

# Reliable Broadcast of Safety Messages in Vehicular Ad Hoc Networks

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**Abstract**—Broadcast communications is critically important in vehicular networks. Many safety applications need safety warning messages to be broadcast to all vehicles present in an area. In this article, we propose a novel repetition-based broadcast protocol based on “optical orthogonal codes.” Optical orthogonal codes are used because of their ability to reduce the possibility of collision. We present a detailed mathematical analysis for obtaining the probability of success and the average delay. We show, by analysis and simulations, that the proposed protocol outperforms existing repetition-based ones and provides reliable broadcast communications and can reliably deliver safety messages under load conditions deemed to be common in vehicular environments.

## I. INTRODUCTION

According to the World Health Organization (WHO), road accidents annually cause approximately 1.2 million deaths and 50 million injuries worldwide [1]. If preventive measures are not taken, traffic accident death is likely to become the third cause of the loss of disability-adjusted life years (DALY) <sup>1</sup> in 2020 from ninth place in 1990 [2].

However, fatalities caused by car crashes are, in principle, avoidable. 21,000 of the annual 43,000 road accident deaths in the US are caused by roadway departures and intersection related incidents [3]. This number can be significantly lowered by deploying local warning systems enabled by vehicular communications.

Unlike conventional safety systems, which try to minimize the casualties of collisions by using devices such as air bags and shock absorbers, active/cooperative safety systems are capable of preventing accidents. Active/cooperative safety systems are part of a broad range of emerging communications, electronics, and informatics technologies, unified under Intelligent Transportation Systems (ITS), being developed to fundamentally enhance safety and productivity in surface transportation. ITS development relies, at its core, on a communication platform enabling fast and reliable communication in vehicular environments. Dedicated Short Range Communication (DSRC) standard, adopted by IEEE and ASTM International<sup>2</sup> (ASTM E 2213-03 [4]), provides the communication platform required by ITS [5].

75MHz bandwidth at 5.9GHz is allocated to public and private vehicular communication applications based on DSRC [6]. The 75MHz bandwidth is divided into seven 10MHz channels. Among the seven designated channels, one channel is the control channel (ch 178) used mainly for broadcast traffic.

Our goal in this work is to provide a Medium Access Control (MAC) protocol in ad hoc mode for broadcast communication. Such a MAC protocol must be able to reliably deliver safety-critical messages. Due to stringent delay requirements of safety traffic, transmission delay of a protocol designed for vehicular communication must be very low. Furthermore, a vehicular MAC must be capable of supporting mobility and effectively coordinating tens of sources of broadcast traffic.

Compared to infrastructure networks, vehicular ad hoc networks can provide communication with lower delay by delivering messages from vehicle to vehicle, eliminating the delay caused by transmitting messages to infrastructure and back to vehicles. Because vehicles moving in the same direction have lower speed relative to each other than to roadside units, problems caused by mobility are also alleviated in ad hoc networks. Nevertheless, in specific situations such as urban intersections, in which line-of-sight communication is not possible and one cannot rely on the presence of other vehicles to relay critical messages, the presence of roadside units may be necessary.

We assume that safety systems installed on each vehicle require a map of relative position of neighboring vehicles. If positions of neighboring vehicles are known to the safety systems, many collisions can be avoided. If the velocity of the neighbors is also known, each vehicle can predict future positions and avoid possible collision-prone situations. Building a local map in each vehicle requires that: 1) each vehicle be able to discover its own absolute or relative position, and 2) vehicles be able to communicate position information. Discovering the position of a vehicle can be done via GPS [7], radio ranging techniques [8], and/or, radar. Our focus in this work is on designing a medium access protocol that is capable of delivering position information messages, as well as other data.

At 100km/hr, a vehicle moves 6m (approximately the accuracy of GPS) in 216ms. Therefore, update frequency of approximately 5 messages/second guarantees accurate and up-to-date maps.

Since vehicular update messages need to deliver limited information such as vehicle ID, message ID, position, velocity, road condition, warning, etc, the size of these messages is under a few hundred bytes. Location information, on the Earth’s surface, in a spherical system with fixed  $r$  coordinate and 1cm resolution can be delivered with  $\log_2(2\pi 6.4 \times 10^6 \text{ m}/10^{-2} \text{ m}) + \log_2(\pi 6.4 \times 10^6 \text{ m}/10^{-2} \text{ m}) = 62.81 \leq 63$  bits where  $6.4 \times 10^6 \text{ m}$  is the Earth’s radius. Relative location information

<sup>1</sup>DALYs are the sum of the years of life lost due to premature mortality and the years lost due to disability.

<sup>2</sup>Originally known as the American Society for Testing and Materials

within 100m (in a 200m×200m square centered at the reference point) in a Cartesian system with 1 cm resolution can be delivered with  $2\log_2(200\text{ m}/10^{-2}\text{ m}) = 28.6 \leq 29$  bits. Assuming each vehicle transmits its position in absolute form and its velocity and the positions and the velocities of vehicles immediately in front, behind, left, and right in relative to the absolute position,  $63 + 29 + 4(29 + 29) = 324$  bits or 41 bytes need to be transmitted. Adding 2 bytes for the ID of each vehicle, in total 51 bytes is needed. If about 49 bytes are allocated for other uses, such as obstacle and its position, emergency car and its position, emergency braking, etc, the length of the safety message is about 100 bytes. Therefore, in vehicular communications, safety messages are short compared to data or multimedia messages.

An automatic safety system is successful if it can recognize a dangerous situation before the driver of a vehicle does. The mental processing time, i.e., the time from the moment an event occurs until the moment a decision is made, is between 500ms to 1.2s, depending on how unexpected the event is [9]. Noting that a warning message alerting a driver, itself needs to be processed, communication delay must not exceed 100-200ms. This value is, henceforth, called the lifetime of a safety message.

The rest of this article is organized as follows. We review related works in Section II. The proposed broadcast protocol is explained in Section III. Analytical performance study and simulation results are presented in Sections IV and V, respectively. Finally, we conclude this paper in Section VI.

