Dynamic Joint Resource Optimization for LTE-Advanced Relay Networks

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***Abstract*—A dynamic optimization algorithm is proposed for the joint allocation of subframes, resource blocks, and power in the Type 1 inband relaying scheme mandatory in the LTE-Advanced standard. Following the general framework of Lyapunov optimization, we decompose the original problem into three sub-problems in the forms of convex programming, linear programming, and mixed-integer programming. We solve the last sub-problem in the Lagrange dual domain, showing that it has zero duality gap, and that a primal optimum can be obtained with probability one. The proposed algorithm dynamically adapts to traffic and channel fluctuations, it accommodates both instantaneous and average power constraints, and it obtains arbitrarily near-optimal sum utility of each user's average throughput. Simulation results demonstrate that the joint optimum can significantly outperform suboptimal alternatives.**

***Index Terms*—LTE-Advanced networks, relay, power, subframe, RB, dynamic optimization.**

I. Introduction

The LTE-Advanced standard has specified the usage of relay nodes (RNs) as a cost efficient means to extend the service coverage area of a base station (termed eNB, for *evolved NodeB*) [1]. Each RN accesses the eNB through a wireless *backhaul link* (BL). It forwards data to and from some user equipment (UE) through a wireless *access link* (AL). The basic resource granularity for transmission is a *resource block* (RB), each consisting of twelve 15 kHz subcarriers and six to seven OFDM symbols. The temporal duration of two RBs is represented by one *subframe* [2].

To accommodate network operators with only one carrier frequency, support for *Type 1* relays is mandatory in the LTE-Advanced standard [1] [3]. In the Type 1 relaying scheme, the backhaul link and the access link share the same carrier frequency, with time-division multiplexing to avoid loop interference between the backhaul link receiving antenna and the access link transmitting antenna. More specifically, in each subframe, simultaneous transmissions in the backhaul link and the access link is not allowed. Therefore, an adaptive subframe allocation scheme between the backhaul and access links is of paramount importance.

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The LTE-Advanced standard supports dynamic scheduling, which concerns the assignment of subframes to the backhaul and access links, and the allocation of RBs to the UEs. In addition, the available transmission power at the eNB and RNs is often limited due to legal and hardware constraints, which presents an additional dimension of flexibility and challenges. The decisions in dynamic scheduling and power allocation are highly correlated. Therefore, joint consideration is necessary to provide a judicious solution to fully exploit the channel and user diversity gain in the system.

In this paper, we study the jointly optimal dynamic allocation of subframes, RBs, and power in a Type 1 relay network. Our objective is to maximize the network utility of the average throughput to all UEs. We consider practical impediments, including instantaneous and average power constraints on the eNB and RNs, as well as the special subframe activation limitations on the backhaul and access links imposed by Type 1 relays. Furthermore, by ignoring subframe allocation, the same solution can also be applied to Type 1a *outband* relaying [1], which deploys the backhaul and access links with different carriers to avoid loop interference.

This three-dimensional joint subframe, RB, and power optimization problem can be expressed as a mixed-integer program, whose solution typically has prohibitive complexity. We explain in Section II the existing methods for resource allocation in OFDMA relay systems. None of them are applicable to our problem, since they either do not accommodate the practical constraints imposed by LTE-Advanced Type 1 relays, or consider only a subset of the three resource allocation problems, leading to suboptimal solutions.

Instead, we propose an asymptotically optimal algorithm for the joint allocation of all three resources. Our main contributions are as follows:

- We adopt a general Lyapunov optimization framework to jointly optimize subframe, RB, and power allocation, which involves the minimization of a Lyapunov drift-plus-penalty function [4]. For the specific setting of our problem, we show that this minimization is decomposable into three sub-problems represented by 1) a convex program, 2) a linear program, and 3) a mixed-integer program. The first two sub-problems are solved using standard approaches.
- For the more challenging third sub-problem, we show that optimality can be preserved by continuity relaxation and Lagrange dual decomposition. We further observe that, after optimizing in the dual domain, the special structure of our solution allows recovering the primal optimal solution with probability one.

- The above solution results in a $1 - O(\frac{1}{V})$ utility and $O(V)$ delay tradeoff for the overall optimization problem, for any arbitrary positive V. The proposed algorithm accounts for both traffic-level variations and channel fading through dynamic adaptation.
- We further show that the proposed algorithm is amenable to practical implementation, and we demonstrate its performance advantage over suboptimal alternatives through simulation.

The organization of the rest of this paper is as follows. In Section II, we discuss the related work. In Section III, we summarize the LTE-Advanced Type 1 inband relay network and present the problem statement. The dynamic joint subframe, RB, and power allocation scheme is presented in Section IV. We give analytical performance bounds and show simulation performance in Section V. Concluding remarks are given in Section VI.

II. Related Work

Various solutions to joint resource allocation in OFDMA relay systems have been proposed in the literature. The link scheduling scheme with equal power allocation in [5] requires reusing RBs between the backhaul and access links, which is not supported in the LTE-Advanced standard. A few recent works in OFDMA relaying study the jointly optimal allocation of power and subchannels in [6]–[9], the joint transmission mode, relay node, and subchannel allocation by enabling the physical layer network coding in [10], and the joint relay strategy, relay node, subchannel, and power allocation in [11]. However, the allocation of subchannels is less dynamic than that of subframes and RBs.

Most studies on resource allocation with LTE-Advanced typed relays consider only a partial set among subframes, RBs and power. For example, [12] proposes a low-complexity link scheduling scheme with performance bound by large deviation theory, while [13] studies the effect of multi-cell interference on resource allocation. Neither work considers adaptive subframe activation between the backhaul and access links.

There are relatively few studies on the subframe allocation of inband relaying. Joint subframe and RB allocation for the Type 1 inband relay network has been studied in [14]–[17]. However, none considers power allocation, even though it is supported in the standard [18]. In addition, by reducing the design complexity in the power dimension, they usually face a linear integer optimization problem, which is generally hard to solve. Worse, the additional consideration for joint power allocation changes the problem to non-linear mixed-integer optimization. In particular, the integer variables would exhibit a multiplication form as shown in Section III, which is combined with the non-linear power term leading to drastically increased computational complexity. To the best of our knowledge, no solution exists in the literature for jointly optimal allocation of all three resources.

Related to Lyapunov-typed optimization with data forwarding by relays, the backpressure-typed algorithms have been proposed to dynamically schedule packet transmission over multiple hops [19]–[21]. The admission control component of our algorithm borrows from this general approach. However, these works generally assume a given function that maps channel state to data rate, without considering how to optimize the actual system details, such as the allocation of subframes, RBs, and power, which we consider in this paper. Other queue-length based scheduling strategies can be found in [22]–[24]. However, the UE is allowed simultaneous reception from multiple RNs in [22], which is not supported in the LTE-Advanced Standard; [23] only focuses on the out-band relaying scheme, without considering the power allocation; In [24], without considering power adaption, equal subframe is allocated to the backhaul link and the access link in the Type 1 inband relaying scheme, leading to a sub-optimal solution.

III. System Model and Problem Statement

We consider the downlink transmission in the LTE-Advanced Type 1 relaying network as illustrated in Fig. 1, with one eNB and K RNs in a specific area. We denote the set containing the eNB and RNs as $\mathcal{B} = \{0, 1, \cdots, K\}$, where 0 refers to the eNB. We denote the RN set as $\mathcal{R} = \{1, \cdots, K\}$. The important notations used throughout this paper are summarized in Table I.

TABLE I
Notation Summary

$\mathcal{B}$	eNB and RN set
$\mathcal{R}$	RN set
$\mathcal{U}_k$	Set of UEs served by RN k or by the eNB when $k=0$
$\mathcal{J}$	RB set
$s_A(t)$	Allocation of subframe t to the AL
$s_B(t)$	Allocation of subframe t to the BL
$b_{mkj}(t)$	Allocation of RB j to UE (m,k) in BL or to UE $(m,0)$ in DL in subframe t
$a_{mkj}(t)$	Allocation of RB j to UE (m,k) in AL in subframe t
$c_j(t)$	Indicate function for the usage of RB j in subframe t
P_k	Average power constraint for RN k or for the eNB when $k=0$
$\hat{P}_k$	Instantaneous power constraint for RN k or for the eNB when $k=0$
$p^B_{mkj}(t)$	Transmission power allocated to UE (m,k) in BL or to the UE $(m,0)$ in DL over RB k in subframe t
$p^A_{mkj}(t)$	Transmission power allocated to UE (m,k) in AL over RB j in subframe t
$p_k(t)$	Instantaneous power of RN k or of the eNB when $k=0$ in subframe t
$g^D_{m0j}(t)$	Channel gain of RB j between UE $(m,0)$ and eNB in DL in subframe t
$g^B_{k0j}(t)$	Channel gain of RB j between RN k and eNB in BL in subframe t
$g^A_{mkj}(t)$	Channel gain of RB j between UE (m,k) and RN k in AL in subframe t
$r^B_{mk}(t)$	Transmission rate for UE (m,k) in BL or for UE $(m,0)$ in DL in subframe t
$r^A_{mk}(t)$	Transmission rate for UE (m,k) in AL in subframe t
$Z_{mk}(t)$	Application layer queue length for UE (m,k) in subframe t
$\beta_{mk}(t)$	Admitted amount of data for UE (m,k) from the application layer queue in subframe t
$\beta_{\max}$	Upper bound for $\beta_{mk}(t)$ regarding each UE in each subframe
$Q_{mk}(t)$	Queue length at the eNB for UE (m,k) in subframe t
$H_{mk}(t)$	Queue length at RN k for UE (m,k) in subframe t
$\Theta_k(t)$	Virtual queue length for the average power constraint in subframe t
$Y_{mk}(t)$	Virtual queue length for the auxiliary variable constraint in subframe t

