International Symposium on Information Theory and its Applications, ISITA2008
Auckland, New Zealand, 7-10, December, 2008

Gaussian Z-Interference Channel with a Relay Link: Achievable Rate Region and Asymptotic Sum Capacity

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Abstract

This paper studies a Gaussian Z-interference channel with a rate-limited digital relay link from the interference-free receiver to the interfered receiver. An achievable rate region is derived based on a combination of the Han-Kobayashi common-private information splitting technique and a partial interference-forwarding relaying strategy in which a part of the interference is decoded then forwarded through the digital link using a binning strategy for interference cancellation. The proposed strategy is shown to be capacity achieving in the strong interference regime, and asymptotically sum capacity approaching in the weak interference regime in the high signal-to-noise ratio and high interference-to-noise ratio limit.

1. INTRODUCTION

The classic interference channel models a communication situation in which two transmitters communicate with their respective receivers while mutually interfering with each other. The largest known achievable rate region for the interference channel is due to Han and Kobayashi [1], where a common-private information splitting technique is used to partially decode and subtract the interfering signal. The Han-Kobayashi scheme has recently been shown to be capacity achieving in a very weak interference regime [2, 3, 4] and to be within one bit of the capacity region in general [5].

This paper considers a novel communication model in which the classic interference channel is augmented by a noiseless relay link between the two receivers, and explores the use of relay techniques for interference mitigation. We focus on the simplest interference channel model, the Gaussian Z-interference channel, and let the direction of the digital relay link go from the interference-free receiver to the interfered receiver. The main result of this paper is a relay strategy called partial interference forwarding, in which the relay link is used for partial interference subtraction at the interfered receiver. The idea is to explore the fact that interference consists of structured codewords, which can be described efficiently using a binning strategy. For the Z-interference channel considered in this paper, partial interference forwarding is shown to achieve the capacity in the strong interference regime, and to achieve the asymptotic sum capacity cut-set bound in the weak interference regime in the high signal-to-noise ratio (SNR) and interference-to-noise ratio (INR) limit.

This work was supported by LG Electronics and by NSERC.

The Gaussian Z-interference channel is one of the few examples of an interference channel (besides the strong interference case [6, 7, 1] and the very weak interference case [2, 3, 4]) for which the sum capacity has been established. The sum capacity of the Gaussian Z-interference channel in the weak interference regime is achieved with both transmitters using Gaussian codebooks and with the interference treated as noise [8, 5].

The fundamental decode-and-forward and quantize-and-forward strategies for the relay channel are due to the classic work of Cover and El Gamal [9]. This paper is motivated by the more recent capacity results for a class of relay channels in which the relay observes the noise in the direct channel [10, 11]. The situation investigated in [10, 11] is similar to the Z-interference channel with a relay link, where one of the receivers observes a noisy version of the interference at the other receiver and helps the other receiver by describing the interference through the noiseless relay link.

The channel model studied in the paper is related to the recent work of Sahin and Erkip [12, 13], Marić et al. [14] and Dabora et al. [15], where the achievable rate regions are derived for an interference channel with an additional relay node. In particular, [14, 15] propose an interference-forwarding strategy which is similar to the one used in this paper. However, the channel model of this paper is considerably simpler. By focusing on a Z-channel with a digital relay, we are able to derive more concrete achievability results and upper bounds.

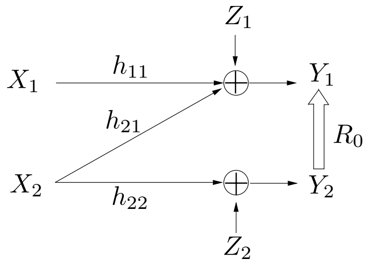

Figure 1: Gaussian Z-interference channel with a digital relay link.

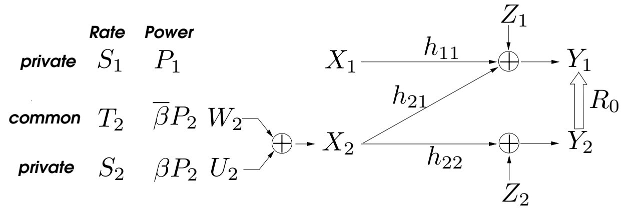

Figure 2: Common-private information splitting.