## II. RELATED WORK

A major difference between an ad hoc network in vehicular environment and a conventional ad hoc network is that in vehicular networks, as discussed earlier, traffic is of broadcast type; routine safety messages are issued from all vehicles several times per second and are intended for all their neighbors. Transmission of safety messages must be reliable and with very low delay. Conventional MAC protocols for ad hoc networks are not designed to handle broadcast traffic from many nodes in the network. For example, in IEEE 802.11 no mechanism exists to reduce the probability of collision for broadcast traffic. In IEEE 802.11, Request To Send (RTS) and Clear To Send (CTS) packets can be transmitted before unicast communications to avoid collisions. It may seem straight forward to add RTS/CTS handshake to broadcast communications as well. However, in vehicular communication, the length of broadcast messages is short and comparable to that of RTS. Therefore, the probability of collision is not significantly lower for RTS packets. The short length of messages also contributes to inefficiency since the payload (safety message) is not significantly larger than the overhead (RTS+CTS). Furthermore, RTS/CTS handshake needs to be performed with more than one receiver to obtain the same reliability as that of unicast communication. Therefore, protocols such as Broadcast Medium Window (BMW) [10], Batch Mode Multicast MAC (BMMM) [11], which rely on RTS/CTS handshake with multiple nodes are not effective

methods for the delivery of short broadcast messages in a vehicular environment. Even unicasting such short messages as in vehicular safety communications, with 802.11 approach is very inefficient. According to a model devised by Bianchi [12], the maximum bandwidth utilization of 802.11a with RTS/CTS handshake, at 54 Mb/s, with payload size of 100 bytes is less than 7% [6]. Multiple RTS/CTS handshakes, as proposed by the above protocols, will further decrease the efficiency.

Synchronous p-Persistent Retransmission (SPR) and Synchronous Fixed Retransmission (SFR) [13], repetition-based protocols proposed to solve said broadcast problems in vehicular environment, are discussed next.

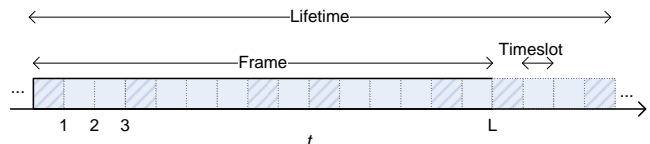


Fig. 1. Division of time into frames and timeslots for repetition-based broadcast with  $L = 15$ , each time unit is equal to the length of one timeslot.

### A. Repetition-based Broadcast Protocols

The fundamental idea behind repetition-based broadcast is repeating a message several times in an interval shorter than or equal to its lifetime to ensure high probability of reception. In repetition-based broadcast protocols, time is divided into frames, the maximum length of which must not be greater than the lifetime of a safety message. Each frame, in turn, is divided into  $L$  timeslots with length equal to the transmission time of a *single packet*. The division of time into frames and timeslots is shown in Fig. 1. Each packet is transmitted a number of times inside the frame according to a transmission pattern. In each timeslot, if a node is not transmitting, it switches to the receive mode. Each pattern can be represented by a binary vector of length  $L$  in which a ‘1’ denotes a transmission and a ‘0’ represents an idle timeslot, as illustrated in Fig. 2. Each of these vectors is called a *codeword* and the set of all codewords is called a *code*. In the following, we only consider synchronous protocols. In a synchronous protocol, timeslots are synchronized. Synchronization can be achieved using a variety of methods. One that is particularly appealing to vehicular communication applications is Global Positioning System (GPS) [7] because many vehicles are already equipped with GPS devices and more will be equipped in the future.

In this section, we study two random protocols proposed in [14], [15], and [13]. In random repetition protocols, the transmission patterns are chosen randomly.

1) *Synchronous p-Persistent Repetition (SPR)*: In SPR, the source node transmits the packet in each timeslot in a frame with probability  $p$  and remains idle with probability  $1-p$ . Note that a packet may be transmitted  $L$  times or not transmitted at all.

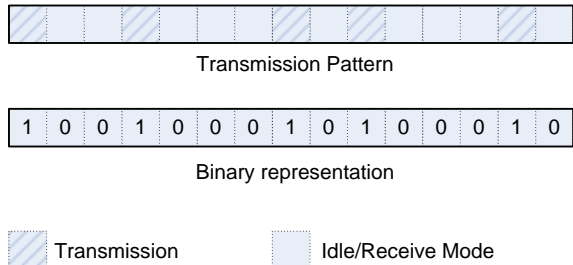


Fig. 2. A transmission pattern and its binary representation.

2) *Synchronous Fixed Repetition (SFR)*: In SFR, each packet is transmitted  $w$  times in each frame, i.e.,  $w$  timeslots are randomly chosen out of the  $L$  available timeslots for repeated transmissions of the packet.

It is shown in [13], via simulation, that SFR decreases the probability of failure by one order of magnitude, compared to IEEE 802.11a. The performance of SPR is worse than SFR but better than IEEE 802.11a.

The above protocols are able to perform well in harsh channel conditions due to multiple transmissions and are robust against mobility because they do not rely on the knowledge of position of nodes in the network, and instead take advantage of the fact that safety messages are intended for all nodes in the neighborhood area. Furthermore, although each message is repeated several times, the overhead is still less than many of the protocols that rely on multiple RTS/CTS handshakes to transmit a broadcast packet because of the short length of safety messages. However, these protocols are not able to combat collision. In SPR and SFR, the timeslots in which a transmission takes place, i.e., the transmission patterns, are chosen randomly. Randomly choosing transmission patterns results in relatively high probability of collision. On the contrary, if transmission patterns are chosen deterministically with the goal of decreasing collision, we are able to mitigate collision among users. In the next section, we propose a repetition-based broadcast protocol in which transmission patterns are deterministically chosen using optical orthogonal codes to reduce the possibility of collision.

### III. BROADCAST USING OPTICAL ORTHOGONAL CODES

One way to choose the transmission patterns is to make sure that —it any two patterns do not collide in more than one timeslot. This is the main idea behind choosing optical orthogonal codes as transmission patterns.

Assuming vectors  $\mathbf{x}$  and  $\mathbf{y}$  are two codewords used as transmission patterns, their cross-correlation, i.e., the inner product  $\langle \mathbf{x}, \mathbf{y} \rangle$ , is the number of collisions in an ideal channel which occur if two users transmit with patterns indicated by  $\mathbf{x}$  and  $\mathbf{y}$ . Therefore, limiting the correlation of two codewords

is equivalent to limiting the possibility of collisions. A synchronous optical orthogonal code,  $C$ , with length  $L$  and weight  $w$  is a code whose codewords are binary vectors with length  $L$  and the number of ‘1’s in each codeword is  $w$ . Furthermore, the codewords satisfy the following condition

$$\langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=1}^L x_i y_i \leq \lambda \quad \forall \mathbf{x}, \mathbf{y} \in C \quad (1)$$

where  $\lambda$  is a fixed integer usually taken to be 1. Choosing OOC with  $\lambda = 1$  as transmission patterns guarantees that any two users have at most one common timeslot, while in random methods, the number of collisions can be up to  $w$  for SFR and up to  $L$  for SPR.