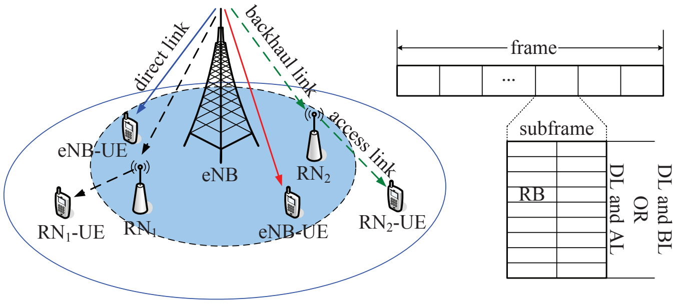

Fig. 1. LTE-Advanced inband relay network and frame architecture

A. LTE-Advanced Type 1 Inband Relaying

The LTE-Advanced standard does not support simultaneous reception of signals from eNB and RN by an UE. The UE can be served either directly by the eNB or by one RN. The UE closer to the eNB directly connects with the eNB, while the UE that is further from the eNB communicates with its closest RN. We denote the set of UEs served by the eNB as $\mathcal{U}_0$ and those served by RN k as $\mathcal{U}_k$ for $k = 1, 2, \cdots, K$. The number of UEs in the set $\mathcal{U}_k$ is $M_k = |\mathcal{U}_k|$, so the total number of UEs is expressed as $M = \sum_{k\in\mathcal{B}} M_k$ and the number of UEs attached to RNs is $M_R = \sum_{k\in\mathcal{R}} M_k$. Furthermore, we use indices of the form (m, k) to specify one UE, indicating that the UE m is attached to the eNB when $k = 0$ or attached to RN k when $k \neq 0$. We term the link between eNB and an UE the *direct link* (DL).

For the reasons explained in Section I, we focus on Type 1 relays in the standard [1]. The backhaul and access links share the same frequency spectrum in Type 1 relays. They operate in the time-division multiplexing mode, such that in each subframe, all RBs are used exclusively either in the direct and backhaul links, or in the direct and access links. We denote the set of RBs as $\mathcal{J}$ and index each RB by j. The total number of RBs is $J = |\mathcal{J}|$. We do not consider RB reuse here, i.e., each RB may be assigned to only one UE for the transmission in any link. Hence, for each subframe, the problem for the scheduler is to decide its activation among the three types of links, and to assign RBs and power to UEs.

The above assumption excludes RB reuse between the direct link and the access link. This benefits interference control in the scenario where the eNB and the RN are near each other. Given the NP hardness of power control in interference-limited systems [25], in this paper, we aim to provide an efficient and optimal network control algorithm for Type 1 relaying LTE-Advanced systems under the orthogonal RB allocation constraint. In addition, if the eNB and the RN are geographically far enough such that their mutual interference can be ignored, our proposed algorithm can be easily extended without changing its structure, by allowing both the eNB-UE links and the RN-UE links to use all RBs in the access link activation subframe, while guaranteeing the RB orthogonality only within the eNB-UE links and within the RN-UE links.

B. Subframe Allocation

Since the backhaul and access links of a Type 1 inband RN use the same frequency spectrum, they cannot be simultaneously active in the same subframe. We use binary variables $s_B(t)$ and $s_A(t)$ as indicate functions for the allocation of subframe t to the backhaul link and the access link, respectively. We have $s_A(t) + s_B(t) \leq 1, \forall t$. Note that the direct link may use any subframe.

C. Resource Block Allocation

In each subframe, either direct link and access link can transmit simultaneously, or direct link and backhaul link can transmit simultaneously, but different RBs must be allocated to these links and to different UEs. We introduce the binary variable $b_{mkj}(t)$ as an indicator function for the allocation of RB j to UE (m, k) in subframe t on the backhaul link, for $k = 1, 2, \ldots, K$. We similarly define $a_{mkj}(t)$ in subframe t for the access link. We also similarly define $b_{m0j}(t)$ in subframe t for the direct link. Then the no-reuse constraint on RBs implies

$$\begin{aligned} c_j(t) \triangleq & s_A(t) \sum_{k\in\mathcal{R}} \sum_{m\in\mathcal{U}_k} a_{mkj}(t) + s_B(t) \sum_{k\in\mathcal{R}} \sum_{m\in\mathcal{U}_k} b_{mkj}(t) \\ & + \sum_{m\in\mathcal{U}_0} b_{m0j}(t) \leq 1, \quad \forall j \in \mathcal{J}. \end{aligned} \tag{1}$$

We note from the above that there is a non-linear relation between the subframe and RB allocation decisions, which brings challenge to joint optimization.

D. Average and Instantaneous Power Constraints

Both the eNB and RNs have average and instantaneous power constraints. We denote the average power constraint for the eNB and RNs as P_k, and the instantaneous power constraint as $\hat{P}_k$, for $k = \{0, 1, \cdots, K\}$

Let the transmission power allocated to RB j in subframe t be $p^B_{m0j}(t)$ for the direct link regarding UE $(m, 0)$, and $p^B_{mkj}(t)$,$k \neq 0$, for the backhaul link regarding UE (m, k). Then the instantaneous power of the eNB can be written as

$$\begin{aligned} p_0(t) \triangleq & s_B(t) \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{R}} \sum_{m\in\mathcal{U}_k} b_{mkj}(t) p^B_{mkj}(t) + \\ & \sum_{j\in\mathcal{J}} \sum_{m\in\mathcal{U}_0} b_{m0j}(t) p^B_{m0j}(t). \end{aligned} \tag{2}$$

It then follows that the instantaneous power constraint for the eNB can be expressed as $p_0(t) \leq \hat{P}_0$, and the average power constraint for the eNB can be expressed as $\bar{p}_0 \triangleq \lim_{u\to\infty} \frac{1}{u} \sum_{t=1}^{u} p_0(t) \leq P_0$.

We similarly define $p^A_{mkj}(t)$ for the access link, so that the instantaneous power of RN k can be written as

$$p_k(t) = s_A(t) \sum_{j\in\mathcal{J}} \sum_{m\in\mathcal{U}_k} a_{mkj}(t) p^A_{mkj}(t). \tag{3}$$

Then the instantaneous power constraint for RN k can be expressed as $p_k(t) \leq \hat{P}_k$, and the average power constraint for the RN k is written as $\bar{p}_k \triangleq \lim_{u\to\infty} \frac{1}{u} \sum_{t=1}^{u} p_k(t) \leq P_k$.

The above formulation corresponds to a fast power control scheme on the subframe basis as specified in the LTE-Advanced standard. Note that a slow power control scheme on the frame basis, where the link adaptation may be achieved through adaptive modulation and coding, can also be adopted in the standard. However, it is outside the scope of this paper.

E. Transmission Rates

In subframe t, let $g^D_{m0j}(t)$ be the channel gain of RB j between the eNB and UE $(m,0)$ in the direct link, $g^B_{k0j}(t)$ be the channel gain of RB j between the eNB and RN k in the backhaul link, and $g^A_{mkj}(t)$ be the channel gain of RB j between RN k and UE (m,k) in the access link. We assume that $g^D_{m0j}(t)$, $g^B_{k0j}(t)$, and $g^A_{mkj}(t)$ are a constant in each subframe but can dynamically change in different subframes. We further suppose that the channel gains are ergodic and follows a certain continuous distribution, but we do not need to know the distribution.

