

# Capacity of Fading Channels Under Spectrum-Sharing Constraints

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**Abstract**—Traditionally, frequency spectrum is licensed to users by government agencies in a fixed manner where licensee has exclusive right to access the allocated band. This policy has been de jure practice to protect systems from mutual interference for many years. However, with increasing demand for the spectrum and scarcity of vacant bands, a spectrum policy reform seems inevitable. On the other hand, recent measurements suggest the possibility of sharing spectrum among different parties subject to interference-protection constraints. In this paper we study spectrum-sharing between a primary licensee and a secondary user in a fading environment. The intuition is that channel fluctuations provide secondary transmitter with opportunities to access the licensed spectrum without inflicting an intolerable interference on the licensee. Capacity of fading channel for secondary user is derived subject to two different spectrum-sharing constraints. We further evaluate and compare the capacity under different fading distributions. Results indicate a significant gain in spectrum access in fading environments.

## I. INTRODUCTION

Recent years have witnessed a dramatic increase in the demand for radio spectrum. This is partly due to the increasing interest of consumers in wireless services which in turn is driving the evolution of wireless networks toward high-speed data networks. With the emergence of new applications and the compelling need for mobile Internet access, demand for the spectrum is expected to grow even more tremendously in the coming years.

Spectrum is inherently a limited natural resource access to which is regulated by government agencies such as Federal Communications Commission (FCC) in the United States. Traditional approach to spectrum management is very inflexible in the sense that frequency bands are exclusively licensed to users and tight limits on the allowed transmitted power are set in order to shield systems from mutual interference all the time. However, with most of the spectrum being already allocated, it is becoming exceedingly hard to find vacant bands to either deploy new services or to enhance the existing ones. On the other hand, recent measurements by Spectrum Policy Task Force (SPTF) within FCC indicate that many portions of the spectrum are either unused or lightly-used for significant periods of time [1]. This finding suggests that currently spectrum scarcity is largely due to the inefficient and rigid regulations rather than physical shortage of the spectrum.

FCC's initiative along with several other projects including Defense Advanced Research Projects Agency (DARPA)'s "Next Generation" (XG) program [2] and national science foundation's "NeTS-ProWiN" project [3] signal a paradigm shift in the spectrum access policy. This raises several technical and regulatory issues to be addressed by the academia as well as the policy-makers. Interested reader is referred to [4]- [6] for general overview of issues associated with the spectrum access policy reform.

In its report to the commission, SPTF proposed secondary access to already-licensed spectrum as a means to mitigate spectrum shortage. However, such *spectrum-sharing* should be carried out in a controlled fashion so that primary licensee's operation in the band is not compromised. Therefore, a secondary user trying to access the licensed spectrum should consider the impact of its transmission on the reception quality of the primary licensee. To this end, *interference temperature* concept has been introduced in [1] which indicates the tolerable interference level at the primary licensee's receiver. From licensee's point of view, secondary access does not affect its operation as long as the total interference power at its receiver remains below a certain threshold. Therefore, power emitted from a secondary transmitter need not be limited as long as the interference inflicted on the primary receiver is below the threshold.

Almost all prior studies on channel capacity assume constraints on the transmitted power. Such constraints are dictated by either hardware limitations or rigid spectrum regulations. In light of the recent developments in spectrum access policy reform and motivated by the interference temperature concept, a constraint on the received-power at the primary receiver seems much more appropriate within the spectrum-sharing context.

Channel capacity under received-power constraint has been first considered by Gastpar [8] who derived the capacity of different AWGN channels with average received-power at a third-party receiver (e.g. licensee's receiver) being constrained. In absence of fading, it was shown that transmitted and received power constraints give rise to very similar capacity formulas for point-to-point AWGN channels [8]. This is due to the fact that for such non-varying channels, received-power at a third-party receiver is merely a scaled version of the

transmitted power. However, this is not the case when channels vary due to fading.

As we shall illustrate later, with the same limit on the received-power level, channel capacity in severe fading (e.g. Rayleigh or Log-normal fading) exceeds that of the non-fading AWGN channel. However, with transmitted-power being constrained instead, capacity is lower in presence of fading (except at low signal-to-noise-ratios) [9]. This discrepancy is due to the involvement of two (rather than one) fading channel-gains in computing the capacity with received-power constraint. In a nutshell, when channel gain of the user toward the third-party receiver is low due to fading, user may *opportunistically* transmit at very high power levels. In severe fading there is a higher likelihood for this channel to be in a deep fade thereby increasing the spectrum access opportunity for the secondary transmitter.