A synchronous optical orthogonal code,  $C$ , with length  $L$ , weight  $w$ , and maximum correlation  $\lambda$  is equivalent to a constant weight code with minimum Hamming distance  $2\delta = 2(w - \lambda)$  and same length and weight. The size of the largest constant-weight code with given values for  $L$ ,  $w$ , and  $2\delta$  is unknown in the general case [16]. Johnson [17] provides an upper bound for the number of codewords in such code

$$\|C\| \leq \left\lfloor \frac{L}{w} \left\lfloor \frac{L-1}{w-1} \dots \left\lfloor \frac{L-w+\delta}{\delta} \right\rfloor \right\rfloor \right\rfloor \quad (2)$$

where  $\lfloor x \rfloor$  is the largest integer less than or equal to  $x$ . In this work, we only consider  $\lambda = 1$ . For example, for  $L = 64$  and  $w = 6$ , code cardinality is bounded by 128 and for  $L = 128$  and  $w = 9$ , code cardinality is bounded by 233. Lower bounds are usually obtained by constructing a code with given parameters. Note that strict orthogonality, i.e.,  $\lambda = 0$ , leads to a very low code cardinality, namely, at most  $L/w$ .

Typical values for frame length,  $L$ , packet length,  $L_p$ , and data rate,  $R$ , can be 100, 100B or 800bits, and 10Mbps, respectively. Therefore, a typical value for the duration of a frame is  $T_f = LL_p/R = 100 \times 800/10M \approx 10ms$ .

#### A. Distributed Code Assignment

We assume the set of codewords is decomposed into two subsets. A subset of codewords in the code is reserved only for network association, denoted by set  $C_a$ . Once a vehicle enters a road, it randomly selects a tentative codeword from the subset reserved for network association. In the network association phase, the vehicle that wants to join the network, can start transmitting its data packet as usual. However, it must also acquire a permanent codeword that is unique within its two-hop communication range. To obtain information about codewords used in the two-hop neighborhood the joining node issues Code Information Requests (CIQ). Every node  $i$  receiving a CIQ transmits Code Information Response (CIR) which contains the index of its codeword and its ID, the codewords of the node’s one-hop neighbors, and the ID of those neighbors. The codewords indicated in the CIR received from node  $i$ , denoted by  $C_i$ , are used by other nodes and hence unusable by the joining node. After receiving several of these packets, the node with the tentative codeword chooses a permanent codeword from the set  $C_p = C \setminus C_a \setminus \cup_i C_i$ . While network association is performed only once when the vehicle

enters the road, each node with a permanent codeword also periodically transmits a CIR with frequency of once every few seconds. This enables the network to adapt to topology changes. If a node with a permanent codeword discovers that its codeword is being used by one of its two-hop neighbors, it releases that codeword and chooses another one from  $C_p$ .

*Code Information Response Window:* When a joining node issues a CIQ, if all neighbors transmit CIR packets in the next frame, the additional load caused by several immediate CIR packets results in performance degradation. To resolve this issue, we introduce the Code Information Response Window (CIRW). Each node that receives a CIQ, sets a counter to a random number uniformly chosen between 1 and CIRW. At the end of each frame, the counter is decreased by one. When the counter reaches zero, CIR is transmitted in the next frame. The joining node determines its permanent codeword after CIRW frames have passed.

The number of permanent codewords required depends on the desired communication range,  $R_c$ . The cardinality of  $C_p$  must be large enough to support all vehicles in length  $4R_c$  of a road. For example, assuming  $R_c = 100m$  and adjacent cars in the same lane are 30m apart, in a four-lane road at least  $4 \times 4 \times 100/30 \approx 53$  permanent codewords are required.

If code cardinality is not large enough to allocate sufficient number of codes for  $C_a$ , more codewords can be added to the code with higher cross-correlation for use in code assignment phase. Since these codes are in use only for a short time, performance degradation caused by their higher cross-correlation is minimal.

### B. Adaptive Elimination

If two users have transmission patterns that include transmission in a common timeslot, a collision is likely to happen. However, if one user has a transmission before the common timeslot in its transmission pattern and includes some information in the transmitted packets in this timeslot with which the second user can identify the codeword used by the first user, provided that the second user successfully receives this transmission, it can prevent the collision simply by not transmitting in the common timeslot. We call this method *Adaptive Elimination*.

Adaptive elimination increases reliability by eliminating transmissions in timeslots that may potentially result in collision. To enable this, the codebook must be stored in all nodes and nodes must transmit the index of their codeword and indicate which timeslots are disabled. Each node adds a *codeword indicator* field with format (index of codeword, enabled/disabled timeslots) to the header of its data packets to inform other nodes of its codeword. Each part of this field has a predetermined length. It can be argued that adaptive elimination adds insignificant overhead. We leave the detail to a later work.

## IV. ANALYTICAL PERFORMANCE STUDY

To gain a better understanding of repetition-based broadcast protocols, in this section, we analytically study the performance of such protocols. Although we only consider SPR,

SFR, and OOC, our analysis is general and can also be applied to other repetition-based protocols. In this section, we do not consider adaptive elimination. Furthermore, we assume a network that is interference limited and an ideal wireless channel, which carries signals with no attenuation and no noise. Hence, we neglect the effect of noise on the performance. As the traffic model, we use the binomial distribution. However, other traffic models can also be used. Furthermore, we assume frames and timeslots are synchronized.

### A. Probability of Success

Transmission in a *timeslot* is successful if only one node transmits in that timeslot. If two nodes transmit in the same timeslot, a collision occurs and both transmissions fail. Since the channel is assumed ideal, all nodes are able to receive a transmission when there is no collision. In each *frame*, each node transmits in several timeslots. The message is successfully transmitted if at least one of the transmissions in the frame is successful. Probability of success is defined as the number of messages successfully transmitted by a node divided by the number of messages that the node has attempted to transmit. Probability of success depends on the number of *interfering users*, i.e., the number of users transmitting in the same frame as the desired user.

