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A NEW EIGENPAIR BASED BEAMFORMING METHOD FOR INTERFERENCE AND NOISE REDUCTION

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ABSTRACT

In this paper, we introduce a generalized principal component (GPC) beamforming technique that allows a trade-off between interference and noise reduction via the introduction of a control parameter, ϵ. With analytical studies and computer simulation we compare the robustness of the GPC beamforming and conventional beamforming against calibration and/or pointing errors.

1. INTRODUCTION

Several algorithms have been proposed in order to maximize the signal to interference plus noise ratio (SINR) at the output of antenna arrays. Multiple sidelobe canceller (MSC) and the minimum variance (MV) methods [1] are examples of such algorithms. To compute the array weight vector using such methods, the signal-free correlation matrix (SFCM) is required to be known.

If the array is perfectly calibrated and the look angle of the desired signal is exactly known, one can instead use the correlation matrix of the received mixture of signal, noise and interferences in these methods. However, small errors in calibration and/or DOA estimation causes a strong destructive effect on the performance of these methods [2, 3].

Diagonal loading is an effective method that makes the MV beamformer relatively robust against such errors [4]. But, the performance of this method varies between that of MV and the conventional beamformer.

In addition, in some systems the performance of detection and demodulation depends on the signal-to-interference ratio (SIR). For example, in spread spectrum communications, penetration of a smart jammer into the system may cause a destructive effect on the system performance.

In compare to the conventional beamforming methods, subspace-based methods offer considerable improvement in signal reconstruction. There exist eigenvalue decomposition (EVD) methods that are able to effectively estimate and track the eigen-subspace of the received signal covariance matrix [5]. Consequently, the subspace-based beamforming methods have attracted more attention for research [6, 7].

In this paper, we obtain a new subspace-based beamforming method, which incorporates a control parameter that allows a proper trade-off between cancellation of interference and noise. We call the proposed method the *generalized principal component* (GPC) beamformer. Then introducing some novel yardstick we study the robustness of GPC beamformer.

2. RECEIVED SIGNAL MODEL

We assume a scenario with p uncorrelated, far-field narrowband signals and an arbitrary geometry antenna array with $L > p$ elements. The received signal vector $\mathbf{x}(k) = [x_1(k), \ldots, x_L(k)]^T$ by the antenna array at the kth snapshot can be expressed as

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{n}(k)\ . \tag{1}$$

Here, $\mathbf{s}(k) = [s_1(k), \ldots, s_p(k)]^T$ represent the p signals, $\mathbf{n}(k)$ is the temporally and spatially white noise with variance σ^2, and

$$\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1)\ \ \mathbf{a}(\theta_2, \phi_2)\ \ \cdots\ \ \mathbf{a}(\theta_p, \phi_p)]. \tag{2}$$

Here θ_n and ϕ_n represent the azimuth and elevation of the nth source, respectively, and $\mathbf{a}_n \stackrel{\text{def}}{=} \mathbf{a}(\theta_n, \phi_n)$ is the array steering vector. Using (1), the autocorrelation matrix of the array output is given by

$$\mathbf{R} = \mathrm{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{A}\mathbf{P}\mathbf{A}^H + \sigma^2\mathbf{I}_L = \sum_{i=1}^{p} p_i \mathbf{a}_i \mathbf{a}_i^H + \sigma^2\mathbf{I}\ . \tag{3}$$

Here, $\mathrm{E}\{.\}$ represents the expected value, superscript H denotes Hermitian transposition, and

$$\mathbf{P} = \mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \mathrm{diag}(p_1,\ \cdots,\ p_p)\ , \tag{4}$$

where p_i is the received power of the ith signal source. Note that the diagonal structure of $\mathbf{P}$ is a result of uncorrelated sources.