2. Achievable Rate Region

2.1. Channel Model

The Gaussian Z-interference channel with a digital relay link is modeled as follows (see Fig. 1):

$$\begin{cases} Y_1 = h_{11}X_1 + h_{21}X_2 + Z_1 \\ Y_2 = h_{22}X_2 + Z_2 \end{cases} \tag{1}$$

where X_1 and X_2 are the transmit signals with power constraints P_1 and P_2 respectively, h_{ij} represents the channel gain from transmitter i to receiver j, and Z_1, Z_2 are independent additive white Gaussian noises (AWGN) with power N. Receiver 2 has a noiseless link of fixed capacity R_0 to Receiver 1

To simplify the notation, the following definitions are used in this paper:

$$\mathsf{SNR}_1 = \frac{|h_{11}|^2P_1}{N} \qquad \mathsf{SNR}_2 = \frac{|h_{22}|^2P_2}{N}$$
$$\mathsf{INR}_2 = \frac{|h_{21}|^2P_2}{N} \qquad \gamma(x) = \frac{1}{2}\log(1+x)$$

where $\log(\cdot)$ is base 2. In addition, denote $\overline{\beta} = 1-\beta$.

2.2. Achievable Rate Region

Given Y_2's observation, how should it utilize the noiseless relay link to help Y_1 decode X_1? Clearly, decode-and-forward is not useful, as X_1 is not observed at Y_2. Instead, Y_2 observes a noisy version of the interference at Y_1. Thus, quantize-and-forward may be a sensible strategy in which Y_2 is described to Y_1 using a rate-constrained quantization codebook. However, quantize-and-forward does not take into account the fact that X_2 is not a Gaussian random noise, but a codeword from a structured codebook. In addition, one may intentionally design X_2 in order to facilitate interference subtraction. This extra degree of freedom allows us to achieve a higher rate than the rate achievable with a quantize-and-forward strategy.

Theorem 1. *For the Gaussian Z-interference channel with a digital link of limited rate R_0 from the interference-free receiver to the interfered receiver as shown in Fig. 1, in the weak interference regime defined by $\mathsf{INR}_2 \le \mathsf{SNR}_2$, the following rate region is achievable:*

$$\bigcup_{0\le\beta\le1} \Bigg\{(R_1,R_2)\,\Bigg|\, R_1 \le \gamma\left(\frac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right), \\ R_2 \le \min\{\gamma(\mathsf{SNR}_2), \gamma(\beta\mathsf{SNR}_2) + \\ \gamma\left(\frac{\overline{\beta}\mathsf{INR}_2}{1+\mathsf{SNR}_1+\beta\mathsf{INR}_2}\right) + R_0\Big\}\Bigg\}. \tag{2}$$

In the strong interference regime, defined by

$$\mathsf{SNR}_2 \le \mathsf{INR}_2 \ \le \max\{\mathsf{SNR}_2, \mathsf{INR}_2^*\}, \tag{3}$$

where

$$\mathsf{INR}_2^* = (1+\mathsf{SNR}_1)(2^{-2R_0}(1+\mathsf{SNR}_2)-1), \tag{4}$$

the capacity region is given by

$$\left\{(R_1,R_2)\,\middle|\, \begin{array}{rcl} R_1 & \le & \gamma(\mathsf{SNR}_1) \\ R_2 & \le & \gamma(\mathsf{SNR}_2) \\ R_1+R_2 & \le & \gamma(\mathsf{SNR}_1+\mathsf{INR}_2)+R_0 \end{array}\right\}. \tag{5}$$

In the very strong interference regime defined by

$$\mathsf{INR}_2 \ge \max\{\mathsf{SNR}_2, \mathsf{INR}_2^*\}, \tag{6}$$

the capacity region is given by

$$\left\{(R_1,R_2)\,\middle|\, \begin{array}{l} R_1 \le \gamma(\mathsf{SNR}_1) \\ R_2 \le \gamma(\mathsf{SNR}_2) \end{array}\right\}. \tag{7}$$

