On Minimizing Multicast Completion Delay for Instantly Decodable Network Coding

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***Abstract*— In this paper, we consider the problem of minimizing the mean completion delay in wireless multicast for instantly decodable network coding. We first formulate the problem as a stochastic shortest path (SSP) problem. Although finding the optimal selection policy using SSP is intractable, we use this formulation to draw the theoretical properties of efficient selection algorithms. Based on these properties, we propose a simple online selection algorithm that efficiently minimizes the mean completion delay of a frame of multicast packets with a similar computational complexity as the random and greedy selection algorithms. Simulation results show that our proposed algorithm indeed outperforms these random and greedy selection algorithms.**

***Index Terms*—Wireless Multicast; Instantly Decodable Network Coding; Online Algorithms; Stochastic Shortest Path Problem; Maximal Weighted Clique Search**

I. Introduction

Network coding (NC) has shown great abilities to substantially improve transmission efficiency, throughput and delay over broadcast erasure channels. These merits can greatly impact the performance of current and emerging applications that require content to be downloaded quickly and reliably from a sender over possibly unknown channels, such as satellite imaging, roadside to vehicle communication, internet TV and wireless downlink broadcasting. In these applications, transmissions are subject to packet losses due to network or channel impairments such as network congestion, wireless fading and interference. These losses are perceived as packet erasures at higher layers, and are usually modeled by independent erasure channels for different receivers. Consequently, network coding can play a big role in exploiting the diversity of received and lost packets by different receivers to improve the system performance.

The design of online NC packet selection algorithms that optimizes throughput and delay performances over single-hop broadcast erasure channels, has been an intensive area of research [1]–[6]. In [1], the authors propose an online selection algorithms for the three-receiver case, prove it is rate-optimal and conjecture that it achieves an asymptotically optimal average delay. In [2], [3], several greedy online NC selection algorithms minimizing a given notion of unordered delay were compared for i.i.d. erasure channels. However, these proposed algorithms performed un-prioritized packet selection for each NC transmission and did not consider the channel conditions in their selection procedures. In [4], a first approach for NC selection algorithms with packet prioritization and channel awareness was proposed. The authors proposed an algorithm that minimizes the same notion of delay in [2], [3] for *instantly decodable network coding (IDNC)* over Markovian ON-OFF channels. In [7], we proposed an ID-NC algorithm based on a maximal clique search over a graph defining all possible instantly decodable packet combinations. However, we assumed that the clique search is done randomly without considering delay minimization nor channel conditions.

On the other hand, [5], [6] employed of Markov Decision Processes (MDP) [8] to find the optimal NC selection policy that minimizes the distortion of video streams over a finite transmission horizon. However the dimensionality of the MDP's state and action spaces makes the computation of these optimal policies intractable. In [6], a simulation based dynamic programming algorithm was proposed to reduce the computational complexity. However, the resulting complexity of the proposed algorithm is still intractable.

In this paper, we address the following question: *Given the knowledge of received and lost packets at different receivers and their packet loss rates, how can we design an efficient online packet selection algorithm for IDNC that can minimize the mean completion delay of a frame of packets?* We consider IDNC for due its desirable property of simple decoding. Indeed, IDNC can be implemented using binary XOR (operations over GF(2)), and thus each receiver can simply cancel out the packets it already knows. This eliminates the need for matrix inversion at the receivers, which is a computational bottleneck in linear network coding [4].

To answer the above question, we first formulate the problem as a *stochastic shortest path (SSP) problem*, which is a special case of MDP that has an absorbing state. Although this formulation suffers from the same curse of dimensionality as in [5], [6], we mainly employ it to draw the theoretical properties of efficient packet selection algorithms. Based on these properties, we propose a simple yet efficient online IDNC selection algorithm that finds the packet selection using a maximum weight vertex search over the graph employed in [7]. Simulation results show that this proposed algorithm achieves a lower mean completion delay compared to the random selection algorithm (that selects served receivers arbitrarily at each step) and the greedy selection algorithm (that serves the maximum number of receivers at each step) while the computational complexity stays similar to the random algorithm.

The rest of the paper is organized as follows. In Section II, we introduce the system model and parameters. The ID-NC graph is illustrated in Section III. We present the problem formulation using SSP in Section IV and draw from it the properties of efficient NC selection algorithms in Section V. Our proposed NC selection algorithm is introduced in Section VI and its performance is compared against random and greed algorithms in Section VII. Finally, Section VIII concludes the paper.