The sample covariance matrix, for a window of N snapshots, is computed as [8]

$$\hat{\mathbf{R}} = \frac{1}{N}\mathbf{X}\mathbf{X}^H\ . \tag{5}$$

where $\mathbf{X}$ is,

$$\mathbf{X} = [\ \mathbf{x}(1) \quad \mathbf{x}(2) \quad \cdots \quad \mathbf{x}(N)\ ]\ . \tag{6}$$

For the positive-definite Hermitian correlation matrix $\mathbf{R}$, there exist a set of L orthonormal eigenvectors $\{\mathbf{q}_1, \ldots, \mathbf{q}_L\}$ and corresponding eigenvalues $\{\lambda_1 \geq \cdots \geq \lambda_L\}$, such that

$$\mathbf{R} = \sum_{i=1}^{L} \lambda_i \mathbf{q}_i \mathbf{q}_i^H\ , \tag{7}$$

$$\mathbf{R}^{-1} = \sum_{i=1}^{L} \lambda_i^{-1} \mathbf{q}_i \mathbf{q}_i^H\ . \tag{8}$$

The following property can be proved for the eigen-pair decomposition of correlation matrix (3) [9],

$$\mathbf{A}^H \mathbf{Q}_s (\mathbf{\Lambda}_s - \sigma^2 \mathbf{I})^{-1} \mathbf{Q}_s^H \mathbf{A} = \mathbf{P}^{-1}\ , \tag{9}$$

where,

$$\mathbf{Q}_s = [\mathbf{q}_1, \cdots, \mathbf{q}_p]\ , \tag{10}$$

$$\mathbf{\Lambda}_s = \mathrm{diag}(\lambda_1, \cdots, \lambda_p)\ . \tag{11}$$

We also define $\mathbf{Q}_n = [\mathbf{q}_{p+1}, \cdots, \mathbf{q}_L]$.

The followings are known,

$$\left.\begin{array}{c} \lambda_k = \sigma^2 \\ \mathbf{A}^H \mathbf{q}_k = \mathbf{0} \end{array}\right\} \quad \text{for} \quad (p+1) \leq k \leq L\ . \tag{12}$$

3. PROPOSED BEAMFORMING METHOD

We propose to find the beamforming weight vector $\mathbf{w}$ in order to maximizes the following ratio when the desired signal arrive from (θ_n, ϕ_n),

$$\mathbf{w}_{n,\epsilon} = \max_{\mathbf{w}} \frac{S_o}{\epsilon I_o + (1-\epsilon) N_o} \quad \text{for} \quad 0 \leq \epsilon \leq 1\ . \tag{13}$$

Here S_o, I_o and N_o, which are functions of weight vector $\mathbf{w}$, respectively denote the desired signal power, the total interference power, and the noise power at the array output. Note that maximization (13) for $\epsilon = 0$, $\epsilon = 0.5$, and $\epsilon = 1$ corresponds to maximizing the array output SNR, SINR, and SIR , respectively.

To maximize the ratio (13), one can equivalently solve the following optimization problem,

$$\begin{array}{l} \min\limits_{\mathbf{w}} \epsilon S_o + \epsilon I_o + (1-\epsilon) N_o\ , \\ \text{s.t.} \quad S_o = c \end{array} \tag{14}$$

where, c is some positive constant. Assuming that the nth point source is the desired one then $S_o = p_n |\mathbf{a}_n^H \mathbf{w}|^2$. We define the Lagrangian $L(\mathbf{w}, \lambda)$ associated with problem (14) as,

$$L(\mathbf{w}, \lambda) = \epsilon S_o + \epsilon I_o + (1-\epsilon) N_o - \lambda(\sqrt{p_n} \mathbf{a}_n^H \mathbf{w} - \sqrt{c}) \tag{15}$$

where λ is the so called Lagrange multiplier associated with equality constraint $S_o = c$.

Taking into account that $S_o + I_o = \mathbf{w}^H (\mathbf{R} - \sigma^2 \mathbf{I}) \mathbf{w}$ and $N_o = \sigma^2 \mathbf{w}^H \mathbf{w}$, (15) can be written as,

$$\begin{aligned} L(\mathbf{w}, \lambda) \ =\ & \epsilon \mathbf{w}^H (\mathbf{R} - \sigma^2 \mathbf{I}) \mathbf{w} + (1-\epsilon) \sigma^2 \mathbf{w}^H \mathbf{w} \\ & - \lambda(\sqrt{p_n} \mathbf{a}_n^H \mathbf{w} - \sqrt{c})\ . \end{aligned} \tag{16}$$

