PARAMETRIC LOCALIZATION OF MULTIPLE INCOHERENTLY DISTRIBUTED SOURCES USING COVARIANCE FITTING

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ABSTRACT

A new algorithm for parametric localization of multiple incoherently distributed sources is proposed. Our algorithm is based on an approximation of the array covariance matrix using central and non-central moments of the source angular power densities. Based on this approximation, a new computationally simple covariance fitting-based technique is proposed to estimate these moments. The source parameters are then obtained from the moment estimates. Compared to earlier algorithms, our technique has lower computational cost and obtains the parameter estimates in a closed form. Also, it can be applied to scenarios with multiple sources that may have different angular power densities while other known methods are not applicable to such scenarios.

1. INTRODUCTION

In most applications of array processing, source localization methods are based on point source modeling, where it is assumed that the energy arriving on a sensor array originates from multiple point sources. In terms of direction finding, this means that the source energy is assumed to be concentrated at discrete angles which are referred to as the Directions-Of-Arrival (DOA's). Based on this assumption, several high-resolution direction finding methods have been proposed to estimate the source DOA's. MUSIC [1] and ESPRIT [2] are representative examples of such methods. However, in numerous applications such as sonar, radar and wireless communications, signal scattering phenomena may cause angular spreading of the source energy. Hence, in such cases the *distributed source* model is more appropriate than the point source one.

Several techniques have been proposed for distributed source localization and DOA estimation of sources with imperfectly coherent (randomly distorted) wavefronts[1], see [3]-[15] and references therein.

In the present paper, we develop a new algorithm for Incoherently Distributed (ID) source localization. We use the Taylor series expansion of the array response vector to approximate the array covariance matrix using the central or non-central moments of the source angular power densities. Based on this approximation, we propose a covariance fitting optimization to estimate these moments. We show that the source central angles and angular spreads can be obtained from the central and non-central moments. Using the second central moment of the source angular power density as a measure of angular spread, we propose a simple way to estimate this parameter in the presence of sources with different angular power densities. The algorithm developed is applicable to the multiple source scenarios and, unlike the DSPE and DISPARE algorithms [3]-[4], it does not require any spectral search. As a result, the proposed method has lower computational cost than these techniques and outperforms the ESPRIT-based estimator presented in [9].

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[1]These two problems are known to be very much related to each other.

2. SIGNAL MODEL

Assume that stationary signals with the same central frequency ω_0 impinge on an array of p sensors from q distributed narrowband far-field sources. The output of the ith sensor of the array is given by

$$x_i(t) = \sum_{m=1}^{q} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} s_m(\theta, \boldsymbol{\psi}_m, t)\, a_i(\theta)\, d\theta + n_i(t) \tag{1}$$

where $s_m(\theta, \boldsymbol{\psi}_m, t)$ is the complex random time-varying angular distribution of the mth source, $a_i(\theta)$ is the response of the ith sensor to the unit energy source emitting from the direction θ, $\boldsymbol{\psi}_m$ is the location parameter vector of the mth source, and $n_i(t)$ is the additive zero-mean spatially white noise in the ith sensor. Examples of the parameter vector $\boldsymbol{\psi}_m$ are the two angular bounds of a uniformly distributed source, or the mean and standard deviation of a source with Gaussian angular distribution. Equation (1) can be rewritten in the vector form as

$$\boldsymbol{x}(t) = \sum_{m=1}^{q} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} s_m(\theta, \boldsymbol{\psi}_m, t)\, \boldsymbol{a}(\theta)\, d\theta + \boldsymbol{n}(t) \tag{2}$$

where $\boldsymbol{x}(t) \triangleq [x_1(t), \ldots, x_p(t)]^T$, $\boldsymbol{n}(t) \triangleq [n_1(t), \ldots, n_p(t)]^T$, and $\boldsymbol{a}(\theta) \triangleq [a_1(\theta), \ldots, a_p(\theta)]^T$ are the array observation, sensor noise, and array response vectors, respectively, and $(\cdot)^T$ denotes the transpose. Assuming that the sources and noise are uncorre-