Then the transmission rate for UE $(m,0)$ in the direct link can be expressed as

$$r^B_{m0}(t) = \sum_{j\in\mathcal{J}} R_b b_{m0j}(t) \log(1+\frac{g^D_{m0j}(t)p^B_{m0j}(t)}{\sigma^2}), \quad (4)$$

where R_b is the symbol rate and σ^2 is the noise power. We can write the time averaged transmission rate as $\bar{r}^B_{m0} \triangleq \lim_{u\to\infty}\frac{1}{u}\sum_{t=1}^{u} r^B_{m0}(t)$. Similarly, the transmission rate in the backhaul link for UE (m,k) can be written as

$$r^B_{mk}(t) = s_B(t)\sum_{j\in\mathcal{J}} R_b b_{mkj}(t) \log(1+\frac{g^B_{k0j}(t)p^B_{mkj}(t)}{\sigma^2}). \quad (5)$$

Its corresponding time averaged transmission rate is $\bar{r}^B_{mk} \triangleq \lim_{u\to\infty}\frac{1}{u}\sum_{t=1}^{u} r^B_{mk}(t)$. The access link transmission rate for the UE (m,k) can also be written as

$$r^A_{mk}(t) = s_A(t)\sum_{j\in\mathcal{J}} R_b a_{mkj}(t) \log(1+\frac{g^A_{mkj}(t)p^A_{mkj}(t)}{\sigma^2}). \quad (6)$$

Its corresponding time averaged transmission rate is $\bar{r}^A_{mk} \triangleq \lim_{u\to\infty}\frac{1}{u}\sum_{t=1}^{u} r^A_{mk}(t)$. Since the data of a RN assisted UE is transmitted on the backhaul and access links, we need to balance the transmission on both links with dynamic subframe and RB allocation.

Under an instantaneous power constraint, we further assume that all transmission rates satisfy the following constraints on their second moments:

$$\mathbb{E}\{r^B_{mk}(t)^2\} \le r^2_{\max}, \forall m\in\mathcal{U}_k, k\in\mathcal{B}, \quad (7)$$
$$\mathbb{E}\{r^A_{mk}(t)^2\} \le r^2_{\max}, \forall m\in\mathcal{U}_k, k\in\mathcal{R}, \quad (8)$$

where $r^2_{\max}$ is a positive constant. Most widely used channel models, e.g., Rayleigh fading, satisfy the above requirement.

F. Traffic Arrival Process and Queue Updating Functions

We denote $\eta_{mk}(t)$ as the amount of data arriving at the application layer of the eNB destined for UE (m,k) in subframe t. The data is first injected into an outgoing queue in the application layer, with queue length $Z_{mk}(t)$ in subframe t. In the case of finite capacity, the overflowed data will be dropped. An admission control scheme will then determine the admitted amount of data for each UE (m,k) in each subframe t, which is denoted as $\beta_{mk}(t)$.

In addition, separate transmission queues are maintained at the eNB and RN k for each UE (m,k), and the queue lengths at the eNB and RN k in subframe t is denoted as $Q_{mk}(t)$ and $H_{mk}(t)$, respectively. The admitted data from the outgoing queue in the application layer are then injected into the queue at the eNB. To avoid queue instability due to an infinity input, we enforce an upper bound, $\beta_{\max}$, on the admitted data for each UE in each subframe t.

Since the admitted data cannot be more than the data in the application layer outgoing queue in subframe t, we in addition have $\beta_{mk}(t) \le Z_{mk}(t)$. Then, the time averaged throughput for UE (m,k) is expressed as $\bar{\beta}_{mk} \triangleq \lim_{u\to\infty}\frac{1}{u}\sum_{t=1}^{u}\beta_{mk}(t)$.

Based on the above, we can express the queue updating functions for UE (m,k) as

$$Z_{mk}(t+1) = Z_{mk}(t) - \beta_{mk}(t) + \eta_{mk}(t), \quad (9)$$
$$Q_{mk}(t+1) = \max\{Q_{mk}(t) - r^B_{mk}(t), 0\} + \beta_{mk}(t), \quad (10)$$

and

$$H_{mk}(t+1) = \max\{H_{mk}(t) - r^A_{mk}(t), 0\} + r^B_{mk}(t). \quad (11)$$

G. Dynamic Joint Resource Optimization Problem

Our objective of joint resource allocation is to maximize the total UE utility, which is a function of the average throughput for each UE. In other words, we want to maximize $\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k} U(\bar{\beta}_{mk})$, where $U(\cdot)$ is some concave non-decreasing function. A typical example of $U(\cdot)$ is $\log(\cdot)$, through maximizing which we can maintain proportional fairness among UEs. Then the optimization problem can be expressed as follows:

$$\max_{\{\boldsymbol{\beta},\mathbf{s},\mathbf{a},\mathbf{b},\mathbf{p}^A,\mathbf{p}^B\}} \Big\{\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k} U(\bar{\beta}_{mk})\Big\} \quad (12)$$

s.t.

$$\begin{aligned}
&\bar{\beta}_{mk} < \bar{r}^B_{mk}, && \forall m\in\mathcal{U}_k, k\in\mathcal{B}, && (13)\\
&\bar{r}^B_{mk} < \bar{r}^A_{mk}, && \forall m\in\mathcal{U}_k, k\in\mathcal{R}, && (14)\\
&\bar{p}_k \le P_k, && \forall k\in\mathcal{B}, && (15)\\
&\beta_{mk}(t) \le Z_{mk}(t), && \forall m\in\mathcal{U}_k, k\in\mathcal{B}, t, && (16)\\
&\beta_{mk}(t) \le \beta_{\max}, && \forall m\in\mathcal{U}_k, k\in\mathcal{B}, t, && (17)\\
&p_k(t) \le \hat{P}_k, && \forall k\in\mathcal{B}, t, && (18)\\
&c_j(t) \le 1, && \forall j\in\mathcal{J}, t, && (19)\\
&b_{mkj}(t) \in \{0,1\}, && \forall m\in\mathcal{U}_k, k\in\mathcal{B}, t, && (20)\\
&a_{mkj}(t) \in \{0,1\}, && \forall m\in\mathcal{U}_k, k\in\mathcal{R}, t, && (21)\\
&s_A(t) + s_B(t) \le 1, && \forall t, && (22)\\
&s_A(t) \in \{0,1\}, && \forall t, && (23)\\
&s_B(t) \in \{0,1\}, && \forall t, && (24)
\end{aligned}$$

where $\boldsymbol{\beta} = [\beta_{mk}(t)]_{1\times M}$, $\mathbf{s} = [s_A(t), s_B(t)]$, $\mathbf{a} = [a_{mkj}(t)]_{M_R\times J}$, $\mathbf{b} = [b_{mkj}(t)]_{M\times J}$, $\mathbf{p}^A = [p^A_{mkj}(t)]_{M_R\times J}$, and $\mathbf{p}^B = [p^B_{mkj}(t)]_{M\times J}$. We denote its optimum as $\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k} U(\bar{\beta}^{\text{opt}}_{mk})$.

The constraints (13) and (14) are to ensure stability of the eNB queues and RN queues, respectively. Equations (15) and (18) are the average and instantaneous power constraints for the eNB and RNs. The constraint (16) ensures that the admitted data cannot be more than the data in the outgoing queue. The constraint (19) ensures that each RB can only be assigned to one UE. And the constraint (22) guarantees that the backhaul and access links cannot simultaneously transmit in the same subframe. It should be noted that $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{s}$ are all binary variables, which increases the complexity of the problem.

IV. Optimal Dynamic Joint Resource Scheduling Strategy

In this section, we present an optimal admission control and dynamic joint subframe, RB, and power allocation strategy (JFRP) for the optimization problem (12)-(24). Its outline is based on a general Lyapunov optimization approach [4].

Since the objective function (12) is a function of the time averaged throughput, to transfer it to an optimization problem with a time averaged objective function, we adopt the standard approach of introducing an auxiliary variable $\alpha_{mk}(t)$ for each UE (m,k) which satisfies $\alpha_{mk}(t) \le \beta_{\max}$. We denote $\bar{\alpha}_{mk} \triangleq \lim_{u\to\infty}\frac{1}{u}\sum_{t=1}^{u}\alpha_{mk}(t)$. Then the above optimization is transferred to the following equivalent problem [4]:

$$\max_{\{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{s},\mathbf{a},\mathbf{b},\mathbf{p}^A,\mathbf{p}^B\}} \quad \sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}\overline{U(\alpha_{mk})} \tag{25}$$

s.t.

$$\bar{\alpha}_{mk} \le \bar{\beta}_{mk}, \qquad \forall m\in\mathcal{U}_k, k\in\mathcal{B}, \tag{26}$$

$$\alpha_{mk}(t) \le \beta_{\max}, \qquad \forall m\in\mathcal{U}_k, k\in\mathcal{B}, t, \tag{27}$$

$$(13)-(24), \tag{28}$$

where $\overline{U(\alpha_{mk})} = \lim_{u\to\infty}\frac{1}{u}\sum_{t=1}^{u}U(\alpha_{mk}(t))$, and $\boldsymbol{\alpha} = [\alpha_{mk}(t)]_{1\times M}$.

