An Adaptive Network Coded Retransmission Scheme for Single-Hop Wireless Multicast Broadcast Services

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***Abstract*—Network coding has recently attracted attention as a substantial improvement to packet retransmission schemes in wireless multicast broadcast services (MBS). Since the problem of finding the optimal network code maximizing the bandwidth efficiency is hard to solve and hard to approximate, two main network coding heuristic schemes, namely opportunistic and full network coding, were suggested in the literature to improve the MBS bandwidth efficiency. However, each of these two schemes usually outperforms the other in different receiver, demand, and feedback settings. The continuous and rapid change of these settings in wireless networks limits the bandwidth efficiency gains if only one scheme is always employed. In this paper, we propose an adaptive scheme that maintains the highest bandwidth efficiency obtainable by both opportunistic and full network coding schemes in wireless MBS. The proposed scheme adaptively selects, between these two schemes, the one that is expected to achieve the better bandwidth efficiency performance. The core contribution in this adaptive selection scheme lies in our derivation of performance metrics for opportunistic network coding, using random graph theory, which achieves efficient selection when compared to appropriate full network coding parameters. To compare between different complexity levels, we present three approaches to compute the performance metric for opportunistic coding using different levels of knowledge about the opportunistic coding graph. For the three considered approaches, simulation results show that our proposed scheme almost achieves the bandwidth efficiency performance that could be obtained by the optimal selection between the opportunistic and full coding schemes.**

***Index Terms*—Chromatic number of random graphs, graph coloring, multicast broadcast services (MBS), opportunistic and full network coding, packet retransmission.**

I. Introduction

MULTICAST broadcast services (MBS) have become essential applications that are greatly considered in the design of all future wireless networks due to the increasing demand on applications that are requested by subsets or all the receivers located in the coverage area of a wireless access node. Examples of such applications are online TV, downloads of new applications, news feeds, and location-based applications such as location-based advertisement and queries for location-based services. In multicast, the receivers are interested in receiving only a subset of the packets transmitted by the access node. Broadcast can then be regarded as a special case of multicast where all receivers are interested in receiving all packets. Due to the high demand on MBS applications and their high bandwidth requirements, it is very important to develop new techniques that can improve the system bandwidth efficiency in order to satisfy these demands with a certain level of quality of service. This motivated several studies to explore more efficient utilization of the scarce wireless bandwidth in wireless MBS.

To achieve a reliable multicast/broadcast, all the receivers must correctly detect all the information packets they requested from the access node. Since wireless communication channels are lossy in general, the guarantee of packet delivery is achieved through packet retransmission using automatic repeat request (ARQ) or its combinations with forward error correction (FEC), known as the hybrid automatic repeat request (HARQ). However, both schemes retransmit lost packets separately, which considerably reduces the number of receivers benefiting from each retransmission. This results in more retransmissions and thus a low system bandwidth efficiency.

Motivated by the significant bandwidth efficiency improvements achieved by network coding [7], several works aimed to exploit it for packet retransmission, as a substitute to ARQ/HARQ in wireless networks. However, it has been proven that achieving both the optimal and k-approximation capacities of network coding is NP-hard [2], [3]. Consequently, several heuristics have been developed to apply network coding in packet retransmission. In [4] and [5], Nguyen *et al.* and Tran *et al.* proposed a packet retransmission scheme that opportunistically combines lost packets of different receivers such that some of them recover one of their missing packets upon correct reception of this combined packet. We refer to this scheme as the *opportunistic network coded retransmission (ONCR) scheme*. Nguyen *et al.* also proposed a *full network coded retransmission (FNCR) scheme* in [6] to improve wireless multimedia broadcast.

It has been shown that both ONCR and FNCR schemes achieve a considerable gain in bandwidth efficiency compared to ARQ. Each of these two schemes usually outperforms the other in different receiver, demand, and feedback settings [1]. The continuous and rapid change of these settings in wireless networks limits the bandwidth efficiency gains if only one scheme is always employed. This fact raises an interesting question. *How can we adaptively select the scheme that is expected to achieve the higher bandwidth efficiency?*

In this paper, *we investigate the answer to this question and propose an efficient and adaptive solution to improve the system*

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bandwidth efficiency in MBS using a combination of opportunistic and full network coded retransmissions. The proposed scheme adaptively selects, between these two schemes, the one that is expected to achieve the better bandwidth efficiency performance. The core contribution in this adaptive selection scheme is our derivation of an ONCR performance metric that achieves efficient selection when compared to an appropriate full network coding parameter. This metric is derived by modeling the ONCR graph representation as a random graph and computing its chromatic number using a famous result from random graph theory [8]. The scheme selection is then done by comparing the computed ONCR performance metric to a corresponding FNCR parameter.

To determine the complexity level required to achieve efficient selection, we present three approaches to compute the ONCR performance metric. The first two approaches employ the known ONCR lossless graph representation [2], [9] and differ from each other by the amount of information considered while randomizing the graph. In the third approach, we first develop an extension of the ONCR graph representation by considering retransmission losses and compute the lossy ONCR metric from its corresponding random graph model. For the three considered approaches, simulation results show that our proposed scheme can almost achieve the bandwidth efficiency performance that could be obtained by the optimal selection between the opportunistic and full schemes.

The rest of this paper is organized as follows. In Section II, we summarize some related works to our problem. The single-hop wireless MBS system model and its parameters are illustrated in Section III. In Section IV, we briefly illustrate the ONCR and FNCR schemes in the general MBS case. We then present this paper's main contribution in Section V by introducing the theoretical foundation and the detailed description of our proposed adaptive scheme. Section VI illustrates the simulation results that justify the merits of our proposed solutions. Section VII concludes the paper.

II. Related Work

A. Network Coding

Since its first introduction in [7], network coding has been a great attraction to numerous studies as a routing and scheduling scheme that attains maximum information flow in a network. The core of network coding is the idea of packet mixing using several techniques such as packet XOR [10] and linear coding [11]. Two trends of network coding can be distinguished in the literature, namely opportunistic [10], [12] and full [13], [14] network coding. Both trends have been proposed for a wide range of applications.

B. Index Coding

The index coding problem was studied in several works [2], [9], [15], [16] motivated by several applications in wireless networking and distributed computing. It includes a sender, a set of receivers, a set of packets, and *lossless channels between the sender and these receivers*. The objective of the index coding problem is to define the packet coding schedule that delivers the requested subsets of packets by each of the receivers with the minimum number of transmissions.

In [2] and [15], it has been shown that finding the optimal solution of the index coding problem is NP-hard. Consequently, different heuristics to solve the index coding problem were proposed in [9]. Most of these heuristics are different simplifications of a suboptimal graph-coloring solution of the index coding problem. To the best of our knowledge, there are no studies showing whether this graph-coloring approach is always the best suboptimal solution to index coding.

C. Network Coded Retransmission

In [4] and [5], the diversity of received and lost packets at different receivers is exploited by using the ONCR scheme instead of ARQ/HARQ, respectively. In [6], a hybrid ARQ-FNCR scheme was proposed for wireless multimedia broadcast.

In [17] and [18], the concept of network coded retransmissions was also studied in the contexts of minimizing the average packet detection delay and the average sender queue size, respectively, in wireless broadcast. In [19], we proved that the FNCR scheme achieves the optimum packet loss rate in a unicast setting given some constraints and an upper bound for the number of retransmissions. All these contexts are different from this paper as we consider maximizing the system bandwidth efficiency in MBS and rateless settings.

