

Using (A2) in (6) yields

$$G(n) = \frac{K(n)X(n)}{Q}. \quad (A3)$$

Then, substituting (A3) into (8) and rearranging terms, we obtain

$$\widehat{W}_{id}(n) = K(n)K^{-1}(n-1)\widehat{W}_i(n-1), \quad i = 1, 2, \dots, V. \quad (A4)$$

From (9), (A2), and (A4), it can be shown that

$$\begin{aligned} \widehat{W}_{id}(n) &= \frac{K(n)K^{-1}(0)\widehat{W}_i(0)}{\left[\widehat{W}_i^H(0)K^{-1}(0)K(n-1)K(n-1)K^{-1}(0)\widehat{W}_i(0)\right]^{1/2}}. \end{aligned} \quad (A5)$$

If $K(0)$ is chosen as an identity matrix, substituting (A2) and (A5) into (9) yields

$$\widehat{W}_i(n) = \frac{\widehat{R}^{-1}(n)\widehat{W}_i(0)}{\|\widehat{R}^{-1}(n)\widehat{W}_i(0)\|}, \quad i = 1, 2, \dots, V. \quad (A6)$$

Thus,

$$\lim_{n \rightarrow \infty} \widehat{W}_i(n) = \frac{R^{-1}\widehat{W}_i(0)}{\|R^{-1}\widehat{W}_i(0)\|}, \quad i = 1, 2, \dots, V \quad (A7)$$

with probability 1 convergence, since $\lim_{n \rightarrow \infty} \widehat{R}(n) = R$.

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Alternative Proofs for "On Unique Localization of Constrained-Signal Sources"

S. Valaee and P. Kabal

Abstract—In this correspondence, we provide simple proofs for the theorems in "On Unique Localization of Constrained-Signal Sources" by M. Wax. The approach is based on the topological dimension of a set. All the possible observation matrices form the legitimate set. The observation matrices that can have nonunique solutions form the ambiguity set. The components of the legitimate and ambiguity set are random matrices. We find the conditions under which the dimensionality of the ambiguity set is smaller than the dimensionality of the legitimate set. In such a case, the probability of the ambiguity set is zero and with probability one, a unique solution can be found for the localization problem.

I. INTRODUCTION

In [1], Wax showed that when the signals are constrained to certain loci in the complex plane, the number of uniquely resolvable signals is larger than the number of sensors. He considered two classes of constraints, and for each class, he found the conditions under which almost surely a unique solution for the localization problem can be found. Here, we give alternative and simple proofs to the theorems in [1]. The same approach is also used to find a sufficient condition for unique localization of unconstrained signals [2].

Assume that an array of p sensors receives the wavefield of q sources in N consecutive snapshots. The sources can be partially or fully correlated. In a noise-free environment, the $p \times N$ observation matrix \mathbf{X} can be shown as

$$\mathbf{X} = \mathbf{A}(\theta)\mathbf{S} \quad (1)$$

where $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_q)]$ is the $p \times q$ location matrix, and \mathbf{S} is the $q \times N$ source signal matrix. Uniqueness can be stated as "Given the observation matrix \mathbf{X} , a unique vector θ can be found that satisfies (1) for any source signal matrix \mathbf{S} ." The following constraints have been imposed on the array location vectors:

- A1) The array manifold $\{\mathbf{a}(\theta) : \theta \in \Omega\}$, where Ω is the field of view, is known.
- A2) Any p distinct steering vectors from the array manifold are linearly independent.

The source signal matrix can be expressed as $\mathbf{S} = [\mathbf{S}_1, \mathbf{S}_2]$, where \mathbf{S}_1 and \mathbf{S}_2 are $q \times \eta$ and $q \times (N - \eta)$ matrices with η being the rank of \mathbf{S} . Similar to [1], we can show that the uniqueness problem needs only to be considered for the observation matrix $\mathbf{X}_1 = \mathbf{A}(\theta)\mathbf{S}_1$. In the sequel, we assume that the dimensionality of \mathbf{X} and \mathbf{S} are equal to $p \times \eta$ and $q \times \eta$, respectively. In this paper, two kinds of constraints are considered for the signals. These constraints will be discussed separately in the following sections. In Section III, we provide a simpler proof for the sufficient condition for unique localization of unconstrained signals [2].

