Chapter 2 Problem Set Supplement: Part B

1. Suppose that $g(t) \rightleftharpoons G(f)$ is a Fourier Transform pair, and let

$$h(t) = g(t) + G(t) + g(-t) + G(-t).$$

   Show that $\mathcal{F}[h(t)] = h(f)$ (here $\mathcal{F}[\cdot]$ represents the Fourier transform).

2. We know that $\mathcal{F}[\exp(-\pi t^2)] = \exp(-\pi f^2)$. Find $\int_{-\infty}^{\infty} \exp(-a(x-m)^2)dx$, where $a > 0$ and m are real values.

3. Consider the signal $g(t) = \text{rect}(t/T)$ with $T = 1\ \mu\text{s} = 10^{-6}$ s where the rectangle function is given by:

$$\text{rect}(t) = \begin{cases} 1 & \text{for } t \leq \frac{1}{2} \\ 0 & \text{elsewhere} \end{cases}.$$

   (a) Sketch the Fourier transform of $g(t)$ denoted $G(f)$. In your sketch please indicate relevant points such as the frequency location of nulls and peak value(s).

   (b) Let $g_1(t) = g(t)\cos(2\pi f_c t)$ where $f_c = 1$ GHz $= 10^9$ Hz. Sketch $G_1(f)$ the Fourier transform of $g_1(t)$. What is its first null bandwidth (i.e., the bandwidth between 0 Hz and the first null)?

   (c) The 60 dB bandwidth is the bandwidth after which the Fourier transform is 60 dB below the peak. Assume that $G(f)$ can be approximated as $\frac{1}{fT}$. What is the 60 dB bandwidth of $g(t)$? What is the 60 dB bandwidth of $g_1(t)$?