II. System Model and Parameters

The model consists of a wireless sender that is required to deliver a frame $\mathcal{N}$ of N packets to a set $\mathcal{M}$ of M receivers. Each receiver is interested in receiving either a subset or all the packets of $\mathcal{N}$. The sender initially transmits the N packets of the frame uncoded in an *initial transmission phase*. Each of the receivers stores all correctly received packets its memory even if it did not request it. For each lost packet, each receiver instantaneously sends a NAK packet to the sender. At the end of the initial transmission phase, 3 sets of packets can be attributed to each receiver i:

- The *Has* set (denoted by $\mathcal{H}_i$) is defined as the set of packets correctly received by i. This set includes both desired and undesired packets by this receiver.
- The *Lost* set (denoted by $\mathcal{L}_i$) is defined as the set of packets that were not correctly received by i whether requested or not by this receiver. In other words, $\mathcal{L}_i = \mathcal{N} \setminus \mathcal{H}_i$.
- The *Wants* set (denoted by $\mathcal{W}_i$) is defined as the set of packets that are both requested and lost by m_i in the initial transmission phase of the current MBS frame.

The sender stores this information in a *state feedback matrix (SFM)* $\mathbf{F} = [f_{ij}]\,,\ \forall\ i \in \mathcal{M}, j \in \mathcal{N}$ such that:

$$f_{ij} = \begin{cases} 0 & j \in \mathcal{H}_i \\ 1 & j \in \mathcal{W}_i \\ 2 & \text{otherwise} \end{cases} \qquad (1)$$

After the initial transmission phase, a recovery transmission phase starts. In this phase, the sender exploits the SFM to transmit network coded combinations of source packets containing at most one source packet from the Wants set of each of the receivers. This NC scheme is referred to as *Instantly decodable network coding*. For each transmission, the receivers send ACK/NAK packets that are used by the sender to update the SFM. This process is repeated until all receivers obtain their demanded packets. We define the *completion delay* of a frame as the number of recovery transmissions required until all receivers obtain all their requested packets.

Finally, let $\mathbf{p} = [p_i]\ i \in \mathcal{M}$ be the packet loss rate vector. We assume that the entries of $\mathbf{p}$ do not change during the frame transmission period. Also, let μ_i be the demand ratio of receiver i defined as the ratio of demanded packets of i in the frame to the total frame size.

III. The Instantly Decodable Network Coding Graph

The IDNC graph is a graph that defines all possible instantly decodable packet combinations. It was first introduced in the context of a heuristic algorithm design solving the index coding problem [9], [10]. The IDNC graph $\mathcal{G}$ is constructed by first generating a vertex v_{ij} in $\mathcal{G}$ for each packet $j \in \mathcal{W}_i$, $\forall\ i \in \mathcal{M}$. Two vertices v_{ij} and v_{kl} in $\mathcal{G}(s)$ are connected if one of the following conditions is true:

- $j = l \Rightarrow$ The two vertices are induced by the request and loss of the same packet j by two different receivers i and k.
- $j \in \mathcal{H}_k$ and $l \in \mathcal{H}_i \Rightarrow$ The requested packet of each vertex is in the Has set of the receiver that induced the other vertex.

let $\mathcal{K}$ be a maximal clique in $\mathcal{G}$ (a maximal clique is a clique that is not a subset of any larger cliques). From the construction of $\mathcal{G}$, it can be easily inferred that any packet, generated by XORing the source packets identified by the vertices of any maximal clique in $\mathcal{G}$, is an instantly decodable packet. Consequently, IDNC packets can be generated by maximal clique search over $\mathcal{G}$.

In [7], we employed an IDNC algorithm that randomly selects a maximal clique for each transmission. When the ACK/NAK of this transmission is received by the sender, it updates the graph and the procedure is repeated until all receivers obtain their requested packets. One can intuitively suppose that the best selection strategy is to select any maximum clique in $\mathcal{G}$ at each step (a maximum clique is a clique that has the largest number of vertices). We refer to this approach as greedy selection. In this paper, we aim to find a more strategic clique selection approach in order to minimize the mean completion delay over both the random and greedy selection algorithms.

