1.7

(a) $F_{max} = 10$ kHz $\Rightarrow F_s > 2F_{max} = 20$kHz. This is based on Sampling Theorem.

(b) Since $F_s = 8$ kHZ < 20kHz, there is aliasing.
Therefore, the frequency content above the sampling rate F_s gets folded about $F_{fold} = \frac{1}{2} F_s = 4$ kHZ. So the frequency $F_1 = 5$ kHz gets folded to $F_a = 3$KHz

Note: We can also calculate the aliasing frequency F_a by using the following equation: $F_a = | k \cdot F_s - F_1 |$, where k is the integer to get minimum F_a.
e.g. in this question, $F_a = | 8 \cdot k - 5 |$, by choosing $k = 1$, we can get $F_a = 3$ kHz.

(c) Following the same method illustrated in (b), we can get the frequency $F_2 = 9$ kHz get folded to $F_a = 1$ KHz.

1.15 P_2

(a) The plot in Fig. 1 is the mirror-image of the plot in Fig. 4.
The plot in Fig. 2 is the mirror-image of the plot in Fig. 3.
When $F_0 = 3$ kHz and 4.5 kHZ, $F_s < 2F_0$, therefore this is aliasing.
$F_{fold} = \frac{F_s}{2} = 2.5$ kHz, thus $F_0 = 3$ kHz is aliased to 0.5 kHz and $F_0 = 4.5$ kHz is aliased to 2 kHz.