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lated, the covariance matrix can be written as

$$\begin{aligned}\boldsymbol{R}_{xx} &\triangleq \mathrm{E}\{\boldsymbol{x}(t)\boldsymbol{x}^H(t)\} \\ &= \sum_{m=1}^{q}\sum_{n=1}^{q}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p_{mn}(\theta,\theta';\boldsymbol{\psi}_m,\boldsymbol{\psi}_n) \\ &\quad \cdot \boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta')\,d\theta\,d\theta' + \sigma^2\boldsymbol{I} \end{aligned} \tag{3}$$

where σ^2 is the unknown noise power, $\boldsymbol{I}$ is the identity matrix, $\mathrm{E}\{\cdot\}$ is the statistical expectation operator, and $(\cdot)^H$ denotes the Hermitian transpose. The function

$$p_{mn}(\theta,\theta';\boldsymbol{\psi}_m,\boldsymbol{\psi}_n) \triangleq \mathrm{E}\{s_m(\theta,\boldsymbol{\psi}_m,t)s_n^*(\theta',\boldsymbol{\psi}_n,t)\} \tag{4}$$

is termed as the *angular cross-correlation kernel*, where $(\cdot)^*$ stands for the complex conjugate.

Throughout the paper, we will consider the ID source model. A distributed source is said to be ID if its components arriving from different directions are uncorrelated. That is, for the mth source we have

$$p_{mm}(\theta,\theta';\boldsymbol{\psi}_m,\boldsymbol{\psi}_m) = \sigma_m^2\,\rho_m(\theta,\boldsymbol{\psi}_m)\,\delta(\theta-\theta') \tag{5}$$

where $\delta(\theta-\theta')$ is the Dirac delta-function, σ_m^2 is the power of the mth source, and $\rho_m(\theta,\boldsymbol{\psi}_m)$ is its *normalized angular power density*. The index m in $\rho_m(\theta,\boldsymbol{\psi}_m)$ is used to emphasize that the sources can have different parameterized angular power densities. Note that

$$\int_{-\pi/2}^{\pi/2} \rho_m(\theta,\boldsymbol{\psi}_m)\,d\theta = 1, \quad m=1,2,\ldots,q \tag{6}$$

Let us assume that all distributed sources are mutually uncorrelated. Then, we can rewrite (4) as

$$p_{mn}(\theta,\theta';\boldsymbol{\psi}_m,\boldsymbol{\psi}_n) = \sigma_m^2\,\rho_m(\theta,\boldsymbol{\psi}_m)\,\delta(\theta-\theta')\,\delta_{mn} \tag{7}$$

where δ_{mn} is the Kronecker delta. Using (7), we can represent (3) as

$$\boldsymbol{R}_{xx} = \sum_{m=1}^{q}\int_{-\pi/2}^{\pi/2}\sigma_m^2\,\rho_m(\theta,\boldsymbol{\psi}_m)\,\boldsymbol{a}(\theta)\,\boldsymbol{a}^H(\theta)\,d\theta + \sigma^2\boldsymbol{I} \tag{8}$$

For further convenience, let us define the *central angle* of the mth source as the mass center of the source angular power density:

$$\theta_{0m} \triangleq \frac{\int_{-\pi/2}^{\pi/2}\theta\,\rho_m(\theta,\boldsymbol{\psi}_m)\,d\theta}{\int_{-\pi/2}^{\pi/2}\rho_m(\theta,\boldsymbol{\psi}_m)\,d\theta} = \int_{-\pi/2}^{\pi/2}\theta\,\rho_m(\theta,\boldsymbol{\psi}_m)\,d\theta \tag{9}$$

The source central angles form the vector

$$\boldsymbol{\theta}_0 = [\theta_{01},\theta_{02},\cdots,\theta_{0q}]^T \tag{10}$$

Next, let us define the nth *non-central moment* of the angular power density of the mth source around $\tilde{\theta}_{0m}$ as

$$M_{nm}(\tilde{\theta}_{0m}) \triangleq \int_{-\pi/2}^{\pi/2}(\theta-\tilde{\theta}_{0m})^n\,\rho_m(\theta,\boldsymbol{\psi}_m)\,d\theta \tag{11}$$

where $\tilde{\theta}_{0m}$ is an arbitrary angle and, for the sake of brevity, we use the notation where the dependence of M_{nm} on $\boldsymbol{\psi}_m$ is not shown explicitly.