A. General Lyapunov Optimization Approach

To ensure the average power constraint (15), we construct a virtual power queue for each of the eNB and RNs, whose queue length in subframe t is denoted as $\Theta_k(t)$, and whose updating function is

$$\Theta_k(t+1) = \max\{\Theta_k(t) - P_k, 0\} + p_k(t). \tag{29}$$

We also construct a virtual queue to satisfy each auxiliary constraint (26), whose queue length in subframe t is denoted as $Y_{mk}(t)$ for the UE (m,k), and whose updating function is

$$Y_{mk}(t+1) = \max\{Y_{mk}(t) - \beta_{mk}(t), 0\} + \alpha_{mk}(t). \tag{30}$$

For simple notation, let us denote

$$\Omega(t) = \big\{Q_{mk}(t), H_{mk}(t), \Theta_k(t), Y_{mk}(t)\big\}. \tag{31}$$

In addition, we define the Lyapunov function as

$$\begin{aligned} L(\Omega(t)) =& \frac{1}{2}\mathbb{E}\Big\{\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k} Q_{mk}(t)^2 + \sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k} H_{mk}(t)^2 + \\ & \sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k} Y_{mk}(t)^2 + \sum_{k\in\mathcal{B}}\Theta_k(t)^2\Big\}. \end{aligned} \tag{32}$$

Then the Lyapunov conditional drift-plus-penalty function can be written as

$$\begin{aligned} &\Delta(\Omega(t)) = \\ &\mathbb{E}\Big\{L\big(\Omega(t+1)\big) - L\big(\Omega(t)\big) - V\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k} U\big(\alpha_{mk}(t)\big)|\Omega(t)\Big\} \\ &\le C + \sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}\Big\{\big(Q_{mk}(t) - Y_{mk}(t)\big)\beta_{mk}(t) + \\ &Y_{mk}(t)\alpha_{mk}(t) - VU(\alpha_{mk}(t))\Big\} - \sum_{m\in\mathcal{U}_0} Q_{m0}(t) r^B_{m0}(t) - \\ &\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k}\Big\{(Q_{mk}(t) - H_{mk}(t)) r^B_{mk}(t) + H_{mk}(t) r^A_{mk}(t)\Big\} \\ &+ \sum_{k\in\mathcal{B}}\Big(\Theta_k(t)p_k(t) - \Theta_k(t)P_k\Big), \end{aligned} \tag{33}$$

where

$$\begin{aligned} C =& \frac{1}{2}\mathbb{E}\Big\{\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}(r^B_{mk}(t)^2 + \alpha_{mk}(t)^2 + 2\beta_{mk}(t)^2) + \\ & \sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k}(r^B_{mk}(t)^2 + r^A_{mk}(t)^2) + \sum_{k\in\mathcal{B}}(P_k^2 + p_k(t)^2)\Big\} \\ <& \frac{3}{2}M(r^2_{\max} + \beta^2_{\max}) + \frac{1}{2}\sum_{k\in\mathcal{B}}\Big\{P_k^2 + \hat{P}_k^2\Big\}. \end{aligned} \tag{34}$$

Following the general Lyapunov optimization approach [4], our dynamic joint resource optimization strategy for the LTE-Advanced Type 1 inband relay network is based on minimizing the RHS of (33). This minimization is non-trivial, however, mainly due to mixed-integer nature of (33). Next, we decompose this problem into three sub-problems and provide an optimal solution to each.

B. Optimal Decision for the Auxiliary Variables

The optimal decision for the auxiliary variables on the RHS of (33) is made based on minimizing $\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}\big(Y_{mk}(t)\alpha_{mk}(t) - VU(\alpha_{mk}(t))\big)$, so we can determine the optimal auxiliary variable for each UE separately. We write the optimization problem for UE (m,k) as

$$\begin{aligned} \min_{\alpha_{mk}(t)} \quad & Y_{mk}(t)\alpha_{mk}(t) - VU\big(\alpha_{mk}(t)\big) \\ \text{s.t.} \quad & 0 \le \alpha_{mk}(t) \le \beta_{\max}. \end{aligned} \tag{35}$$

It is a convex optimization problem since the objective function is the summation of a linear function $Y_{mk}(t)\alpha_{mk}(t)$ and a convex function $-VU\big(\alpha_{mk}(t)\big)$. By differentiating, the optimal solution can be easily derived. In a special case of $U(\cdot) = \log(\cdot)$, we can obtain the optimal solution as $\alpha^{\text{JFRP}}_{mk}(t) = \min\{\frac{V}{Y_{mk}(t)}, \beta_{\max}\}$. Clearly, a larger $Y_{mk}(t)$ will decrease $\alpha_{mk}^{\text{JFRP}}(t)$ in the current subframe t, which in turn avoids the further increase of $Y_{mk}(t)$.

C. Optimal Admission Control

To minimize the RHS of (33), we also need to minimize $\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}\big(Q_{mk}(t)-Y_{mk}(t)\big)\beta_{mk}(t)$, which may be viewed as an optimal admission control problem. Clearly, we may consider each UE separately. The optimization problem for UE (m,k) in each subframe t can be expressed as follows:

$$\begin{aligned}\min_{\beta_{mk}(t)} \quad & \Big(Q_{mk}(t)-Y_{mk}(t)\Big)\beta_{mk}(t)\\ \text{s.t.} \quad & \beta_{mk}(t)\le Z_{mk}(t),\\ & \beta_{mk}(t)\le\beta_{\max}.\end{aligned} \tag{36}$$

This is a linear problem with the following solution:

$$\beta_{mk}^{\text{JFRP}}(t)=\begin{cases}\min\{Z_{mk}(t),\beta_{\max}\}, & \text{if } Q_{mk}(t)-Y_{mk}(t)<0\\ 0, & \text{otherwise}\end{cases}. \tag{37}$$

D. Optimal Subframe, RB, and Power Allocation

The final, and most challenging, task is to minimize the remaining terms in the RHS of (33). This involves joint allocation of subframes, RBs, and power. We express the optimization problem as follows:

$$\begin{aligned}\min_{\{\mathbf{s},\mathbf{a},\mathbf{b},\mathbf{p}^A,\mathbf{p}^B\}} & \Big\{-\sum_{m\in\mathcal{U}_0}Q_{m0}(t)r_{m,0}^B(t)+\sum_{k\in\mathcal{B}}\Theta_k(t)p_k(t)\\ & -\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k}\big\{\big(Q_{mk}(t)-H_{mk}(t)\big)r_{mk}^B(t)+H_{mk}(t)r_{mk}^A(t)\big\}\Big\}\end{aligned} \tag{38}$$

s.t. (18) − (24).

The above objective function is weighted by the virtual power queue lengths and the packet queue lengths. This reflects a balance between the average power constraint and the transmission rate requirement. Note that $r_{mk}^A(t)$, $r_{mk}^B(t)$, and $p_k(t)$ are functions of the subframe allocation binary variable, $s_A(t)$ and $s_B(t)$, and the RB allocation binary variables, $a_{mkj}(t)$ and $b_{mkj}(t)$. Combining this with the power allocation, this is a mixed-integer problem, which usually is prohibitively hard to solve. Moreover, the relation between the subframe allocation variables and the RB allocation variables is multiplicative, which further adds to the problem complexity.

However, we show that in the case of (38) the problem can be exactly and efficiently solved through continuity relaxation and Lagrange dual decomposition. Our proposed solution consists of the following four steps:

1) **Removal of Binary Multiplication:** To make the this problem tractable, we first remove the multiplicative binary variables by introducing auxiliary variables $x_{mkj}=s_A(t)a_{mkj}(t)$ and $y_{mkj}=s_B(t)b_{mkj}(t)$, $\forall k\in\mathcal{R}$. Clearly, $x_{mkj}\in\{0,1\}$ and $y_{mkj}\in\{0,1\}$. After substituing x_{mkj} and y_{mkj} into $p(t)$, $r_{mk}^A(t)$, $r_{mk}^B(t)$, and $c_j(t)$, we can rewrite the optimization problem (38) as

$$\begin{aligned}\min_{\{\mathbf{s},\mathbf{x},\mathbf{y},\mathbf{b}_0,\mathbf{p}^A,\mathbf{p}^B\}} & \Big\{-\sum_{m\in\mathcal{U}_0}Q_{m0}(t)r_{m0}^B(t)+\sum_{k\in\mathcal{B}}\Theta_k(t)\tilde{p}_k(t)\\ & -\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k}\big\{\big(Q_{mk}(t)-H_{mk}(t)\big)\tilde{r}_{mk}^B(t)+H_{m,k}(t)\tilde{r}_{m,k}^A(t)\big\}\Big\}\end{aligned} \tag{39}$$

s.t.