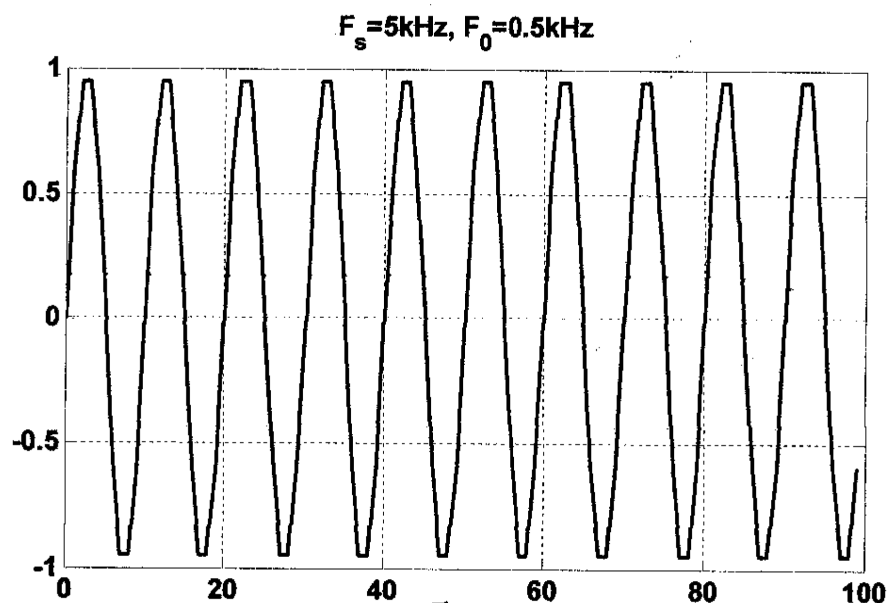


Fig. 1

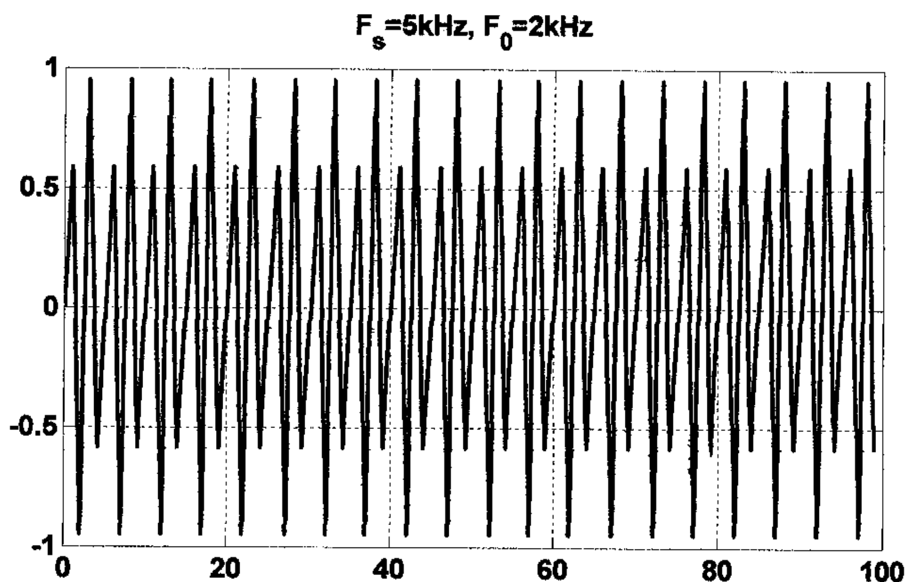