III. System Model and Parameters

Our model consists of a wireless access node, such as a base station in a 4G or WiMAX cell, responsible for delivering multicast or broadcast packets to a set $\mathcal{R} = \{R_1, \ldots, R_M\}$ of M receivers. The access node initially transmits a MBS frame consisting of a set $\mathcal{P} = \{P_1, \ldots, P_N\}$ of N packets in an *initial transmission phase*. During this phase, each receiver listens to the packets it requested as well as the other packets requested by other receivers, and all correctly received packets are stored in its memory. For each lost packet, each receiver sends a NAK packet to the access node. The access node keeps a table of received and lost packets by all receivers that we will refer to as the feedback table. At the end of the initial transmission phase, three sets of packets can be associated with each receiver R_i.

- The *Has* set (denoted by $\mathcal{H}_i$) is defined as the set of packets correctly received by R_i. This set includes both desired and undesired packets by this receiver.
- The *Complementary* set (denoted by $\mathcal{C}_i$) is defined as the set of packets that were not correctly received by R_i whether requested or not by this receiver. In other words, $\mathcal{C}_i = \mathcal{P} \setminus \mathcal{H}_i$.
- The *Wants* set (denoted by $\mathcal{W}_i$) is defined as the set of packets that are both requested and lost by R_i in the initial transmission phase of the current MBS frame.

At the end of the initial transmission phase, a packet retransmission scheme is employed to deliver the lost packets to the receivers that requested them. Afterwards, the whole procedure is reexecuted for a new MBS frame.

We define the demand ratio μ_i of receiver R_i as the ratio of the number of packets requested by this receiver in each MBS frame to the MBS frame size N. According to this definition, we can infer that broadcast can be regarded as a special case of multicast when the demand ratios of all receivers are equal to 1. We also assume that each packet is subject to loss by receiver R_i with probability p_i during the initial transmission and retransmission phases. Let μ be the average demand ratio of all receivers expressed as $\mu = (1/M)\sum_{i=1}^{M}\mu_i$. Finally, we assume that all packets have a fixed length in both initial transmission and retransmission phases. Consequently, we define the bandwidth efficiency as the ratio of the MBS frame size to the total number of initial transmissions and retransmissions until all receivers obtain their requested packets.

IV. ONCR and FNCR Schemes

A. ONCR Scheme

The ONCR scheme exploits the diversity of received and lost packets at different receivers in opportunistically combining them for retransmission using network coding. Each packet combination is performed so as to maximize the number of receivers that directly recover one of their requested and lost packets upon correct reception of this coded packet.

Assuming lossless retransmissions, it is clear from [9] that obtaining the opportunistic packet coding sequence to minimize the number of retransmissions is equivalent to solving the corresponding index coding problem. Since solving index coding problems is NP-hard, the graph-coloring approximation, proposed in [9], can be used to efficiently implement the ONCR scheme in case of lossless retransmissions. The graph-coloring implementation of the ONCR scheme starts by generating a graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, in which each packet $j \in \mathcal{W}_i \forall i$ induces a vertex v_{ij} in the graph. Two vertices v_{ij} and v_{kl} in $\mathcal{G}$ are connected if one of the following is true:

- $j = l$ (i.e., vertices represent the same lost packet from two receivers i and k);
- $j \in \mathcal{H}_k$ and $l \in \mathcal{H}_i$ (i.e., the requested packet of each vertex is in the Has set of the receiver that induced the other vertex).

After the construction of the graph, clique partitioning is performed on it. For each clique $\mathcal{K}_n$, a coded packet XORing all the packets $\{j | v_{ij} \in \mathcal{K}_n\}$ is generated and transmitted. Since clique partitioning of a graph is equivalent to the coloring of its complementary graph, the minimum achievable number of retransmissions (T_{O}) using this technique is equal to

$$T_{\mathrm{O}} = \chi(\mathcal{G}^{\mathrm{c}}) \tag{1}$$

where $\chi(\mathcal{G}^{\mathrm{c}})$ is the chromatic number of graph $\mathcal{G}^{\mathrm{c}}(\mathcal{V},\mathcal{E}^{\mathrm{c}})$, such that $\mathcal{E}^{\mathrm{c}} = \mathcal{V} \times \mathcal{V} \setminus \mathcal{E}$.

For the more realistic case of lossy retransmissions, a dynamic retransmission algorithm can be developed using the above graph-based approach as follows. After the initial transmission phase, the access node constructs graph $\mathcal{G}$ as described, finds a maximal clique in it, and broadcasts an XOR of all the packets represented in its vertices. Each receiver sends a NAK packet to the access node if it lost this retransmission packet. These resulting NAK packets are used by the access node to update the feedback table, which is then used to construct a new graph, and the aforementioned process is reexecuted. This process continues until each receiver correctly receives its requested packets. For this described algorithm, it is difficult to derive an expression for the number of ONCR retransmissions $(\widetilde{T}_{\mathrm{O}})$. However, it is clear that the larger T_{O}, the larger $\widetilde{T}_{\mathrm{O}}$.

B. FNCR Scheme

Full network coding has been proposed in the literature for different wireless applications [13], [14]. In [6], the FNCR scheme has been proposed for packet retransmission to improve wireless multimedia broadcast.

In general, the FNCR scheme combines all the MBS frame packets in each retransmission using linear network coding. Coding coefficients can be either deterministic or selected from a large field such that a large number of coded packets are guaranteed to be linearly independent almost surely. The retransmission procedure continues until all receivers get enough packets to decode all packets of the MBS frame. One drawback of the FNCR scheme for wireless multicast is that it necessitates the delivery of all packets of the MBS frame to all receivers regardless of their needs.

Note that, assuming lossless retransmissions, the number of retransmission packets needed by receiver R_i to correctly decode all the packets is equal to the cardinality of its complimentary set $|\mathcal{C}_i|$. Consequently, the number of lossless retransmissions (T_{F}) is equal to

$$T_{\mathrm{F}} = \max_{i \in \mathcal{R}} |\mathcal{C}_i|. \tag{2}$$

In case of lossy retransmissions, the number of FNCR retransmissions $(\widetilde{T}_{\mathrm{F}})$ is equal to the maximum of M negative binomial random variables $\mathrm{NegBin}(|\mathcal{C}_i|, 1-p_i)$ [20]. It is clear that the larger T_{F}, the larger $\widetilde{T}_{\mathrm{F}}$.

V. Adaptive Network Coded Retransmission (ANCR) Scheme

In this section, we aim to design an efficient and adaptive scheme that can adaptively select the network coded retransmission scheme for each wireless MBS frame. We will refer to this scheme as the ANCR scheme. The ANCR scheme should select the retransmission scheme that is expected to achieve the smaller number of retransmissions according to the system, demand, and feedback parameters. For the broadcast case, it has been proven that the FNCR scheme is optimal [21]. Therefore, our focus will be on the multicast case.

For each MBS frame, the ANCR scheme selects one of the two schemes by comparing metrics representing the number of retransmissions for each of them. The scheme having the lower metric is selected to be executed for this MBS frame. The metrics used for selection will be the focus of the rest of this section. We will first present the theoretical foundation of our proposed selection method and then introduce three approaches to compute the selection metrics.

A. Theoretical Foundation

In order to determine the better scheme for packet retransmission in each frame, we should compute an *a priori* estimate of the number of retransmissions for each scheme. Since it is very difficult to find analytical expressions for the exact number of retransmissions of both ONCR and FNCR schemes in case of lossy retransmissions ($\widetilde{T}_{\mathrm{O}}$ and $\widetilde{T}_{\mathrm{F}}$, respectively), we propose two methods to estimate their performances.