$$\tilde{p}_k(t)\le\hat{P}_k, \tag{40}$$

$$\tilde{c}_j(t)\le 1, \tag{41}$$

$$s_A(t)+s_B(t)\le 1, \tag{42}$$

$$s_A(t)\in\{0,1\}, \tag{43}$$

$$s_B(t)\in\{0,1\}, \tag{44}$$

$$x_{mkj}\in\{0,1\}, \tag{45}$$

$$b_{m0j}(t)\in\{0,1\}, \tag{46}$$

$$y_{mkj}\in\{0,1\}, \tag{47}$$

where $\mathbf{x}=[x_{mkj}]_{M_R\times J}$, $\mathbf{y}=[y_{mkj}]_{M_R\times J}$, $\mathbf{b}_0=[b_{m0j}(t)]_{M_0\times J}$,

$$\tilde{r}_{mk}^B(t)=\sum_{j\in\mathcal{J}}R_by_{mkj}\log(1+\frac{g_{k0j}^B(t)p_{mkj}^B(t)}{\sigma^2}),\forall k\in\mathcal{R}, \tag{48}$$

$$\tilde{r}_{mk}^A(t)=\sum_{j\in\mathcal{J}}R_bx_{mkj}\log(1+\frac{g_{mkj}^A(t)p_{mkj}^A(t)}{\sigma^2}), \tag{49}$$

$$\tilde{p}_0(t)=\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k}y_{mkj}p_{mkj}^B(t)+\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{U}_0}b_{m0j}(t)p_{m0j}^B(t), \tag{50}$$

$$\tilde{p}_k(t)=\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{U}_k}x_{mkj}p_{mkj}^A(t),\forall k\in\mathcal{R}, \tag{51}$$

and

$$\tilde{c}_j(t)=\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k}x_{mkj}+\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k}y_{mkj}+\sum_{m\in\mathcal{U}_0}b_{m0j}(t). \tag{52}$$

Note that $Q_{mk}(t)-H_{mk}(t)$ is the queue difference between the eNB and RN k with respect to UE (m,k). It can be less than zero for some UE (m,k), and in such a case, any RB allocation to the UE will make the term $\Theta_k(t)x_{mkj}p_{mkj}^A(t)-(Q_{mk}(t)-H_{mk}(t))R_bx_{mkj}\log(1+\frac{g_{mkj}^A(t)p_{mkj}^A(t)}{\sigma^2})>0$. Therefore, no RB will be allocated to such UE. Intuitively, by stopping the transmission in the backhaul link, the queue in the RN will be decreased, and this help balance the transmission between the backhaul link and access link. Hence, we will only focus on the UEs with $Q_{mk}(t)-H_{mk}(t)>0$. Without loss of generality, we assume that all $Q_{mk}(t)-H_{mk}(t)$ are positive in the rest of this section.

2) **Continuity Relaxation and Convexification:** The above optimization problem is still a mixed-integer nonlinear programming problem, which is typically computational intractable. Here we propose a novel algorithm that can efficiently optimize the joint allocation of subframes, RBs, and power.

To derive the solution to (39)-(47), we introduce auxiliary variables $h_{mkj}=y_{mkj}p_{mkj}^B(t)$, $\forall k\in\mathcal{R}$, $w_{m0j}=b_{m0j}(t)p_{m0j}^B(t)$, and $q_{mkj}=x_{mkj}p_{mkj}^A(t)$. In addition, we relax the binary variables $b_{m0j}(t)$, $s_A(t)$, and $s_B(t)$ to take value continuously in $[0,1]$. Since $x_{mkj}=s_A(t)a_{mkj}(t)$

and $y_{mkj} = s_B(t)b_{mkj}(t)$, we then have $x_{mkj} \in [0,1]$ and $y_{mkj} \in [0,1]$. Then we can rewrite (39)-(47) as

$$\min_{\{\mathbf{s},\mathbf{x},\mathbf{y},\mathbf{b}_0,\mathbf{h},\mathbf{q},\mathbf{w}\}} \Big\{ \sum_{j\in\mathcal{J}} \sum_{m\in\mathcal{U}_0} \{\Theta_0(t)w_{m0j} - Q_{m0}(t)R_b b_{m0j}(t)\log(1+\frac{g^D_{m0j}(t)w_{m0j}}{b_{m0j}(t)\sigma^2})\} + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k} \{\Theta_0(t)h_{mkj} + \Theta_k(t)q_{mkj} - \big(Q_{mk}(t)-H_{mk}(t)\big)R_b y_{mkj}\log(1+\frac{g^B_{k0j}(t)h_{mkj}}{y_{mkj}\sigma^2}) - H_{mk}(t)R_b x_{mnj}\log(1+\frac{g^A_{mkj}(t)q_{mkj}}{x_{mkj}\sigma^2})\}\Big\} \quad (53)$$

s.t.

$$\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{U}_0} w_{m0j} + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k} h_{mkj} \le \hat{P}_0, \quad (54)$$

$$\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{U}_k} q_{mkj} \le \hat{P}_k, \forall k\in\mathcal{R}, \quad (55)$$

$$\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k}\big(x_{mkj}+y_{mkj}\big) + \sum_{m\in\mathcal{U}_0} b_{m0j}(t) \le 1, \forall j\in\mathcal{J}, \quad (56)$$

$$x_{mkj}\in[0,1], y_{m0j}\in[0,1], b_{m0j}(t)\in[0,1], \quad (57)$$

$$s_A(t)+s_B(t)\le 1, \quad (58)$$

$$s_A(t)\in[0,1], s_B(t)\in[0,1], \quad (59)$$

$$h_{m0j}\ge 0, q_{mkj}\ge 0, w_{mkj}\ge 0, \quad (60)$$

where $\mathbf{h} = [h_{mkj}]_{M_R\times J}$, $\mathbf{q} = [q_{mkj}]_{M_R\times J}$, and $\mathbf{w} = [w_{m0j}]_{M_0\times J}$.

Note that the term $-Q_{m0}(t)R_b b_{m0j}(t)\log(1+\frac{g^D_{m0j}(t)w_{m0j}}{b_{m0j}(t)\sigma^2})$ is the perspective function of the convex function $-Q_{m0}(t)R_b\log(1+\frac{g^D_{m0j}(t)w_{m0j}}{\sigma^2})$ given that $Q_{m0}(t)\ge 0$ always holds. Therefore, $-Q_{m0}(t)R_b b_{m0j}(t)\log(1+\frac{g^D_{m0j}(t)w_{m0j}}{b_{m0j}(t)\sigma^2})$ is also a convex function. Using a similar conclusion for the other term in (53), we observe that (53) is a convex function. In addition, since all constraints are linear, the above optimization problem is a convex optimization problem, and the Slater's condition is satisfied. Thus a zero Lagrange duality gap is guaranteed [26].

3) **Dual Decomposition:** We relax the constraints (54) and (55) by introducing the dual variables λ and μ_k. The Lagrangian can be written as

$$F(\lambda,\boldsymbol{\mu}) = (53) + \sum_{k\in\mathcal{R}}\mu_k(\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{U}_k} q_{mkj} - \hat{P}_k) + \lambda\big(\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{U}_0} w_{m0j} + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k} h_{mkj} - \hat{P}_0\big), \quad (61)$$

where $\boldsymbol{\mu} = [\mu_k]_{1\times K}$. Then the dual function can be derived through

$$D(\lambda,\boldsymbol{\mu}) = \min_{\{\mathbf{b}_0,\mathbf{x},\mathbf{y},\mathbf{s}\}} F(\lambda,\boldsymbol{\mu}) \quad \text{s.t. } (56)-(60). \quad (62)$$

To derive an optimal solution to the above optimization problem, we write part of its KKT conditions as follows:

$$\Theta_0(t) - \frac{Q_{m0}(t)R_b b_{m0j}(t) g^D_{m0j}(t)}{(g^D_{m0j}(t)w_{m0j} + b_{m0j}(t)\sigma^2)\ln 2} + \lambda + \xi_{m0j} = 0, \quad (63)$$

$$\Theta_0(t) - \frac{\big(Q_{mk}(t)-H_{mk}(t)\big)R_b y_{mkj} g^B_{k0j}(t)}{\big(g^B_{k0j}(t)h_{mkj} + y_{mkj}\sigma^2\big)\ln 2} + \lambda + \varphi_{mkj} = 0, \quad (64)$$

$$\Theta_k(t) - \frac{H_{mk}(t)R_b x_{mkj} g^A_{mkj}(t)}{(g^A_{mkj}(t)q_{mkj} + x_{mkj}\sigma^2)\ln 2} + \mu_k + \theta_{mkj} = 0, \quad (65)$$

$$\xi_{m0j}w_{m0j} = 0, \quad (66)$$

$$\varphi_{mkj}h_{mkj} = 0, \quad (67)$$

$$\theta_{mkj}q_{mkj} = 0, \quad (68)$$

$$w_{m0j}\ge 0, \quad h_{l0j}\ge 0, \quad q_{lnj}\ge 0, \quad (69)$$

$$\varphi_{mkj}\ge 0, \quad \theta_{mkj}\ge 0, \quad \xi_{m0j}\ge 0. \quad (70)$$

From (63), we can write

$$w_{m0j} = \Big(\frac{Q_{m0}(t)R_b}{(\Theta_0(t)+\lambda-\xi_{m0j})\ln 2} - \frac{\sigma^2}{g^D_{m0j}(t)}\Big) b_{m0j}(t). \quad (71)$$