In order to obtain the probability of success, we introduce the following events.  $\mathcal{S}$  is the event that at least one transmission is successful in a frame. When discussing protocols with exactly  $w$  transmissions in a frame, such as OOC and SFR,  $\mathcal{S}_i$  denotes the event that the  $i$ th transmission among  $w$  transmissions is successful and  $\hat{\mathcal{S}}_i$  denotes the event that the  $i$ th transmission is the first successful transmission. When discussing SPR,  $\mathcal{S}_i$  is the event that the transmission in the  $i$ th timeslot is successful and  $\hat{\mathcal{S}}_i$  is the event that the first successful transmission occurs in the  $i$ th timeslot.

Assume that the desired user is transmitting a message. The probability of success,  $P_s$ , can be written as

$$P_s = \sum_{M=0}^{N-1} P_M(\mathcal{S})P(M=M) \quad (3)$$

where,  $M$  is the random variable denoting the number of interfering users,  $P_M(\mathcal{S})$  is the probability of the event  $\mathcal{S}$  given that there are  $M$  interfering users, and  $N$  is the total number of users in the network.

The probability mass function of  $M$  depends on the traffic model. The traffic model is discussed in Section IV-C. In this section, we focus on obtaining  $P_M(\mathcal{S})$ .

1) *Probability of Success for SPR:* For SPR, the desired transmitter is successful in the  $i$ th timeslot if it transmits in that timeslot and all other users are silent. Assuming each user transmits with probability  $p$  in each timeslot, the probability of success in a timeslot,  $s$ , is

$$s \triangleq P_M(\mathcal{S}_i) = p(1-p)^M \quad 1 \leq i \leq L. \quad (4)$$

The desired user fails to transmit its packet successfully if it fails in all  $L$  timeslots. The probability of failure in all

timeslots is  $(1 - s)^L$ . Therefore, the probability of success,  $P_M(\mathcal{S})$ , is

$$P_M^{(SPR)}(\mathcal{S}) = 1 - (1 - s)^L = 1 - \left(1 - p(1 - p)^M\right)^L. \quad (5)$$

2) *Probability of Success for SFR and OOC*: In SFR and OOC, since there are exactly  $w$  repetitions, different timeslots are not independent. Therefore, the probability of success cannot be obtained as easily as that of SPR. SFR and OOC are two special cases of repetition-based broadcast schemes with exactly  $w$  transmissions.

In a repetition-based protocol with exactly  $w$  transmissions, the probability that at least one transmission is successful among  $w$  transmissions,  $P_M(\mathcal{S}_1 \cup \dots \cup \mathcal{S}_w)$ , can be written as

$$\sum_{k=1}^w (-1)^{k+1} \sum_{\{a_1, \dots, a_k\} \in \binom{w}{k}} P_M(\mathcal{S}_{a_1} \cap \dots \cap \mathcal{S}_{a_k}) \quad (6)$$

where  $\binom{w}{k}$  is the set of all  $k$ -subsets of  $\{1, \dots, w\}$ . As we will see,  $P_M(\mathcal{S}_{a_1} \cap \dots \cap \mathcal{S}_{a_k})$  does not depend on  $a_1, \dots, a_k$  but rather only on  $k$ . Therefore, by defining

$$\gamma_k = P_M(\mathcal{S}_{a_k} \cap \dots \cap \mathcal{S}_{a_1}), \quad \{a_1, \dots, a_k\} \in \binom{w}{k} \quad (7)$$

we have

$$P_M(\mathcal{S}) = P_M(\mathcal{S}_1 \cup \dots \cup \mathcal{S}_w) = \sum_{k=1}^w (-1)^{k+1} \binom{w}{k} \gamma_k \quad (8)$$

Next, we find  $\gamma_k$  for SFR and OOC and substitute it in (8) to obtain success probability for SFR and OOC.

a) *SFR*: The probability that a certain interfering user, i.e., a user that transmits in the same frame, does not transmit in timeslots  $a_k, a_{k-1}, \dots, a_1$  with the desired user is equal to  $\binom{L-k}{w} / \binom{L}{w}$  where the transmission pattern of the interfering user can be any of the  $\binom{L}{w}$  patterns with equal probability. Among the possible patterns,  $\binom{L-k}{w}$  pattern does not include transmission in the prohibited timeslots  $a_k, a_{k-1}, \dots, a_1$ .

Since the  $M$  interfering users are independent,

$$\gamma_k = P_M(\mathcal{S}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \dots \cap \mathcal{S}_{a_1}) = \left( \frac{\binom{L-k}{w}}{\binom{L}{w}} \right)^M \quad (9)$$

Therefore, as claimed earlier,  $P_M(\mathcal{S}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \dots \cap \mathcal{S}_{a_1})$  does not depend on  $a_1, \dots, a_k$  but only on  $k$ . Hence,

$$P_M^{(SFR)}(\mathcal{S}) = \sum_{k=1}^w (-1)^{k+1} \binom{w}{k} \left( \frac{\binom{L-k}{w}}{\binom{L}{w}} \right)^M. \quad (10)$$

b) *OOC*: In OOC with  $\lambda = 1$ , by definition, an interfering user may transmit in only one timeslot in which the desired user also transmits. Consider the  $a_j$ th transmission of the desired user. Let  $p_1$  be the probability that a certain interfering user transmits in the same timeslot as the  $a_j$ th transmission of the desired user. The probability that the interfering user does not transmit at the same time as any of the transmission timeslots  $a_k, a_{k-1}, \dots, a_1$  of the desired user is  $1 - kp_1$ . Considering  $M$  independent interfering users, we have

$$\gamma_k = P_M(\mathcal{S}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \dots \cap \mathcal{S}_{a_1}) = (1 - kp_1)^M. \quad (11)$$