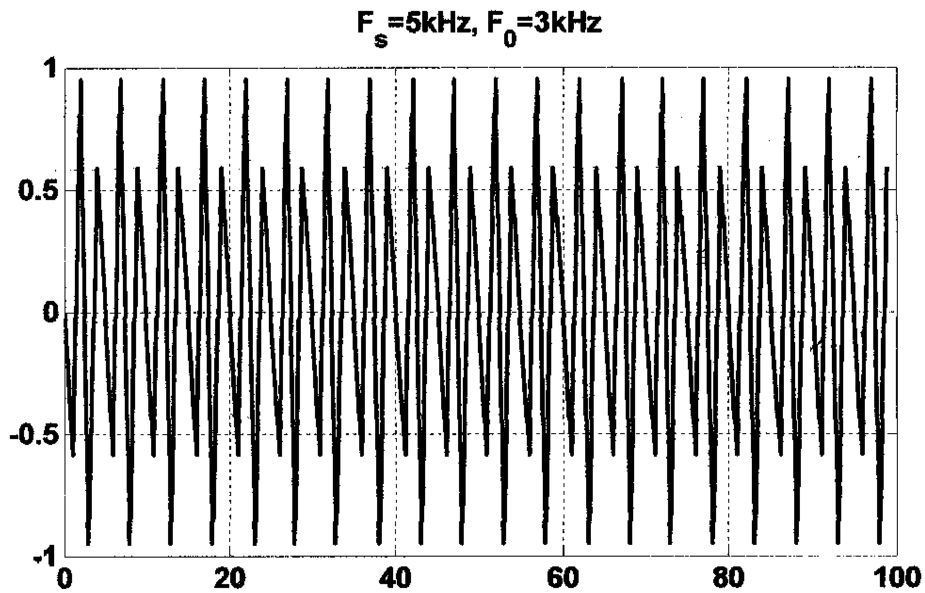
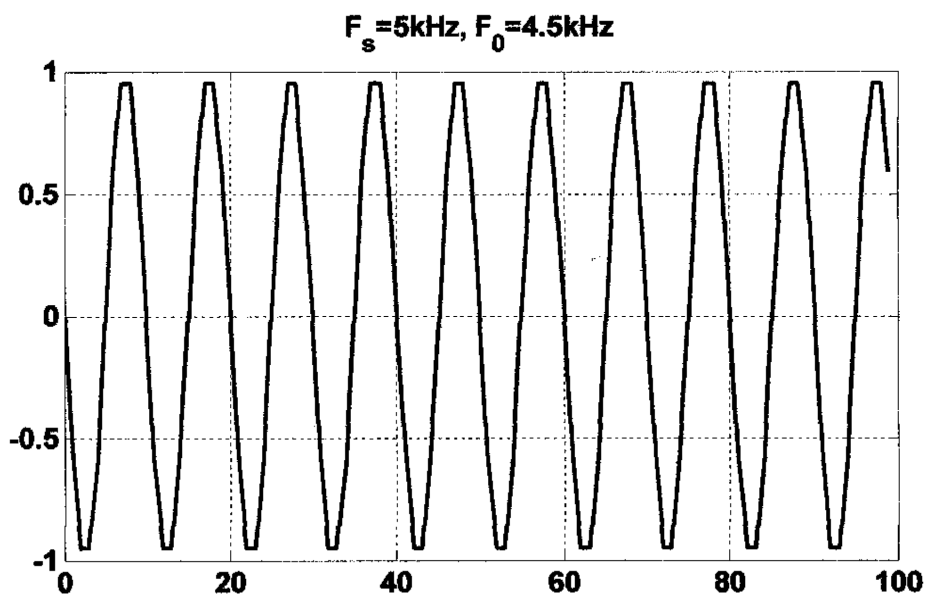