Based on (66), we have that if $w_{m0j} > 0$, then $\xi_{m0j} = 0$. Otherwise, if $w_{m0j} = 0$, then $\xi_{m0j}\ge 0$, and it follows that $\frac{Q_{m0}(t)R_b}{(\Theta_0(t)+\lambda)\ln 2} \le \frac{\sigma^2}{g^D_{m0j}(t)}$. Based on the above, we have

$$w_{m0j} = \Big[\frac{Q_{m0}(t)R_b}{(\Theta_0(t)+\lambda)\ln 2} - \frac{\sigma^2}{g^D_{m0j}(t)}\Big]^+ b_{m0j}(t), \quad (72)$$

where $[x]^+ \triangleq \max\{x,0\}$. Similarly, we can derive

$$h_{mkj} = \Big[\frac{\big(Q_{mk}(t)-H_{mk}(t)\big)R_b}{(\Theta_0(t)+\lambda)\ln 2} - \frac{\sigma^2}{g^B_{k0j}(t)}\Big]^+ y_{mkj}, \quad (73)$$

and

$$q_{mkj} = \Big[\frac{H_{mk}(t)R_b}{(\Theta_k(t)+\mu_k)\ln 2} - \frac{\sigma^2}{g^A_{mkj}(t)}\Big]^+ x_{mkj}. \quad (74)$$

For notation simplicity, we denote

$$\Lambda_{m0j}(\lambda) = \Big[\frac{Q_{m0}(t)R_b}{(\Theta_0(t)+\lambda)\ln 2} - \frac{\sigma^2}{g^D_{m0j}(t)}\Big]^+, \quad (75)$$

$$\Xi_{mkj}(\lambda) = \Big[\frac{\big(Q_{mk}(t)-H_{mk}(t)\big)R_b}{(\Theta_0(t)+\lambda)\ln 2} - \frac{\sigma^2}{g^B_{k0j}(t)}\Big]^+, \quad (76)$$

and

$$\Upsilon_{mkj}(\mu_k) = \Big[\frac{H_{mk}(t)R_b}{(\Theta_k(t)+\mu_k)\ln 2} - \frac{\sigma^2}{g^A_{mkj}(t)}\Big]^+. \quad (77)$$

Substituting them into (62) and denoting

$$\Gamma_{m0j}(\lambda) = (\Theta_0(t)+\lambda)\Lambda_{m0j}(\lambda) - Q_{m0}(t) R_b \log(1+\frac{g^D_{m0j}(t)\Lambda_{m0j}(\lambda)}{\sigma^2}), \tag{78}$$

$$\Psi_{mkj}(\lambda) = (\Theta_0(t)+\lambda)\Xi_{mkj}(\lambda) - (Q_{mk}(t)-H_{mk}(t)) R_b \log(1+\frac{\Xi_{mkj}(\lambda)g^B_{k0j}(t)}{\sigma^2}), \tag{79}$$

and

$$\Phi_{mkj}(\mu_k) = (\Theta_k(t)+\mu_k)\Upsilon_{mkj}(\mu_k) - H_{mk}(t) R_b \log(1+\frac{g^A_{mkj}(t)\Upsilon_{mkj}(\mu_k)}{\sigma^2}), \tag{80}$$

we can rewrite (62) as

$$D(\lambda,\boldsymbol{\mu}) = \min_{\{\mathbf{b}_0,\mathbf{x},\mathbf{y}\}} \Big\{ \sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{U}_0} \Gamma_{m0j}(\lambda)b_{m0j}(t) + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k} \big\{\Psi_{mkj}(\lambda)h_{mkj} + \Phi_{mkj}(\mu_k)q_{mkj}\big\}\Big\} \tag{81}$$
$$\text{s.t. } (56)-(59).$$

Thus, by using the KKT conditions, we have converted the optimization problem (62) into a linear problem. Combining this with the constraints (56)-(57), the optimal x^*_{mkj}, y^*_{mkj}, and $b^*_{m0j}(t)$ can only be among the extreme points in the constraint set, i.e., 0 or 1. Thus, after continuity relaxation on x_{mkj}, y_{mkj}, and $b_{m0j}(t)$, we can still obtain optimal solutions that are binary. The following lemma formalizes this observation.

Lemma 1: The optimal solution, $s^*_A(t)$, $s^*_B(t)$, x^*_{mkj}, y^*_{mkj}, and $b^*_{m0j}(t)$ to (81) satisfies the constraints (41)-(47).

Proof: Note that, in the optimization problem (53)-(60), we have only relaxed $s_A(t)$ and $s_B(t)$ to be in $[0,1]$. We still have $a_{mkj}(t)\in\{0,1\}$ and $b_{mkj}(t)\in\{0,1\}, k\in\mathcal{R}$. For the optimal solution x^*_{mkj}, y^*_{mkj}, $b^*_{m0j}(t)$ to (81), it can be easily verified that $x^*_{mkj}\in\{0,1\}$, $y^*_{mkj}\in\{0,1\}$, and $b^*_{m0j}(t)\in\{0,1\}$ from the constraints (56)-(57). Since $x^*_{mkj}=s^*_A(t)a^*_{mkj}(t)$ and $y^*_{mkj}=s^*_B(t)b^*_{mkj}(t)$, we have $s^*_A(t)\in\{0,1\}$ and $s^*_B(t)\in\{0,1\}$. In addition, we have $s^*_A(t)+s^*_B(t)\le 1$ in (58). It then follows that $s^*_A(t)$ and $s^*_B(t)$ cannot both be 1. Thus, either $s^*_A(t)=1$ and $s^*_B(t)=0$, or $s^*_A(t)=0$ and $s^*_B(t)=1$. Therefore, the optimal $s^*_A(t)$, $s^*_B(t)$, x^*_{mkj}, y^*_{mkj}, and $b^*_{m0j}(t)$ satisfy the constraints (41)-(47). ■

Based on Lemma 1, we have either $s^*_A(t)=1$ and $s^*_B(t)=0$, or $s^*_A(t)=0$ and $s^*_B(t)=1$. We next design a scheme to actually derive $s^*_A(t)$, $s^*_B(t)$, x^*_{mkj}, y^*_{mkj}, and $b^*_{m0j}(t)$. We consider the following two possible actions:

- $s_A(t)=1$ and $s_B(t)=0$: Only the direct link and access link transmission is allowed. For each RB j, we define a minimum as $W_j \triangleq \min\{\Gamma_{m0j}(\lambda), m\in\mathcal{U}_0; \Phi_{mkj}(\mu_k), k\in\mathcal{R}, m\in\mathcal{U}_k\}$. If $W_j<0$, we set $b_{l0j}(t)=1$ for UE $(l,0)$ with $\Gamma_{l0j}(\lambda)=W_j$, and set $x_{lnj}=1$ for UE (l,n) with $\Phi_{lnj}(\mu_k)=W_j$.
- $s_A(t)=0$ and $s_B(t)=1$: Only the direct link and backhaul link transmission is allowed. For each RB j, we define a minimum as $W_j \triangleq \min\{\Gamma_{m0j}(\lambda), m\in\mathcal{U}_0; \Psi_{mkj}(\lambda), k\in\mathcal{R}, m\in\mathcal{U}_k\}$. If $W_j<0$, we set $b_{l0j}(t)=1$ for UE $(l,0)$ with $\Gamma_{l0j}(\lambda)=W_j$, and set $y_{lnj}=1$ for UE (l,n) with $\Psi_{lnj}(\lambda)=W_j$.

We choose the action that gives a smaller $D(\lambda,\boldsymbol{\mu})$ and use the subframe and RB allocation solution therein. It can be easily verified that we can minimize (81) while satisfying the constraints (41)-(47) through the above scheme.

Once the dual function $D(\lambda,\boldsymbol{\mu})$ is derived, the dual optimization problem can be written as

$$\begin{aligned} &\max_{\lambda,\boldsymbol{\mu}} D(\lambda,\boldsymbol{\mu}) \\ &\text{s.t. } \lambda\ge 0, \\ &\quad\ \ \mu_k\ge 0, \forall k\in\mathcal{B}. \end{aligned} \tag{82}$$

Using the subgradient method, we can iteratively compute the optimal solution to the above problem as follows:

$$\lambda(\nu+1) = \Big[\lambda(\nu) + \epsilon(\nu)\big(\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{U}_0} w_{m0j} + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{R}}\sum_{m\in\mathcal{U}_k} h_{mkj} - \hat{P}_0\big)\Big]^+, \tag{83}$$

$$\mu_k(\nu+1) = \Big[\mu_k(\nu) + \epsilon(\nu)\big(\sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{U}_k} q_{mkj} - \hat{P}_k\big)\Big]^+, \tag{84}$$

where ν is the iteration index and $\epsilon(\nu)$ is the step size at the νth iteration. By choosing a proper sequence for $\epsilon(\nu)$, the optimum of (82) can be derived [27].