Since $L(\mathbf{w}, \lambda)$ is a convex quadratic function of $\mathbf{w}$, the solution can be find by putting $\nabla_{\mathbf{w}} L(\mathbf{w}, \lambda) = \mathbf{0}$ as,

$$\begin{aligned} \nabla_{\mathbf{w}} L(\mathbf{w}, \lambda) \ =\ & \epsilon \mathbf{w}^H (\mathbf{R} - \sigma^2 \mathbf{I}) + \mathbf{w}^H (1-\epsilon) \sigma^2 \\ & - \lambda \sqrt{p_n} \mathbf{a}_n^H\ . \end{aligned} \tag{17}$$

Thus,

$$\mathbf{w} = \lambda \sqrt{p_n} (\epsilon \mathbf{R} + (1-2\epsilon) \sigma^2 \mathbf{I})^{-1} \mathbf{a}_n\ , \tag{18}$$

where we have to adjust λ to satisfy the constraint $S_o = c$. However, for our problem we can ignore the constant coefficient $\lambda \sqrt{p_n}$. Thus, the optimum weight vector for optimization (14), which is a function of ϵ and $\mathbf{a}_n$, can be written as

$$\mathbf{w}_{n,\epsilon} = [\epsilon \mathbf{R} + (1-2\epsilon) \sigma^2 \mathbf{I}]^{-1} \mathbf{a}_n\ . \tag{19}$$

Note that, similar to (8), the following is valid,

$$[\epsilon \mathbf{R} + (1-2\epsilon) \sigma^2 \mathbf{I}]^{-1} = \sum_{i=1}^{L} \frac{\mathbf{q}_i \mathbf{q}_i^H}{f(\lambda, \epsilon, \sigma)}\ , \tag{20}$$

where

$$f(\lambda, \epsilon, \sigma) = \epsilon \lambda + (1-2\epsilon) \sigma^2\ . \tag{21}$$

Now, with the assumption that all $p-1$ interferers are also point sources, we can use (20) and (12), to write (19) in term of signal subspace eigen-pairs (i.e. $(\lambda_i, \mathbf{q}_i)$ for $i = 1, ..., p$), as

$$\mathbf{w}_{n,\epsilon} = \sum_{i=1}^{p} \frac{\mathbf{q}_i \mathbf{q}_i^H}{f(\lambda_i, \epsilon, \sigma)} \mathbf{a}_n\ . \tag{22}$$

We call (22) the weight vector for GPC beamformer.

Let us assume that the nth signal arriving from look angle (θ_n, ϕ_n) is the desired signal and GPC beamformer (22) is used for signal extraction. In this case, the output signal power is,

$$\begin{aligned} S_o \ &=\ p_n |\mathbf{w}_{n,\epsilon}^H \mathbf{a}(\theta_n, \phi_n)|^2 \\ &=\ p_n \left| \sum_{i=1}^{p} \frac{\mathbf{a}(\theta_n, \phi_n)^H \mathbf{q}_i \mathbf{q}_i^H \mathbf{a}(\theta_n, \phi_n)}{f(\lambda_i, \sigma, \epsilon)} \right|^2 \end{aligned} \tag{23}$$

and the output noise power is,

$$\begin{aligned} N_o \ &=\ \sigma^2 \mathbf{w}_{n,\epsilon}^H \mathbf{w}_{n,\epsilon} \\ &=\ \sigma^2 \sum_{i=1}^{p} \sum_{j=1}^{p} \frac{\mathbf{a}(\theta_n, \phi_n)^H \mathbf{q}_i \mathbf{q}_i^H \mathbf{q}_j \mathbf{q}_j^H \mathbf{a}(\theta_n, \phi_n)}{f(\lambda_i, \sigma, \epsilon) f(\lambda_j, \sigma, \epsilon)}. \end{aligned} \tag{24}$$

Knowing that $\mathbf{q}_i$'s are orthonormal and $\lambda_1 \geq \cdots \geq \lambda_L$, it is straightforward to show,

$$p_n \frac{Q_a^2(\theta_n, \phi_n)}{f^2(\lambda_1, \sigma, \epsilon)} \leq \quad S_o \quad \leq p_n \frac{Q_a^2(\theta_n, \phi_n)}{f^2(\lambda_p, \sigma, \epsilon)} , \quad (25)$$

$$\sigma^2 \frac{Q_a(\theta_n, \phi_n)}{f^2(\lambda_1, \sigma, \epsilon)} \leq \quad N_o \quad \leq \sigma^2 \frac{Q_a(\theta_n, \phi_n)}{f^2(\lambda_p, \sigma, \epsilon)} , \quad (26)$$

where $Q_a(\theta_n, \phi_n) = \sum_{i=1}^{p} |\mathbf{q}_i^H \mathbf{a}(\theta_n, \phi_n)|^2$.