We quantify this opportunistic spectrum access gain by evaluating the capacity of a fading channel under spectrum-sharing constraint. This constraint in turn is imposed on either average or peak received-power at a third-party receiver, modelling different situations where receiver's operation is limited by either average or instantaneous interference level.

We study both of the above scenarios by formulating the channel capacity as a maximization problem and solving for the optimum solutions in both settings. In each case, closed-form capacity formulas under different fading channels are also provided where possible. Our results suggest that a significant spectrum access gain may be achieved in fading environments.

Remainder of this paper is organized as follows. In the next section we describe our system model and main assumptions. Capacity under average and peak received-power constraints will be derived in sections 3 and 4 respectively. Extension of the results to the scenario with multiple primary receivers will be discussed in section 5. Finally, concluding remarks are provided in section 6.

## II. SYSTEM MODEL

We consider a point-to-point AWGN fading channel with perfect channel side information (CSI) available to both receiver and transmitter. The capacity of this channel is well-known under both peak and average input power constraints [9], [10]. However, instead of a constraint on the transmitted power, we assume a constraint on the power received at a third-party receiver [8]. As stated earlier, this model is mainly motivated by the interference temperature concept which allows a secondary user to operate within a licensee's spectrum as long as the power received at licensee's receiver does not exceed a certain threshold,  $Q$ .

Let  $g_0$  and  $g_1$  denote the instantaneous channel gains from the secondary transmitter to primary and secondary receivers respectively as shown in Fig. 1. Both gains are assumed to be stationary and ergodic with probability density functions (pdf)  $f_0(g_0)$  and  $f_1(g_1)$  respectively. Without loss of generality, both channel gains are assumed to be unit-mean<sup>1</sup>. Since the

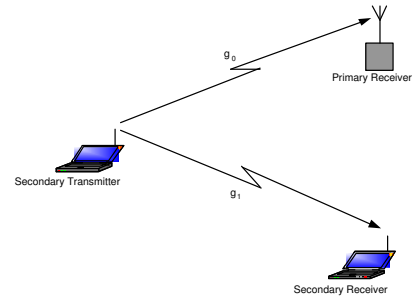


Fig. 1. A secondary user sharing the spectrum with the primary user

constraint is imposed on the received power, transmitter should have perfect knowledge of  $g_0$  as well as  $g_1$  (i.e. a two-dimensional CSI). Thus, perfect CSI assumption in this context is different from that of [9] which only includes  $g_1$ . Feeding back of  $g_0$  to secondary transmitter may be carried out directly by the licensee or indirectly through a *band manager* which mediates between the two parties [4].

We consider two different spectrum-sharing constraints, namely peak and average received-power constraints. On the other hand, for clarity of presentation no constraints on the transmitted power are assumed. In practice, transmitted power is limited by hardware capabilities. Thus, capacities derived in this paper serve as upper-bounds for the capacity of fading channels under received-power constraints.

## III. CAPACITY UNDER AVERAGE RECEIVED-POWER CONSTRAINT

Average received-power constraint has been considered in [8] where author derives the capacity for different AWGN channels. In particular for a single AWGN channel, capacity formula was shown to be very similar to the case with average transmitted-power being constrained. This may be attributed to the fact that in AWGN case, received power is merely a *deterministic* scaled version of the transmitted power. Thus, a constraint on the received-power is fundamentally equivalent to one on the transmitted-power in this setting. However, as we shall demonstrate shortly, this result does not generalize to the fading AWGN channel. Now, assuming  $g_0$  and  $g_1$  are independent<sup>2</sup>, the capacity of the channel is given by [9],

$$C = \max_{P(g_0, g_1) \geq 0} \iint B \log \left( 1 + \frac{g_1 P(g_0, g_1)}{N_0 B} \right) \times f_1(g_1) f_0(g_0) dg_1 dg_0 \quad (1)$$

$$\text{s.t.} \quad \int_{g_0} \int_{g_1} g_0 P(g_0, g_1) f_1(g_1) f_0(g_0) dg_1 dg_0 \leq Q \quad (2)$$

where  $Q$  is the maximum average power that the primary user can tolerate at its receiver,  $P$  and  $B$  are the transmitted power and total available bandwidth respectively and finally  $N_0$  is the power spectral density of the AWGN noise at the secondary receiver. Note that transmitted power,  $P$ , is in general a mapping from the joint fading state  $(g_0, g_1)$  to the set of non-negative real numbers,  $\mathbb{R}_+$ . Thus, optimization is over

<sup>1</sup>Mean of  $g_0$  and  $g_1$  are captured into other parameters as discussed later on.

