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Reducing Symbol Loss Probability in the Downlink of an OFDMA Based Wireless Network

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***Abstract*—This paper studies the problem of *minimizing symbol loss probability while keeping the system throughput above a certain threshold* in downlink transmission of future OFDMA based wireless networks that rely on *imperfect one-bit channel state feedback*. To solve this problem, we study different precoding classes and propose a new class of precoding matrices that can gain a better result. This work is different from previous *OFDM precoding* literature in two main aspects. First, it addresses a more practical case where *one-bit channel state feedback* is available at the base station. Second, it compares precoding classes and proposes a new one. We prove analytically that our proposed precoding class has a lower symbol loss probability than the existing classes. Numerical evaluations show that a large gain in symbol loss probability is achieved by our class in comparison with the other classes.**

***Index Terms*—Symbol Loss Probability, Throughput, OFDMA, Pre-coded OFDM, One-bit Channel State Feedback.**

I. Introduction

Orthogonal frequency division multiple access (OFDMA) has emerged as one of the prime multiple access schemes for broadband wireless networks (e.g. IEEE 802.16 Mobile WiMax, DVB-RCA, etc.). In OFDMA, the whole spectrum is divided into a number of subcarriers for parallel transmission of signals that belong to different users [1]. In the 802.16 standards and the 3GPP-LTE pre-standards, the number of these subcarriers can be as large as 2048 [2], [3].

In the base station (BS), subcarriers are allocated to users if their measured SNRs at the receiver are above a certain threshold. Therefore, users should feedback to the BS the measured SNR at the receiver end for all subcarrier. Unfortunately, reporting the measured SNRs for all subcarriers by all users causes a huge overhead. As a solution, consecutive carriers are partitioned into groups that are generally termed *channels*. For each channel, the receiver reports the average value of its measured SNRs for all the subcarriers in that channel [4], [5].

Unfortunately, because of the limitation on the feedback channel capacity, reporting the exact value of the average SNR for each channel by all users is still not practical [6]. Therefore, in practice the *one-bit channel state feedback* is usually used. In this scheme, the BS broadcasts a threshold signal-to-noise ratio (SNR_{th}). This threshold is generally determined in order to achieve a certain per subcarrier symbol error rate using a certain modulation and coding scheme. For each channel, users with the average measured SNRs above the threshold report this channel as being in a "good" condition by sending only one bit as 1. If the average SNR is below the SNR_{th}, then the reported bit is 0 and the channel is considered to be in a "bad" condition [4], [6].

In the conventional scheduling schemes, based on the reported channel status and the MAC layer information, the BS allocates each channel to a user, from among the ones with good channel condition. Since the exact value of the measured SNR per subcarrier is not available, the BS assumes that all the subcarriers have the same SNR level (equal to SNR_{th}).

In this paper, we study the problem of *reducing the symbol loss probability of the transmitted signals while keeping the throughput above a certain threshold* in the downlink of an OFDMA based wireless network and in the presence of imperfect one-bit channel state feedback. It has been shown that precoding can reduce symbol error probability [7]–[10]. A precoder maps a vector of data symbols to a number of output signals that are transmitted on the OFDMA subcarriers. The receiver reconstructs the data vector by inverting the precoding operation. In this paper, we study four different types of precoding techniques: *diversity precoding*, *full coding*, and also two proposed precoding techniques, called Class A and Class B. We show that the Class A precoder has the smallest symbol error probability.

The rest of the paper is organized as follows. Section II illustrates the formulation of our optimization problem. In Section III, we describe different precoding classes to solve this problem, propose two new classes and compare their performance. Numerical evaluations are shown in Section IV. Finally, we conclude this paper in Section V.

II. Problem Formulation

In our analysis, we consider one *channel* in the downlink of an OFDMA based wireless network, in the presence of one-bit channel state feedback. We define the throughput T as the number of *information bits* sent in each *transmission slot* through the F available subcarriers of one *channel* $(f_1, \cdots, f_F)$ and assume that the network has to satisfy a

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minimum throughput T_{th} over this *channel*. To satisfy this condition on T, $U = \left\lceil \frac{T_{th}}{\log_2(M)} \right\rceil$ symbols $(u_1, \cdots, u_U)$ have to be transmitted in each *transmission slot* through the F carriers, where M is the modulation level. We first assume that $d = \frac{F}{U}$ is an integer number but it is shown at the end of this paper that this condition on d can be easily relaxed. A symbol is lost if it cannot be correctly detected at the receiver. To the symbol loss event, we attribute the symbol loss probability P_{ei} which is defined as the probability of loosing symbol i.