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II. REAL CONSTRAINTS

Assume that the signals are constrained to the loci [1]

$$f_k(s_k(t_i), \lambda_k^{(\mu)}) = 0, \quad k = 1, \dots, q \quad (2)$$

where

- $s_k(t_i)$ signal of the k th source at time instant t_i
- f_k smooth map from the complex plane to the real line
- $\lambda_k^{(\mu)}$ vector of parameters with μ real unknowns.

Theorem 1: Let θ be fixed, and let \mathbf{S} be a $q \times \eta$ random matrix drawn from the set of all rank- η matrices having elements constrained by (2) and jointly distributed according to some absolutely continuous distribution. A general array satisfying A1) and A2) can then almost always, with the exception of a set of signals of measure zero, uniquely localize q sources, provided that

$$q < \frac{\eta}{\eta + \mu + 1} 2p. \quad (3)$$

Proof: The location matrix $\mathbf{A}(\theta)$ is uniquely defined by the parameter vector θ . Thus, the number of free parameters to describe $\mathbf{A}(\theta)$ is q . The source signal matrix \mathbf{S} , is a $q \times \eta$ -dimensional complex matrix. In general, a $q \times \eta$ complex matrix is described by $2q\eta$ real parameters. Since the signals are constrained by (2), the number of free parameters is reduced by the number of constraints $q\eta$. However, each constraint adds μ new unknowns. Thus, the total number of free parameters of \mathbf{S} is equal to $(2q\eta - q\eta + q\mu)$.

The $p \times \eta$ matrices \mathbf{X} , which satisfy (1) and (2), form the "legitimate set," which is denoted by \mathcal{G} . The legitimate set is a subset of the subspace of $p \times \eta$ -dimensional complex matrices. Since θ and \mathbf{S} can be chosen independently in (1), the dimensionality of the legitimate set is equal to $(q + q\eta + q\mu)$.

A nonunique solution appears for the localization problem if

$$\mathbf{X} = \mathbf{A}(\theta)\mathbf{S} = \mathbf{A}(\theta')\mathbf{S}' \quad (4)$$

for different θ and θ' . Let us define

$$\mathbf{C} = \mathbf{A}(\theta)\mathbf{S} - \mathbf{A}(\theta')\mathbf{S}'. \quad (5)$$

Note that \mathbf{C} is a $p \times \eta$ complex matrix and can be uniquely described by $2(q + q\eta + q\mu)$ real parameters.

The "ambiguity set" is defined by the $p \times \eta$ complex matrices that satisfy

$$\mathcal{D} = \{\mathbf{A}(\theta)\mathbf{S} : \mathbf{A}(\theta)\mathbf{S} - \mathbf{A}(\theta')\mathbf{S}' = 0, \text{ for } \theta, \theta' \in \Omega, \text{ and } \mathbf{S}, \mathbf{S}' \text{ satisfying (2)}\}. \quad (6)$$

The constraints of the ambiguity set can be shown as

$$\mathbf{C} = 0. \quad (7)$$

In such a case, the number of free parameters will be reduced by the number of constraints in (7). Since \mathbf{C} is a complex $p \times \eta$ matrix, the number of free parameters is reduced by $2p\eta$. Thus, the number of free parameters to describe (4) is equal to $2(q + q\eta + q\mu) - 2p\eta$. For uniqueness, this number, which is the dimensionality of the ambiguity set, should be strictly smaller than the dimensionality of the legitimate set. Hence

$$2(q + q\eta + q\mu) - 2p\eta < q + q\eta + q\mu \quad (8)$$

which is the same result as

$$q < \frac{\eta}{\eta + \mu + 1} 2p. \quad (9)$$

In such a case, the ambiguity set is a proper subset of the legitimate set. Since the ambiguity set is a set of random matrices, its conditional probability given the legitimate set is zero. Hence, if the number of signals satisfies (9) everywhere except in a set with probability zero, a unique solution for the localization problem exists.