In what follows, $\tilde{\theta}_{0m}$ will be viewed as a coarse initialization of the true central angle θ_{0m}. If $\tilde{\theta}_{0m}=\theta_{0m}$ then $M_{nm}(\tilde{\theta}_{0m})$ becomes the nth *central moment* $M_{nm}(\theta_{0m})$ of the mth source angular power density. The following Lemma is of key importance for our subsequent derivations.

Lemma 1: For the mth source, the value of the first non-central moment around an arbitrary angle $\tilde{\theta}_{0m}$ determines the deviation of $\tilde{\theta}_{0m}$ with respect to the central angle θ_{0m}.

Proof: See [16] □

Therefore, *given some estimate for the first non-central moment, we are able to estimate the source central angle.*

In what follows, we assume that the angular distribution of each source is determined by the normalized angular power density which is a non-negative function parameterized by two parameters: the *central angle* and the *angular spread.* We also assume that different sources may have different shapes of their angular distribution function. However, for each source, we assume that we know the shape of the angular power density function (for example, we know whether it is Gaussian or uniform), but we do not know the parameters of this shape which have to be estimated.

3. COVARIANCE MATRIX APPROXIMATION

In this section, we show that the array covariance matrix can be approximated using a few non-central moments of the source angular power densities.

Consider an I-term Taylor series approximation of $\boldsymbol{a}(\theta)$ around $\tilde{\theta}_{0m}$

$$\boldsymbol{a}(\theta) \simeq \sum_{i=0}^{I-1}\frac{1}{i!}(\theta-\tilde{\theta}_{0m})^i\boldsymbol{a}^{(i)}(\tilde{\theta}_{0m}) \tag{12}$$

where $\boldsymbol{a}^{(0)}(\tilde{\theta}_{0m}) = \boldsymbol{a}(\tilde{\theta}_{0m})$ and

$$\boldsymbol{a}^{(i)}(\tilde{\theta}_{0m}) = \frac{\partial^i\boldsymbol{a}(\theta)}{\partial\theta^i}\Big|_{\theta=\tilde{\theta}_{0m}} \tag{13}$$

Inserting (12) into (2), we obtain the following approximation of the snapshot vector:

$$\boldsymbol{x}(t) \simeq \sum_{m=1}^{q}\sum_{i=0}^{I-1}\frac{1}{i!}\,\alpha_{im}(\tilde{\theta}_{0m},\boldsymbol{\psi}_m,t)\boldsymbol{a}^{(i)}(\tilde{\theta}_{0m}) + \boldsymbol{n}(t) \tag{14}$$

where

$$\alpha_{im}(\tilde{\theta}_{0m},\boldsymbol{\psi}_m,t) \triangleq \int_{-\pi/2}^{\pi/2}(\theta-\tilde{\theta}_{0m})^i s_m(\theta,\boldsymbol{\psi}_m,t)\,d\theta \tag{15}$$

The following Lemma holds for $\alpha_{im}(\tilde{\theta}_{0m},\boldsymbol{\psi}_m,t)$.