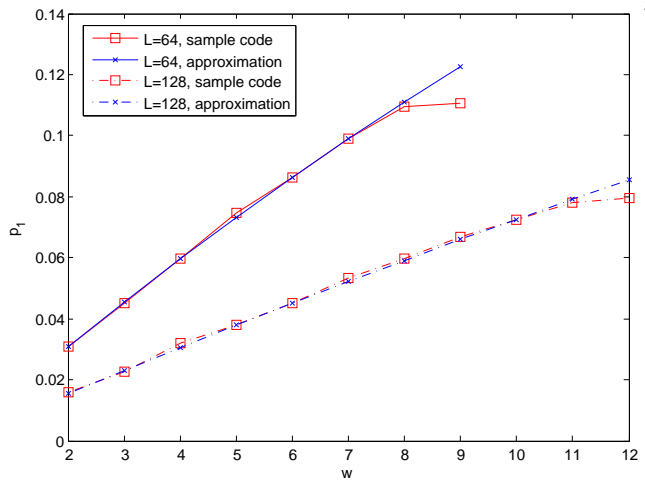


Fig. 3. Approximate and sample values of  $p_1$  for OOC

Note that  $P_M(\mathcal{S}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \dots \cap \mathcal{S}_{a_1})$  only depends on  $k$ . Substitution in (??) yields

$$P_M^{(OOC)}(\mathcal{S}) = \sum_{k=1}^w (-1)^{k+1} \binom{w}{k} (1 - kp_1)^M. \quad (12)$$

we should now find  $p_1$ . Assume that by reordering the codeword of the desired user is written as  $\overleftarrow{1111} \dots \overleftarrow{11000} \dots \overleftarrow{0}$ . Then,  $p_1$  is equal to the number of codewords with form  $\overleftarrow{1000} \dots \overleftarrow{00xxx} \dots \overleftarrow{x}$  after reordering, divided by the total number of possible codewords. Codewords with this form are common in the first timeslot. Therefore, they cannot have any other common timeslot among the  $L - w$  timeslots denoted by  $x$ . Since  $w - 1$  ones must be placed in the  $L - w$  timeslots denoted by  $x$  with no overlap, there are at most  $\frac{L-w}{w-1}$  codewords with this form. From (2), the total number of codewords is at most  $\left\lfloor \frac{L}{w} \left\lfloor \frac{L-1}{w-1} \right\rfloor \right\rfloor$ . Therefore,  $p_1$  may be approximated by

$$p_1 \approx \frac{w(L-w)}{L(L-1)}. \quad (13)$$

$p_1$  can also be empirically obtained from a sample generated code using

$$p_1 \approx \frac{\sum_{i=1}^{\|C\|} \sum_{j=i+1}^{\|C\|} \langle \mathbf{c}_i, \mathbf{c}_j \rangle}{w \binom{\|C\|}{2}} \quad (14)$$

where  $C$  is the generated code and vectors  $\mathbf{c}_i$  are codewords and  $\|C\|$  is the size of the code (number of codewords). The division by  $w$  is because  $p_1$  corresponds only to one transmission among  $w$  transmissions. Comparison of approximate values and sample values of  $p_1$  is presented in Fig. 3. It is observed that the approximation (13) provide values close to empirical results, especially when  $w$  is not too large.

## B. Average Delay

When a packet is transmitted several times in a frame, the delay,  $\mathbf{D}$ , is defined as the first timeslot in which the packet is successfully received. The average delay,  $D_s$ , is defined as

$$D_s \triangleq E(\mathbf{D}|\mathcal{S}) = \sum_{M=0}^{N-1} D_s(M)P(M=M) \quad (15)$$

where  $D_s(M)$  is the average delay of a successful transmission when there are  $M$  interfering users. Note that  $\mathbf{D}$  is not defined when all transmissions in a frame are unsuccessful.

1) *SPR*: The average delay for SPR, conditioned on successful transmission when there are  $M$  interfering users, can be obtained as

$$\begin{aligned} D_s(M) &\triangleq E_M[\mathbf{D}|\mathcal{S}] = \sum_{i=1}^L iP_M(\hat{\mathcal{S}}_i|\mathcal{S}) \\ &= \sum_{i=1}^L \frac{iP_M(\hat{\mathcal{S}}_i \cap \mathcal{S})}{P_M(\mathcal{S})} = \frac{\sum_{i=1}^L iP_M(\hat{\mathcal{S}}_i)}{P_M(\mathcal{S})}. \end{aligned} \quad (16)$$

Timeslot  $i$  is the first successful timeslot with probability

$$\begin{aligned} P_M(\hat{\mathcal{S}}_i) &= P_M(\mathcal{S}_i \cap \bar{\mathcal{S}}_{i-1} \cap \cdots \cap \bar{\mathcal{S}}_1) \\ &= P_M(\mathcal{S}_i)P_M(\bar{\mathcal{S}}_{i-1}) \cdots P_M(\bar{\mathcal{S}}_1) \\ &= s(1-s)^{i-1} \end{aligned} \quad (17)$$

where we have used (4). By substituting (17) in (16) we obtain

$$D_s^{(SPR)}(M) = \frac{1}{s} - \frac{L(1-s)^L}{1-(1-s)^L}. \quad (18)$$

Note that the right-hand-side of (18) implicitly depends on  $M$  through  $s$ .

2) *SFR and OOC*: In a repetition-based protocol with exactly  $w$  transmissions we have

$$\begin{aligned} D_s(M) &\triangleq E_M[\mathbf{D}|\mathcal{S}] = \sum_{i=1}^w P_M(\hat{\mathcal{S}}_i|\mathcal{S})E_M[\mathbf{D}|\hat{\mathcal{S}}_i] \\ &= \sum_{i=1}^w \frac{P_M(\hat{\mathcal{S}}_i)}{P_M(\mathcal{S})} \sum_{j=i}^w jP_M(\mathbf{D} = j|\hat{\mathcal{S}}_i) \end{aligned} \quad (19)$$

where  $P_M(\mathbf{D} = j|\hat{\mathcal{S}}_i)$  is the probability that the delay is equal to  $j$  when the  $i$ th transmission is the first successful transmission. In other words, this is the probability that the  $i$ th transmission takes place in the  $j$ th timeslot given that the  $i$ th transmission is the first successful transmission. To calculate (19) we need to find  $P_M(\hat{\mathcal{S}}_i)$  and  $P_M(\mathbf{D} = j|\hat{\mathcal{S}}_i)$ .