In the present schemes, *frequency diversity* is used to reduce the symbol loss probability by transmitting each of the U symbols on d different subcarriers [11]–[13]. However, there are other possible loading alternatives that can achieve less symbol loss probability. To illustrate our idea, let $\overline{s} = [s_1, \cdots, s_F]$ be the vector of signals to be transmitted through $f_1, \cdots, f_F$ in one *transmission slot*, one signal on each carrier. We call $\overline{s}$ the vector of *transmitted signals*. Now, each element s_i of this vector can convey the information about either one specific symbol or a linear combination of some symbols chosen from the original symbol vector $\overline{u} = [u_1, \cdots, u_U]$, that is for $1 \leq j \leq F$,

$$s_j = \sum_{i=1}^{U} \alpha_{ji} u_i$$

where α_{ji} are called the precoder *coefficients*.

As an example, suppose $F = 6$ and U is calculated to be be 3 ($d = 2$). In the frequency diversity scheme [12]:

$$\begin{aligned} s_1 &= s_4 = u_1 \\ s_2 &= s_5 = u_2 \\ s_3 &= s_6 = u_3. \end{aligned}$$

In this case, we have a diversity of 2 and the probability of loss of symbol i, denoted as (P_{ei}), $i = 1, 2, 3$, is P_{sc}^2(or more generally P_{sc}^d), where P_{sc} is the signal error probability in each subcarrier. We have assumed that all subcarriers have the same signal error probability.

Another possible scheme is that each element of the vector $\overline{s}$ be a linear combination of all symbols as follows:

$$s_j = \sum_{i=1}^{3} \alpha_{ji} u_i, \quad 1 \leq j \leq 6.$$

where the coefficient vectors $\overline{\alpha_j} = [\alpha_{j1}\, \alpha_{j2}\, \alpha_{j3}]\,, 1 \leq j \leq 6$, are chosen such that any three of them are independent and none of the α_{ji} is zero. We refer to this scheme as *full coding*. At the receiver side, if any set of three or more signals is correctly received, then all of the symbols can be recovered. Therefore,

$$P_{ei} = \sum_{j=0}^{2} \binom{6}{j} (1 - P_{sc})^j P_{sc}^{(6-j)},\ 1 \leq i \leq 3$$

Obviously, one of these two schemes, diversity or full coding, has a smaller P_{ei} and therefore is a preferred scheme.

To generalize this example, we can model the relation between the original signal vector $\overline{u}$ and the transmission symbol vector $\overline{s}$ as follows:

$$\overline{s} = \overline{u} \cdot A$$

where A is a $U \times F$ coefficient matrix whose elements α_{ij} take values of a finite field (all the operations are in the finite field). The finite field can be selected in accordance with the modulation. For example, F_{2^2} for Quadrature Phase Shift Keying (QPSK), F_{2^4} for 16-Quadrature Amplitude Modulation (QAM), F_{2^6} for 64-QAM, and so on. Note that any $U \times U$ submatrix of A has to be full rank in order for the receiver to decode the received signals. Note that the above property may not be easily satisfied for the finite fields with a small degree. However, because we only consider one OFDMA channel with practically 25 or 26 subcarriers [5] [6], this problem is easily solved with careful design of the coefficient matrix. The matrix A is pre-known to both the BS and the receiver end.

In this paper, we do not study or optimize the elements of the coefficient matrix A, but rather focus on the form of this matrix. To this aim, we define a characteristic function $\mathcal{X}$ with argument $(A = [\alpha_{ij}])$ and output $(C = [c_{ij}])$ defined for $1 \leq i \leq U$, and $1 \leq j \leq F$ as follows:

$$C = \mathcal{X}(A) \quad \Rightarrow \quad \begin{cases} c_{ij} = 1 & \text{if} \quad \alpha_{ij} \neq 0 \\ c_{ij} = 0 & \text{if} \quad \alpha_{ij} = 0 \end{cases}$$

Clearly, this function transforms the details of the matrix coefficients into a matrix with the elements 1 or 0. This matrix C will be called the *characteristic matrix*.