Now, using inequalities (25) and (26), we would find the following bounds for the output SINR,

$$\left(\frac{S}{N}\right)_o \leq \left(\frac{S}{N}\right)_i \frac{f^2(\lambda_1, \sigma, \epsilon)}{f^2(\lambda_p, \sigma, \epsilon)} Q_a(\theta_n, \phi_n) \qquad (27)$$

$$\left(\frac{S}{N}\right)_o \geq \left(\frac{S}{N}\right)_i \frac{f^2(\lambda_p, \sigma, \epsilon)}{f^2(\lambda_1, \sigma, \epsilon)} Q_a(\theta_n, \phi_n) \qquad (28)$$

In the sequel, we study the properties of this beamformer for three special values of ϵ (i.e. for $\epsilon = 1$, $\epsilon = 0.5$ and $\epsilon = 0$).

3.1. Type–1 (T1) beamformer

For this beamformer, we use $\epsilon = 1$ in (22) and compute the beamformer weight vector as

$$\mathbf{w}_{n,1} = \sum_{i=1}^{p} \frac{\mathbf{q}_i \mathbf{q}_i^H}{\lambda_i - \sigma^2} \mathbf{a}_n = \mathbf{Q}_s (\mathbf{\Lambda}_s - \sigma^2 \mathbf{I}_p)^{-1} \mathbf{Q}_s^H \mathbf{a}_n. \qquad (29)$$

Theorem 1 *The pattern of T1 beamformer has nulls in the direction of interferers and its gain in the direction of the desired signal is equal to the inverse of the received power from the target source. That is [9],*

$$\mathbf{a}_m^H \mathbf{w}_{n,1} = \sum_{i=1}^{p} \mathbf{a}_m^H \frac{\mathbf{q}_i \mathbf{q}_i^H}{\lambda_i} \mathbf{a}_n = \frac{\delta_{mn}}{p_n}. \qquad (30)$$

From (30), it is seen that the array produces exact nulls in the direction of interference (that is for θ_m whenever $m \neq n$), and the array gain in the direction of the desired source is equal to the inverse of the received signal power p_n.

The desired signal power at the array output, say S_o, is equal to the desired source power, p_n, multiplied by the array power gain in the direction of the desired signal. Therefore, for T_1 beamformer, the output power of the desired signal is,

$$S_o = p_n |\mathbf{a}_n^H \mathbf{w}_{n,1}|^2 = p_n |\frac{1}{p_n}|^2 = \frac{1}{p_n}. \qquad (31)$$

3.2. Type–2 (T2) beamformer

For this beamformer, we use $\epsilon = 0.5$ in (22) and obtain[1]

$$\mathbf{w}_{n,0.5} = \sum_{i=1}^{p} \frac{\mathbf{q}_i \mathbf{q}_i^H}{\lambda_i} \mathbf{a}_n = \mathbf{Q}_s \mathbf{\Lambda}_s^{-1} \mathbf{Q}_s^H \mathbf{a}_n \ , \qquad (32)$$

[1]For sake of simplicity , a coefficient 0.5 in the denominator of (32) has been ignored.

It is interesting to remind that (32) is the conventional reduced-rank principal component beamforming method method [6].

Using the Karhunen-Loève expansion (8), the following relation, which is the MV beamformer would be written,

$$\mathbf{w}_{n,0.5} = \mathbf{R}^{-1} \mathbf{a}_n. \qquad (33)$$

It is well known in the array processing literature that the MV beamformer is sensitive to signal DOA uncertainty and array calibration error [4].

However, it can be shown that the T2 beamformer is less sensitive to these errors when compared to the MV technique.