- **Method 1**: We can estimate their performance through their number of lossless retransmissions $T_{\mathrm{O}} = \chi(\mathcal{G}^{\mathrm{c}})$ and

$T_{\mathrm{F}} = \max_{i\in\mathcal{R}} |\mathcal{C}_i|$. Since finding the chromatic number of a graph is NP-hard, we will need to find an approximation for $\chi(\mathcal{G}^{\mathrm{c}})$.

- **Method 2**: We can extend the lossless ONCR graph representation by including loss pattern information in it, thus generating a lossy ONCR graph model $\widetilde{\mathcal{G}}^{\mathrm{c}}$. We then can estimate the chromatic number of this new graph and compare it to a lossy approximation of the FNCR scheme performance.

In both methods, we need to estimate the chromatic number of a graph. To do so, we propose the modeling of $\mathcal{G}^{\mathrm{c}}$ or $\widetilde{\mathcal{G}}^{\mathrm{c}}$ as a random graph $\mathcal{G}_{\nu,\pi}$ having the same vertex set size (that we will denote by ν) as $\mathcal{G}^{\mathrm{c}}$ or $\widetilde{\mathcal{G}}^{\mathrm{c}}$ and a vertex connectivity probability π. If we can find this model, then we can apply the result in the following lemma, proved in [8], to approximate the chromatic number of $\mathcal{G}^{\mathrm{c}}$ or $\widetilde{\mathcal{G}}^{\mathrm{c}}$.

Lemma 1: Almost every random graph $\mathcal{G}_{\nu,\pi}$, with ν vertices and a fixed probability π $(0 < \pi < 1)$ that any two vertices are connected, has a chromatic number that can be expressed as

$$\chi(\mathcal{G}_{\nu,\pi}) = \left(\frac{1}{2} + o(1)\right) \log\left(\frac{1}{1-\pi}\right) \frac{\nu}{\log \nu}. \quad (3)$$

Several approaches can be used to model the vertex connectivity of $\mathcal{G}^{\mathrm{c}}$ by a fixed probability π using the connectivity conditions in $\mathcal{G}^{\mathrm{c}}$. Also, according to the design of the lossy graph model, an expression for π could be computed. To compare between the two methods and decide the complexity level needed to obtain an efficient algorithm, we first propose two approaches to derive π in $\mathcal{G}^{\mathrm{c}}$, then propose a design for $\widetilde{\mathcal{G}}^{\mathrm{c}}$ and compute its π accordingly.

B. Approach I

In this approach, we propose to ignore both the vertices' identities (their ij indices) and the content of the Has, Complementary, and Wants sets of all receivers. This approach considers only the graph vertex set size (ν), the system parameters (M, N), and the cardinalities of the different sets that can be extracted from the feedback table. We consider this approach as the number of retransmissions mostly depends on the cardinalities of the sets and not their content.

Define X_i, Y_i, and $\overline{Y}_i$ as the cardinalities of $\mathcal{W}_i$, $\mathcal{H}_i$, and $\mathcal{C}_i$, respectively. Also, let Z_j be the number of receivers that requested and lost packet P_j in the initial transmission phase. Thus, there will exist Z_j vertices in $\mathcal{G}^{\mathrm{c}}$ induced by packet P_j. Let $\mathbf{x} = [X_1, \ldots, X_M]$, $\mathbf{y} = [Y_1, \ldots, Y_M]$, $\overline{\mathbf{y}} = [\overline{Y}_1, \ldots, \overline{Y}_M]$, and $\mathbf{z} = [Z_1, \ldots, Z_N]$. Finally, let $\mathfrak{D}$ be the status descriptor of each MBS frame after the initial transmission phase, such that $\mathfrak{D} = \{\mathbf{x}, \mathbf{y}, \overline{\mathbf{y}}, \mathbf{z}, \nu\}$. Given this status descriptor, we derive an expression for π in the following theorem.

Theorem 1: Given $\mathfrak{D}$, the probability π of having any two vertices v and w connected in $\mathcal{G}^{\mathrm{c}}$ can be expressed as

$$\pi = \frac{\mathbf{x}\overline{\mathbf{y}}^T}{N\nu}\left(2 - \frac{\mathbf{x}\overline{\mathbf{y}}^T}{N\nu}\right)\left(1 - \frac{\mathbf{z}(\mathbf{z}-\mathbf{1})^T}{\nu(\nu-1)}\right) \quad (4)$$

where $\mathbf{1}$ is the all ones row vector of appropriate dimensions.

Proof: The proof is in Appendix A. ■

Having the probability π computed, the ANCR algorithm can be easily implemented as follows. For each frame, the access node computes the status descriptor parameters from its feedback table, then computes π from (4). The ONCR performance metric is then computed from (3) and compared to T_{F}. Finally, the access node employs the retransmission scheme having the smaller metric. The value of the $o(1)$ term employed in (3) will be empirically determined in Section VI-A in order to achieve the best ANCR performance.

C. Approach II

One drawback of the previous approach is the need to compute π for each MBS frame since it depends on the cardinalities of the feedback table sets. In this section, we consider a simpler approach that ignores these cardinalities in addition to the vertices' identities. Consequently, this approach considers only the vertex set size ν, the system parameters (M, N), and the parameters of the packet request–loss random process (μ_i, p_i). Although this simplification in the model should result in a lower performance, simulation results in Section VI show that this degradation is very small. For simplicity of the analysis in this section, we will assume that all receivers have an equal average packet loss probability p and a demand ratio equal to the average demand ratio μ.

Given that we ignore all the feedback table information, including the set cardinalities, and only consider ν, $\mathbf{z}$ becomes a random vector where $Z_N = \nu - \sum_{v=1}^{N-1} Z_v$. Any realization $\mathbf{z}' = \{z_1, \ldots, z_N\}$ of $\mathbf{z}$ is just a random subset of ν vertices from MN candidate vertices. This random subset results from the packet request–loss random process in the initial transmission phase. Consequently, we can introduce the following lemma.

Lemma 2: Given ν, $\mathbf{z}$ is a multivariate hypergeometric distributed random vector. In other words

$$\mathbb{P}(\mathbf{z} = \mathbf{z}'|\nu) = \prod_{u=1}^{N-1} \binom{M}{z_u} \cdot \binom{M}{\nu - \sum_{v=1}^{N-1} z_v} \cdot \binom{MN}{\nu}^{-1} \quad (5)$$

where $z_u \in \{0, 1, \ldots, M\}$ for all $u \in \{1, \ldots, N-1\}$ and

$$\binom{n}{k} = \begin{cases} \frac{n!}{k!(n-k)!}, & 0 \le k \le n \\ 0, & \text{otherwise} \end{cases}. \quad (6)$$

Proof: The proof is in Appendix B. ■

Based on the above lemma, we can introduce the following theorem.

Theorem 2: Given ν, the probability π of having two vertices connected in $\mathcal{G}^{\mathrm{c}}$ can be expressed as

$$\pi = p(2-p)\frac{M(N-1)}{MN-1}. \quad (7)$$

Proof: The proof is in Appendix C. ■

We can see that the obtained expression of π, using this approach, depends only on M, N, and p. These parameters generally vary with much smaller rate compared to the MBS frame rate. This results in a lower computational rate of π compared to that achieved by Approach I.