Fig. 2.



F_s=5kHz, F_0=3kHz
1
0.5
0
-0.5
-1
0
20
40
60
80
100

Fig. 3



F_s=5kHz, F_0=4.5kHz
1
0.5
0
-0.5
-1
0
20
40
60
80
100

Fig. 4

1.15

(b) 1. $x(n) = \sin(2\pi \frac{F_0}{F_s} n) = \sin(2\pi \frac{2}{50} n) = \sin(\frac{2\pi}{25} n)$

Thus $f_0 = \frac{1}{25}$ and the plot is shown in Fig. 5.

2. By taking the even numbered samples, the sampling frequency is reduced to half of F_s, i.e. 25Hz, which is still larger than the Nyquist rate $2F_0$. Thus, there is no aliasing, and $y(n)$ is still a sinusoidal signal.
$y(n) = \sin(2\pi \frac{F_0}{F_s'} n) = \sin(2\pi \frac{2}{25} n) = \sin(\frac{4\pi}{25} n)$ is plotted in Fig. 6, and its frequency is $f_0 = \frac{2}{25}$.

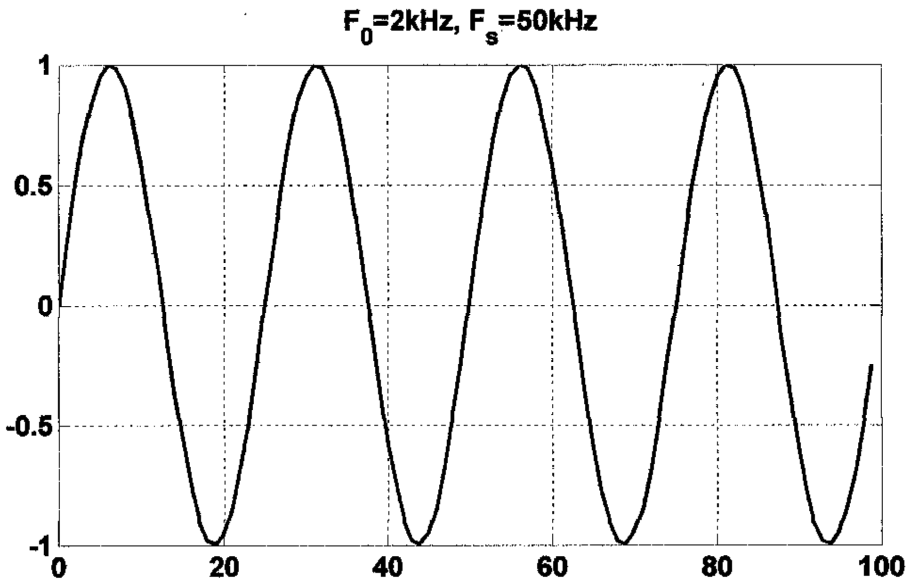


Fig. 5

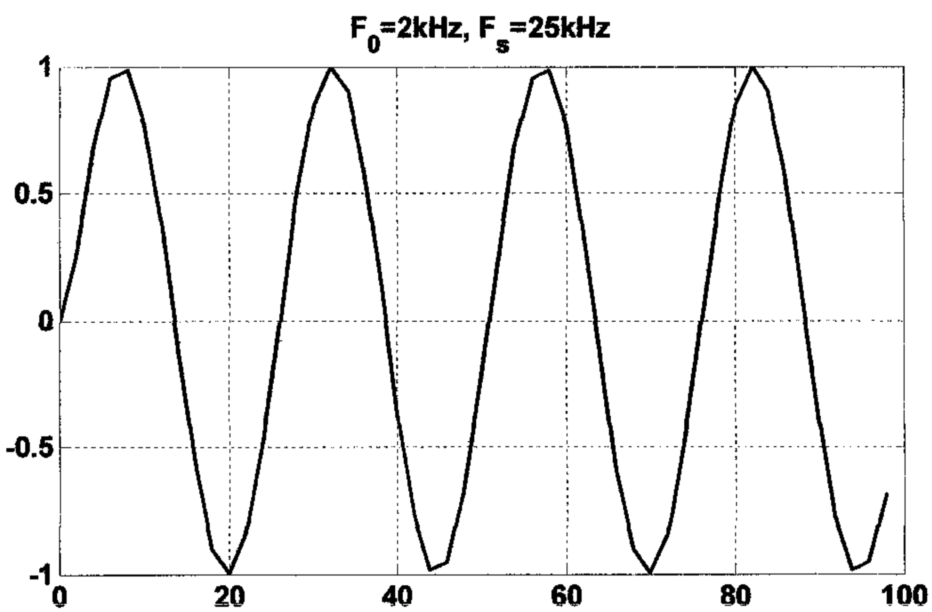


Fig. 6

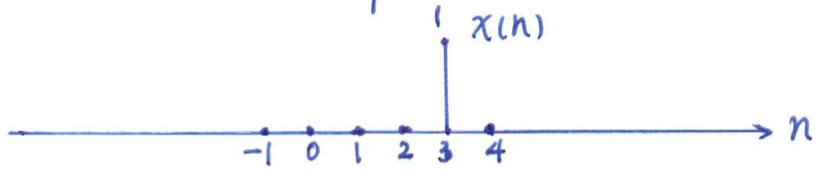
2.2

(e) $x(n-1)\,\delta(n-3)$

$$\delta(n-3) = \{\cdots \underset{\uparrow}{0}, 0, 0, 1, 0 \cdots\}$$

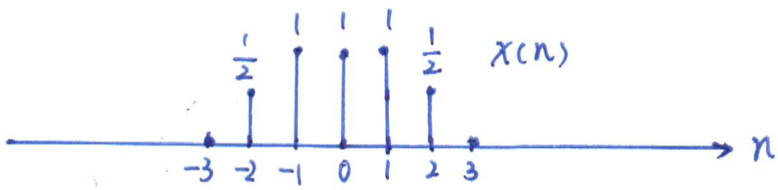
$$x(n-1) = \{\cdots 0, 0, \underset{\uparrow}{1}, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0 \cdots\}$$

$$x(n-1)\,\delta(n-3) = \{\cdots \underset{\uparrow}{0}, 0, 0, 1, 0, \cdots\}$$



(f) $x(n^2) = \{\cdots 0, x(4), x(1), x(0), x(1), x(4), 0, \cdots\}$

$$= \{\cdots 0, \tfrac{1}{2}, 1, \underset{\uparrow}{1}, 1, \tfrac{1}{2}, 0 \cdots\}$$



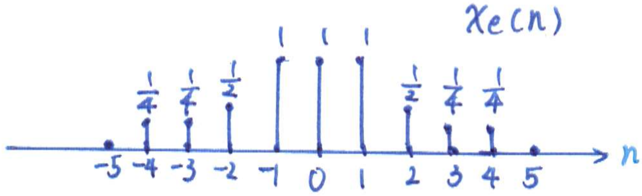
(g)(h) $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