To obtain  $P_M(\hat{\mathcal{S}}_i)$ , we first find the probability that ‘‘transmission  $i$  is not successful but at least one previous transmission is successful’’,  $P_M(\bar{\mathcal{S}}_i \cap (\mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1))$ . We have

$$\begin{aligned} P_M(\bar{\mathcal{S}}_i \cap (\mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1)) &= P_M\left(\bigcup_{l=1}^{i-1} (\bar{\mathcal{S}}_i \cap \mathcal{S}_l)\right) \\ &= \sum_{k=1}^{i-1} (-1)^{k+1} \sum_{\substack{\{a_1, \dots, a_k\} \in \\ (i-1)_k}} P_M(\bar{\mathcal{S}}_i \cap \mathcal{S}_{a_1} \cap \cdots \cap \mathcal{S}_{a_k}). \end{aligned} \quad (20)$$

Let

$$\begin{aligned} \gamma_k &\triangleq P_M(\mathcal{S}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \cdots \cap \mathcal{S}_{a_1}) \\ \eta_k &\triangleq P_M(\bar{\mathcal{S}}_{a_k} \cap \mathcal{S}_{a_{k-1}} \cap \cdots \cap \mathcal{S}_{a_1}). \end{aligned} \quad (21)$$

Therefore,

$$\begin{aligned} P_M(\bar{\mathcal{S}}_i \cap (\mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1)) &= \sum_{k=1}^{i-1} (-1)^{k+1} \binom{i-1}{k} \eta_{k+1} \\ &= \sum_{k=1}^{i-1} (-1)^k \binom{i-1}{k} (\gamma_{k+1} - \gamma_k) \end{aligned} \quad (22)$$

As discussed earlier, because the probability of an interference pattern only depends on the number of timeslots in which two users transmit simultaneously and not the position of those timeslots,  $\gamma_k$  and  $\eta_k$  only depend on  $k$ .

Using (22) we can write

$$\begin{aligned} P_M(\hat{\mathcal{S}}_i) &= P_M(\mathcal{S}_i \cap \bar{\mathcal{S}}_{i-1} \cap \cdots \cap \bar{\mathcal{S}}_1) \\ &= 1 - P_M(\bar{\mathcal{S}}_i \cup \mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1) \\ &= 1 - P_M(\bar{\mathcal{S}}_i) - P_M(\mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1) \\ &\quad + P_M(\bar{\mathcal{S}}_i \cap (\mathcal{S}_{i-1} \cup \cdots \cup \mathcal{S}_1)) \\ &= \sum_{k=0}^{i-1} (-1)^k \binom{i-1}{k} \gamma_{k+1} \\ &= \sum_{k=1}^i (-1)^{k+1} \binom{i-1}{k-1} \gamma_k \end{aligned} \quad (23)$$

where in the fourth step we have used (8).

Note that (23) can be used to obtain  $P_M(\mathcal{S})$ . As seen below, the result is the same as in (8).

$$\begin{aligned} P_M(\mathcal{S}) &= \sum_{i=1}^w P_M(\hat{\mathcal{S}}_i) = \sum_{i=1}^w \sum_{k=1}^i (-1)^{k+1} \binom{i-1}{k-1} \gamma_k \\ &= \sum_{k=1}^w (-1)^{k+1} \gamma_k \sum_{i=k}^w \binom{i-1}{k-1} \\ &= \sum_{k=1}^w (-1)^{k+1} \binom{w}{k} \gamma_k \end{aligned} \quad (24)$$

Next, we obtain  $P_M(\mathbf{D} = j|\hat{\mathcal{S}}_i)$  for SFR and OOC. Let  $\mathbf{T}_i$  be the timeslot in which the  $i$ th transmission takes place.

$$P_M(\mathbf{D} = j|\hat{\mathcal{S}}_i) = P_M(\mathbf{T}_i = j|\hat{\mathcal{S}}_i) = P_M(\mathbf{T}_i = j) \quad (25)$$

The last equality holds because the position of the  $i$ th transmission is independent of it being the first successful transmission.

For SFR, since the position of transmissions in the frame is strictly random, we have

$$P_M(\mathbf{D} = j|\hat{\mathcal{S}}_i) = P_M(\mathbf{T}_i = j) = \frac{\binom{j-1}{i-1} \binom{L-j}{w-i}}{\binom{L}{w}}. \quad (26)$$

For OOC,  $P_M(\mathbf{T}_i = j)$  may depend on the code. Assuming ‘1’s are distributed evenly in each codeword, we can use

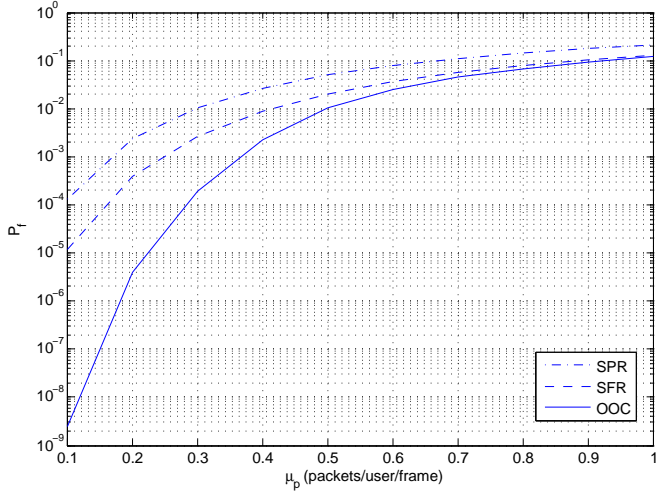


Fig. 4. Optimum probability of failure for OOC, SFR, and, SPR, versus load, for  $N = 31$ ,  $L = 128$ .

an expression identical to that of SFR as an approximation. Finally, substituting (23) and (26) in (19), for OOC and SFR, gives  $D_s(M)$ . On average, messages wait  $L/2$  timeslots in a buffer from their arrival at the network interface until the beginning of the next frame. If delay is defined from the moment that a packet arrives at the network interface,  $L/2$  must be added to  $D_s(M)$ .

### C. Numerical Results

In this section, we present numerical results for Sections IV-A and IV-B. Let  $L$  and  $N$  be 128 and 31 respectively. The value of  $p_1$  for OOC is calculated from (13). We assume each vehicle independently makes a local decision, whether or not to transmit its location to neighbor vehicles. Furthermore, we assume these periodical updates are generated according to a Bernoulli model in each frame with probability  $\mu_p$ . Since the decisions for data transmission are independent, the number of nodes with an active packet in each frame is a Binomial random variable with parameters  $N$  and  $\mu_p$ , where  $N$  is the total number of cars in a (loosely defined) cluster.