Note that we assumed equal packet loss probabilities and demand ratios to be able to exploit the result in [8]. However, these assumptions are not true in practice. Consequently, when this approach is employed, we will approximate the fixed packet loss probability using the average of the receivers' packet loss probabilities. Although the above approximation should affect the accuracy of the selection, we will show in the simulation section that the algorithm performance is still satisfactory. Having π computed and ν obtained from feedback, the ANCR algorithm computes the ONCR and FNCR metrics from (3) and (2) and selects the scheme having the smaller metric. The best $o(1)$ term value for this approach will be empirically determined in Section VI-A.

D. Approach III

In the previous two approaches, we ignored retransmission packet losses that might occur at different receivers when estimating the ONCR and FNCR performances. In this section, we aim to consider these loss possibilities in the estimation model to test whether this achieves a better performance than the previous two approaches. Since we do not know the loss realization that will occur during the retransmission phase at the selection time, we will assume that an average number of loss events will occur at each of the receivers. Thus, we need to find approximations for the expectations of both $\widetilde{T}_{\rm O}$ and $\widetilde{T}_{\rm F}$. In this section, we say that a retransmitted packet *addresses* a receiver if the corresponding clique includes a vertex belonging to this receiver.

Since we assume average losses at each of the receivers, we will set the performance metric of the FNCR scheme to the maximum of the expected completion times of all receivers (i.e., $\max_{i\in\mathcal{R}}\{\overline{Y}_i/(1-p_i)\}$).

On the other hand, it is difficult to compute $\mathbb{E}(\widetilde{T}_{\rm O})$ for the lossy ONCR algorithm described in Section IV-A using the lossless ONCR graph representation and lossy channels. However, we can model the average packet loss events by incorporating them inside the ONCR graph representation and maintaining the lossless channel assumption. We know from the packet loss probability p_i of receiver R_i that it requires on average $a_i = 1/(1-p_i)$ transmissions to get one packet. Consequently, receiver R_i is expected to detect all its requested and lost packets after a_iX_i retransmissions addressing this receiver. Thus, we will design our lossy graph model $\widetilde{\mathcal{G}}$ (corresponding to $\mathcal{G}$ in Section IV-A) such that it has on average a_iX_i vertices induced by R_i. These vertices should not be connected in the graph so that they represent different retransmissions.

Based on the above description, we can build graph $\widetilde{\mathcal{G}}(\widetilde{\mathcal{V}},\widetilde{\mathcal{E}})$ by replacing each vertex v_{ij} in $\mathcal{G}$ with $\widetilde{X}_{ij}$ vertices with the same identity ij, where

$$\mathbb{P}\left(\widetilde{X}_{ij}=\lfloor a_i\rfloor\right)=\lceil a_i\rceil-a_i \tag{8}$$

$$\mathbb{P}\left(\widetilde{X}_{ij}=\lceil a_i\rceil\right)=a_i-\lfloor a_i\rfloor \tag{9}$$

where $\lfloor a_i\rfloor$ and $\lceil a_i\rceil$ are the floor and ceiling values of a_i, respectively. It is clear that $\mathbb{E}(\widetilde{X}_{ij})=a_i$. Consequently, the expected number of vertices induced by R_i in $\widetilde{\mathcal{G}}$ is equal to a_iX_i. The connectivity conditions in $\widetilde{\mathcal{G}}$ are similar to that of $\mathcal{G}$ except that vertices of the same identity must not be connected. Thus, we modify the connectivity conditions such that two vertices v_{ij} and v_{kl} are connected in $\widetilde{\mathcal{G}}$ if only one of the two following conditions holds:

- $\widetilde{\rm C}1: j=l$ AND $i\neq k$;
- $\widetilde{\rm C}2: j\in\mathcal{H}_k$ AND $l\in\mathcal{H}_i$.

Having the graph constructed, we can obtain the complimentary graph $\widetilde{\mathcal{G}}^{\rm c}(\widetilde{\mathcal{V}},\widetilde{\mathcal{E}}^{\rm c})$ (where $\widetilde{\mathcal{E}}^{\rm c}=\widetilde{\mathcal{V}}\times\widetilde{\mathcal{V}}\setminus\widetilde{\mathcal{E}}$), randomize it as we did in the two previous approaches, and employ the chromatic number of the corresponding random graph as an approximation of $\mathbb{E}(\widetilde{T}_{\rm O})$.

To randomize $\widetilde{\mathcal{G}}^{\rm c}$, we will ignore the vertices' identities and the content of the Has and Complimentary sets only. In other words, the content of the Wants sets will be considered in our computations. Defining $\widetilde{\nu}$ as the number of vertices in $\widetilde{\mathcal{G}}^{\rm c}$, the status descriptor $\widetilde{\mathfrak{D}}=\{\widetilde{\mathbf{X}}=[\widetilde{X}_{ij}],\mathbf{y},\widetilde{\nu}\}$, where

$$\widetilde{X}_{ij}=\begin{cases}\lfloor a_i\rfloor \text{ or } \lceil a_i\rceil, & \forall\, i,j\in\mathcal{W}_i\\ 0, & \forall\, i,j\notin\mathcal{W}_i\end{cases}. \tag{10}$$

Given this status descriptor, we present an expression for π in the following theorem.

Theorem 3: Given $\widetilde{\mathfrak{D}}$, the probability π of having any two vertices v and w connected in $\widetilde{\mathcal{G}}^{\rm c}$ can be expressed as

$$\pi=1-\left(\frac{\mathbf{Tr}\left[(\widetilde{\mathbf{X}}^T\boldsymbol{\Theta})\widetilde{\mathbf{X}}\right]}{\widetilde{\nu}(\widetilde{\nu}-1)}+\left(\frac{(\mathbf{1}\widetilde{\mathbf{X}}^T)\mathbf{y}^T}{N\widetilde{\nu}}\right)^2\right) \tag{11}$$

where $\mathbf{Tr}[\mathbf{A}]$ is the trace of matrix $\mathbf{A}$ and $\boldsymbol{\Theta}=[\theta_{ij}]$ is defined as

$$\theta_{ij}=\begin{cases}0, & i=j\\ 1, & i\neq j\end{cases}. \tag{12}$$

Proof: The proof is in Appendix D. ■

Using the derived expression of π, the ANCR computes the ONCR metric from (3) and compares it to the FNCR metric $\max_{i\in\mathcal{R}}\{\overline{Y}_i/(1-p_i)\}$ to select the scheme with the smaller metric. The best $o(1)$ term value for this approach will be empirically determined in Section VI-A. Note that this approach involves more information and requires more complexity to construct the lossy ONCR graph model $\widetilde{\mathcal{G}}^{\rm c}$.

VI. Simulation Results

In this section, we test the performance of our proposed ANCR algorithm in wireless MBS through simulations. The simulation scenario consists of an access node that transmits MBS frames of size $N=30$ packets to M receivers. The packet loss probability p_i of each receiver changes per frame during the simulation time taking values from 0.1 to 0.3. Also, the demand ratio μ_i of each receiver changes with time while maintaining the average demand ratio μ constant. The results obtained in the following figures are computed over 2000 frames for each reading.