Lemma 2: For uncorrelated ID sources,

$$\begin{aligned}E\{\alpha_{im}(\tilde{\theta}_{0m},\boldsymbol{\psi}_m,t)\alpha_{ln}^*(\tilde{\theta}_{0n},\boldsymbol{\psi}_n,t)\} \\ = \sigma_m^2\delta_{mn}M_{(i+l)m}(\tilde{\theta}_{0m})\end{aligned} \tag{16}$$

Proof: See [16]. □

Using the results of Lemma 2, and neglecting the terms which contain moments with the orders higher than $I-1$, we get

$$\boldsymbol{R}_{xx} \simeq \tilde{\boldsymbol{R}} + \sigma^2 \boldsymbol{I} \tag{17}$$

where

$$\begin{aligned}\tilde{\boldsymbol{R}} &= \sum_{m=1}^{q}\sum_{i=0}^{I-1}\sum_{r=i}^{I-1}\sigma_m^2 M_{rm}(\tilde{\theta}_{0m})\frac{\boldsymbol{a}^{(i)}(\tilde{\theta}_{0m})}{i!}\frac{\boldsymbol{a}^{(r-i)H}(\tilde{\theta}_{0m})}{(r-i)!}\\ &= \sum_{m=1}^{q}\sum_{r=0}^{I-1}\sum_{i=0}^{r}\sigma_m^2 M_{rm}(\tilde{\theta}_{0m})\frac{\boldsymbol{a}^{(i)}(\tilde{\theta}_{0m})}{i!}\frac{\boldsymbol{a}^{(r-i)H}(\tilde{\theta}_{0m})}{(r-i)!}\\ &= \sum_{m=1}^{q}\sum_{r=0}^{I-1}\sigma_m^2 M_{rm}(\tilde{\theta}_{0m})\boldsymbol{C}_{rm}(\tilde{\theta}_{0m}) \end{aligned}\tag{18}$$

and

$$\begin{aligned}\boldsymbol{C}_{rm}(\tilde{\theta}_{0m}) &\triangleq \frac{1}{r!}\sum_{i=0}^{r}\frac{r!}{i!(r-i)!}\boldsymbol{a}^{(i)}(\tilde{\theta}_{0m})\boldsymbol{a}^{(r-i)H}(\tilde{\theta}_{0m})\\ &= \frac{1}{r!}\frac{\partial^r\{\boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta)\}}{\partial\theta^r}\Big|_{\theta=\tilde{\theta}_{0m}}\end{aligned}\tag{19}$$

In (19), we use the following property of the derivative operator: $(uv)^{(r)} = \sum_{i=0}^{r} C_r^i u^{(i)} v^{(r-i)}$ where $C_r^i \triangleq \frac{r!}{i!(r-i)!}$.

4. COVARIANCE FITTING

The covariance fitting scheme (sometimes referred to as the *covariance matching* approach) has been frequently used for distributed source localization (see [5], [7], [8], [10], [12] and references therein). In this section, we derive a new computationally simple covariance fitting-based direction finding algorithm.

Using the LS criterion and equations (17) and (18), let us minimize the function

$$\begin{aligned}f(\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0),\tilde{\boldsymbol{\theta}}_0) &\triangleq \|\hat{\boldsymbol{R}}_{xx} - \tilde{\boldsymbol{R}} - \sigma^2\boldsymbol{I}\|^2\\ &= \|\hat{\boldsymbol{R}}_{xx} - \sum_{m=1}^{q}\sum_{r=0}^{I-1}\sigma_m^2 M_{rm}(\tilde{\theta}_{0m})\boldsymbol{C}_{rm}(\tilde{\theta}_{0m})\\ &\quad -\sigma^2\boldsymbol{I}\|^2\end{aligned}\tag{20}$$

where $\hat{\boldsymbol{R}}_{xx} = \frac{1}{N}\sum_{t=1}^{N}\boldsymbol{x}(t)\boldsymbol{x}^H(t)$ is the sample covariance matrix, and the vectors $\tilde{\boldsymbol{\theta}}_0 \triangleq [\tilde{\theta}_{01},\tilde{\theta}_{02},\cdots,\tilde{\theta}_{0q}]^T$, $\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0) \triangleq [\boldsymbol{m}_1^T(\tilde{\theta}_{01}),\boldsymbol{m}_2^T(\tilde{\theta}_{02}),\cdots,\boldsymbol{m}_q^T(\tilde{\theta}_{0q}),\sigma^2]^T$, and $\boldsymbol{m}_m(\tilde{\theta}_{0m}) \triangleq \sigma_m^2[1, M_{1m}(\tilde{\theta}_{0m}), M_{2m}(\tilde{\theta}_{0m}),\cdots,M_{(I-1)m}(\tilde{\theta}_{0m})]^T$ contain the model parameters.