Proof. We use a combination of the Han-Kobayashi [1] common-private information splitting scheme and a bin-and-forward strategy to prove the achievability of the rate regions (2), (5) and (7). The encoding procedure is as depicted in Fig. 2. User 1's signal X_1 is intended for decoding at Y_1 only. User 2's signal X_2 is the superposition of private message U_2 and common message W_2. The private message can only be decoded by the intended receiver Y_2, while the common message can be decoded by both receivers. Independent Gaussian codebooks of sizes 2^{nS_1}, 2^{nS_2} and 2^{nT_2} are generated according to i.i.d. Gaussian distributions $X_1 \sim \mathcal{N}(0, P_1)$, $U_2 \sim \mathcal{N}(0, \beta P_2)$, and $W_2 \sim \mathcal{N}(0, \overline{\beta} P_2)$, respectively, where $0 \le \beta \le 1$.

Decoding takes place in two steps. First, (W_2, U_2) are decoded at Y_2. The set of achievable rates (T_2, S_2) is the capacity region of a Gaussian multiple-access channel, denoted here by $\mathcal{C}_2$, where

$$\left\{ \begin{array}{rcl} T_2 & \le & \gamma(\overline{\beta}\mathsf{SNR}_2) \\ S_2 & \le & \gamma(\beta\mathsf{SNR}_2) \\ S_2 + T_2 & \le & \gamma(\mathsf{SNR}_2). \end{array} \right. \tag{8}$$

After (W_2, U_2) are decoded at Y_2, (X_1, W_2) are then decoded at Y_1 with U_2 treated as noise, but with the help of the relay link. This is a multiple-access channel with a rate-limited relay Y_2, who has complete knowledge of W_2. Denote the capacity region of such a multiple-access channel with a digital relay link R_0 by $\mathcal{C}_1$. Using the degraded relay channel capacity result [9], one can prove that a bin-and-forward relay strategy is capacity achieving in this case. The capacity region $\mathcal{C}_1$ is the set of (S_1, T_2) for which

$$\left\{ \begin{array}{rcl} S_1 & \le & \gamma\left(\dfrac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right) \\ T_2 & \le & \gamma\left(\dfrac{\overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) + R_0 \\ S_1 + T_2 & \le & \gamma\left(\dfrac{\mathsf{SNR}_1 + \overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) + R_0. \end{array} \right. \tag{9}$$

An achievable rate region of the Gaussian Z-interference channel with a relay link is then the set of all (R_1, R_2) such that $R_1 = S_1$ and $R_2 = S_2 + T_2$ for some $(S_1, T_2) \in \mathcal{C}_1$ and $(S_2, T_2) \in \mathcal{C}_2$. Using Fourier-Motzkin elimination (see e.g. [16].), it is possible to show that for each fixed β, the set of achievable rates (R_1, R_2) is a pentagon denoted here as $\mathcal{R}_\beta$:

$$\left\{ (R_1, R_2) \left| \begin{array}{l} R_1 \le \gamma\left(\dfrac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right) \\ R_2 \le \min\{\gamma(\mathsf{SNR}_2), \gamma(\beta\mathsf{SNR}_2) + \\ \qquad \gamma\left(\dfrac{\overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) + R_0\} \\ R_1 + R_2 \le \gamma(\beta\mathsf{SNR}_2) + \\ \qquad \gamma\left(\dfrac{\mathsf{SNR}_1 + \overline{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) + R_0 \end{array} \right. \right\}. \tag{10}$$

To complete the proof, we need to show that the convex hull of the union of these pentagons reduces to the regions (2), (5) and (7).

In the weak interference regime, it is possible to show that the union of the pentagons (10) but without

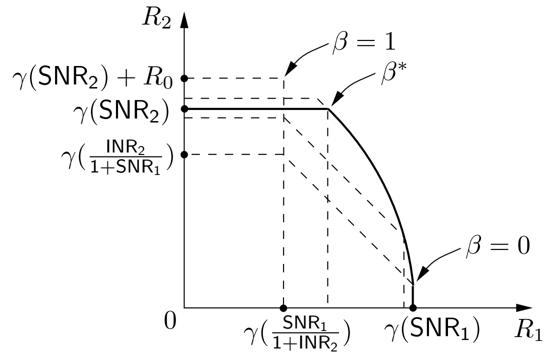

Figure 3: The union of rate region pentagons in the weak interference regime.