III. COMPLEX CONSTRAINTS

In this case, the signals are constrained to the loci [1]

$$g_k(s_k(t_i), \lambda_k^{(\mu)}) = 0, \quad k = 1, \dots, q \quad (10)$$

where g_k is a smooth map from the complex plane to itself, and $\lambda_k^{(\mu)}$ is a vector of parameters with μ real unknowns.

Theorem 2: Let θ be fixed, and let \mathbf{S} be a $q \times \eta$ random matrix drawn from the set of all rank- η matrices having elements constrained by (10) and jointly distributed according to some absolutely continuous distribution. A general array satisfying A1) and A2) can then almost always, with the exception of a set of signals of measure zero, uniquely localize q sources, provided that

$$q > \frac{\eta}{\mu + 1} 2p. \quad (11)$$

Proof: With a similar argument, the number of free parameters of \mathbf{X} is $q + q\mu$, which is the dimensionality of the legitimate set. Here, the constraint functions are complex, and therefore, the number of free parameters of the ambiguity set is $2(q + q\mu) - 2p\eta$. This number should be smaller than the total number of free parameters of the legitimate set

$$2(q + q\mu) - 2p\eta < q + q\mu. \quad (12)$$

Thus, the number of uniquely localizable sources is bounded by

$$q < \frac{\eta}{\mu + 1} 2p. \quad (13)$$

If the number of signals satisfies (13), a unique solution for the localization problem exists with probability one.

IV. UNCONSTRAINED SIGNALS

The method of the preceding sections can also be used to find the maximum number of uniquely localizable unconstrained signals. This problem has been discussed in [2]. There, it has been proved that with probability one a unique solution for the localization problem can be found if

$$q < \frac{2\eta}{2\eta + 1} p. \quad (14)$$

In the present section, we give a simpler proof for this condition.

Similar to previous sections, we define the legitimate set by the set of all observation matrices that satisfy (1). Here, no constraints are imposed on the signal matrix \mathbf{S} . The dimensionality of the legitimate set is equal to the free real parameters that are used to describe \mathbf{X} . The signal matrix \mathbf{S} is a $q \times \eta$ dimensional and can be uniquely described by $2q\eta$ real parameters. Since θ and \mathbf{S} can be chosen independently in (1), the dimensionality of the legitimate set is equal to $q + 2q\eta$.

The ambiguity set is defined by

$$\mathcal{D} = \{\mathbf{A}(\theta)\mathbf{S} : \mathbf{A}(\theta)\mathbf{S} - \mathbf{A}(\theta')\mathbf{S}' = 0, \text{ for } \theta, \theta' \in \Omega\}. \quad (15)$$

With a similar argument, we can show that the dimensionality of the ambiguity set is equal to $2(q + 2q\eta) - 2p\eta$. With probability one, a unique solution for the localization problem can be found if

$$\dim \mathcal{D} < \dim \mathcal{G}. \quad (16)$$

Using the dimensionality of \mathcal{D} and \mathcal{G} , this condition can be shown as

$$q < \frac{2\eta}{2\eta + 1} p. \quad (17)$$

Thus, if q satisfies (17), almost surely, a unique solution exists for the localization problem.

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Efficient Scheduling and Instruction Selection for Programmable Digital Signal Processors

Kin H. Yu and Yu Hen Hu

Abstract—We present an efficient method for optimized instruction selection and scheduling for Programmable Digital Signal Processors. Our approach uses artificial intelligence techniques to yield code that is comparable to that of hand-written assembly codes by DSP experts. Several examples which demonstrate the feasibility of the approach, targeted to the TMS32020/50 architecture, are presented.