Let us define the sensitivity of array output SINR with respect to the array steering vector error ($\Delta \mathbf{a} = \tilde{\mathbf{a}} - \mathbf{a}$) by [2]

$$\mathcal{S}^{\mathbf{w}}_{\mathrm{SINR}_o, \mathbf{a}} = \lim_{\Delta \mathbf{a} \to 0} \frac{|\Delta \mathrm{SINR}_o|}{||\Delta \mathbf{a}||^2} \ , \qquad (34)$$

where $\mathbf{w}$ is the array weight vector and

$$\Delta \mathrm{SINR}_o = \mathrm{SINR}_o|_{\Delta \mathbf{a} = 0} - \mathrm{SINR}_o|_{\Delta \mathbf{a} \neq 0} \ .$$

Eq. 34 is a novel and suitable measure of beamformer sensitivity to array steering vector errors, that can be used to show the robustness of T2 beamformer.

Theorem 2 *The sensitivity of output SINR to the array steering vector error in T2 beamformer is smaller than that for MV beamformer [9].*

3.3. Type–3 (T3) beamformer

For this reduced-rank beamformer, we use (22) with $\epsilon = 0$, that is,

$$\mathbf{w}_{n,0} = \sum_{i=1}^{p} \frac{\mathbf{q}_i \mathbf{q}_i^H}{\sigma^2} \mathbf{a}_n = \frac{1}{\sigma^2} \mathbf{Q}_s \mathbf{Q}_s^H \mathbf{a}_n. \qquad (35)$$

Knowing that $\mathbf{Q}_s \mathbf{Q}_s^H = \mathbf{I}_L - \mathbf{Q}_n \mathbf{Q}_n^H$ and $\mathbf{Q}_n^H \mathbf{a}_n = \mathbf{0}$, (35) can be written as

$$\mathbf{w}_{n,0} = \frac{1}{\sigma^2} \mathbf{a}_n, \qquad (36)$$

which is the well-known conventional beamformer (note that (35) and (36) are identical if $\mathbf{a}_n$ is exactly known, otherwise they are different). However, as we shall show, the T3 beamformer is less sensitive to steering vector errors than the conventional beamformer.

We define the sensitivity of array output SNR with respect to the array steering vector error ($\Delta \mathbf{a} = \tilde{\mathbf{a}} - \mathbf{a}$) as

$$\mathcal{S}^{\mathbf{w}}_{\mathrm{SNR}_o, \mathbf{a}} = \lim_{\Delta \mathbf{a} \to 0} \frac{|\Delta \mathrm{SNR}_o|}{||\Delta \mathbf{a}||^2} \qquad (37)$$

[2]The steering vector mismatch ($\tilde{\mathbf{a}} \neq \mathbf{a}$) may be due to system calibration error, pointing error, and/or unsynchronized A/D converters.

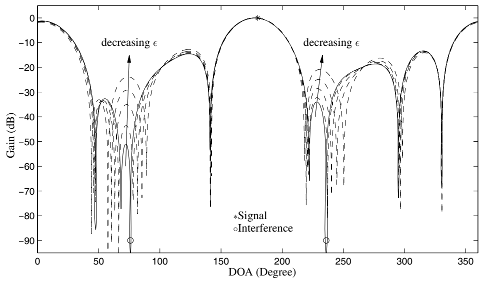


Figure 1. Plots of the beampatterns for GPC beamforming method (ϵ =1, 0.5, 0.2, 0.1, 0.05).

where

$$\Delta\text{SNR}_o = \text{SNR}_o|_{\Delta\mathbf{a}=0} - \text{SNR}_o|_{\Delta\mathbf{a}\neq 0}$$

and $\mathbf{w}$ is the array weight vector.

Theorem 3 *The sensitivity of output SNR of T3 beamformer to the array steering vector error is smaller than that for the conventional beamformer[9].*

4. SIMULATION RESULTS

For simulation, an 8-element uniform circular array with half a wavelength inter-element spacing is considered. The power of the received noise is estimated based on the average of the $L-p$ smallest eigenvalues of the correlation matrix and received power of all source are assumed to be the same.

The patterns of GPC beamformer (22) for various ϵ's are plotted in Fig. 1. Here, the desired signal look angle is 180° and the interferences arrive from 76° and 236°.