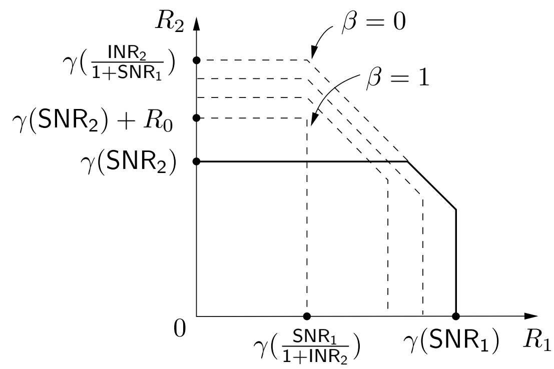

Figure 4: The union of rate region pentagons in the strong interference regime.

the constraint $R_2 \le \gamma(\mathsf{SNR}_2)$ reduces to a region defined by $R_1 \le \gamma(\mathsf{SNR}_1)$, $R_2 \le \gamma(\mathsf{SNR}_2) + R_0$, and the lower-right corner points of the pentagons with

$$\left\{ \begin{array}{rcl} R_1 & = & \gamma\left(\dfrac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right) \\ R_2 & = & \gamma(\beta\mathsf{SNR}_2) + \gamma\left(\dfrac{\overline{\beta}\mathsf{INR}_2}{1+\mathsf{SNR}_1+\beta\mathsf{INR}_2}\right) + R_0 \end{array} \right. \tag{11}$$

where $0 \le \beta \le 1$, as shown in Fig. 3. One can verify that such a region is convex when $\mathsf{INR}_2 \le \mathsf{SNR}_2$. Then, incorporating the constraint $R_2 \le \gamma(\mathsf{SNR}_2)$ gives the weak-interference achievable region (2).

In the strong interference regime, where $\mathsf{INR}_2 \ge \mathsf{SNR}_2$, one can show that $\bigcup_{0 \le \beta \le 1} \mathcal{R}_\beta = \mathcal{R}_0$, as illustrated in Fig. 4. Thus, the achievable rate region simplifies to

$$\left\{ (R_1, R_2) \left| \begin{array}{l} R_1 \le \gamma(\mathsf{SNR}_1) \\ R_2 \le \min\{\gamma(\mathsf{SNR}_2), \gamma(\mathsf{INR}_2) + R_0\} \\ R_1 + R_2 \le \gamma(\mathsf{SNR}_1 + \mathsf{INR}_2) + R_0 \end{array} \right. \right\} \tag{12}$$

which is equivalent to (5) by noting that

$$\gamma(\mathsf{INR}_2) + R_0 \ge \gamma(\mathsf{SNR}_2) \tag{13}$$

when $\mathsf{INR}_2 \geq \mathsf{SNR}_2$.

In the very strong interference regime where $\mathsf{INR}_2 \geq \max\{\mathsf{SNR}_2, \mathsf{INR}_2^*\}$ and $\mathsf{INR}_2^* = (1+\mathsf{SNR}_1)(2^{-2R_0}(1+\mathsf{SNR}_2)-1)$, the constraint on $R_1 + R_2$ in (5) becomes redundant and the rate region reduces to (7).

The converse in the strong and very strong interference regimes uses a technique similar to that of [1] and [7] for proving the converse for the strong interference channel without the relay link. The details are omitted due to space constraint. $\square$

2.3. Discussion

The achievability results for the classic Gaussian interference channel are categorized into the weak interference, the strong interference, and the very strong interference regimes. The result of the previous section shows that the capacity region of the Gaussian Z-interference with a relay link follows the same pattern. It is interesting to note that the relay link does not change the boundary between the weak and the strong interference regimes, while it does change the boundary between the strong and the very strong interference regimes. In other words, receiver-side relaying may potentially turn a strong interference channel into a very strong interference channel, but it never turns a weak interference channel into a strong interference channel.