Assuming some initial value $\tilde{\boldsymbol{\theta}}_0$ for the vector $\boldsymbol{\theta}_0$, we find the estimate of $\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0)$ as

$$\hat{\boldsymbol{m}}(\tilde{\boldsymbol{\theta}}_0) = \arg\min_{\boldsymbol{m}} \operatorname{tr}\{(\hat{\boldsymbol{R}}_{xx} - (\tilde{\boldsymbol{R}} + \sigma^2\boldsymbol{I}))^2\} \tag{21}$$

where $\operatorname{tr}\{\cdot\}$ denotes the trace operator.

Differentiating $f(\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0),\tilde{\boldsymbol{\theta}}_0)$ with respect to the ith ($i = 1,2,\cdots,qI$) element of $\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0)$, we get

$$\begin{aligned}f_i' &\triangleq \frac{\partial f(\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0),\tilde{\boldsymbol{\theta}}_0)}{\partial[\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0)]_i}\\ &= 2\sum_{m=1}^{q}\sum_{r=0}^{I-1}\operatorname{tr}\{\boldsymbol{C}_{rm}(\tilde{\theta}_{0m})\boldsymbol{C}_{kl}(\tilde{\theta}_{0l})\}[\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0)]_{(m-1)I+r+1}\\ &\quad +2\operatorname{tr}\{\boldsymbol{C}_{kl}(\tilde{\theta}_{0l})\}[\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0)]_{qI+1} - 2\operatorname{tr}\{\hat{\boldsymbol{R}}_{xx}\boldsymbol{C}_{kl}(\tilde{\theta}_{0l})\}\end{aligned}\tag{22}$$

where $[\cdot]_i$ denotes the ith element of a vector. Here, it is assumed that $i = (l-1)I+k+1$ $(0 \le k < I;\ 1 \le l \le m)$. Differentiating $f(\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0),\tilde{\boldsymbol{\theta}}_0)$ with respect to the $(qI+1)$st element of $\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0)$, we have

$$\begin{aligned}f_{qI+1}' &\triangleq \frac{\partial f(\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0),\tilde{\boldsymbol{\theta}}_0)}{\partial[\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0)]_{qI+1}}\\ &= 2\sum_{m=1}^{q}\sum_{r=0}^{I-1}\operatorname{tr}\{\boldsymbol{C}_{rm}(\tilde{\theta}_{0m})\}[\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0)]_{(m-1)I+r+1}\\ &\quad +2p[\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0)]_{qI+1} - 2\operatorname{tr}\{\hat{\boldsymbol{R}}_{xx}\}\end{aligned}\tag{23}$$

Then, equating (22) and (23) to zero and rewriting these equations in the vector form as $[f_1', f_2', \ldots, f_{qI+1}']^T = \boldsymbol{0}$, after straightforward manipulations, we have

$$\boldsymbol{Q}(\tilde{\boldsymbol{\theta}}_0)\boldsymbol{m}(\tilde{\boldsymbol{\theta}}_0) = \boldsymbol{p}(\tilde{\boldsymbol{\theta}}_0) \tag{24}$$

where

$$[\boldsymbol{Q}(\tilde{\boldsymbol{\theta}}_0)]_{ij} \triangleq \operatorname{tr}\{\boldsymbol{C}_{kl}(\tilde{\theta}_{0l})\boldsymbol{C}_{rm}(\tilde{\theta}_{0m})\} \tag{25}$$

$$[\boldsymbol{p}(\tilde{\boldsymbol{\theta}}_0)]_i \triangleq \operatorname{tr}\{\boldsymbol{C}_{kl}(\tilde{\theta}_{0l})\hat{\boldsymbol{R}}_{xx}\} \tag{26}$$