Fig. 2 illustrates the effect of ϵ on the array output SINR and SIR for up to 7 sources (i.e. for $p = 2, ..., 7$), where sources have 10dB SNR. Here, each point of simulation result is the average of 2000 Monte-Carlo runs.

In our next experiment, we assume an array consists of one directional antenna and 9 omnidirectional antennas. The directional antenna is located at the origin of the xy plane and points toward z^+. This directional antenna pattern is depicted in Fig. 4(a). The omnidirectional elements are arranged in a crossed form in the xy plane and located at the center distance $d = 5\lambda$ from the directional antenna. The spacing of the omnidirectional elements on each arm of the cross is $\lambda/2$ (see Fig. 3).

The desired signal is at $(\theta = 0°, \phi = 90°)$ and two interferers are assumed at $(\theta = 0°, \phi = 62°)$ and $(\theta = 90°, \phi = 62°)$. Received SINR is 15 dB and the total output power of the main antenna is 10.1 dBm. Fig. 4(b) shows the resulting pattern using the GPC

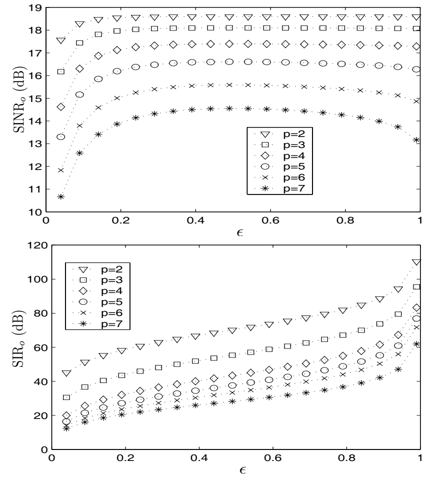


Figure 2. Array output SINR and SIR as a function of ϵ for $p = 2, ..., 7$.

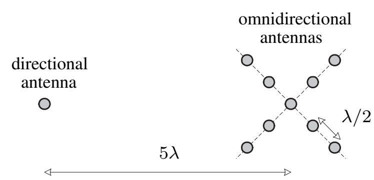


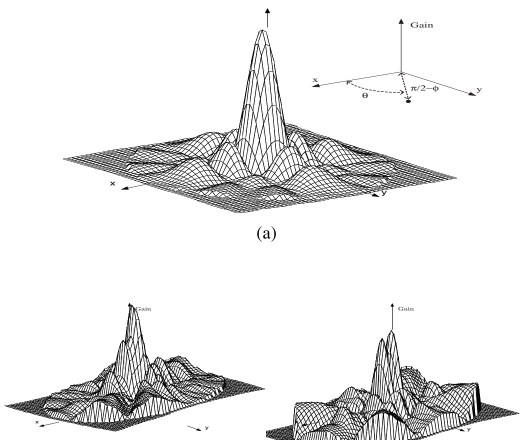
Figure 3. Antenna array with one directional antenna and 9 omnidirectional elements used in simulation.

with $\epsilon = 0.5$ in the case of a $\phi = 0.01°$ pointing error. Fig. 4(c) shows the produced pattern using MV beamforming method for the same scenario. The output SINR for the GPC and MV methods are 37.1dB and 15.8dB, respectively. The lower output SINR for MV beamformer is a result of split mainlobe and high sidelobes.

5. CONCLUSION

We introduced the GPC beamforming technique that uses a weighted combination of the signal subspace eigenvalues and eigenvectors of the received signal correlation matrix. The beamformer weight vector is parameterized with respect to a variable, ϵ to allow a tradeoff between noise and interference reduction.

Three special cases of the beamformer called $T1$, $T2$, and $T3$ were discussed. If the array is calibrated and the autocorrelation matrix is perfectly known, it was shown that T2 and T3 coincide with the MV and conventional beamformers, respectively. However, using analytical studies and computer experiments, we shown that T2 and T3 beamformer outperforms the MV and conventional beamforming methods.



(a) (b) (c)

Figure 4. (a) The beampattern of the directional antenna, (b) The produced beampattern for GPC method with $\epsilon = 0.5$, (c) The produced beampattern with MV method.

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