The optimum probabilities of failure, for SPR, SFR, and OOC are plotted in Fig. 4. For OOC,  $w$  ranges from 2 to 12. OOC for  $w = 1$  is a trivial case and the code cardinality for  $w > 12$  is not big enough to accommodate 31 users. It is observed that, for probability of user activity below 0.4, OOC can offer a performance advantage of multiple orders of magnitude.

Fig. 5 shows the average delay of successful transmissions calculated using (15). It is observed that the delay is more or less the same for different protocols. This fact will also be observed in simulation results.

## V. SIMULATION RESULTS

In Section IV, we discussed the theoretical performance of our proposed repetition-based broadcast protocol as well as similar protocols. As mentioned earlier, for obtaining analytical results, we have assumed nodes communicate in an ideal

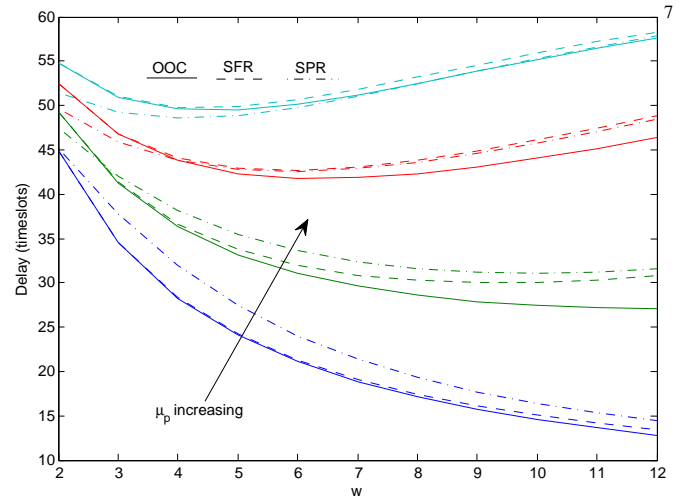


Fig. 5. Delay of successful transmissions for OOC, SFR, and, SPR, for  $N = 31$ ,  $L = 128$ ,  $w$  ranging from 2 to 12 and  $\mu_p = 0.1, 0.4, 0.7$ , and 1 (packets/user/frame).

channel in which every node receives a signal from every other transmitting node. Furthermore, the capture effect and adaptive elimination are neglected in the analytical study.

In an ideal channel, all simultaneous transmissions result in collision. In a non-ideal channel with capture, however, one of the many simultaneous transmissions may be successful. Note that since we are studying a multiple access system, we neglect the effect of noise in the system. Therefore, collision is the only contributor to packet loss.

In this section, assuming a Rician channel with capture effect, we present simulation results for different protocols. We also consider the effect of adaptive elimination in the performance of the protocol in the simulation.

### A. Channel Model

In a Rician fading channel with Rice factor  $K$ , the pdf of the received power,  $P$ , is

$$f_P(P) = \frac{2K}{A^2} \exp\left(-K\left(1 + \frac{2P}{A^2}\right)\right) I_0\left(\sqrt{\frac{8K^2P}{A^2}}\right) \quad (27)$$

where  $A$  is the amplitude of the line-of-sight component, which is inversely proportional to the  $n$ th power of distance from transmitter where  $n$  is a constant called the path loss exponent.

In timeslot  $m$ , the desired receiver, denoted by  $u_0$ , receives the power  $P_i^{(m)}$  from user  $u_i$ . In an interference limited network, the desired transmitter,  $u_j$ , is successful in sending its packet to  $u_0$  in the  $m$ th timeslot if

$$\begin{cases} u_j \in \mathbf{T}^{(m)}, u_0 \notin \mathbf{T}^{(m)} \\ P_j^{(m)} > \frac{1}{\beta} \sum_{i: u_i \in \mathbf{T}^{(m)} \setminus \{u_j\}} P_i^{(m)} \end{cases} \quad (28)$$

where  $\mathbf{T}^{(m)}$  is the set of transmitting users in the  $m$ th timeslot and  $\beta$  is the capture ratio. A message transmitted by  $u_j$  is successfully delivered if (28) is satisfied at least for one timeslot in that frame.

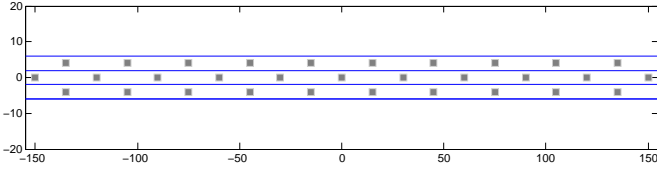


Fig. 6. Map of roadway and cars.

## B. Protocol Performance

1) *Simulation Setup*: In the simulation setup, cars are placed on a three-lane road with 4m lane separation and the distance between two adjacent cars in the same lane is 30m, as illustrated in Fig. 6. The received power by a vehicle from any other vehicle is randomly driven according to the Rician distribution with  $K = 3$  and  $n = 2$ . The capture ratio,  $\beta$  is 0.2 unless otherwise stated. The number of cars,  $N$ , is 31 which occupy 300m of road. Frame length,  $L$ , is 64. Data rate is 5Mbps and safety message size, after adding the overheads of different layers, is 200 bytes. When 200B is transmitted in each timeslot, the length of each timeslot is  $320\mu\text{s}$  and each frame is 20.48ms. In an actual implementation, timeslots must be longer to compensate for non-ideal synchronization. The traffic model is binomial as shown in (??); in each frame, a message arrives at each node with probability  $\mu_p$ .

## C. Probability of Success

An important metric in the simulation results in this work is the probability that more than 90% of the nodes successfully receive the transmitted message, denoted by  $P_s^{(0.9)}$ . Fig. 7 shows  $P_f^{(0.1)} = 1 - P_s^{(0.9)}$ , i.e., the probability that more than 10% of the nodes in the network fail to receive a transmitted message successfully, for  $w$ 's from 2 to 8. Quadratic curves are fitted to the simulation results. Average load,  $\mu_p$ , is 0.2 (messages/user/frame); on average each car produces a 200B message every  $20.48\text{ms}/0.2=102.4\text{ms}$ . This figure indicates that by choosing a good value for  $w$ , all protocols are capable of delivering messages reliably while OOC performs better than the other protocols.