<sup>2</sup>For the general non-independent case,  $f_1(g_1)f_0(g_0)$  should be replaced by the joint pdf of  $g_0$  and  $g_1$ .

the set of all such mappings having an average received-power of at most  $Q$  at the primary receiver.

The capacity defined by (1)-(2) is equivalent to the capacity of an AWGN fading channel with gain  $g_1/g_0$  and transmitted power equal to  $g_0P$ , for which the channel coding theorem and the converse have been proved in [9]. Thus,  $C$  defined by (1)-(2) is indeed the capacity of an AWGN fading channel under average received-power constraint.

To find the optimal power allocation,  $P(g_0, g_1)$ , we form the Lagrangian,

$$L(P, \lambda) = \iint \log \left( 1 + \frac{g_1 P(g_0, g_1)}{N_0 B} \right) f_1(g_1) f_0(g_0) dg_1 dg_0 - \lambda \left( \iint g_0 P(g_0, g_1) f_1(g_1) f_0(g_0) dg_1 dg_0 - Q \right) \quad (3)$$

For the optimization problem defined in (1)-(2), the first-order Kuhn-Tucker conditions are necessary and sufficient for optimality [7]. Thus, optimal power allocation should satisfy the following,

$$\frac{\partial L(P, \lambda)}{\partial P} = \left( \frac{g_1}{g_1 P(g_0, g_1) + N_0 B} - \lambda g_0 \right) f_1(g_1) f_0(g_0) = 0 \quad (4)$$

Solving (4) with the constraint that  $P(g_0, g_1) \geq 0$  yields,

$$P(g_0, g_1) = \left( \frac{1}{\lambda_0 g_0} - \frac{N_0 B}{g_1} \right)^+ \quad (5)$$

where  $(\cdot)^+$  denotes  $\max\{\cdot, 0\}$  and  $\lambda_0$  is determined such that the average received power in (2) is equal to  $Q$  i.e.,

$$\int_{g_0} \int_{g_1} \left( \frac{1}{\lambda_0} - N_0 B \frac{g_0}{g_1} \right)^+ f_1(g_1) f_0(g_0) dg_1 dg_0 = Q \quad (6)$$

The optimal power allocation obtained in (5) is in accordance with our intuition. That is, more transmission power is used when either  $g_1$  increases (cf. [9]) or  $g_0$  decreases which is a manifestation of the two-fold opportunistic spectrum access gain in fading channels. Substituting (5) in (1) results in the following formula for channel capacity,

$$C = \int_{\frac{g_1}{g_0} \geq \frac{1}{\gamma_0}} \int B \log \left( \gamma_0 \frac{g_1}{g_0} \right) f_1(g_1) f_0(g_0) dg_1 dg_0 = \int_{\frac{1}{\gamma_0}}^{\infty} B \log(\gamma_0 g_{10}) f_{\frac{g_1}{g_0}}(g_{10}) dg_{10} \quad (7)$$

where  $g_{10}$  and  $f_{\frac{g_1}{g_0}}(\cdot)$  denote the random variable  $g_1/g_0$  and its pdf respectively and  $\gamma_0 = 1/(\lambda_0 N_0 B)$ . It can be seen from (6) that  $\lambda_0$  (and thus  $\gamma_0$ ), depends on the joint channel statistics only through the first-order pdf of  $g_1/g_0$ ,  $f_{\frac{g_1}{g_0}}(\cdot)$ .

In what follows, we shall study the effect of fading characteristics on the gain of opportunistic spectrum access by evaluating the capacity in (7) under different fading statistics.