It is instructive to compare the achievable regions of the Gaussian Z-interference channel with and without the relay link. In the strong and very strong interference regimes, the capacity region is achieved by transmitting common information only at X_2. In the very strong interference regime, the relay link does not increase capacity, because the interference can already be subtracted completely without the help of the relay.

In the strong interference regime, the relay link increases the capacity by helping the common information decoding at Y_1. In fact, a relay link of rate R_0 increases the sum capacity by exactly R_0 bits. Further, the increase in sum capacity can be arbitrarily divided between the two users, as long as the individual rates are below their respectively interference-free upper bound. This is because the relay link can either increase the common information rate (which improve the rate at the interference-free receiver), or increase the power of the common information component of X_2 (which decreases the noise at the interfered receiver), or increase a combination of both.

As a numerical example, Fig. 5 shows the capacity region of a Gaussian Z-interference channel in the strong interference regime with and without the relay link. The channel parameters are set to be $\mathsf{SNR}_1 = \mathsf{SNR}_2 = 25$dB, $\mathsf{INR}_2 = 30$dB. The capacity region without the relay is the dash-dotted pentagon. With $R_0 = 2$ bits, the capacity region expands to the dashed pentagon region, which represents an increase in sum rate of exactly 2 bits. As R_0 increases to 4 bits, the channel falls into the very strong interference regime. The capacity region becomes the solid rectangular region.

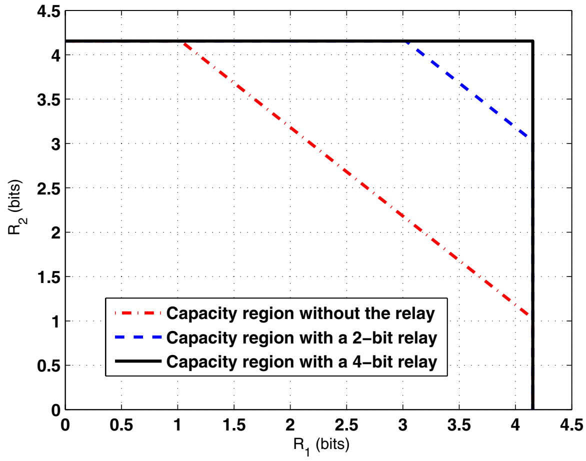

Figure 5: Capacity region of the Gaussian Z-interference channel in the strong interference regime with and without the digital relay link.

The situation in the weak interference regime is more interesting. When $\mathsf{INR}_2 \leq \mathsf{SNR}_2$, the achievable rate region (2) is obtained by a Han-Kobayashi common-private power splitting scheme. By inspection, the effect of a relay link is to shift the rate region curve upward by R_0 bits while limiting R_2 by its single-user bound $\gamma(\mathsf{SNR}_2)$. Interestingly, although the relay link of rate R_0 is provided from the receiver 2 to the receiver 1, it can help R_2 by exactly R_0 bits, while it can only help R_1 by strictly less than R_0 bits!

As a numerical example, Fig. 6 shows the achievable rate region of a Gaussian Z-interference channel in the weak interference regime with $\mathsf{SNR}_1 = \mathsf{SNR}_2 = 25$dB and $\mathsf{INR}_2 = 20$dB. The solid curve represents the rate region achieved without the relay link. The dashed rate region is with a relay rate of $R_0 = 2$ bits. For most part of the curve, R_0 can be used to provide 2-bit increase in R_2, but less than 2-bit increase in R_1.

It is illustrative to identify the optimal common-private information splitting for maximizing the sum rate. Point A corresponds to $\beta = 1$. This is where the entire X_2 is private message. As β decreases, more private message is converted into common message, which means that less interference is seen at receiver 1. As a result, R_1 increases, R_2 is kept at a constant, and the achievable rate pair moves horizontally from point A