for $i = (l-1)I+k+1;\ j = (m-1)I+r+1;\ 1 \le l,m \le q;\ 0 \le k,i < I$; and

$$[\boldsymbol{Q}(\tilde{\boldsymbol{\theta}}_0)]_{qI+1,j} \triangleq \operatorname{tr}\{\boldsymbol{C}_{rm}(\tilde{\theta}_{0m})\} \tag{27}$$

$$[\boldsymbol{Q}(\tilde{\boldsymbol{\theta}}_0)]_{j,qI+1} \triangleq \operatorname{tr}\{\boldsymbol{C}_{rm}(\tilde{\theta}_{0m})\} \tag{28}$$

$$[\boldsymbol{Q}(\tilde{\boldsymbol{\theta}}_0)]_{qI+1,qI+1} \triangleq p \tag{29}$$

$$[\boldsymbol{p}(\tilde{\boldsymbol{\theta}}_0)]_{qI+1} \triangleq \operatorname{tr}\{\hat{\boldsymbol{R}}_{xx}\} \tag{30}$$

The solution to (24) is given by[2]

$$\hat{\boldsymbol{m}}(\tilde{\boldsymbol{\theta}}_0) = \boldsymbol{Q}^{-1}(\tilde{\boldsymbol{\theta}}_0)\boldsymbol{p}(\tilde{\boldsymbol{\theta}}_0) \tag{31}$$

Using (31), we estimate the non-central moments, and then, using Lemma 1, the central angles can be estimated as

$$\hat{\theta}_{0m} = \hat{M}_{1m}(\tilde{\theta}_{0m}) + \tilde{\theta}_{0m} \tag{32}$$

where $\hat{M}_{1m}(\tilde{\theta}_{0m})$ is the first estimated non-central moment of the angular density of the mth source.

Note that $\tilde{\theta}_{0m}$ is an arbitrary angle. However, it is important to select $\tilde{\theta}_{0m}$ as close as possible to θ_{0m} to maintain the estimation errors reasonably small.

When the central angles are estimated, we can obtain the estimates of the central moments, $\hat{M}_{nm}(\hat{\theta}_{0m})$ $(m = 1,2,\ldots,q)$, by means of solving the system (24) again with $\tilde{\theta}_{0m}$ replaced by $\hat{\theta}_{0m}$. Hence, the estimation algorithm should involve two stages. In the first stage the non-central moments and, consequently, the central angles are estimated, while in the second stage the central moments can be obtained using previously estimated central angles.

According to our assumptions, the angular power density of each source is determined by its central angle and angular spread.

[2] If the matrix $\boldsymbol{Q}(\tilde{\boldsymbol{\theta}}_0)$ is singular or ill-conditioned, one can replace its inverse by pseudoinverse. However, note that in our simulations (based on multiple simulation runs, see below) there was no single run where this matrix became singular or ill-conditioned.

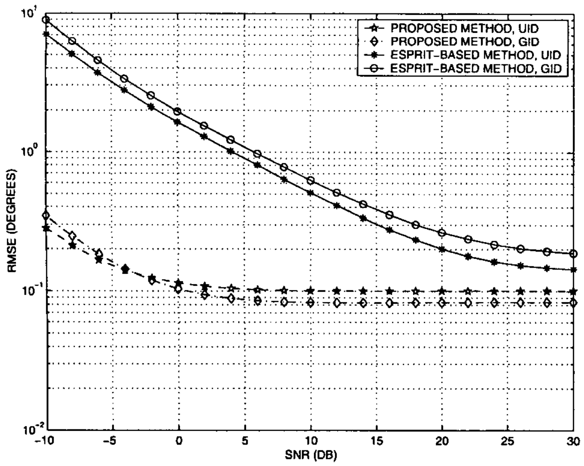

Fig. 1. RMSE of the central angle estimates versus the SNR; first example.