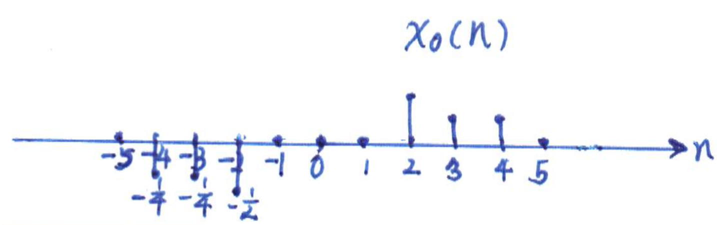
$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$x(-n) = \{\cdots 0, \tfrac{1}{2}, \tfrac{1}{2}, 1, 1, \underset{\uparrow}{1}, 1, 0, \cdots\}$$

So $x_e(n) = \{\cdots 0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}, 1, \underset{\uparrow}{1}, 1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, 0, \cdots\}$

$$x_o(n) = \{\cdots 0, -\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{2}, 0, \underset{\uparrow}{0}, 0, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, 0, \cdots\}$$





2.3

(a) $u(n) \equiv \begin{cases} 1, & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases} = \begin{cases} 1, & \text{for } n > 0 \\ 1, & \text{for } n = 0 \\ 0, & \text{for } n < 0 \end{cases}$

Thus $u(n-1) = \begin{cases} 1, & \text{for } n \geq 1 \\ 0, & \text{for } n < 1 \end{cases} = \begin{cases} 1, & \text{for } n > 0 \\ 0, & \text{for } n = 0 \\ 0, & \text{for } n < 0 \end{cases}$

Thus $u(n) - u(n-1) = \begin{cases} 0, & \text{for } n > 0 \\ 1, & \text{for } n = 0 \\ 0, & \text{for } n < 0 \end{cases} = \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{for } n \neq 0 \end{cases} = \delta(n)$

(b) $\delta(n) \equiv \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{for } n \neq 0 \end{cases}$

Thus $\sum_{k=-\infty}^{n} \delta(k) = \begin{cases} 0, & \text{for } n < 0 \\ 1, & \text{for } n \geq 0 \end{cases} = u(n)$

Let $m = n-k$, we can get $\sum_{k=0}^{\infty} \delta(n-k) = \sum_{m=-\infty}^{n} \delta(m)$.

Based on the result we get above, we have $\sum_{m=-\infty}^{n} \delta(m) = u(n)$.

Therefore $\sum_{k=0}^{\infty} \delta(n-k) = u(n)$.

2.4 (1) Let $x(n)$ be any signal, then we have:

$x(n) = \frac{1}{2}[x(n)+x(-n)] + \frac{1}{2}[x(n)-x(-n)] = x_1(n)+x_2(n)$,

where $x_1(n) = \frac{1}{2}[x(n)+x(-n)]$, and $x_2(n)=\frac{1}{2}[x(n)-x(-n)]$.

$x_1(-n) = \frac{1}{2}[x(-n)+x(n)] = x_1(n)$, therefore $x_1(n)$ is even.

$x_2(-n) = \frac{1}{2}[x(-n)-x(n)] = -\frac{1}{2}[x(n)-x(-n)]$, therefore $x_2(n)$ is odd.

Thus any signal can be decomposed into an even and an odd component.

(2) The decomposition is unique. The proof is as follows:

Assume that there exits another different decomposition $x(n)=x_3(n)+x_4(n)$ such that $x_3(n)$ is even and $x_4(n)$ is odd.

Defining $a(n) = x_3(n)-x_1(n)$, we have $x_4(n)-x_2(n) = -a(n)$, and $a(n)$ is not a zero function.

Since $x_1(n)$ and $x_3(n)$ are both even, we have as follows:

$a(-n) = x_3(-n)-x_1(-n) = x_3(n)-x_1(n) = a(n)$, thus $a(n)$ is even.