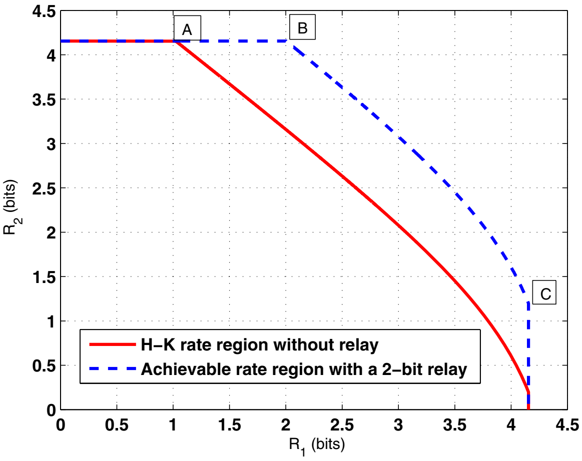

Figure 6: Achievable rate region of the Gaussian Z-interference channel in the weak interference regime with and without the digital relay link.

to the right until it reaches point B, corresponding to some β^*, which is a critical point where

$$\gamma(\mathsf{SNR}_2) = \gamma(\beta^*\mathsf{SNR}_2) + \gamma\left(\frac{\overline{\beta^*}\mathsf{INR}_2}{1+\mathsf{SNR}_1+\beta^*\mathsf{INR}_2}\right) + R_0.$$

Point B is the maximum sum-rate point with

$$\beta^* = \frac{(1+\mathsf{SNR}_1)(1+\mathsf{SNR}_2) - 2^{2R_0}(1+\mathsf{SNR}_1+\mathsf{INR}_2)}{2^{2R_0}\mathsf{SNR}_2(1+\mathsf{SNR}_1+\mathsf{INR}_2) - \mathsf{INR}_2(1+\mathsf{SNR}_2)}. \tag{14}$$

As β decreases further from β^*, more private message is converted into common message, which makes R_1 larger at the expense of R_2.

3. Asymptotic Sum Capacity

In this section, we investigate the asymptotic sum capacity of the Gaussian Z-interference channel with a relay link in the weak interference regime when

$$\min\{\mathsf{SNR}_1, \mathsf{SNR}_2, \mathsf{INR}_2\} \gg 1. \tag{15}$$

More precisely, we let noise power $N \to 0$, while keeping power constraints P_1, P_2, channel parameters h_{ij}, and the digital relay link rate R_0 fixed. In other words, $\mathsf{SNR}_1, \mathsf{SNR}_2, \mathsf{INR}_2 \to \infty$, while their ratios are kept fixed.

Denote the sum capacity of a Gaussian Z-interference channel with a relay link of rate R_0 by $C_{sum}(R_0)$. Without the digital relay link, the asymptotic high SNR/INR sum capacity of the classic Gaussian Z-interference channel in the weak interference regime can be derived using results from [8, 5] as follows:

$$\begin{aligned} C_{sum}(0) &= \gamma(\mathsf{SNR}_2) + \gamma\left(\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) \\ &\approx \frac{1}{2}\log\left(\frac{\mathsf{SNR}_2(\mathsf{SNR}_1+\mathsf{INR}_2)}{\mathsf{INR}_2}\right) \end{aligned} \tag{16}$$

where the notation $f(x) \approx g(x)$ means that $\lim f(x) - g(x) = 0$, and the limit is taken as $N \to 0$.

With a digital relay link of finite capacity R_0, how many bits can it contribute to the sum rate? Intuitively, the sum rate increase due to the relay link must be bounded by R_0. In the following, we show that in the high SNR/INR limit, the asymptotic sum capacity increase is in fact R_0 in the weak-interference regime.

Theorem 2. *For the Gaussian Z-interference channel with a relay link of capacity R_0 as shown in Fig. 1, when $\mathsf{INR}_2 \le \mathsf{SNR}_2$, the asymptotic sum capacity is given by*

$$C_{sum}(R_0) \approx C_{sum}(0) + R_0. \tag{17}$$

Proof. We first prove the achievability. The sum rate (16) is achieved at point B in Fig. 6, which corresponds to $\beta = \beta^*$ as derived in (14). In the high SNR/INR limit, we have

$$\lim_{N\to 0} \beta^* = \frac{2^{-2R_0}}{1+(1-2^{-2R_0})\frac{\mathsf{INR}_2}{\mathsf{SNR}_1}}. \tag{18}$$