Using this concept, the characteristic matrix for the frequency diversity scheme in the above example is

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

which can also be written as $C = [I\,I]$, where I is the identity matrix. The characteristic matrix for full coding is

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

which can also be written as $C = [E]$, where E is the all ones matrix of corresponding dimension.

As another example, assume that the relation between $\overline{u}$ and $\overline{s}$ is as follows

$$s_i = \begin{cases} u_i & \text{for } i \leq U \\ \sum_{j=1}^{U} \alpha_{ji} u_j & \text{for } i > U \end{cases}$$

where all α_{ji}'s are nonzero. Then

$$A = \begin{bmatrix} 1 & 0 & \cdots & 0 & \alpha_{1(U+1)} & \cdots & \alpha_{1F} \\ 0 & 1 & \cdots & 0 & \alpha_{2(U+1)} & \cdots & \alpha_{2F} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & \alpha_{U(U+1)} & \cdots & \alpha_{UF} \end{bmatrix}$$

and therefore

$$C = [I \; E].$$

Note that all the above three forms of C have a common property: that is they all yield equal recovery chances for all the original symbols and thus the transmitted symbols have the same loss probability, P_{ei}. We refer to the characteristic matrices that satisfy this condition as *symmetric* matrices. Note that, given that P_{sc} is the same across all subcarriers inside the channel, any permutation of the columns or the rows of C, in the above examples will not alter the symbol loss probability.

We note that there are other possible forms of C in which different symbols have different P_{ei}. For instance, for $F = 4$ and $U = 2$, we might have:

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

which transmits u_1 on f_1 and three different linear combinations of u_1 and u_2 on each of f_2, f_3 and f_4. In this case, chances of correct detection of the symbols are different, that is $P_{e1} \neq P_{e2}$.

As we have shown, different forms of the characteristic matrix produce different symbol loss probabilities, P_{ei}. This paper selects C so as to minimize the maximum symbol loss probability $(\max_i P_{ei})$ while achieving a minimum throughput (T_{th}) in the channel. Obviously, sacrificing one user to give a better probability of success to another user may not be fair. Therefore, the "min-max" objective is desirable.

This min-max problem can be formulated as the optimization

$$\begin{aligned} &\min_{C \in \mathbb{C}} \max_{i=1,\cdots,U} P_{ei}(C) \\ &s.t. \quad T \geq T_{th} \end{aligned} \tag{1}$$

where $P_{ei}(C)$ is the loss probability of symbol u_i using the characteristic matrix C, and $\mathbb{C}$ is the set of all characteristic matrices of dimension $U \times F$. Note that $P_{ei}(C)$ is a function of C, P_{sc}, and F. Further note that F and P_{sc} are fixed parameters where P_{sc} is calculated based on the modulation scheme and SNR_{th} (broadcast by the BS). Since the modulation level is fixed, the constraint $T \geq T_{th}$ in (1) tacitly determines U. Note that we minimize the maximum of P_{ei} since in general C is not necessarily symmetric.

III. Mathematical Analysis

We will first simplify the optimization problem defined in (1) by grouping the characteristic matrices into classes. For instance, suppose a matrix C_1 can be obtained by applying any permutation on rows or columns of matrix C_2. Then, it is obvious that $\max_{i=1,\cdots,U} P_{ei}(C_1) = \max_{i=1,\cdots,U} P_{ei}(C_2)$. Therefore, we can partition the set $\mathbb{C}$ into different classes so that all members of a specific class have the same value for $\max_{i=1,\cdots,U} P_{ei}$ and as a result it is enough to evaluate $\max_{i=1,\cdots,U} P_{ei}$ only on one member of each class that we term as the *class representative*. In this partitioning, the following properties should hold for all classes:

1) If matrices C' and C'' are members of class i, then C' and C'' are isomorphic.
2) If C' is a member of class i, then all its isomorphic matrices are also members of this class.

For example, $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ are all members of the same class. After class partitioning, class representatives (C_l) are grouped in the set $\mathbb{C}'$ Based on the above classification and after calculating U from the constraint on the throughput, the optimization problem (1) can be simplified to the following problem:

$$\min_{C_l \in \mathbb{C}'} \max_{i=1,\cdots,U} P_{ei}(C_l) \tag{2}$$

where the optimization is over the set of class representatives.