4) **Primal Recovery:** Let λ^* and $\boldsymbol{\mu}^*$ be the optimizers for (82). We then need to recover the optimal solution for the primal problem (53)-(60). However, we observe that the Lagrangian $F(\lambda,\boldsymbol{\mu})$ is not a strictly convex function in terms of $b_{m0j}(t)$, x_{mkj}, and y_{mkj}. Hence, given λ^* and $\boldsymbol{\mu}^*$, we may not uniquely recover the optimal $b_{m0j}(t)$, x_{mkj}, and y_{mkj}. Here we use the allocation of RB j as an example to illustrate this issue. Let $\psi_j = \min\{\Gamma_{m0j}(\lambda), m\in\mathcal{U}_0; \Psi_{mkj}(\lambda), \Phi_{mkj}(\mu_k), k\in\mathcal{R}, m\in\mathcal{U}_k\}<0$. If more than one $\Gamma_{m0j}(\lambda)$, $\Psi_{mkj}(\lambda)$, or $\Phi_{mkj}(\mu_k)$ is equal to ψ_j, then we cannot decide how to assign RB j among the UEs, since in that case there will be an infinite number of choices for the primal variables to minimize (81), not all of which are optimal or even feasible.

Fortunately, the probability of the above case is zero, as stated in Theorem 1. Hence, we can almost always uniquely recover the optimal x^*_{mkj}, y^*_{mkj}, and $b^*_{m0j}(t)$ given λ^* and $\boldsymbol{\mu}^*$.

Theorem 1: Given the optimal λ^* and $\boldsymbol{\mu}^*$, for each RB j, the probability that more than one $\Gamma_{m0j}(\lambda)$, $\Psi_{mkj}(\lambda)$, and $\Phi_{mkj}(\mu_k)$ is equal to ψ_j in the continuous fading channel model is zero. Thus we can recover the optimal primal variable $b^*_{m0j}(t)$, x^*_{mkj}, and y^*_{mkj} uniquely with probability 1.

Proof: Without loss of generality, we only discuss the case that more than one $\Gamma_{m0j}(\lambda)$ is equal to ψ_j for the RB j. A similar conclusion can be applied to the discussion of $\Psi_{mkj}(\lambda)$, $\Phi_{mkj}(\mu_k)$, and the relation among them.

Suppose $\Gamma_{m0j}(\lambda)=\psi_j$. We inspect the probability that there exist $l\ne m$ such that $\Gamma_{l0j}(\lambda)=\psi_j$. Since $\psi_j<0$, from the expression of $\Gamma_{l0j}(\lambda)$, we must have $\Lambda_{l0j}(\lambda)>0$. Substituting $\Lambda_{l0j}(\lambda)$ into $\Gamma_{l0j}(\lambda)$, we can easily see that $\Gamma_{l0j}(\lambda)=$

Algorithm 1: Joint optimization for (53)-(60)

Output: Optimal allocation decisions

while $\|\lambda(\nu+1)-\lambda(v)\|+\|\boldsymbol{\mu}(\nu+1)-\boldsymbol{\mu}(\nu)\|>\epsilon$ **do**

  Set $x_{mkj}=0$, $y_{mkj}=0$, $b_{m0j}(t)=0$, $w_{m0j}=0$, $h_{mkj}=0$, and $q_{mkj}=0$;

  Solve (81) by comparing the two actions derived from Lemma 1;

  Determine optimal w_{m0j}, h_{mkj}, and q_{mkj} with (72), (73), and (74);

  Updating $\lambda(\nu)$ and $\mu_k(\nu)$ through (83) and (84);

end

$\frac{Q_{l0}(t)R_b}{\ln 2}-\frac{(\Theta_0(t)+\lambda)\sigma^2}{g^D_{l0j}(t)}-Q_{l0}(t)R_b\log(\frac{g^D_{l0j}(t)Q_{l0}(t)R_b}{(\Theta_0(t)+\lambda)\sigma^2\ln 2})$. Now we inspect the relation between $\Gamma_{l0j}(\lambda)$ and the channel state $g^D_{l0j}(t)$. It is obvious that $\Gamma_{l0j}(\lambda)$ is a non-flat function regarding $g^D_{l0j}(t)$, i.e., it does not contain any horizontal line segments. Therefore, there is at most a countable set of values for $g^D_{l0j}(t)$ to allow $\Gamma_{l0j}(\lambda)=\Gamma_{m0j}(\lambda)$. Let us denote the set of those points as $\mathcal{N}_{l0j}$. Since the continuous fading channel state can take an uncountable number of values, we have $\Pr\{g^D_{l0j}(t)\in\mathcal{N}_{l0j}\}=0$. Hence, we have $\Pr\{\Gamma_{l0j}(\lambda)=\Gamma_{m0j}(\lambda)=\psi_j\}=0$. ∎

5) **Overall Algorithm and Optimality:** In Algorithm 1, we summarize the overall procedure above to solve the optimization problem (53)-(60). We next show how this can directly lead to an optimal solution to (38).

Using Algorithm 1, we can derive the optimal $\mathbf{s}^*$, $\mathbf{x}^*$, $\mathbf{y}^*$, $\mathbf{b}_0^*$, $\mathbf{q}^*$, $\mathbf{h}^*$, and $\mathbf{w}^*$ to the continuity relaxed problem (53)-(60). We then recover the optimal $\mathbf{s}^{\text{JFRP}}$, $\mathbf{a}^{\text{JFRP}}$, $\mathbf{b}^{\text{JFRP}}$, $\mathbf{p}^{A^{\text{JFRP}}}$, and $\mathbf{p}^{B^{\text{JFRP}}}$ based on the following: $\mathbf{s}^{\text{JFRP}}=\mathbf{s}^*$, $\mathbf{a}^{\text{JFRP}}=\mathbf{x}^*$, $\mathbf{b}^{\text{JFRP}}=\mathbf{y}^*$, $\mathbf{b}_0^{\text{JFRP}}=\mathbf{b}_0^*$, $\mathbf{p}^{A^{\text{JFRP}}}=\mathbf{q}^*$, $\mathbf{p}^{B^{\text{JFRP}}}=\mathbf{h}^*$, and $\mathbf{p}_0^{B^{\text{JFRP}}}=\mathbf{w}_0^*$. The reason that we can recover the optimal solution to (38) is that the optimal solution to the continuity relaxed problem (53)-(60) turn out to be binary. This is a main highlight of this algorithm and is summarized in the following theorem.

***Theorem** 2:* With Algorithm 1 and the above recovering scheme , we can derive the optimal subframe, RB, and power allocation, $\mathbf{s}^{\text{JFRP}}$, $\mathbf{a}^{\text{JFRP}}$, $\mathbf{b}^{\text{JFRP}}$, $\mathbf{p}^{A^{\text{JFRP}}}$, and $\mathbf{p}^{B^{\text{JFRP}}}$, to the optimization problem (38).

Proof: From Lemma 1, we know that the optimal solution to (53)-(60) satisfies the constraints (40)-(47). By the above recovering scheme, we can easily derive that the optimal $\mathbf{s}^{\text{JFRP}}$, $\mathbf{a}^{\text{JFRP}}$, $\mathbf{b}^{\text{JFRP}}$, $\mathbf{p}^{A^{\text{JFRP}}}$, and $\mathbf{p}^{B^{\text{JFRP}}}$ satisfy the constraints (18)-(24). Since the optimization (53)-(60) is a relaxed version of the optimization problem (39)-(47) which is equivalent to the optimization problem (38) by introducing the auxiliary variables, the optimum of (53)-(60) should be no more than that of (38). In addition, the optimal solution to the continuity relaxed problem (53)-(60) satisfy the constraints (18)-(24) with the above recovering scheme. We then have the optimum of (53)-(60) is equal to that of (38). ∎

6) **Implementation:** We briefly discuss the implementation of Algorithm 1 in an LTE-Advanced Type 1 inband relay network. To reduce system complexity, the LTE-Advanced standard advocates that, instead of centralized control by the eNB, a Type 1 relay could implement its own radio resource management (RRM) for transmissions over the access link. To follow Algorithm 1 under such a condition, the system may adopt an iterative Network Utility Maximization approach. The RN first decides on a suboptimal $a_{mkj}(t)$ with its own RRM. It then sends $a_{mkj}(t)$ and the corresponding $\Phi_{mkj}(\mu_k)$ to the eNB. The eNB is responsible for resolving the conflict among UEs at different links. It sends its optimal control decision to the RN, which is then used by the RN to update its dual valuable $\mu_k(\nu)$. Such a procedure requires tight cooperation between the eNB and an RN, which may be supported by a relay-dedicated control channel termed R-PDCCH specified in [28].