#### A. AWGN Channel

When there is no channel fading,  $g_0$  and  $g_1$  are equal to 1 all the time (recall that we have assumed unit-mean channel

gains). In this case  $P = Q$  and channel capacity is simply given by,

$$C_{awgn} = B \log(1 + \alpha) \quad (8)$$

where<sup>3</sup>  $\alpha = Q/N_0 B$  is the average signal-to-noise-ratio (SNR) for the AWGN channel.

#### B. Log-normal Shadowing

Empirical measurements suggest that medium-scale variations of the received-power, when represented in dB units, follow a normal distribution (see e.g. [13]). In other words, the linear (as opposed to dB) channel gain may be modelled by a log-normal random variable (r.v.)  $e^X$  where  $X$  is a zero-mean Gaussian r.v. with variance  $\sigma^2$ . Log-normal shadowing is usually characterized in terms of its dB spread which is related to  $\sigma$  by  $\sigma = 0.1 \log(10) \sigma_{dB}$ .

Now let's assume  $g_0 = e^{X_0}$  and  $g_1 = e^{X_1}$ , where  $X_0$  and  $X_1$  are two independent zero-mean Gaussian random variables with equal variance of  $\sigma^2$ . Then  $g_1/g_0 = e^Y$  is a log-normal r.v. with  $Y = X_1 - X_0$  being a zero-mean Gaussian r.v. with a variance of  $2\sigma^2$ . Now rewriting (6) using  $g_1/g_0 = e^Y$ ,

$$\int_{-\infty}^{\log \gamma_0} \frac{(\gamma_0 - e^y)}{2\sqrt{\pi}\sigma} e^{-\frac{y^2}{4\sigma^2}} dy = \frac{\gamma_0}{2} \left[ \operatorname{erf} \left( \frac{\log \gamma_0}{2\sigma} \right) + 1 \right] - \frac{e^{\sigma^2}}{2} \left[ \operatorname{erf} \left( \frac{\log \gamma_0}{2\sigma} - \sigma \right) + 1 \right] = \alpha \quad (9)$$

where  $\alpha = Q/N_0 B$  as defined before and  $\operatorname{erf}(x)$  is the error function defined as  $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ . Thus, for a given  $\alpha$ ,  $\gamma_0(\alpha)$  may be obtained from (9) and capacity is given by,

$$C_{\lognormal} = \int_{-\log \gamma_0(\alpha)}^{\infty} B \log(\gamma_0(\alpha) e^y) \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{y^2}{4\sigma^2}} dy = B \left[ \frac{\log \gamma_0(\alpha)}{2} \operatorname{erf} \left( \frac{y}{2\sigma} \right) - \frac{\sigma e^{-\frac{y^2}{4\sigma^2}}}{\sqrt{\pi}} \right]_{-\log \gamma_0(\alpha)}^{\infty} = B \left[ \frac{\log \gamma_0(\alpha)}{2} \left( 1 + \operatorname{erf} \left( \frac{\log \gamma_0(\alpha)}{2\sigma} \right) \right) + \frac{\sigma}{\sqrt{\pi}} e^{-\frac{\log^2 \gamma_0(\alpha)}{4\sigma^2}} \right] \quad (10)$$

Capacity under log-normal fading for different values of  $\sigma_{dB} = 10\sigma/\log(10)$  is shown in Fig. 2. Results indicate that the capacity grows with increasing variance of the log-normal fading. This is mainly due to the higher probability of  $g_0$  being in deep fades when  $\sigma^2$  becomes larger, thereby allowing very high transmit power levels. Moreover, with a dB spread of 12 dB, capacity under log-normal fading is consistently higher than that of the AWGN channel (cf. [9]).

#### C. Rayleigh Fading

In this case, both  $g_0$  and  $g_1$  are exponentially distributed with unit-mean. Thus, it may be shown that  $g_1/g_0$  has a log-logistic distribution defined as follows,

$$f_{\frac{g_1}{g_0}}(x) = \frac{1}{(1+x)^2}, \quad x \geq 0 \quad (11)$$

<sup>3</sup>Mean of  $g_0$  and  $g_1$  may be captured in  $Q$  and  $N_0 B$  respectively