IV. Problem Formulation using SSP

A. The SSP Problem

The stochastic shortest path (SSP) problem is a special case of the infinite horizon MDP that can model decision based stochastic dynamic systems with a terminating state. In SSP, the different possible situations that the system could encounter are modeled as states $s \in \mathcal{S}$ (where $\mathcal{S}$ denotes the state space of SSP). In each state $s \in \mathcal{S}$, the system can take different actions $a \in \mathcal{A}(s) \subseteq \mathcal{A}$ that will charge it an immediate cost $c(s, a)$ (where $\mathcal{A}$ denotes the action space of SSP). The terminating condition of the system can be thus represented as a zero-cost absorbing goal state (s_g). Once an action a is taken at state s, the system can move to a state s' with probability $P_a(s, s')$, which only depends on the current state and the taken action. An SSP policy $\pi = [\pi(s)]$ is a mapping from $\mathcal{S} \rightarrow \mathcal{A}$ that specifies a given action to each of the states. The optimal policy π^* of an SSP is the one that minimizes the cumulative mean cost until the goal state is reached.

The algorithms solving SSPs define a value function $V_\pi(s)$ as the expected cumulative cost until absorption when the system starts at state s and follows policy π. It can be recursively expressed $\forall\ s \in \mathcal{S}$ as:

$$V_\pi(s) = c(s, \pi(s)) + \sum_{s' \in \mathcal{S}(s,a)} P_{\pi(s)}(s, s')\, V_\pi(s') \tag{2}$$

where $\mathcal{S}(s, a)$ is the set of successor states to s when action a is taken (i.e. $\mathcal{S}(s, a) = \{s' | P_a(s, s') > 0\}$). Consequently, the optimal policy at state s can be defined $\forall\ s \in \mathcal{S}$ as:

$$\pi^*(s) = \arg \min_{a \in \mathcal{A}(s)} \left\{ c(s, a) + \sum_{s' \in \mathcal{S}(s,a)} P_a(s, s')\, V_{\pi^*}(s') \right\} \tag{3}$$

B. Problem Formulation

The packet selection problem that minimizes the mean completion delay for IDNC can be formulated as an SSP problem as follows:

1) State Space $\mathcal{S}$:
States are defined by all possibilities of SFM $\mathbf{F}(s)$ that may occur during the recovery transmission phase. For state s, the matrix represents the content of Has and Wants sets in s (i.e. $\mathcal{H}_i(s)$ and $\mathcal{W}_i(s)\ \forall\ i \in \mathcal{M}$) as defined by (1). For each state s, we define the wants vector $\lambda(s) = [\lambda_i(s)]$, such that $\lambda_i(s) = |\mathcal{W}_i(s)|$. Note that, based on this state definition, the cardinality of the state space $|\mathcal{S}| = O\left(2^{MN}\right)$.

2) Action Spaces $\mathcal{A}(s)$:
For each state s, the action space $\mathcal{A}(s)$ consists of all possible maximal clique in the graph $\mathcal{G}(s)$ constructed from the Has and Wants sets of different receivers in state s. We define $\mathcal{C}(s)$ as the set of maximal cliques in $\mathcal{G}(s)$ and $K(s) = |\mathcal{C}(s)|$. Consequently, the cardinality $|\mathcal{A}(s)| = K(s)$.

3) State-Action Transition Probabilities:
To define the state-action transition probability $P_a(s, s')$ for an action $a = \mathcal{K}(s) \in \mathcal{C}(s)$, we first introduce the two following sets:

$$\mathcal{X} = \{i \in \mathcal{M} | \lambda_i(s) > \lambda_i(s')\} \tag{4}$$

$$\mathcal{Y} = \{i \in \mathcal{M} | \lambda_i(s) = \lambda_i(s') \text{ and } \exists\ v_{ij} \in k(s)\} \tag{5}$$

Based on the definitions of these sets, $P_a(s, s')$ can be expressed for action as follows:

$$P_a(s, s') = \prod_{i \in \mathcal{X}} (1 - p_i) \cdot \prod_{i \in \mathcal{Y}} p_i \tag{6}$$

4) State-Action Costs:
The mean completion delay is defined in SSP terms as the expected number of transitions in the process before arriving to the goal state. Since any transition (due to any action) take one packet transmission, the cost payed by the process is one time-slot. Consequently, the costs of all actions in all sets should be set to 1. In other words, $c(s, a) = 1\ \forall\ a \in \mathcal{A}(s), s \in \mathcal{S}$.