Substituting this β^* into the achievable rate pair in (2), we obtain an asymptotic sum rate at point B:

$$\begin{aligned} R_{sum} &\approx \frac{1}{2}\log\left(\frac{\mathsf{SNR}_2(\mathsf{SNR}_1+\mathsf{INR}_2)}{\mathsf{INR}_2}\right) + R_0 \\ &\approx C_{sum}(0) + R_0. \end{aligned} \tag{19}$$

The converse starts with Fano's inequality. Denote the output of the relay link by V^n. Since the relay link has a capacity R_0, we have $H(V^n) \le nR_0$. Thus, for any codebook of block length n, we have

$$\begin{aligned} & n(R_1+R_2) \\ \le\ & I(X_1^n; Y_1^n, V^n) + I(X_2^n; Y_2^n) + n\epsilon_n \\ =\ & I(X_1^n; Y_1^n) + I(X_1^n; V^n|Y_1^n) + I(X_2^n; Y_2^n) + n\epsilon_n \\ \le\ & I(X_1^n; Y_1^n) + H(V^n|Y_1^n) + I(X_2^n; Y_2^n) + n\epsilon_n \\ \overset{(a)}{\le}\ & I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n) + nR_0 + n\epsilon_n \\ =\ & h(Y_1^n) - h(Z_2^n) + h(h_{22}X_2^n + Z_2^n) - \\ & h(h_{21}X_2^n + Z_1^n) + nR_0 + n\epsilon_n \\ \overset{(b)}{=}\ & h(Y_1^n) - h(Z_2^n) + h(X_2^n + \tilde{Z}_2^n) - h(X_2^n + \tilde{Z}_1^n) + \\ & n\log\frac{|h_{22}|^2}{|h_{21}|^2} + nR_0 + n\epsilon_n \end{aligned} \tag{20}$$

where (a) is due to $H(V^n|Y_1^n) \leq H(V^n) \leq nR_0$, and in (b) we introduce definitions $\tilde{Z}_1^n = Z_1^n/h_{21}$, $\tilde{Z}_2^n = Z_2^n/h_{22}$. Since Z_1^n and Z_2^n are i.i.d. Gaussian, when $\mathsf{INR}_2 \leq \mathsf{SNR}_2$ (i.e. $h_{21} \leq h_{22}$), $\tilde{Z}_1$ has a larger variance than $\tilde{Z}_2$. We can then use an extremal inequality due to Liu and Viswanath [17] to show that under an average power constraint, $h(X_2^n + \tilde{Z}_2^n) - h(X_2^n + \tilde{Z}_1^n)$ is maximized when X_2^n is an i.i.d. Gaussian random vector with distribution $\mathcal{N}(0, P_2)$. (This technique was used earlier in [3, Lemma 1].) Now, since $Y_1^n = h_{11}X_1^n + h_{21}X_2^n + Z_1^n$, $h(Y_1^n)$ is maximized when X_1^n and X_2^n are i.i.d. Gaussian. Since the same Gaussian distribution for X_1^n maximizes $h(Y_1^n)$ and $h(X_2^n + \tilde{Z}_2^n) - h(X_2^n + \tilde{Z}_1^n)$ simultaneously, it must maximize the entire right-hand side of (20). Evaluating (20) with $X_1 \sim \mathcal{N}(0, P_1)$ and $X_2 \sim \mathcal{N}(0, P_2)$, we get

$$\begin{aligned} R_1 + R_2 &\leq \gamma(\mathsf{SNR}_2) + \gamma\left(\frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_2}\right) + R_0 + \epsilon_n \\ &= C_{sum}(0) + R_0 + \epsilon_n \end{aligned} \tag{21}$$

where $\epsilon_n \to 0$ as n goes to infinity. $\square$

4. Concluding Remarks

This paper shows that a receiver-side digital relay link can significantly enlarge the achievable rate region of a Gaussian Z-interference channel. The main insight is that part of the interference may be decoded at one receiver, then forwarded to the other receiver using a binning strategy for interference subtraction. For the channel model considered in this paper, this partial-interference-forwarding technique is capacity achieving in the strong interference regime and asymptotically sum-capacity achieving in the weak interference regime.

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