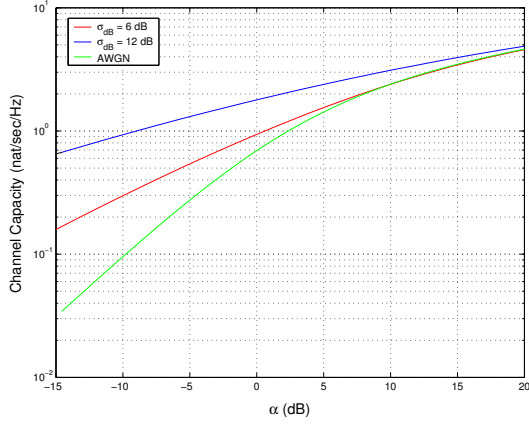


Fig. 2. Capacity under log-normal fading and average received-power constraint vs.  $\alpha$

Now  $\gamma_0$  may be obtained by combining (6) and (11) as follows. Recall that  $g_0$  and  $g_1$  are independent and identically distributed. Therefore,  $g_0/g_1$  and  $g_1/g_0$  have the same pdf. Now, rewriting (6) using the pdf of  $g_0/g_1$  given in (11) yields,

$$\begin{aligned} \int_0^{\gamma_0} (\gamma_0 - x) \frac{1}{(1+x)^2} dx &= \left[ -\frac{1+\gamma_0}{1+x} - \log(1+x) \right]_0^{\gamma_0} \\ &= \gamma_0 - \log(1+\gamma_0) \\ &= \frac{Q}{N_0 B} \end{aligned}$$

Thus  $\gamma_0$  satisfies the following,

$$\gamma_0 - \log(1+\gamma_0) = \alpha \quad (12)$$

It is worth noting that under average received-power constraint and Rayleigh fading,  $\gamma_0$  is determined from (12) which obviates the need for numerical integration (cf. [9]). Now from (7) channel capacity may be written as follows,

$$\begin{aligned} C_{\text{rayleigh}} &= \int_{\frac{1}{\gamma_0(\alpha)}}^{\infty} B \log(\gamma_0(\alpha)x) \frac{1}{(1+x)^2} dx \\ &= B \left[ \log x - \log(1+x) - \frac{\log(\gamma_0 x)}{1+x} \right]_{\frac{1}{\gamma_0(\alpha)}}^{\infty} \quad (13) \\ &= B \log(1+\gamma_0(\alpha)) \end{aligned}$$

where  $\gamma_0(\alpha)$  is the solution of (12) for a given  $\alpha$ . It can be seen from (12) that  $\gamma_0(\alpha) > \alpha$  for all  $\alpha > 0$ . Therefore,

$$C_{\text{rayleigh}} = B \log(1+\gamma_0(\alpha)) > B \log(1+\alpha) = C_{\text{awgn}}$$

Thus, under average received power constraint, capacity of a Rayleigh fading channel is greater than that of the pure AWGN channel for all values of the average SNR,  $\alpha$ . However, with average transmitted power being constrained, this statement holds only at low SNR levels [9].

#### D. Nakagami Fading

A more versatile fading model is the Nakagami- $m$  distribution which better fits a wide range of empirical data by adjusting a single parameter. This parameter,  $m$ , measures the

ratio of line-of-sight (LOS) signal power to that of the multipath component. For a unit-mean channel gain, the distribution is given by,

$$f_{\sqrt{g}}(y) = \frac{2m^m y^{2m-1}}{\Gamma(m)} e^{-my^2}, \quad m \geq 0.5 \quad (14)$$

where  $\Gamma(\cdot)$  is the gamma function defined as,  $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$ . It can be seen that Rayleigh distribution is a special case of the Nakagami distribution with  $m = 1$ .

For a Nakagami fading channel, channel power gain  $g$  is distributed according to the following Gamma distribution,

$$f_g(x) = \frac{m^m x^{m-1}}{\Gamma(m)} e^{-mx}, \quad x \geq 0 \quad (15)$$

With  $g_0$  and  $g_1$  distributed according to (15) with their  $m$  parameter being  $m_0$  and  $m_1$  respectively,  $g_1/g_0$  may be shown to have the following probability distribution (see e.g. [14]),

$$f_{\frac{g_1}{g_0}}(x) = \left( \frac{m_0}{m_1} \right)^{m_0} \frac{x^{m_1-1}}{\mathcal{B}(m_0, m_1) \left(x + \frac{m_0}{m_1}\right)^{m_0+m_1}}, \quad x \geq 0 \quad (16)$$

where  $\mathcal{B}(a, b)$  is the beta function defined as,

$$\mathcal{B}(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

This distribution is known as the beta-prime distribution or beta distribution of the second kind (see p. 248 of [11]). When  $m_0 = m_1 = m$ , (16) reduces to,

$$f_{\frac{g_1}{g_0}}(x) = \frac{x^{m-1}}{\mathcal{B}(m, m) (x+1)^{2m}}, \quad x \geq 0 \quad (17)$$