C. SSP Solution Complexity

The optimal policy of an SSP problem can be computed using the famous policy iteration and value iteration algorithms. The complexity of these algorithms are $\Theta(|\mathcal{S}|^n |\mathcal{A}(s_s))$, where s_s is the starting state and the value of n depends on employed version of the algorithms. According to the dimensions of $\mathcal{S}$ and $\mathcal{A}(s)$ described in Section IV-B, we conclude that obtaining the optimal policy is impossible in real-time for typical values of M and N. Even the simulation based technique proposed in [6] will not be able to compute the optimal policy in real-time since its complexity still scales with $|\mathcal{S}|$.

V. Properties of Efficient IDNC Selection Policy

In this section, we explore the properties of the SSP formulation described in Section IV-B and draw some properties that characterize an efficient policy minimizing the mean completion delay. From Section IV-B, we can infer that the SSP formulation has the following two properties:

- *Non-singleton acyclicity*: This property arises from the fact that no state can be revisited once the process moves to a next state. Indeed, if some packets are received by some receivers when an action is taken at a given state, there is no means of going back with these receivers not having these packets. However, a state can revisit itself (singleton cycles) if all the receivers targeted by the taken action do not receive the IDNC packet.
- *Non-increasing successor value functions*: Since there are no cycles of size more than one, successor states of a given state are all closer to the goal state than itself. Consequently, the expected cost to absorption starting from a given state is always greater than or equal to the expected costs to absorption starting from all its successor states.

These two properties can be employed to draw the properties of the optimal policy π^* minimizing the mean completion delay as follows. If the system is at state s, we have:

$$\begin{aligned} \pi^*(s) &= \arg \min_{a \in \mathcal{A}(s)} \left\{ 1 + \sum_{s' \in \mathcal{S}(s,a)} P_a(s, s')\, V_{\pi^*}(s') \right\} \\ &= \arg \min_{a \in \mathcal{A}(s)} \left\{ \sum_{s' \in \mathcal{S}(s,a)} P_a(s, s')\, V_{\pi^*}(s') \right\} \\ &= \arg \min_{a \in \mathcal{A}(s)} \left\{ \mathbb{E}_a \left(V_{\pi^*}(s') \right) \right\}\ , \end{aligned} \tag{7}$$

where $\mathbb{E}_a$ is the expectation operator over the different transition possibilities when action a is taken. Thus, the optimal action at state s is the action minimizing the expectation of the optimal value functions of the successor states. Due to the two properties of the SSP explained above, all successor states are closer to the goal state (thus having less mean completion delays) expect for itself. Thus, the optimal action at state s is the one that has high probabilities of moving to states with the least expected residual completion delay. To evaluate the right hand expressions, we can exploit the results is in [11], [12] to find expressions for the expected residual completion delays of successor states. In these references, the mean completion time for a set of receivers to successfully receive a sufficiently large number N of packets using instantly decodable network coding can be expressed as:

$$MCT = \frac{N}{1 - \max_{i \in \mathcal{M}} \{p_i\}} \ . \tag{8}$$

This expression was derived in [11], [12] while assuming that, for large N, the receiver with the worst channel condition will always have the largest Wants set over the whole recovery transmission phase. The derivation also assumed that this worst channel receiver is addressed by one of its missing packets in each recovery transmission. In other words, if the receiver was not addressed in each recovery retransmission, the mean completion time would have been larger. Consequently, this expression is approximately the optimal mean completion time for large N and one worst channel receiver, which is achieved when the worst channel receiver is addressed in each instantly decodable packet.

Extending this argument to the case of multiple worst channel receivers, an efficient policy would be the one addressing the maximum number of these receivers in each instantly decodable packet. Now extending this argument to the case when N is not large, an efficient policy would be the one addressing the maximum number of receivers, having the largest individual mean completion times, in each instantly decodable packet. We define the individual mean completion time ($\tau_i(s)$) for receiver i at state s as the expected time for this receiver to receive all its missing packets if considered in all future transmissions. Based on this definition, $\tau_i(s)$ can be expressed as:

$$\tau_i(s) = \frac{\lambda_i(s)}{1 - p_i} \ . \tag{9}$$

Having this argument illustrated, we are now ready to present our proposed packet selection algorithm to minimize the multicast mean completion delay.