A. Study of the $o(1)$ Term

In the chromatic number expression in (3), the term $o(1)$ tends to zero as the number of vertices ν tends to infinity. Since our designed graphs have finite numbers of vertices, we run a study on the value of the $o(1)$ term that achieves a better performance

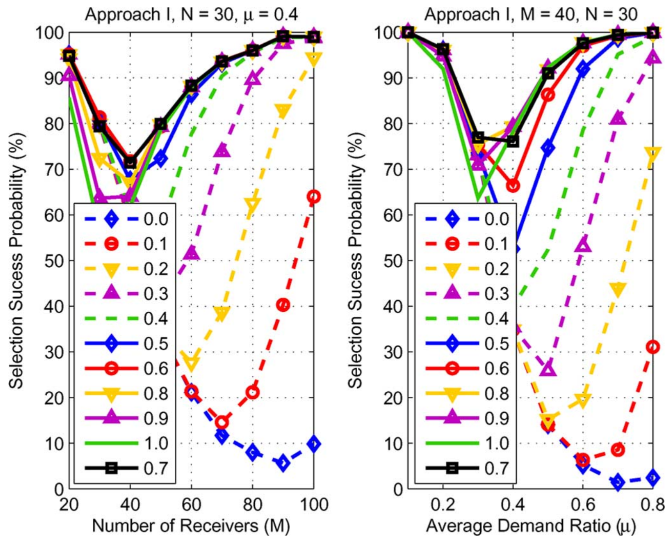

Fig. 1. Study of the $o(1)$ term for Approach I.

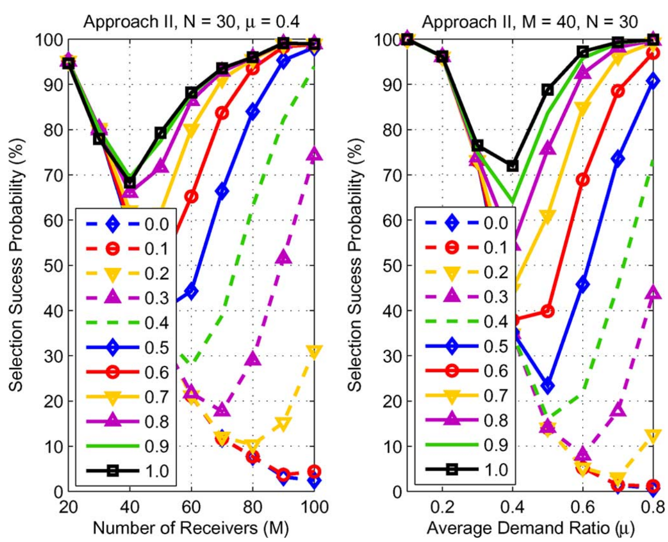

Fig. 2. Study of the $o(1)$ term for Approach II.

for our ANCR algorithm. The metric employed to evaluate different values of this term is the *selection success probability* defined as the number of trials in which the ANCR algorithm succeeds in selecting the scheme with lower number of retransmissions, divided by the total number of trials (which is equal to 2000 trials).

Figs. 1–3 depict the selection success probabilities for Approaches I–III, respectively. The left subfigure depicts the selection success probability against M for $N = 30$ and $\mu = 0.4$, whereas the right subfigure depicts the same metric against μ for $M = 40$ and $N = 30$. For each subfigure, we test different values of $o(1)$ ranging from 0 to 1.

For Approach I, we can see from the left subfigure of Fig. 1 that the highest selection success probabilities are obtained at $o(1) = 0.7$ for all values of M, except for $M = 30$ at which $o(1) = 0.6$ slightly outperforms. From the right subfigure, we can see that $o(1) = 0.7$ still dominates the others for all values of μ except at $\mu = 0.4$, where $o(1) = 0.8$ and 0.9 dominate. However, the testing of the bandwidth efficiency shows that the best performance is already achieved at $o(1) = 0.7$ for all values of M and μ, which makes it the best value to employ for Approach I.

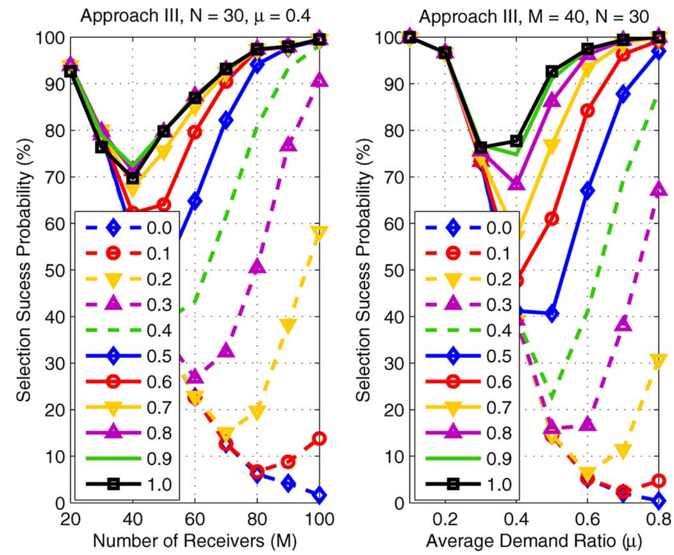

Fig. 3. Study of the $o(1)$ term for Approach III.

For Approach II, we can see that $o(1) = 1$ achieves the best performance for all values of M and μ, except for $M = 30$, where $o(1) = 0.8$ slightly dominates. However, $o(1) = 0.8$ achieves noticeably worse performance for several values of μ, which excludes it from our selection. Thus, $o(1) = 1$ is the best value to employ for Approach II.

For Approach III, $o(1) = 0.9$ and 1 clearly dominate the performance for all values of M and μ, respectively, in Fig. 3. We then ran a more focused simulation in this range and found that the best performance for all values of M and μ is achieved at $o(1) = 0.94$, which makes it the best choice to employ for Approach III.

We finally note that the selection success probabilities drops to around 70% at the middle range of M and μ. As will be shown in the next section, this drop occurs when the ONCR and FNCR schemes achieve close performances. Consequently, the overall average performance of the ANCR scheme is not affected by this drop.

B. *Performance Testing*

In this section, we compare the performance of our three proposed ANCR approaches to both ONCR and FNCR schemes for different numbers of receivers and demand ratios. As a comparison reference, we define the optimal selection scheme (denoted by OPT in the figure legends) as the one that always employs the network coded retransmission scheme that achieves the smaller number of retransmissions.

For Approaches I–III, Figs. 4–6 depict, respectively, the average and standard deviation of bandwidth efficiency achieved by the ONCR, FNCR, optimal selection, and ANCR schemes against the number of receivers M for $N = 30$ and $\mu = 0.4$. Also, Figs. 7–9 depict, respectively, the same performance comparison against the average demand ratio μ for $M = 40$ and $N = 30$.

We can observe from all figures that all our proposed ANCR approaches achieve an average performance that is always above

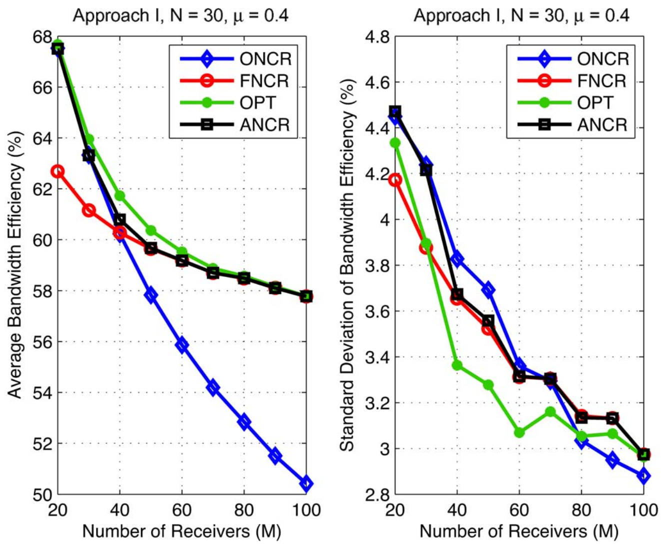

Fig. 4. Performance comparison for Approach I versus M for $o(1) = 0.7$.