I. INTRODUCTION

To fully utilize the available computing power of the modern programmable digital signal processor (PDSP) [1], software designers must face the difficult task of programming in assembly language. Although high-level language (HLL) compilers for PDSP's exist [2], most of them are based on technology developed with general purpose applications in mind. Genin *et al.* [3] estimated that assembly codes written by human DSP experts perform 5 to 50 times faster than those obtained from conventional HLL compilers of just a few years ago. Although the performance of current optimizing compilers has improved significantly, for real-time DSP applications with stringent constraints on execution time and/or code size, careful manual coding, typically with several fine-tuning iterations, is still the only effective approach.

This correspondence deals with code generation for PDSP's with nonuniform register sets. More specifically, we focus on the subproblems of *scheduling* and *instruction selection*. Other issues such as the handling of conditionals and branches, the use of circular buffers and special instructions are under study and will be reported in the future. In this correspondence, we describe an approach in which scheduling and instruction selection are handled concurrently, instead of in separate passes as often implemented in conventional compilers. Our measure of efficiency is the size and execution time of the generated assembly code. Many embedded applications in the DSP area depend directly on the efficiency of the executable code.

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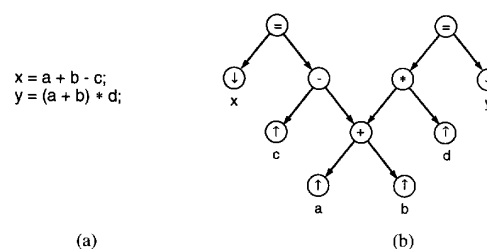


Fig. 1. (a) A fragment of C code; (b) its DAG representation. Up and down arrows represent fetch and write operations, respectively.

Such applications may require a minimum code execution speed (for example, a certain computation must be finished before the next input data arrive) and/or a maximum code size (for example, to fit in the limited on-chip memory of the PDSP). For such applications, longer compilation time and larger memory requirement can still be more attractive than manual assembly coding. A prototype code generator targeted for a subset of the TMS3202x/C5x architecture and instruction set has been implemented. Experimental results reveal that our approach can yield codes that are comparable to that of hand-written assembly codes by DSP experts.

II. PROBLEM FORMULATION

An HLL program can be represented by a directed acyclic graph (DAG). Fig. 1 depicts a fragment of C code and its corresponding DAG. Nodes in the DAG represent operations to be performed and arcs indicate the data or control dependency among those operators. Similarly, each assembly instruction can be represented by a DAG-like pattern. Each pattern specifies, among other information, which registers it needs to read from and what state flags it will use before execution of the associated instruction. Each pattern also specifies which registers it will write to and what state flags it will modify after execution. We call the former *pre-conditions* and the latter *post-conditions*. In this context, it is easy to see that each pattern can "cover" part of the program DAG.

Code generation can be described as determining a sequence of instructions and their ordering, with compatible *pre-conditions* and *post-conditions*, from the given instruction set to realize all the operations specified in the DAG, subject to the DAG data dependency constraints. If each instruction also carries an associated cost, optimal code generation can be defined as determining the minimum cost cover for the program DAG. Unfortunately, it has been shown that optimal code generation is NP-complete [4].

Two important subproblems of code generation are *scheduling* and *instruction selection*. For example, the DAG in Fig. 1 indicates that the "+" node must be covered before either "-" or "*" nodes can be evaluated. However, there are no constraints on the covering order of the latter two nodes. Determining a valid evaluation order for all nodes, consistent with the partial order specified by the arcs, is known as *scheduling*.

Many PDSP's are based on the CISC (complex instruction set computing) concept, characterized by a nonorthogonal set of complex instructions. Consequently, certain parts of the DAG may be coverable by more than one instruction. Determining a set of instructions that can completely cover the program DAG, and for those parts that are coverable by many instructions, determining which instruction to