VI. Proposed Algorithm

We previously stated that the IDNC packet selection in state s is performed by a maximal clique selection from the state's IDNC graph $\mathcal{G}(s)$. According to the properties illustrated in the previous section, an efficient IDNC packet selection algorithm should select, at each visited state, the maximal clique that includes the maximum number of vertices belonging to receivers having the largest $\tau_i(s)$ values. One way to do so is to list all maximal cliques in $\mathcal{G}(s)$ and select the clique satisfying this property. However, maximal clique listing is in general computationally complex. Consequently, it is preferable to design a simpler algorithm that performs a maximal clique selection through a maximum weighted vertex search. In this search, the weights of vertices must reflect the required property illustrated in Section V.

To design the vertices' weights, we first define $a_{ij,kl}(s)$ as the adjacency indicator of vertices v_{ij} and v_{kl} in $\mathcal{G}(s)$ such that:

$$a_{ij,kl}(s) = \begin{cases} 1 & v_{ij} \text{ is connected to } v_{kl} \text{ in } \mathcal{G}(s) \\ 0 & \text{otherwise} \ . \end{cases} \tag{10}$$

Algorithm 1 Maximum Weight Vertex Search Algorithm

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Require: $\mathbf{F}(s)$ and $\tau_i(s)$
  Initialize $\mathcal{K}^*(s) = \varnothing$.
  Construct $\mathcal{G}(s)$ and $\mathbf{A}(s)$.
  while $\mathcal{G}(s) \neq \varnothing$ do
    Compute $\Delta_{ij}(s)$ and $w_{ij}(s)$ using (11) and (12).
    Select $v^* = \arg\max_{v_{ij} \in \mathcal{G}(s)} \{w_{ij}(s)\}$.
    Add $v^*$ to $\mathcal{K}^*(s)$
    Set $\mathcal{G}(s) \leftarrow \mathcal{G}_{v^*}(s)$ and $\mathbf{A}(s) \leftarrow \mathbf{A}_{v^*}(s)$
  end while
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We then define the weighted degree $\Delta_{ij}(s)$ of vertex v_{ij} as:

$$\Delta_{ij}(s) = \sum_{\forall v_{kl} \in \mathcal{G}(s)} a_{ij,kl}(s) \, \tau_k(s) \ . \tag{11}$$

Thus, a large weighted vertex degree reflects its connection to a large number of vertices belonging receivers with large values of $\tau_i(s)$. We finally define the vertex weight $w_{ij}(s)$ as:

$$w_{ij}(s) = \tau_i(s) \, \Delta_{ij}(s) \ . \tag{12}$$

Consequently, a vertex v_{ij} has a large weight when it both belongs to a receiver with large $\tau_i(s)$ value and is connected to a large number of vertices having large $\tau_i(s)$ values. Let $\mathcal{G}_v(s)$ be the subgraph in $\mathcal{G}(s)$ including only vertices connected to vertex v. We finally define $\mathbf{A}(s)$ and $\mathbf{A}_v(s)$ as the adjacency matrices of $\mathcal{G}(s)$ and $\mathcal{G}_v(s)$, respectively.

Based on these definitions, we can introduce our proposed packet selection algorithm as follows. The algorithm operates only for visited states. In each visited state s, the algorithm computes a maximal clique $\mathcal{K}^*(s)$ in $\mathcal{G}(s)$ as depicted in Algorithm 1. At first, $\mathcal{K}^*(s)$ is an empty set. The algorithm starts by selecting the maximum weight vertex in $\mathcal{G}(s)$ to be the source node in $\mathcal{K}^*(s)$. For each of the following iterations, the algorithm first recomputes the new vertex weights within the subgraph connected to all previously selected vertices in $\mathcal{K}^*(s)$, then adds the new maximum weight vertex to it. The algorithm stops when there is no further vertex connected to all vertices in $\mathcal{K}^*(s)$. We refer to this algorithm as the *maximum weight vertex search algorithm*. Once the clique is computed, the sender forms and sends an IDNC packets by XORing the source packets identified by the vertices in $\mathcal{K}^*(s)$. According to the received feedback, a new state is visited and the process is re-executed until the absorbing goal state is reached.

VII. Simulation Results

In this section, we compare thought simulations the performance of our proposed algorithm to the random and greedy clique selection algorithms. In our simulations, we assume that the receivers have packet loss rates that change from frame to frame in the range [0.05,0.3]. We also assume the demand ratios of different receives change while maintaining the average demand ratio (μ) constant.