The optimization problem (2) is still a difficult problem to solve analytically over all classes. Consequently, we will focus in this paper on the most important symmetric classes which are already in common use (full coding and frequency diversity) and propose two other classes, namely class A and class B, which also have the symmetric property. We prove that our proposed class A has a better performance than the other ones. The classes considered in this paper are as follow:

1) **Frequency diversity class**: $C_l = [I_U \cdots I_U]$ where I_U is the $U \times U$ identity matrix and is repeated d times in C_l. This class is the famous *frequency diversity* that has been studied in various works and is applied in different networks [12].
2) **Full coding class**: $C_l = [E_{U \times F}]$ where $E_{U \times F}$ is the $U \times F$ all ones matrix. In this case, each subcarrier transmits a different independent linear combination of all the symbols.
3) **Proposed class A**: $C_l = [I_U \; E_{U \times (F-U)}]$. This scheme transmits original U symbols on U subcarriers and $F - U$ different independent linear combinations of all the symbols on the remaining $F - U$ carriers.
4) **Proposed class B**: $C_l = [I_U \, I_U \, ... \, I_U \, E_{U \times (F-tU)}]$ where I_U is repeated t times, $1 < t < d$. Hereafter, we refer to this method by $B(t)$.

In the rest of the paper, we prove that our *proposed class A* achieves a lower symbol loss probability compared to three other classes for practical values of P_{sc}. This means that this scheme outperforms the *frequency diversity* scheme.

Since the characteristic matrices that belong to these four classes are all symmetric, we can substitute $\max_{i=1,\cdots,U} P_{ei}$ by P_{ei} in (2). In the following analysis, we define p such that $p = 1 - P_{sc}$, and also define P_{eD}, P_{eF}, P_{eA}, $P_{eB}(t)$ as the symbol loss probability using the frequency diversity class, full coding class, our proposed class A, and the proposed class B.

Let $p^*(d)$ be the solution of the following equation:

$$(1-p)^{(d-1)} \Big(1 + p\big(d(\frac{d}{d-1})^{d-1} - 2\big)\Big) = 1 \tag{3}$$

Proposition 1. *Given $d \geq 2$, if $p \geq p^*(d)$ then: $P_{eA} < P_{eD}$.*

Proof: This proposition is proved in [14]. ∎

As it is shown in Figure 1, $p^*(d)$ is a decreasing function of d which has the maximum of 0.5 at $d = 2$, and therefore the condition in the Proposition 1 is satisfied for practical values of P_{sc}.

Proposition 2. $P_{eA} < P_{eF}$.

PROOF: For $C = [I\ E]$, a symbol cannot be correctly detected if it is lost when it is transmitted separately, and if from the remaining $F - 1$ subcarriers at most $U - 1$ are correctly received. Since the two events are independent,

$$\begin{aligned} P_{eA} &= (1-p)\sum_{i=0}^{U-1}\binom{F-1}{i}p^i(1-p)^{(F-1-i)} \\ &= \sum_{i=0}^{U-1}\binom{F-1}{i}p^i(1-p)^{(F-i)} \\ &\leq \sum_{i=0}^{U-1}\binom{F}{i}p^i(1-p)^{F-i} \\ &= P_{eF} \end{aligned}$$

■

Proposition 3. *if* $p \geq p^*(t)$, P_{eA} *is less than* $P_{eB}(t)$ *where* $1 < t < d$.

PROOF: Suppose $C_l = [I_U\, I_U \ldots I_U\, E_{U\times(F-tU)}]$, where I_U is repeated t times. Considering symbol u_i, the events leading to recovery of this symbol from this class can be categorized into three groups:

1) Events that use subcarriers only from $[E_{U\times(F-tU)}]$ part of C_l. These events can be used to recover symbol u_i if we use class $C_l' = [I_U\, E_{U\times(F-U)}]$ instead of class C_l.
2) Events that use subcarriers only from the $[I_U I_U \ldots I_U]$ part of C_l. Substituting d and F by t and $t \times U$, respectively in Proposition 1, one can conclude that if $p > p^*(t)$ then $[I_U\ E_{U\times(t-1)U}]$ outperforms $[I_U\ I_U\ \ldots\ I_U]$ where I_U is repeated t times. Therefore, the probability of loss using subcarriers from $[I_U I_U \ldots I_U]$ is more than the loss probability using subcarriers from $[I_U\ E_{U\times(t-1)U}]$.
3) Events that necessarily use subcarriers from parts $[E_{U\times(F-tU)}]$ and $[I_U I_U \ldots I_U]$, simultaneously. All these events can also lead to the recovery of symbol u_i if we use class $C_l' = [I_U\ E_{U\times(F-U)}]$ instead of class C_l.