Fig. 8 shows the average delay versus  $w$ . As mentioned earlier, the delay is defined as the first timeslot, in which a packet is received successfully by a certain user. The average delay of all protocols is more or less the same, as previously seen in Section IV-C. For all protocols, the average delay is less than 24 timeslots or approximately 8ms. One may also consider the time that a message is buffered until the beginning of the frame by adding  $20.48/2=10.24\text{ms}$  to the above values.

In the simulation results we have considered Rician channel with capture and adaptive elimination while the analytical results correspond to a case in which the wireless channel is perfect and capture and adaptive elimination are disabled. If capture and adaptive elimination are disabled, the analytical results and the simulation results agree. This is shown in Fig. 9, for the probability of success while  $w$  changes and in Fig. 10, for the delay while the average load changes. Irregularities in the curves for OOC can be observed more often because

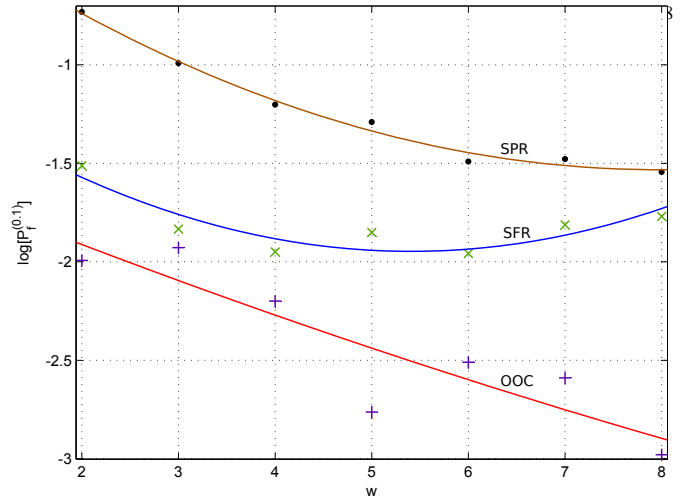


Fig. 7. Probability of failure versus  $w$ , for  $\mu_p = 0.2$ .

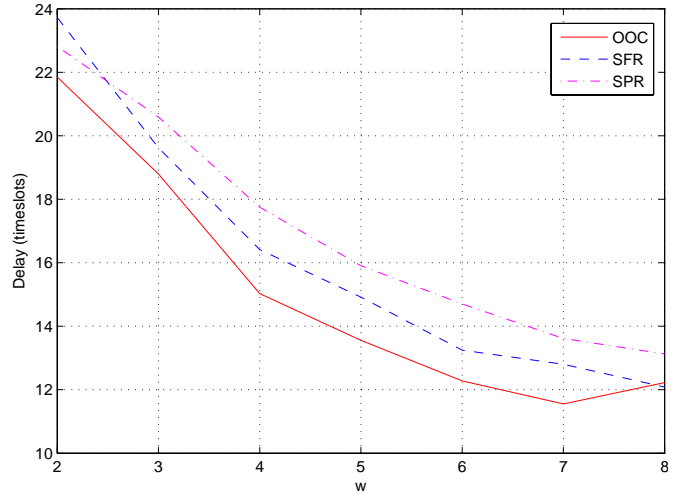


Fig. 8. Average delay versus  $w$ , for  $\mu_p = 0.2$ .

OOC has less intrinsic randomness compared to the other two methods.

## VI. CONCLUSION

In most parts of the simulations, messages with length 200B are issued from each vehicle approximately 5 times per second. As explained in Section I, message frequency of approximately 5 messages with length 100B (per second per user) is sufficient for communicating position and other useful information. After adding different overheads, the message length will not exceed 200B assumed in the simulations. With the described load characteristics, we have shown that OOC-based broadcast can reliably deliver safety messages with low delay. Furthermore, OOC-based broadcast performs noticeably better than random repetition broadcast protocols. We conclude that OOC-based repetition broadcast provides good performance in vehicular environments.



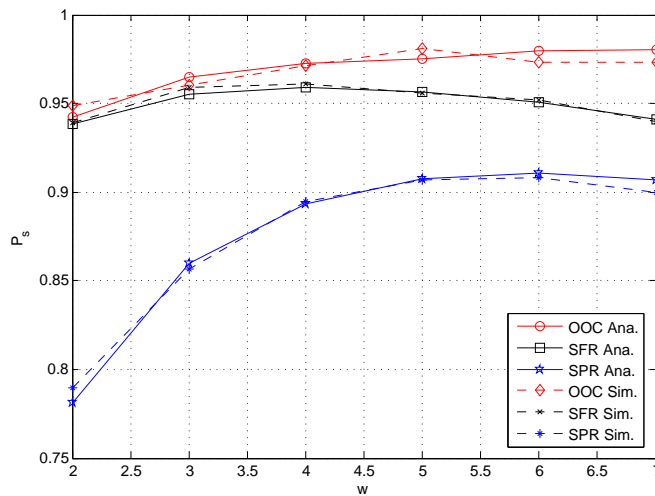


Fig. 9. Comparison of analytical and simulation results:  $P_s$  vs  $w$ , for  $\mu_p = 0.3$ .

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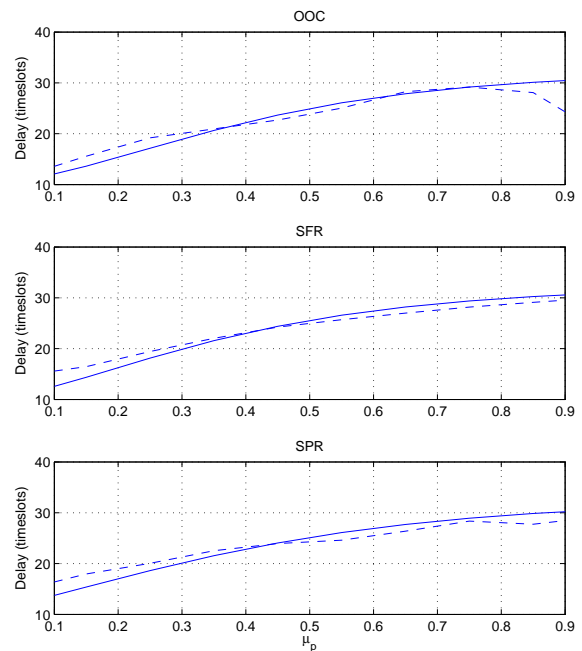


Fig. 10. Comparison of analytical and simulation results: Delay vs Average load, for  $w = 6$  and  $\beta = 0$ .

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