Similarly, we can get

$$
\begin{aligned}
a(-n) &= x_4(-n)-x_2(-n) \\
&= [-x_4(n)]-[-x_2(n)] \\
&= x_2(n)-x_4(n) \\
&= -[x_4(n)-x_2(n)] \\
&= -a(n).
\end{aligned}
$$

Thus $a(n)$ is odd.

Since $a(n)$ is not a zero function, it is impossible to be both even and odd. Therefore, there is a contradiction and we have proved that the decomposition is unique.

(3). $x[n] = [2,3,\underset{\uparrow}{4},5,6]$

$x[-n] = [6,5,\underset{\uparrow}{4},3,2]$

Thus the even component $x_1[n] = \frac{1}{2}[x(n)+x(-n)] = [4,4,\underset{\uparrow}{4},4,4]$,

and the odd component $x_2(n) = \frac{1}{2}[x(n)-x(-n)] = [-2,-1,\underset{\uparrow}{0},1,2]$.

2-7. P_7

(a) $y(n) = \cos[x(n)]$

(1) Since $y(n)$ depends only on $x(n)$, it is static.

(2) Since $\cos[a_1 x_1(n)] \neq a_1 \cos[x(n)]$ where $a_1 \in \mathbb{R}$, we can get $y(n)$ is not Homogeneity, and thus it is nonlinear.

(3) $\cos[x(n-k)] = y(n-k)$. Thus it is time-invariant.

(4) Since $y(n)$ only depends on present input, it is causal.

(5) Since $|\cos[x(n)]| \leq 1$, $y(n)$ is always bounded. Thus $y(n)$ is stable.

(b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

(1) Since $y(n)$ does not only depend on $x(n)$, it is dynamic.

(2)
$$
\begin{aligned}
T[a_1 x_1(n) + a_2 x_2(n)] &= \sum_{k=-\infty}^{n+1} [a_1 x_1(k) + a_2 x_2(k)] \\
&= \sum_{k=-\infty}^{n+1} [a_1 x_1(k)] + \sum_{k=-\infty}^{n+1} [a_2 x_2(k)] \\
&= a_1 \sum_{k=-\infty}^{n+1} [x_1(k)] + a_2 \sum_{k=-\infty}^{n+1} [x_2(k)] \\
&= a_1 y_1(n) + a_2 y_2(n) \\
&= a_1 T[x_1(n)] + a_2 T[x_2(n)]
\end{aligned}
$$
Thus $y(n)$ is linear.

(3) $T[x(n-m)] = \sum_{k=-\infty}^{n+1} x(k-m) = y(n-m)$. Thus $y(n)$ is time-invariant.

(4) Since $y(n)$ also depends on the future input $x(n+1)$, it is noncasual.

(5) $y(n)$ is unstable.

Counter-example: $x(n) = u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0. \end{cases}$ is bounded.

$y(n) = \sum_{k=-\infty}^{n+1} x(k) = \sum_{k=-\infty}^{n+1} u(k) = \begin{cases} 0, & \text{for } n < -1 \\ n+2, & \text{for } n \geq -1. \end{cases}$ is unbounded.

(c) $y(n) = x(n)\cos(\omega_0 n)$

(1) Static

(2)
$$
\begin{aligned}
T[a_1 x_1(n) + a_2 x_2(n)] &= [a_1 x_1(n) + a_2 x_2(n)]\cos(\omega_0 n) \\
&= a_1[x_1(n)\cos(\omega_0 n)] + a_2[x_2(n)\cos(\omega_0 n)] \\
&= a_1 T[x_1(n)] + a_2 T[x_2(n)]
\end{aligned}
$$

Thus $y(n)$ is linear.

2.7 (c) $y(n) = x(n)\cos(w_0 n)$

(3) $T[x(n-k)] = x(n-k)\cos(w_0 n) \neq y(n-k)$
Thus $y(n)$ is time-varying.

