$ or $\boldsymbol{\lambda}^o \succ \mathbf{0}$, there exists $\alpha_m^o > 0$ such that (17a) is satisfied. Note that the objective of the dual problem increases as α_m increases. If $\boldsymbol{\mu}_m^o = 0$ and there exists $\lambda_i^o = 0$ for some i, then α_m^o can be zero. ■

APPENDIX C
PROOF OF PROPOSITION 3

Proof: Suppose the necessary condition in Lemma 1 is satisfied for all $m \in \mathcal{M}$, *i.e.,* $\boldsymbol{\alpha}^o \succ \mathbf{0}$. Then we have $\mathbf{K}_m^o \succ \mathbf{0}$ for all $m \in \mathcal{M}$. Using [9, Lemma 1] and rewriting the expression of the matrix inequality (17a), the dual problem D1 can be expressed as

$$\max_{\boldsymbol{\lambda},\boldsymbol{\mu}}\max_{\boldsymbol{\alpha}} \sum_{m=1}^{M}\alpha_m\sigma_d^2 - P_r(\sum_{i=1}^{N}\lambda_i) \quad \text{(C.1)}$$
$$\text{subject to } \frac{\alpha_m P_m}{\gamma_m}\mathbf{f}_m^H\mathbf{K}_m^{-1}\mathbf{f}_m \leq 1,\ m \in \mathcal{M} \quad \text{(C.2)}$$
$$\text{(17b), and (16a).}$$

Since the optimal beam vector solution of the SIMO beamforming problem is known, in the following, we establish the duality between (C.1) and SIMO beamforming problem similar to [9]. Considering the dual problem (C.1) and the optimization problem

$$\max_{\boldsymbol{\lambda},\boldsymbol{\mu}}\min_{\boldsymbol{\alpha}} \sum_{m=1}^{M}\alpha_m\sigma_d^2 - P_r(\sum_{i=1}^{N}\lambda_i) \quad \text{(C.3)}$$
$$\text{subject to } \frac{\alpha_m P_m}{\gamma_m}\mathbf{f}_m^H\mathbf{K}_m^{-1}\mathbf{f}_m \geq 1,\ m \in \mathcal{M} \quad \text{(C.4)}$$
$$\text{(17b), and (16a),}$$

we have the inner maximization in (C.1) becomes minimization in (C.3) and the SNR inequality is reversed. Substituting (15) into LHS of (C.2), we define

$$\Phi_m(\alpha_m) = \frac{P_m}{\gamma_m}\mathbf{f}_m^H\Big(\frac{1}{\alpha_m}(\mathbf{R}_m\mathbf{D}_{\boldsymbol{\lambda}} + \sum_{j\in\mathcal{B}}\mu_{m,j}\tilde{\mathbf{B}}_{m,j}) + \mathbf{G}_m\Big)^{-1}\mathbf{f}_m \quad \text{(C.5)}$$

which is a monotonically increasing function of α_m. Hence, both (C.2) and (C.4) are met with equality at optimality for a given $\{\boldsymbol{\lambda}^o, \boldsymbol{\mu}^o, \boldsymbol{\alpha}^o\}$ satisfying Lemma 1. Furthermore, problems (C.1) and (C.3) have the same solution $\boldsymbol{\alpha}^o$ satisfying $\Phi_m(\alpha_m^o) = 1$ for $m \in \mathcal{M}$. This implies that the optimization problems (C.1) and (C.3) are equivalent.

Consider the following optimization problem

$$\max_{\boldsymbol{\lambda},\boldsymbol{\mu}}\min_{\mathbf{w}_m,\boldsymbol{\alpha}} \sum_{m=1}^{M}\alpha_m\sigma_d^2 - P_r(\sum_{i=1}^{N}\lambda_i) \quad \text{(C.6)}$$
$$\text{subject to } \frac{\alpha_m P_m|\mathbf{w}_m^H\mathbf{f}_m|^2}{\|\mathbf{K}_m^{\frac{1}{2}}\mathbf{w}_m\|^2} \geq \gamma_m,\ m \in \mathcal{M} \quad \text{(C.7)}$$
$$\text{(17b), and (16a).}$$

The inner minimization of problem (C.6) is the receive SIMO beamforming for power minimization problem where M receivers each are equipped with N antennas. The transmit power and noise covariance matrix for receiver m are $\sum_{m=1}^{M}\alpha_m\sigma_d^2 - P_r(\sum_{i=1}^{N}\lambda_i)$ and $\bar{\mathbf{K}}_m \triangleq$

$\frac{\sum_{m=1}^{M}\alpha_m\sigma_d^2-P_r(\sum_{i=1}^{N}\lambda_i)}{\alpha_m P_m}\mathbf{K}_m$, respectively. The solution of the SIMO beamforming problem, *i.e.*, the inner minimization of problem (C.6), is given by $\bar{\mathbf{w}}_m^o = \bar{\mathbf{K}}_m^{-1}\mathbf{f}_m$. Substituting $\bar{\mathbf{w}}_m^o = \frac{\alpha_m P_m}{\sum_{m=1}^{M}\alpha_m\sigma_d^2-P_r(\sum_{i=1}^{N}\lambda_i)}\mathbf{K}_m^{-1}\mathbf{f}_m$ into problem (C.6), we have problem (C.3). Note that the optimal $\bar{\mathbf{w}}_m^o$ can be scaled by any non-zero coefficient ξ such that $\xi\bar{\mathbf{w}}_m^o$ is also an optimal solution. Hence, the dual problem D1 is equivalent to the SIMO beamforming problem (C.6), and we can use the solution of (C.6) to obtain $\mathbf{w}_m^o$ in the min-max interference problem P1.

Since P1 has zero duality gap as shown in Proposition 1 and $\bar{\mathbf{w}}^o$ is unique up to a scale factor, the optimal beam vector $\mathbf{w}_m^o$ is given by $\mathbf{w}_m^o = \zeta_m\mathbf{K}_m^o\mathbf{f}_m$. In order to obtain ζ_m, note that the SNR constraint (11b) is met with equality based on the slackness condition. Substituting $\mathbf{w}_m^o$ into the equation $\frac{P_m\mathbf{w}_m^H\mathbf{F}_m\mathbf{w}_m}{\mathbf{w}_m^H\mathbf{G}_m\mathbf{w}_m+\sigma_d^2} = \gamma_m$ and after some manipulations, (20) is obtained and the proof is complete. ∎

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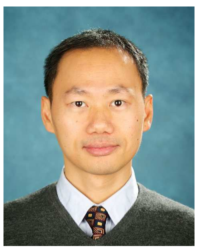

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