From the above discussion, the proof is concluded. ■

From Propositions 1, 2, 3 and considering practical values of P_{sc}, we have the following:

$$P_{eA} \leq \min\{P_{eD}, P_{eF}, P_{eB}\}.$$

A. Relaxing the Condition on d

Now let us consider d as a value which is not necessarily integer, then considering *frequency diversity* scheme, it is clear that $\max_{i=1,\cdots,U} P_{ei}(C_l)$ is the same for both $F = d \times U$ and $F = \lfloor d \rfloor \times U$. This is while, using other schemes, $\max_{i=1,\cdots,U} P_{ei}(C_l)$ for $F = d \times U$ is less than the symbol loss probability obtained when $F = \lfloor d \rfloor \times U$ because the arguments used in the proofs of Propositions 2 and 3 are still valid. Therefore, even for the cases that $d = \frac{F}{U}$ is not an integer value, all the above propositions still apply.

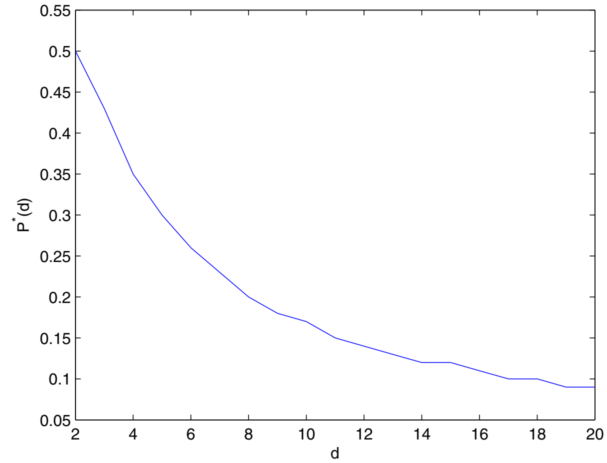

Fig. 1. P^* as a function of d.

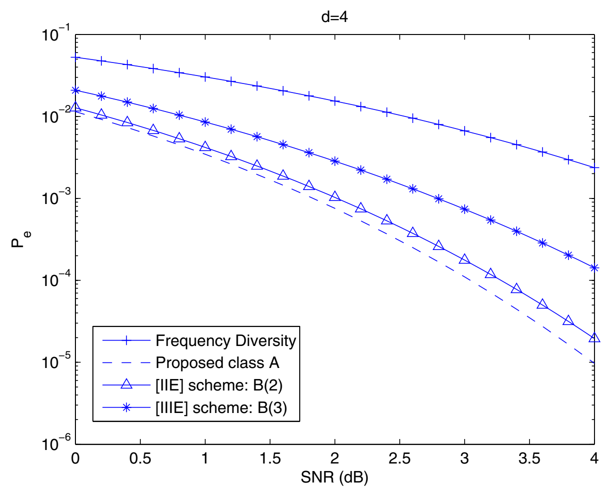

Fig. 2. Comparison of P_e using frequency diversity scheme of order 4, [IIIE], [IIE] and the proposed scheme for 16 QAM modulation.

IV. Numerical Evaluation

In Figure 2, we compare the symbol loss probabilities P_e obtained using frequency diversity and the proposed classes A and B for the modulation level of 16 QAM and for the case of $d = 4$. As we see, our proposed class A has the lowest symbol loss probability among the studied classes. Considering this figure, [IIE] has a better performance compared to [IIIE], and [IIIE] has a better one compared to [IIII] (frequency diversity).

Figure 3 compares P_{eA} with P_{eD}, the conventional scheme of frequency diversity, for different values of U. By increasing U the proposed class A leads to a greater gain in P_e reduction. The figure also shows that as SNR increases, the reduction in P_e becomes more.