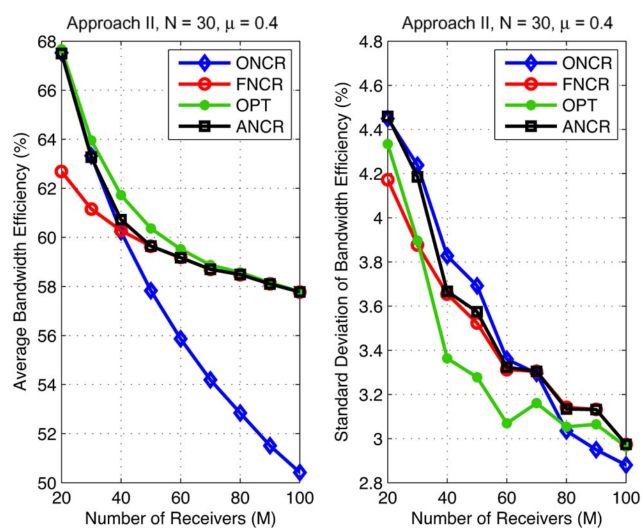

Fig. 5. Performance comparison for Approach II versus M for $o(1) = 1$.

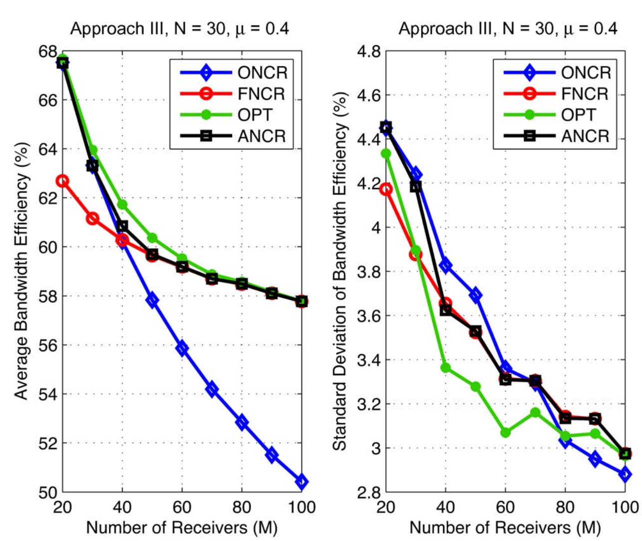

Fig. 6. Performance comparison for Approach III versus M for $o(1) = 0.94$.

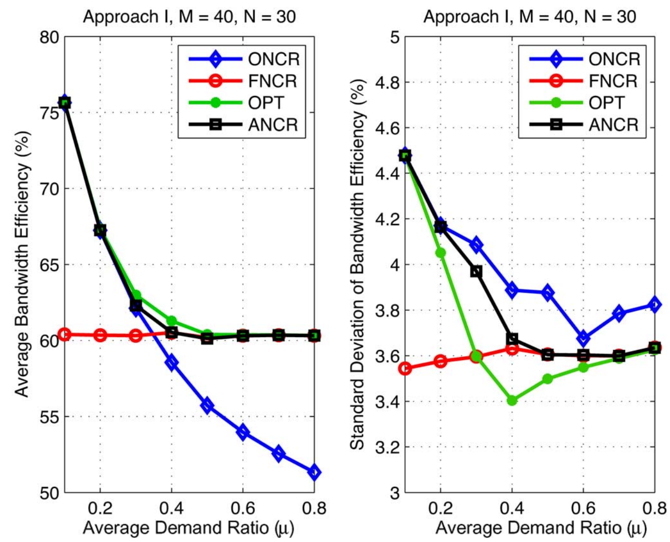

Fig. 7. Performance comparison for Approach I versus μ for $o(1) = 0.7$.

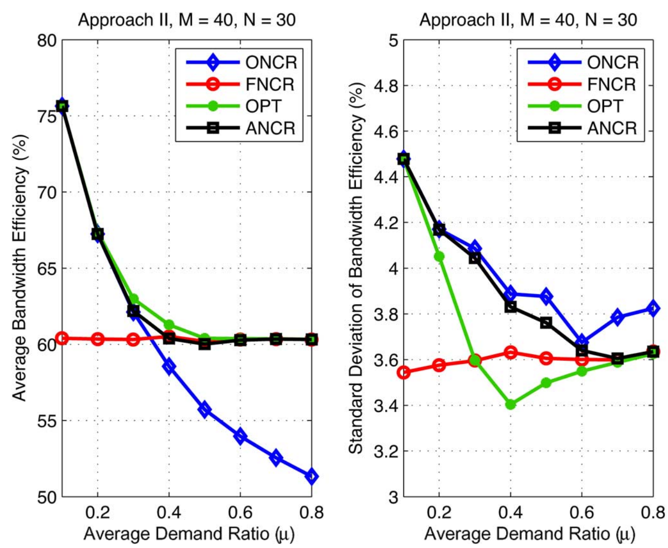

Fig. 8. Performance comparison for Approach II versus μ for $o(1) = 1$.

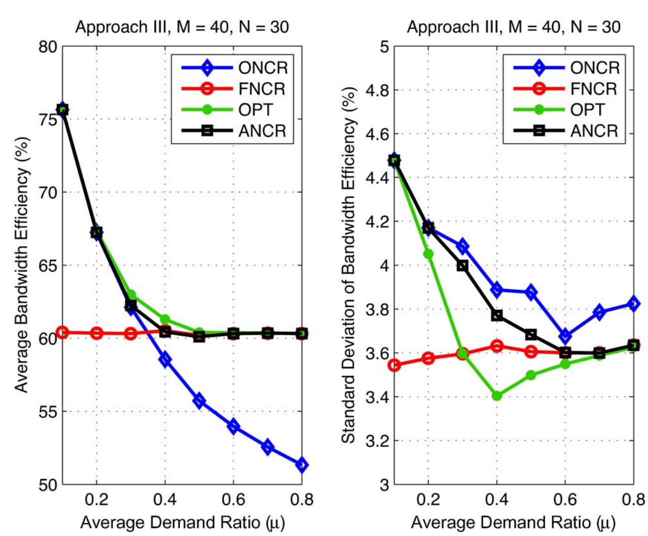

Fig. 9. Performance comparison for Approach III versus μ for $o(1) = 0.94$.

or equal to the average performance of the better scheme. The only exceptions are found in Figs. 8 and 9, in which the average performances of Approaches II and III slightly drop by 0.2% and 0.1%, respectively, below the performance of the better scheme at $\mu = 0.4$ and 0.5. Obviously, this can be considered as no degradation at all. We can also observe that all our proposed approaches achieve a similar standard deviation of the bandwidth efficiency with the selected original ONCR and FNCR schemes, even at the points at which the performance of both schemes are very close and thus a degradation was expected. This result shows that our proposed selection metrics do not add extra fluctuation in the achieved bandwidth efficiency.

Moreover, we can observe that the performances of our proposed ANCR approaches do not degrade much at the points with lower selection success probabilities shown in Figs. 1–3. This result is explained by the fact that the performances of both schemes are very close at these points, which greatly reduces the effect of wrong selection on the overall average performance. We can also see that our proposed ANCR approaches almost achieve the optimal selection performance with maximum degradation between 0.5% to 1%.