V. Performance Bounds and Numerical Evaluation

In this section, we first quantify the performance of our proposed dynamic joint resource optimization algorithm (JFRP), and then evaluate its performance through simulation.

A. Utility and Queue Bounds

***Theorem** 3:* The proposed dynamic joint subframe, RB, and power allocation strategy provides the following performance guarantees:

$$\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}U(\bar{\beta}^{\text{JFRP}}_{mk})\geq\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}U(\bar{\beta}^{opt}_{mk})-\frac{C}{V}-\delta, \tag{85}$$

$$\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t}\mathbb{E}\Big\{\sum_{m\in\mathcal{U}_k}\Big(\sum_{k\in\mathcal{B}}Q_{mk}(u)^{\text{JFRP}}+\sum_{k\in\mathcal{R}}H_{mk}(u)^{\text{JFRP}}\Big)\Big\}$$
$$\leq\frac{C}{\delta}+V, \tag{86}$$

$$\lim_{t\to\infty}\frac{1}{t}\mathbb{E}\{\Theta_k(t)^{\text{JFRP}}\}=0,\quad\forall k\in\mathcal{B}, \tag{87}$$

where $\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}U(\bar{\beta}^{opt}_{mk})$ is the optimal solution to (12)-(24), and δ is an arbitrarily small positive constant.

Proof: The proposed control algorithm minimizes the RHS of (33) via the three sub-problems presented in the last section. Then for the optimization problem (25)-(28), we have

$$\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}\overline{U(\alpha^{\text{JFRP}}_{mk})}\geq\sum_{k\in\mathcal{B}}\overline{U(\alpha^{opt}_{mk})}-\frac{C}{V}-\delta, \tag{88}$$

$$\lim_{t\to\infty}\frac{1}{t}\mathbb{E}\{Y_{mk}(t)^{\text{JFRP}}\}=0,\quad\forall m\in\mathcal{U}_k,k\in\mathcal{B}, \tag{89}$$

along with (86) and (87), where $\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}\overline{U(\alpha^{opt}_{m,k})}$ is the optimum of (25).

Then, Theorem 3 follows from the general derivation in Chapter 5 of [4]. The details are omitted to avoid redundancy. ∎

Note that (86) implies the queues in the eNB and the RNs are stable and upper bounded by $\frac{C}{\delta}+V$. With a large V, we can force $\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}U(\bar{\beta}^{\text{JFRP}}_{mk})$ arbitrarily close to $\sum_{k\in\mathcal{B}}\sum_{m\in\mathcal{U}_k}U(\bar{\beta}^{opt}_{mk})$, but paying a cost of linearly increasing transmission delay.

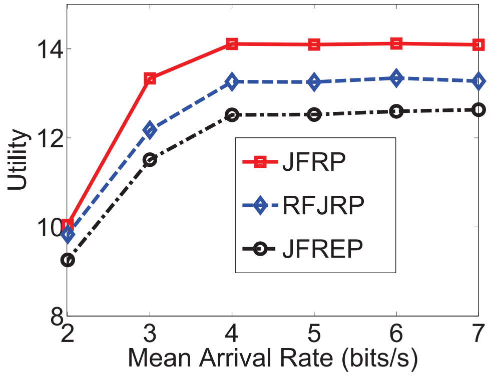

Fig. 2. Sum utility vs. mean arrival rate.

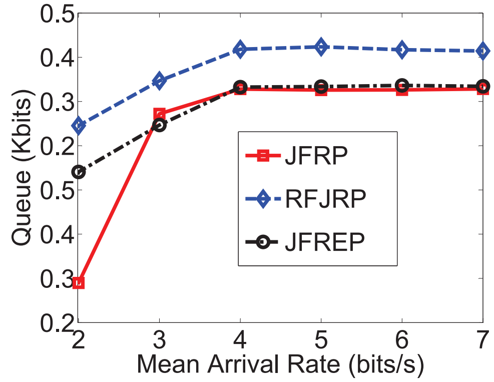

Fig. 3. Sum queue length vs. mean arrival rate.

B. Numerical Evaluation

We then evaluate the performance of the proposed dynamic joint resource allocation algorithm (JFRP) in an example OFDMA based network with Type 1 relaying. The network consists of one eNB and one Type 1 RN. The number of eNB-UEs and RN-UEs is both set to five, and we specifically take $U(\cdot) = \log(\cdot)$ to maintain proportional fairness among UEs. We model the channel amplitudes as i.i.d. Rayleigh random variables with unit average power for the direct, backhaul, and access links. We normalize the noise power to one and set the average and instantaneous maximal power of the eNB to 20 and 40. In addition, we set the average and instantaneous maximal power of the RN to 10 and 20. The number of RBs J is set to twenty in each subframe, and the maximal admitted data $\beta_{\max}$ is given as seven for each UE. Without loss of generality, we normalize the symbol rate R_b to one. The application layer traffic follows the Poisson distribution with a predetermined mean arrival rate.

We compare our algorithm with the Random-subFrame-Joint-RB-and-Power allocation (RFJRP) scheme, where subframes for the backhaul link and access link are randomly assigned, and RBs and power are jointly optimized using our algorithm. Note that the RB and power allocation scheme in RFJRP is applicable to the OFDMA based Type 1a outband relay network. Furthermore, we also compare with the Joint-subFrame-and-RB-Equal-Power allocation (JFREP) scheme as in [15], where the joint subframe and RB allocation is considered and the maximal power is set to the average power and is equally allocated among the RB.

Setting $V = 60$, we first show the throughput utility and the time averaged sum queue length versus the UE traffic mean arrival rate in Fig. 2 and Fig. 3. We observe that the JFRP scheme achieves a higher utility than both RFJRP and JFREP under all arrival rates. Noting that the throughput utility is a log function, the increase in throughput by JFRP is substantial. We also observe that JFRP achieves smaller queue sizes, and hence lower delay, than RFJRP at all arrival rates and JFREP when the arrival rate is small. Overall, by jointly considering subframe, RB, and power allocation, JFRP can significantly improve over RFJRP in both throughput utility and delay, and over JFREP in throughput utility.

We also study the algorithm performance versus the tuning parameter V in Fig. 4 and Fig. 5, where the mean arrival rate is set to 4bits/slot. We can see that the utility achieved by JFRP outperforms that of RFJRP for all V, and it is higher than the utility of JFREP under $V \geq 30$. Note that when $V < 30$, the power in JFRP is under utilized, so a proper operating point for JFRP should have $V \geq 30$. We also notice that, with $V > 50$, both JFRP and RFJRP outperform JFREP, which implies the importance of the adaptive power allocation over subframe and RB allocation. We can also see from Fig. 5 that the time averaged sum queue length in JFRP is smaller than that of RFJRP and JFREP given $V > 50$, which indicates it has a lower delay.

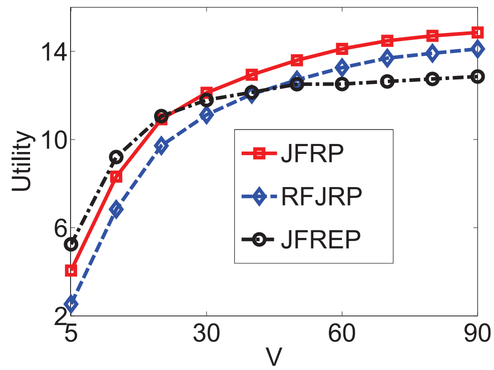

Fig. 4. Sum utility vs. tuning parameter V

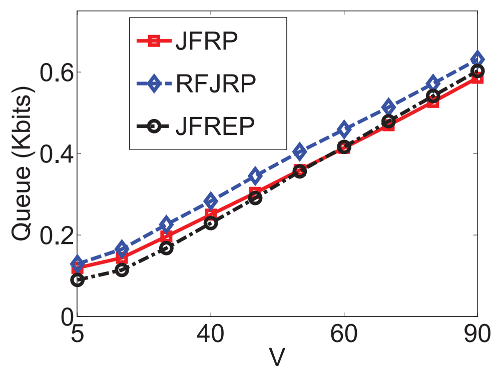

Fig. 5. Sum queue length vs. tuning parameter V

VI. Conclusion

We study the dynamic joint resource optimization problem in LTE-Advanced networks with Type 1 relaying scheme. To adaptively accommodate the network dynamics, we utilize the general framework of Lyapunov optimization. Our main contribution is in proposing a novel algorithm to minimize the drift-plus-penalty function. Taking advantage of continuity relaxation and Lagrange dual decomposition, the joint subframe, RB, and power allocation problem is efficiently solved. We further quantify the performance of our strategy, showing that a tradeoff between $1 - O(\frac{1}{V})$ utility and $O(V)$ delay can be achieved, and that the joint optimization can lead to substantial performance improvements over suboptimal alternatives.

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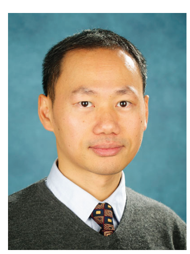

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