Figures 5 and 4 compare the proposed class A with the full coding class for $F = 20$ and $F = 25$. As it is seen in the figures, the proposed class A has a lower symbol loss probability.

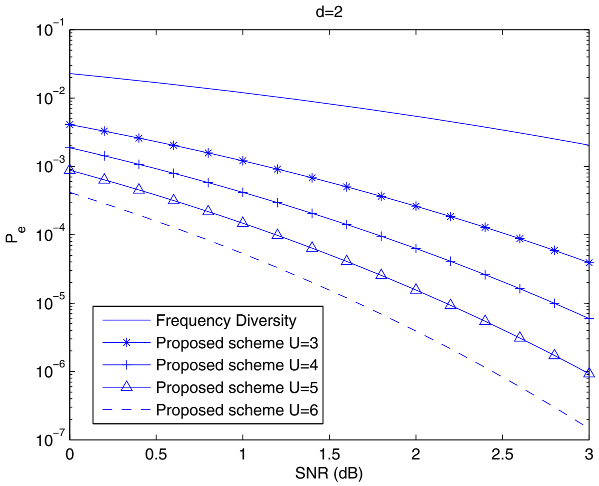

Fig. 3. Comparison of P_e using frequency diversity of order 2 and the proposed scheme for 4 QAM modulation.

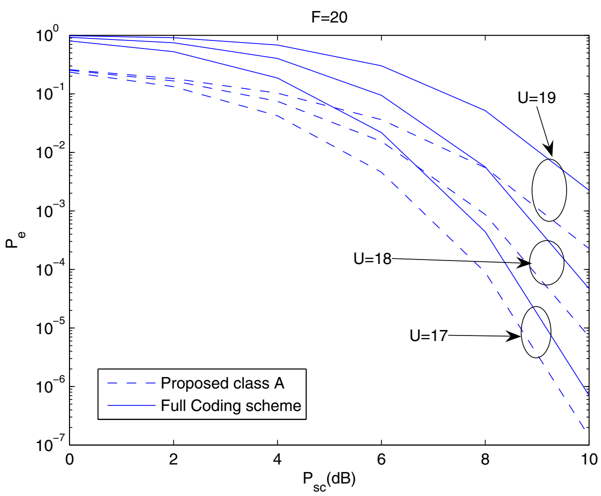

Fig. 4. Comparison of P_e using Full coding and the proposed class A for 16 QAM modulation and F=20.

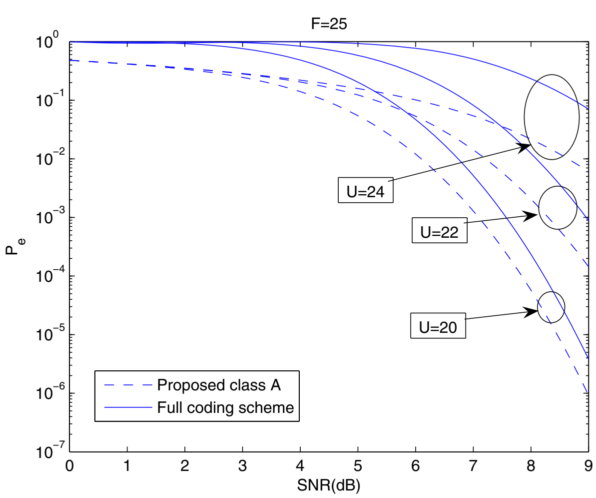

Fig. 5. Comparison of P_e using Full coding and the proposed class A for 16 QAM modulation and F=25.

V. Conclusion

In this paper, we have studied the problem of minimizing the symbol loss probability of downlink transmission in OFDMA based wireless networks, relying on one-bit channel state feedback mechanism, while keeping the throughput above a certain threshold. We have proposed a precoding scheme that outperforms the frequency diversity scheme, currently explored in wireless networks, as well as two other schemes. A precoder linearly combines the data symbols and transmits on multiple subcarriers. A proper combination of the data symbols is of paramount importance. In this paper, we have proposed two precoding schemes and have compared their symbol loss probability with the frequency diversity precoder and full coding precoder. Both the mathematical analysis and the numerical evaluation of our proposed scheme for different SNRs and modulation levels confirm our proposition.

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