Finally, we conclude from all the figures that considering loss patterns in the selection process of the ANCR scheme does not add much value as the lossless approximations achieve the same or slightly better performance. Moreover, the results show that we do not need to compute π in every frame, as was suggested in Approach I, since Approach II almost achieves the same performance with lower computation rate of π.

VII. Conclusion

In this paper, we designed an adaptive scheme for packet retransmissions to improve the bandwidth efficiency in wireless MBS using a combination of opportunistic and full network coding. The proposed scheme selects, between these two schemes, the one that is expected to achieve the better bandwidth efficiency performance. To compare between different complexity levels, we presented three selection approaches. In the first two approaches, we derived ONCR performance metrics by modeling its lossless graph representation by a random graph, then using a famous result in random graph theory. These metrics are then compared to the lossless FNCR performance expression in order to perform scheme selection.

To test the effect of loss patterns on our proposed scheme, we proposed a third approach in which we first designed a new lossy ONCR graph representation by incorporating an average level of packet losses inside the graph. We then derived a lossy ONCR performance metric that is compared to a lossy approximation of the FNCR performance to perform scheme selection. For the three considered approaches, simulation results showed that our proposed scheme almost achieves the bandwidth efficiency performance that could be obtained by the optimal selection between the ONCR and FNCR schemes. They also showed that this result can be achieved without adding extra performance fluctuation, without considering packet losses, and with a low parameter computational rate.

Appendix A
Proof of Theorem 1

Without loss of generality, we assume that v is drawn from the graph vertex set before w. From Section IV-A, we know that two vertices v_{ij} and v_{kl} are connected in $\mathcal{G}^c$ if and only if both of the following conditions hold.

- C1: $j \neq l \Rightarrow$ The two vertices do not represent the request of the same packet.
- C2: $j \notin \mathcal{H}_k$ OR $l \notin \mathcal{H}_i \Rightarrow$ At least one of the two vertices requests a packet that is in the Complementary set of the other.

Since C1 and C2 are independent, we can express the vertex connectivity probability π as

$$\pi = \mathbb{P}(\mathrm{C1}|\mathfrak{D})\mathbb{P}(\mathrm{C2}|\mathfrak{D}) = \left(1 - \mathbb{P}(\overline{\mathrm{C1}}|\mathfrak{D})\right)\mathbb{P}(\mathrm{C2}|\mathfrak{D}). \quad (13)$$

where $\overline{\mathrm{C1}}$ is the opposite condition of C1. For any two vertices v and w, since we ignore the vertices' identities, we get

$$\begin{aligned}\mathbb{P}(\overline{\mathrm{C1}}|\mathfrak{D}) &= \sum_{j=1}^{N} \mathbb{P}(v \to P_j|\mathfrak{D})\mathbb{P}(w \to P_j|\mathfrak{D}) \\ &= \sum_{j=1}^{N} \frac{Z_j}{\nu}\frac{Z_j - 1}{\nu - 1} = \frac{\mathbf{z}(\mathbf{z}-\mathbf{1})^T}{\nu(\nu-1)}\end{aligned} \quad (14)$$

where "$v \to u$" means "vertex v is induced by entity u."

Define event A as the event representing the request of v for a packet that is in the Complimentary set of w. Also define event B as the vice versa of event A. Since we ignore the vertices' identities and the contents of different sets of all receivers, we can derive $\mathbb{P}(\mathrm{A}|\mathfrak{D})$ as follows:

$$\begin{aligned}\mathbb{P}(\mathrm{A}|\mathfrak{D}) &= \sum_{k=1}^{M} \mathbb{P}(\mathrm{A}|\mathfrak{D}, w \to R_k)\mathbb{P}(w \to R_k|\mathfrak{D}) \\ \mathbb{P}(w \to R_k|\mathfrak{D}) &= \sum_{i=1}^{M} \mathbb{P}(w \to R_k|\mathfrak{D}, v \to R_i)\mathbb{P}(v \to R_i|\mathfrak{D}) \\ &= \sum_{\substack{i=1 \\ i \neq k}}^{M} \frac{X_k}{\nu - 1}\frac{X_i}{\nu} + \frac{X_k - 1}{\nu - 1}\frac{X_k}{\nu} \\ &= \frac{X_k}{\nu(\nu-1)}\left(\sum_{i=1}^{M} X_i - 1\right) \\ &= \frac{X_k}{\nu(\nu-1)}(\nu - 1) = \frac{X_k}{\nu} \\ \Rightarrow \mathbb{P}(\mathrm{A}|\mathfrak{D}) &= \sum_{k=1}^{M} \frac{\overline{Y}_k}{N}\frac{X_k}{\nu} = \frac{\mathbf{x}\overline{\mathbf{y}}^T}{N\nu}.\end{aligned} \quad (15)$$

The same result can be derived for event B using a similar approach. Since packet losses at different receivers are independent, the two events A and B are independent of each other. Now, from the definition of C2, we get

$$\mathbb{P}(\mathrm{C2}|\mathfrak{D}) = \mathbb{P}(\mathrm{A} \cup \mathrm{B}|\mathfrak{D}) = \frac{\mathbf{x}\overline{\mathbf{y}}^T}{N\nu}\left(2 - \frac{\mathbf{x}\overline{\mathbf{y}}^T}{N\nu}\right). \quad (16)$$

The theorem follows from substituting (14) and (16) in (13).

Appendix B
Proof of Lemma 2

During the initial transmission phase, packet P_j will induce one vertex in $\mathcal{G}^c$ if a receiver has requested and lost this packet. This event might occur for each of the M receivers with probability μp. Since Z_j is the number of vertices in $\mathcal{G}^c$ induced by P_j, Z_j is the sum of M Bernoulli trials and is thus a binomial random variable $\mathrm{Bin}(M, \mu p)$. This applies for all $j \in \{1, \ldots, N\}$. Given that the total number of vertices in $\mathcal{G}^c$ is equal to $\nu = \sum_{v=1}^{N} Z_v$, we can consider without loss of generality that $Z_N = \nu - \sum_{v=1}^{N-1} Z_v$.

Based on these facts, and given the definition of the binomial coefficient in (6), the probability $\mathbb{P}(\mathbf{z} = \mathbf{z}' | \sum_{v=1}^{N} Z_v = \nu)$ can be expressed as follows:

$$\begin{aligned}
&\mathbb{P}\left(\mathbf{z} = \mathbf{z}' | \sum_{v=1}^{N} Z_v = \nu\right) \\
&= \frac{\mathbb{P}\left(\mathbf{z} = \mathbf{z}', \sum_{v=1}^{N} Z_v = \nu\right)}{\mathbb{P}\left(\sum_{v=1}^{N} Z_v = \nu\right)} \\
&= \frac{\mathbb{P}\left(Z_1 = z_1, \ldots, Z_{N-1} = z_{N-1}, Z_N = \nu - \sum_{v=1}^{N-1} z_v\right)}{\mathbb{P}\left(\sum_{v=1}^{N} Z_v = \nu\right)} \\
&= \prod_{u=1}^{N-1} \binom{M}{z_u} (\mu p)^{z_u} (1-\mu p)^{M-z_u} \binom{M}{\nu - \sum_{v=1}^{N-1} z_v} \\
&\quad \times (\mu p)^{\nu - \sum_{v=1}^{N-1} z_v} (1-\mu p)^{M-\nu+\sum_{v=1}^{N-1} z_v} \\
&\quad \times \left(\binom{MN}{\nu} (\mu p)^{\nu} (1-\mu p)^{MN-\nu}\right)^{-1} \\
&= \prod_{u=1}^{N-1} \binom{M}{z_u} \cdot \binom{M}{\nu - \sum_{v=1}^{N-1} z_v} \cdot \binom{MN}{\nu}^{-1}.
\end{aligned}$$

This expression is the probability mass function of a multivariate hypergeometric distribution, which concludes the proof.