Applying (6),  $\gamma_0(\alpha)$  is the solution to the following equation,

$$\frac{1}{\mathcal{B}(m, m)} \int_0^{\gamma_0} (\gamma_0 - x) \frac{x^{m-1}}{(1+x)^{2m}} dx = \alpha \quad (18)$$

and channel capacity is given by,

$$C_{\text{nakagami}} = B \int_{\frac{1}{\gamma_0}}^{\infty} \frac{\log(\gamma_0(\alpha)x) x^{m-1}}{\mathcal{B}(m, m) (1+x)^{2m}} dx, \quad m \geq 0.5 \quad (19)$$

For general  $m$ , (18) and (19) do not admit simple closed-form formulas and capacity should be evaluated numerically. However, for  $m = 2$ , some manipulation of (6) yields the following relationship between  $\alpha$  and  $\gamma_0$ ,

$$\frac{\gamma_0^3}{(1+\gamma_0^2)} = \alpha \quad (20)$$

By substituting the  $\gamma_0$  satisfying (20), denoted by  $\gamma_0(\alpha)$ , into (19), capacity may be derived as below,

$$C_{\text{nakagami}} = B \left[ \log(1+\gamma_0(\alpha)) - \frac{\gamma_0(\alpha)}{(1+\gamma_0(\alpha))^2} \right] \quad (21)$$

Fig. 3 compares channel capacity under average received-power constraint for different fading distributions as obtained in (8), (13) and (21). Evidently capacity under Rayleigh fading is higher than that of AWGN channel for all values of  $\alpha$  (cf. [9]). For less severe fading channels, capacity reduces and may fall below AWGN channel's capacity at relatively higher values of  $\alpha$ .

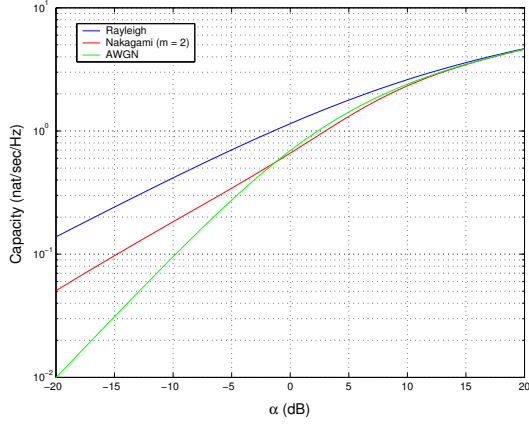


Fig. 3. Capacity under average received-power constraint vs.  $\alpha$

#### IV. CAPACITY UNDER PEAK RECEIVED-POWER CONSTRAINT

From spectrum-sharing point of view, an average received-power constraint as defined in (2) is reasonable when primary licensee's QoS is determined by average SNR (e.g. for a delay-insensitive service). However, in many situations licensee's QoS would be limited by the instantaneous SNR at the receiver which renders a peak received-power constraint more appropriate. Thus, in this section we study the fading channel capacity defined in (1) with (2) being replaced by the following constraint,

$$g_0 P(g_0, g_1) \leq Q \quad (22)$$

where, with slight abuse of notation,  $Q$  represents the *peak* received-power this time. Assuming no other limitation on the power of secondary transmitter, capacity is maximized by transmitting at maximum instantaneous power allowed by the licensee,  $Q/g_0$ . Thus,

$$\begin{aligned} C &= \int_{g_0} \int_{g_1} B \log \left( 1 + \frac{g_1}{g_0} \frac{Q}{N_0 B} \right) f_1(g_1) f_0(g_0) dg_1 dg_0 \\ &= \int_{\frac{g_1}{g_0}} B \log (1 + \alpha x) f_{\frac{g_1}{g_0}}(x) dx \end{aligned} \quad (23)$$

where  $\alpha = Q/N_0 B$  as defined before. Clearly for the AWGN case, peak and average received-power constraints give rise to same capacity as given by (8).