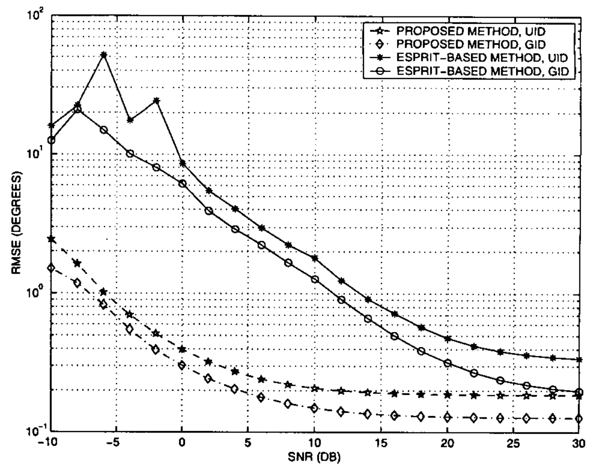

Fig. 2. RMSE of the angular spread estimates versus the SNR; first example.

It is clear that all central moments are related to the second parameter. The functional form of the angular power density determines this relationship, see [16] for more details. Hence, having one of the even-indexed central moments and assuming a certain parametric angular power density, we can estimate the angular spread. For example, using the estimate of the second central moment we have that $\hat{\Delta}_m = \sqrt{3\hat{M}_{2m}(\hat{\theta}_{0m})}$ and $\hat{\Delta}_m = \sqrt{\hat{M}_{2m}(\hat{\theta}_{0m})}$ for Uniformly Incoherently Distributed (UID) and Gaussian Incoherently Distributed (GID) sources, respectively [16].

The estimates of the source central angles can be refined by an iterative algorithm in which the estimates of the source central angles are used in (31) instead of $\tilde{\theta}_{0m}$, $m = 1, 2, \ldots, q$. This refinement procedure can be iterated a few times to improve the estimates.

Now, we can summarize our algorithm as follows:

- Step 1. Compute the sample covariance matrix $\hat{\boldsymbol{R}}_{xx}$ and specify the initial values of $\tilde{\theta}_{0m}$, $m = 1, 2, \ldots, q$.
- Step 2. Compute $\hat{\boldsymbol{m}}(\tilde{\boldsymbol{\theta}}_0)$ from (31) and find the estimates $\hat{M}_{1m}(\tilde{\theta}_{0m})$, $m = 1, 2, \ldots, q$ from the proper elements of the vector $\hat{\boldsymbol{m}}(\tilde{\boldsymbol{\theta}}_0)$.
- Step 3. Update $\tilde{\theta}_{0m} = \tilde{\theta}_{0m} + \hat{M}_{1m}(\tilde{\theta}_{0m})$ and set $\hat{\theta}_{0m} = \tilde{\theta}_{0m}$.
- Step 4. Repeat steps 2 and 3 a few times. Compute $\boldsymbol{Q}(\hat{\boldsymbol{\theta}}_0)$ and $\boldsymbol{p}(\hat{\boldsymbol{\theta}}_0)$ from (25)-(30). Then, using (31), calculate the vector $\hat{\boldsymbol{m}}(\hat{\boldsymbol{\theta}}_0)$ and obtain $\hat{M}_{2m}(\hat{\theta}_{0m})$ from the proper elements of this vector.
- Step 5. Estimate the source angular spread from the previously estimated second central moments $\hat{M}_{2m}(\hat{\theta}_{0m})$, $m = 1, 2, \ldots, q$.

5. SIMULATION RESULTS

We assume a ULA of $p = 11$ omnidirectional sensors spaced half a wavelength apart and $N = 500$. 100 independent simulation runs are performed to obtain each simulated point. Our algorithm is implemented with $I = 3$ and using three iterations of steps 2 and 3. All initial values of $\tilde{\theta}_{0m}$, $m = 1, \ldots, q$ have been chosen far enough from the true central angles, so that the difference between these initial values and the true central angles is larger than that between the true central angles and their estimates obtained by means of conventional beamformer.