Figure 1 depicts the comparison of mean completion delays achieved by the random, greedy and proposed clique selection algorithms against M, for $N = 40$ and $\mu = 0.6$. Results show

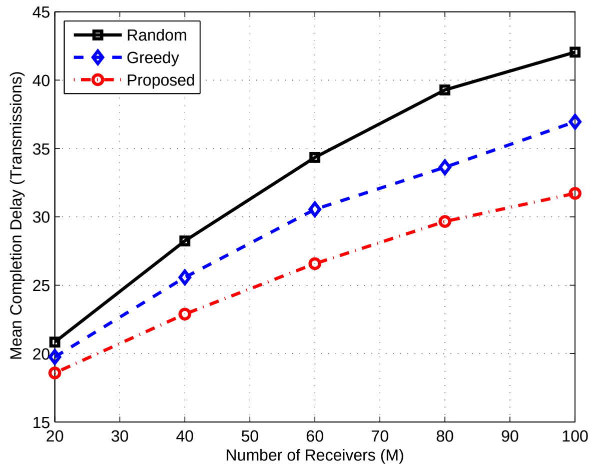

Fig. 1. Comparison of Mean Completion Delays against M

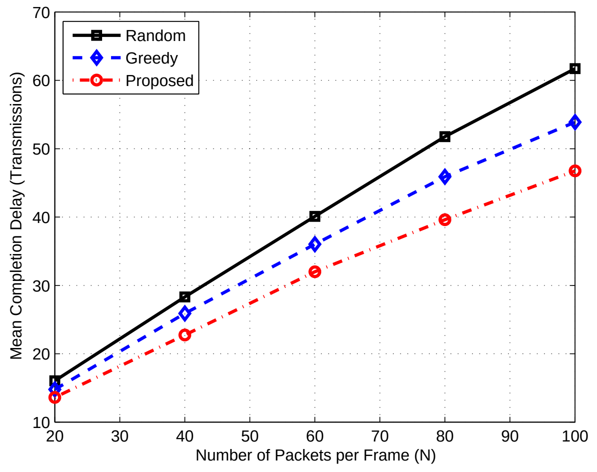

Fig. 2. Comparison of Mean Completion Delays against N

that our proposed algorithms achieves a percentage improvements of $11\% - 24\%$ and $6\% - 14\%$ compared to the random and greedy algorithms, respectively, as M increases. Figure 2 depicts the same comparison against N for $M = 40$ and $\mu = 0.6$. Results show that our proposed algorithms achieves a percentage improvements of $15\% - 24\%$ and $8\% - 13\%$ compared to the random and greedy algorithms, respectively, as N increases. Finally, Figure 3 depicts the same comparison against μ for $M = 40$ and $N = 40$. Results show that our proposed algorithms achieves a percentage improvements of $9\% - 30\%$ and $3\% - 20\%$ compared to the random and greedy algorithms, respectively, as μ increases.

VIII. Conclusion

In this paper, we aimed to design an IDNC packet selection algorithm that minimizing the mean completion delay in wireless multicast. We first formulated the problem as an SSP problem and showed that an efficient algorithm would service

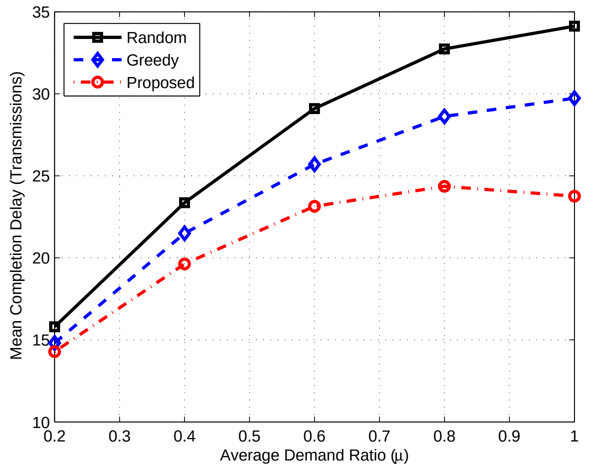

Fig. 3. Comparison of Mean Completion Delays against μ

the maximum number of receivers with larger individual mean completion times. Based on this property, we proposed a simple online selection algorithm that employs a maximum weight vertex search approach over the IDNC graph to determine the best packets to encode at each state. The vertices' weights were designed to reflect the property inferred from SSP. Simulation results show that our proposed algorithm indeed outperforms these random and greedy selection algorithms.

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