In what follows, we derive the capacity for Rayleigh fading channels. Capacity under log-normal and general Nakagami fading should be obtained numerically.

Under Rayleigh fading,  $g_1/g_0$  has a log-logistic distribution as defined in (11). Thus, channel capacity is given by,

$$\begin{aligned} C_{\text{rayleigh}} &= B \int_0^{\infty} \log(1 + \alpha x) \frac{1}{(1+x)^2} dx \\ &= B \frac{\alpha \log \alpha}{\alpha - 1} \end{aligned} \quad (24)$$

Capacity under peak received-power constraint is plotted in Fig. 4 for different fading distributions. It is interesting to note that capacity under both Rayleigh and Nakagami ( $m = 2$ ) fading is consistently higher than that of the AWGN case.

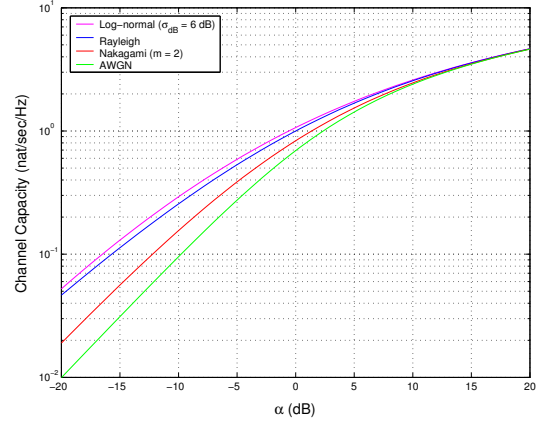


Fig. 4. Capacity under peak received-power constraint vs.  $\alpha$

Moreover, comparing figures 3 and 4 for the same  $\alpha$ , capacities in the former case are generally higher than those of the latter which is due to the more restrictive nature of the peak (as opposed to average) received-power constraint.

#### V. MULTIPLE PRIMARY USERS

Previous results were derived under the assumption that spectrum is being shared between the secondary user and one primary receiver. When more primary receivers are present, received (and hence transmitted) power of the secondary user would be subject to additional constraints resulting in a reduced capacity. To quantify this effect, in this section we derive the capacity subject to peak received-power constraint when  $n$  primary receivers are present. For mathematical tractability only Rayleigh fading case is considered however, following the same procedure, capacity for other fading distributions subject to peak/average received-power constraint may be evaluated numerically.

Let  $g_{0i}$  denote channel gain of the secondary transmitter to  $i$ th primary receiver. Then constraint in (22) is replaced by the following  $n$  constraints,

$$g_{0i} P(g_{01}, \dots, g_{0n}, g_1) \leq Q_i, \quad i = 1, \dots, n$$

or equivalently,

$$P(g_{01}, \dots, g_{0n}, g_1) \leq \min_i \frac{Q_i}{g_{0i}}, \quad i = 1, \dots, n \quad (25)$$

where  $P(g_{01}, \dots, g_{0n}, g_1)$  is the optimal power allocation when the joint channel state is  $(g_{01}, \dots, g_{0n}, g_1)$ . In absence of any other constraints, capacity is achieved when maximum allowed power according to (25) is transmitted. For simplicity let's assume that  $Q_i = Q$ ,  $i = 1, \dots, n$ . Thus, capacity is given by,

$$C = \int_X B \log \left( 1 + x \frac{Q}{N_0 B} \right) f_X(x) dx \quad (26)$$

where  $X = g_1 / \max_i g_{0i}$ . When  $g_1, g_{01}, \dots, g_{0n}$  are i.i.d. unit-mean exponential random variables, pdf of  $X$  is given by (see Appendix A),

$$f_X(x) = n \sum_{k=0}^{n-1} (-1)^k \frac{\binom{n-1}{k}}{(1+x+k)^2} \quad (27)$$

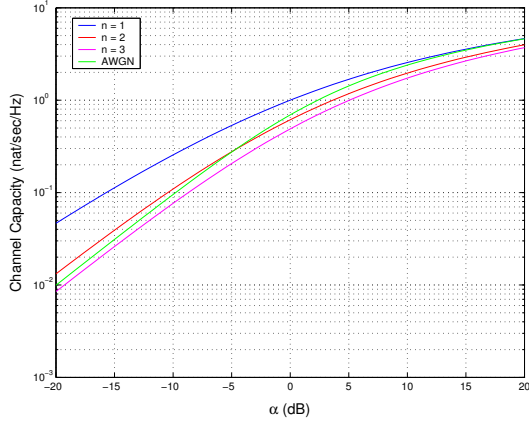


Fig. 5. Capacity under peak received-power constraint and Rayleigh fading for different numbers of primary users