Appendix C
Proof of Theorem 2

Since the two connectivity conditions in $\mathcal{G}^c$ (C1 and C2) are independent, we can express the vertex connectivity probability π, given ν as

$$\pi = \mathbb{P}(\mathrm{C1}|\nu)\mathbb{P}(\mathrm{C2}|\nu) = \left(1 - \mathbb{P}(\overline{\mathrm{C1}}|\nu)\right)\left(1 - \mathbb{P}(\overline{\mathrm{C2}}|\nu)\right). \tag{17}$$

where $\overline{\mathrm{C1}}$ and $\overline{\mathrm{C2}}$ are the opposite conditions of C1 and C2, respectively. $\overline{\mathrm{C1}}$ expresses the request of the two vertices for the same packet. Since we ignored the vertices' identities, we have

$$\mathbb{P}(\overline{\mathrm{C1}}|\nu, \mathbf{z} = \mathbf{z}') = \sum_{m=1}^{N} \frac{z_m(z_m-1)}{\nu(\nu-1)}$$

where $z_N = \nu - \sum_{v=1}^{N-1} z_v$. Thus, we can express $\mathbb{P}(\overline{\mathrm{C1}}|\nu)$ as follows:

$$\begin{aligned}
\mathbb{P}(\overline{\mathrm{C1}}|\nu) &= \mathbb{E}_{\mathbf{z}|\nu}\left(\sum_{m=1}^{N} \frac{z_m(z_m-1)}{\nu(\nu-1)}\right) \\
&= \sum_{m=1}^{N} \mathbb{E}_{\mathbf{z}|\nu}\left(\frac{z_m(z_m-1)}{\nu(\nu-1)}\right).
\end{aligned} \tag{18}$$

From Lemma 2, we have $\forall m \in \{1, \ldots, N-1\}$

$$\begin{aligned}
&\mathbb{E}_{\mathbf{z}|\nu}\left(\frac{z_m(z_m-1)}{\nu(\nu-1)}\right) \\
&= \sum_{\substack{z_u=0 \\ \forall u}}^{M} \frac{z_m(z_m-1)}{\nu(\nu-1)} \times \frac{\prod_{u=1}^{N-1}\binom{M}{z_u}\binom{M}{\nu-\sum_{v=1}^{N-1} z_v}}{\binom{MN}{\nu}} \\
&= \frac{M(M-1)}{MN(MN-1)} \\
&\quad \times \sum_{\substack{z_u=0 \\ \forall\ u\neq m}}^{M} \sum_{z_m-2=0}^{M-2} \frac{\prod_{\substack{u=1 \\ u\neq m}}^{N-1}\binom{M}{z_u}\binom{M-2}{z_m-2}\binom{M}{\nu-\sum_{v=1}^{N-1} z_v}}{\binom{MN-2}{\nu-2}} \\
&= \frac{M-1}{N(MN-1)}.
\end{aligned} \tag{19}$$

The last line in (19) is obtained since the preceding line is a summation of a new multivariate hypergeometric probability mass function over all its sample space. A similar result can be derived for z_N. Substituting in (18), we get

$$\mathbb{P}(\overline{\mathrm{C1}}|\nu) = \frac{M-1}{MN-1}. \tag{20}$$

$\overline{\mathrm{C2}}$ means that both vertices have correctly received the packet requested by each other. Since the reception of a packet from a receiver is independent of the reception of another packet by another receiver, and since we ignored the vertices' identities and the details of the feedback table, we get

$$\mathbb{P}(\overline{\mathrm{C2}}|\nu) = 1 - (1-p)^2 = p(2-p). \tag{21}$$

The theorem follows from substituting (20) and (21) in (17).

Appendix D
Proof of Theorem 3

Without loss of generality, we assume that v is drawn from the graph vertex set before w. Since the events $\widetilde{\mathrm{C}}1$ and $\widetilde{\mathrm{C}}2$ are mutually exclusive, we can express π as

$$\pi = 1 - \mathbb{P}(\widetilde{\mathrm{C}}1 \cup \widetilde{\mathrm{C}}2 | \widetilde{\mathfrak{D}}) = 1 - \left(\mathbb{P}(\widetilde{\mathrm{C}}1|\widetilde{\mathfrak{D}}) + \mathbb{P}(\widetilde{\mathrm{C}}2|\widetilde{\mathfrak{D}})\right). \tag{22}$$

Since we ignore the vertex identities, we can derive $\mathbb{P}(\widetilde{\mathrm{C}}1|\widetilde{\mathfrak{D}})$ as follows:

$$\begin{aligned}\mathbb{P}(\widetilde{\mathrm{C}}1|\widetilde{\mathfrak{D}}) &= \sum_{i=1}^{M}\sum_{j=1}^{N}\mathbb{P}\left(v\rightarrow(R_i,P_j), w\rightarrow(\mathcal{R}\setminus R_i,P_j)|\widetilde{\mathfrak{D}}\right)\\ &= \sum_{i=1}^{M}\sum_{j=1}^{N}\mathbb{P}\left(w\rightarrow(\mathcal{R}\setminus R_i,P_j)|\widetilde{\mathfrak{D}}, v\rightarrow(R_i,P_j)\right)\\ &\quad\times\mathbb{P}\left(v\rightarrow(R_i,P_j)|\widetilde{\mathfrak{D}}\right)\\ &= \sum_{i=1}^{M}\sum_{j=1}^{N}\frac{\sum_{\substack{k=1\\k\neq i}}^{M}X_{kj}}{\widetilde{\nu}-1}\frac{X_{ij}}{\widetilde{\nu}} = \frac{\mathbf{Tr}\left[(\widetilde{\mathbf{X}}^T\mathbf{\Theta})\widetilde{\mathbf{X}}\right]}{\widetilde{\nu}(\widetilde{\nu}-1)}. \quad (23)\end{aligned}$$

Define event A as the event representing the request of v for a packet that is in the Has set of w. Also, define event B as the vice versa of event A. Since we ignore vertices identities, we can easily show, using similar derivations as in (15), that

$$\mathbb{P}(\mathrm{A}|\widetilde{D}) = \mathbb{P}(\mathrm{B}|\widetilde{D}) = \frac{(\mathbf{1}\widetilde{\mathbf{X}}^T)\mathbf{y}^T}{N\widetilde{\nu}}. \quad (24)$$

Since we ignore vertices identities and the content of the Has and Complimentary sets, it is easy to infer that events A and B are independent. Now, $\mathbb{P}(\widetilde{\mathrm{C}}2|\widetilde{\mathfrak{D}})$ can be expressed as

$$\mathbb{P}(\widetilde{\mathrm{C}}2|\widetilde{\mathfrak{D}}) = \mathbb{P}(\mathrm{A}\cap\mathrm{B}|\widetilde{\mathfrak{D}}) = \left(\frac{(\mathbf{1}\widetilde{\mathbf{X}}^T)\mathbf{y}^T}{N\widetilde{\nu}}\right)^2. \quad (25)$$

The theorem follows from substituting (23) and (25) in (22).

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