In the first example, we assume two distributed sources. One of them is UID with the central angle $\theta_{01} = 10°$ and the angular spread $2\Delta_1 = 5°$. The second source is GID with the central angle $\theta_{02} = 30°$ and the angular spread $2\Delta_2 = 3°$. In this example, we compare our method (with the initial values $\tilde{\theta}_{01} = 5°$ and $\tilde{\theta}_{02} = 35°$) with the ESPRIT-based method [9]. To simulate the ESPRIT-based algorithm in a proper way, two identical 11-element ULA's with the half-wavelength interelement spacing have been assumed and the inter-subarray displacement $\lambda/10$ has been chosen, where λ is the wavelength. Figures 1 and 2 display the RMSE's of the estimates of the central angle and angular spread, respectively. From these figures, we see that our method essentially outperforms the ESPRIT-based approach.

In our second example, we consider the case of two sources but, in contrast to the previous example, these sources are closely spaced. The first source was modeled as a UID source with $\theta_{01} = 10°$ and $2\Delta_1 = 3°$, while the second source was GID with $\theta_{02} = 17°$ and $2\Delta_2 = 2°$. Similar to the previous example, our method (with the initial values $\tilde{\theta}_{01} = 5°$ and $\tilde{\theta}_{02} = 22°$) and the ESPRIT-based algorithm are compared. Figures 3 and 4 show the RMSE's of the estimates of the central angle and the angular spread, respectively. Again, our technique substantially outperforms the ESPRIT-based algorithm.

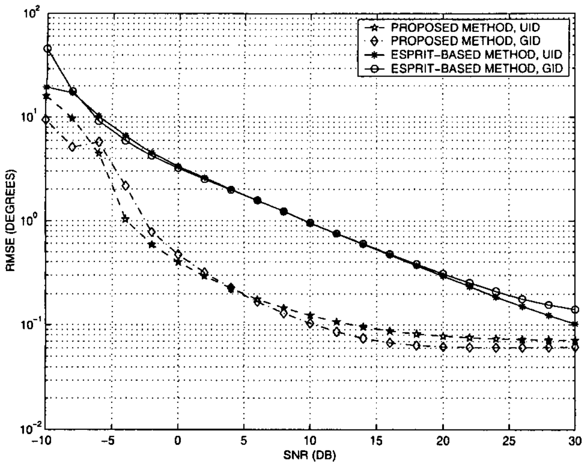

Fig. 3. RMSE of the central angle estimates versus the SNR; second example.

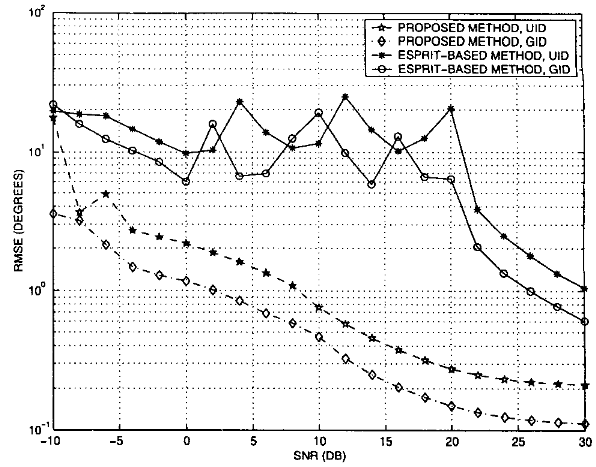

Fig. 4. RMSE of the angular spread estimates versus the SNR; second example.

6. CONCLUSIONS

We have presented a new parametric approach to localization of multiple incoherently distributed sources in sensor array. Our algorithm approximates the covariance matrix using central and non-central moments of the source angular power densities. Based on this approximation, a simple covariance fitting optimization technique is proposed to estimate these moments. Then, the source parameters are obtained from the moment estimates. Compared to the existing methods, our approach has a reduced computational cost and is applicable to the multiple source scenarios with different angular power densities.

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