Substituting the pdf of  $X$  from (27) into (26) yields the capacity under peak received-power constraint and Rayleigh fading when  $n$  primary users are present,

$$\begin{aligned}
C &= B \int_X \log(1 + \alpha x) f_X(x) dx \\
&= B \int_0^\infty \log(1 + \alpha x) n \sum_{k=0}^{n-1} (-1)^k \frac{\binom{n-1}{k}}{(1+x+k)^2} dx \\
&= nB \sum_{k=0}^{n-1} (-1)^k \binom{n-1}{k} \int_0^\infty \frac{\log(1 + \alpha x)}{(1+x+k)^2} dx \\
&= nB \sum_{k=0}^{n-1} (-1)^k \binom{n-1}{k} \frac{\alpha \log\{(1+k)\alpha\}}{(1+k)\alpha - 1} \quad (28)
\end{aligned}$$

where  $\alpha = Q/N_0B$  as before. Simple inspection verifies that (28) boils down to (24) when  $n = 1$ . Fig. 5 shows the plot of capacity in (28) for  $n = 1, 2, 3$ . Capacity of non-fading AWGN channel is also plotted for comparison. As expected, capacity degrades with increasing number of primary receivers. In particular, for a wide range of  $\alpha$ , capacity with  $n = 3$  is smaller compared to the non-fading AWGN case. That is, fading has a detrimental effect on capacity as  $n$  gets larger since there is a higher probability of having a large channel gain to at least one primary receiver.

## VI. CONCLUDING REMARKS

Under a dynamic spectrum-sharing scenario with conventional regulatory constraints on the transmitted-power either relaxed or removed, a constraint on the received-interference seems more appropriate. Therefore, we investigated capacity of fading channels subject to received-power constraints at a third-party (primary) receiver. Derivations provided earlier in this paper serve as a first step toward quantifying fundamental limits of spectrum sharing in fading environments. Our results indicate that in many cases, significant capacity gains may be achieved if the channels are varying due to fading.

## APPENDIX A.

Let  $g_{0i}$  ( $i = 1, \dots, n$ ) be i.i.d. unit-mean exponential random variables. Also assume that  $g_1$  is independent of all

$g_{0i}$  and has the same distribution. Now define  $g_0$  as,

$$g_0 = \max_i g_{0i}, \quad i = 1, \dots, n$$

Then cumulative distribution function (CDF) of  $g_0$  is given by,

$$F_{g_0}(z) = \prod_{i=1}^n F_{g_{0i}}(z) = (1 - e^{-z})^n$$

After differentiation, pdf of  $g_0$  may be written as follows,

$$f_{g_0}(z) = ne^{-z}(1 - e^{-z})^{n-1}, \quad z \geq 0$$

Defining  $X = g_1/g_0$  and  $Y = g_0 + g_1$ , pdf of  $g_1/g_0$  is given by the following integral,

$$\begin{aligned}
f_X(x) &= \int_0^\infty e^{-\frac{xy}{1+x}} n e^{-\frac{y}{1+x}} (1 - e^{-\frac{y}{1+x}})^{n-1} \frac{y}{(1+x)^2} dy \\
&= n \int_0^\infty e^{-y} (1 - e^{-\frac{y}{1+x}})^{n-1} \frac{y}{(1+x)^2} dy \\
&= \frac{n}{(1+x)^2} \int_0^\infty e^{-y} y \sum_{k=0}^{n-1} (-1)^k \binom{n-1}{k} e^{-\frac{ky}{1+x}} dy \\
&= n \sum_{k=0}^{n-1} (-1)^k \frac{\binom{n-1}{k}}{(1+x)^2} \int_0^\infty y e^{-\left(1+\frac{k}{1+x}\right)y} dy \\
&= n \sum_{k=0}^{n-1} (-1)^k \frac{\binom{n-1}{k}}{(1+x+k)^2}
\end{aligned}$$

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