(4) Since $y(n)$ only depends on the present input, it is causal.

(5) Let $x(n)$ is bounded by A, where $0 \leq A < \infty$, i.e. $|x(n)| \leq A$.
Thus $|y(n)| = |x(n)\cos(w_0 n)|$

$$= |x(n)|\,|\cos(w_0 n)|$$

$$\leq A\,|\cos(w_0 n)|$$

Since $|\cos(w_0 n)| \leq 1$, we have $|y(n)| \leq A$.
Since $y(n)$ is also bounded, it is stable.

(d) $y(n) = x(-n+2)$

(1) Dynamic. (2) Linear

(3) $T[x(n-k)] = x[-n+2-k] \neq y(n-k)$, thus $y(n)$ is time-varying

(4) noncausal, counter example: for $n=0$, $y(0) = x(2)$, thus $y(0)$ depends on the future input $x(2)$.

(5) Stable.

(e) $y(n) = \mathrm{Trun}[x(n)]$

(1) Static. (3) Time-invariant (4) Causal.

(2) nonlinear. Counter-example: $x_1(n) \equiv 1.6$, $x_2(n) \equiv 0.1$, $a_1 = 2$, $a_2 = 1$.
Thus $T[a_1x_1(n) + a_2x_2(n)] = \mathrm{Trun}[2\times 1.6 + 1\times 0.1] = \mathrm{Trun}[3.4] = 3$

$$a_1T[x_1(n)] + a_2T[x_2(n)] = 2\cdot\mathrm{Trun}[1.6] + 1\cdot\mathrm{Trun}[0.1]$$

$$= 2\times 1 + 1\times 0 = 2$$

Thus $T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$

(3) Let $x(n)$ is bounded by $0 \leq A < \infty$, we have $|x(n)| \leq A$
Thus $y(n) = \mathrm{Trun}[x(n)] \leq \mathrm{Trun}[A] \leq A$, we can get $y(n)$ is stable.

2.7 Pg

(f) $y(n) = Round[x(n)]$

(1) Static (3) time-invariant (4) Causal.

(2) Nonlinear, counter-example: $x_1(n) \equiv 1.6$, $x_2(n) \equiv 0.1$, $a_1 = 2$, $a_2 = 1$

$$T[a_1x_1(n) + a_2x_2(n)] = Round[a_1x_1(n) + a_2x_2(n)]$$
$$= Round[2\times1.6 + 0.1] = Round[3.3] = 3$$

$$a_1T[x_1(n)] + a_2T[x_2(n)] = a_1Round[x_1(n)] + a_2Round[x_2(n)]$$
$$= 2Round[1.6] + 1\cdot Round[0.1]$$
$$= 2\times2 + 1\times0 = 4$$

Thus $T[a_1x_1(n) + a_2x_2(n)] \neq a_1T[x_1(n)] + a_2T[x_2(n)]$.

(5) Let $|x(n)| \leq A$, where $0 \leq A < \infty$

$|y(n)| = |Round[x(n)]| \leq |Round[A]| \leq A+1$

Thus $y(n)$ is stable.

(g) $y(n) = |x(n)|$

(1) Static (3) time-invariant (4) Causal (5) Stable

(2) Nonlinear. Counter-example: $x_1(n) \equiv -2$, $x_2(n) \equiv 3$, $a_1 = 5$, $a_2 = 3$

Then $T[a_1x_1(n) + a_2x_2(n)] = |a_1x_1(n) + a_2x_2(n)| = |5\times(-2) + (3)\times3|$
$$= 1$$

$$a_1T[x_1(n)] + a_2T[x_2(n)] = a_1|x_1(n)| + a_2|x_2(n)|$$
$$= 5\times|-2| + 3\times|3|$$
$$= 19$$

Thus $T[a_1x_1(n) + a_2x_2(n)] \neq a_1T[x_1(n)] + a_2T[x_2(n)]$

(h) $y(n) = x(n)u(n) = \begin{cases} 0, & \text{for } n<0 \\ x(n), & \text{for } n \geq 0 \end{cases}$

(1) Static (2) Linear (3) time-invariant (4) Causal (5) Stable.

2.7 (i) $y(n) = x(n) + n\,x(n+1)$

(1) Dynamic (2) Linear (4) noncausal

(3) $T[x(n-k)] = x(n-k) + n\,x(n+1-k)$

$$= [x(n-k) + (n-k)\,x(n+1-k)] + k\,x(n+1-k)$$
$$= y(n-k) + k\,x(n+1-k)$$

Thus $T[x(n-k)] \neq y(n-k)$, $y(n)$ is time-variant.

(5) Unstable. Counter-example. $x(n) = u(n)$ is bounded

$$y(n) = x(n) + n\,x(n+1) = u(n) + n\,u(n+1) = \begin{cases} n+1, & \text{for } n \geq 0 \\ n, & \text{for } n = -1 \\ 0, & \text{for } n < -1 \end{cases}$$

Thus $y(n)$ is unbounded.

(j) $y(n) = x(2n)$

(1) Dynamic (2) Linear (4) noncausal (5) Stable

(3) $T[x(n-k)] = x(2n-k)$

$y(n-k) = x[2(n-k)] = x(2n-2k) \neq T[x(n-k)]$

Thus $y(n)$ is time variant.

(k) $y(n) = \begin{cases} x(n) & \text{if } x(n) \geq 0 \\ 0 & \text{if } x(n) < 0 \end{cases}$

(1) Static (3) Time-invariant (4) Causal (5) Stable.

(2) Nonlinear. Counter-example: $x_1(n) \equiv 6$, $x_2(n) \equiv -2$, $a_1 = 1$, $a_2 = 2$

Thus $T[a_1 x_1(n) + a_2 x_2(n)] = T[6 \times 1 + (-2) \times 2] = T[2] = 2$

$a_1 T[x_1(n)] + a_2 T[x_2(n)] = 1 \times T[6] + 2 \times T[-2] = 6.$

$T[a_1 x_1(n) + a_2 x_2(n)] \neq a_1 T[x_1(n)] + a_2 T[x_2(n)]$

2.7 (l) $y(n) = x(-n)$

(1) Dynamic, (2) Linear (3) Time-invariant (5) Stable

(4) Noncausal, counter-example: Let $n=-1$, then $y(-1) = x(1)$
$y(-1)$ depends on the future input $x(1)$

(m) $y(n) = \mathrm{sign}[x(n)] = \begin{cases} -1 & \text{if } x(n) < 0 \\ 0 & \text{if } x(n) = 0 \\ 1 & \text{if } x(n) > 0 \end{cases}$

(1) Static (3) Time-invariant

(4) Causal (5) Since $|y(n)| \le 1$, $y(n)$ is stable.

(2) Nonlinear. Counter example: $x_1(n) = u(n)$, $x_2(n) = u(n)$, $a_1 = 2$, $a_2 = 3$.
Thus $\mathcal{T}[a_1 x_1(n) + a_2 x_2(n)] = \mathrm{sign}[5u(n)] = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{if } n \geq 0 \end{cases}$

$a_1 \mathcal{T}[x_1(n)] + a_2 \mathcal{T}[x_2(n)] = 2\,\mathrm{sign}[u(n)] + 3\,\mathrm{sign}[u(n)]$

$= 5\,\mathrm{sign}[u(n)] = \begin{cases} 0 & \text{if } n < 0 \\ 5 & \text{if } n \geq 0 \end{cases}$

Thus $\mathcal{T}[a_1 x_1(n) + a_2 x_2(n)] \neq a_1 \mathcal{T}[x_1(n)] + a_2 \mathcal{T}[x_2(n)]$.

(n) $\mathcal{T}[x_a(t)] = x_a(nT)$

(1) Static (2) Linear (3) time-invariant (4) causal (5) Stable