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ECE241 – Digital Systems
Finite State Machines (review)

Fall 2023 – Bruno Korst, P.Eng.

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Definitions

- **Synchronous Sequential Machine**
  - Machine whose present outputs are a **function of the present state**
    OR
  - Machine whose present outputs are a **function of the present inputs and the present state**.
- Synchronous → "responds to a clock"
- Sequential logic system → combinational logic + storage

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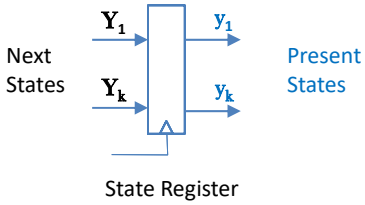
Definitions

- State is "*a set of values that is measured at different locations within the machine*" (stored, in clocked D-FF, registers)
  - **Present state**: state of the system at the present
  - **Next state**: state **to which the system will enter** on the next clock edge.



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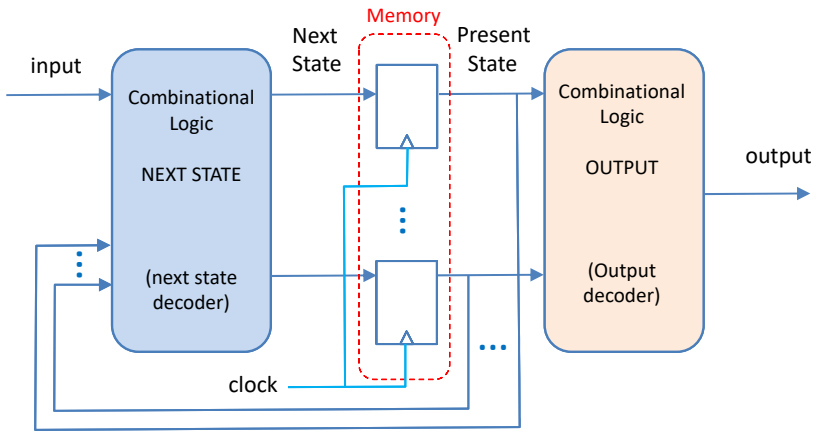
Definitions



- Note: the **clock is is not an input**, it is a "heartbeat"
- Rising edge determines when state transitions occur → **"next" becomes "present"**

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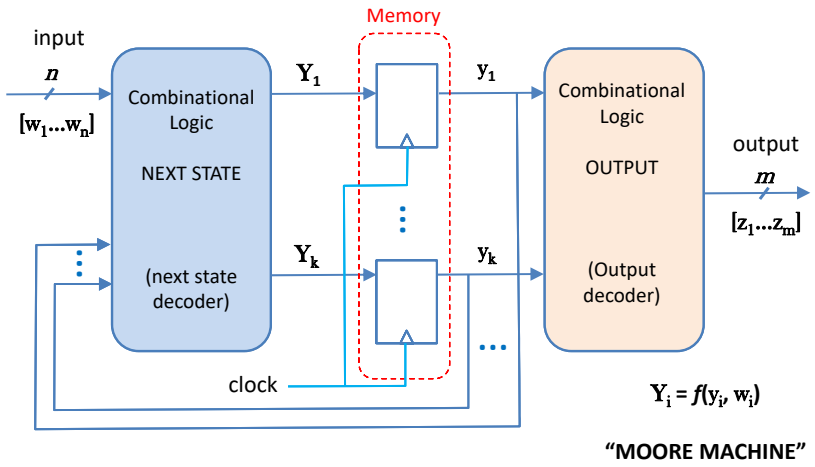
Definitions



- k state variables (number of FF), make possible to represent 2^k states
- On clock edge, Y_i becomes y_i

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Traffic Light FSM

- **The problem**
  - There is an intersection of two busy roads which needs traffic lights to prevent collisions.
  - These lights are to be controlled by traffic sensors, which indicate whether there are cars present or not.

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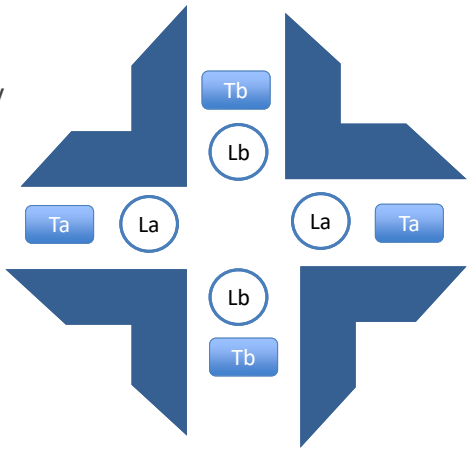
Traffic Light FSM

- Ta, Tb – traffic sensors
  - TRUE if cars present
  - FALSE if street empty
- La, Lb – lights
  - Green
  - Yellow
  - Red



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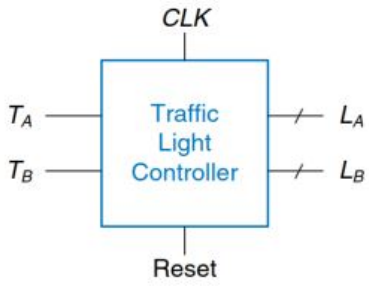
Traffic Light FSM

- Two inputs – Ta, Tb
- Two outputs – La, Lb
- Clock
  - Say, at 5 sec period
  - @ Positive edge
    - Lights Change



- Reset brings it to a known initial state (asynchronously)

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Traffic Light FSM

- **A systematic approach to the design**
  1. Draw the state transition diagram
  2. Create the state transition table
  3. Encode the states on the state transition table
  4. Derive the logic expressions, minimize them
  5. Create the output table, encode it
  6. Derive the logic expressions, minimize them
  7. Draw the circuit, implement it
     - On “old-school” hardware, in code, on HDL

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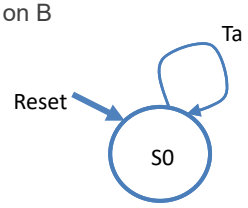
Traffic Light FSM

- State Transition Diagram
  - States are represented by circles, transitions by arcs
  - Reset – green on “A” street, red on “B” street
  - Every 5 seconds (clock), examine sensors (Ta, Tb) and decide:
    - Is traffic present on A – if so, lights do not change
    - When no more traffic on A for 5 seconds – light turns yellow on A
    - 5 seconds later, red on A, green on B



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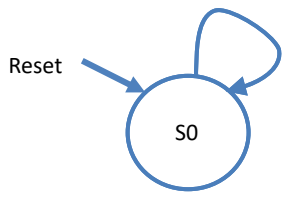
Traffic Light FSM

Reset does not depend on the initial transition



While there IS traffic, there is no change. (Ta is TRUE)

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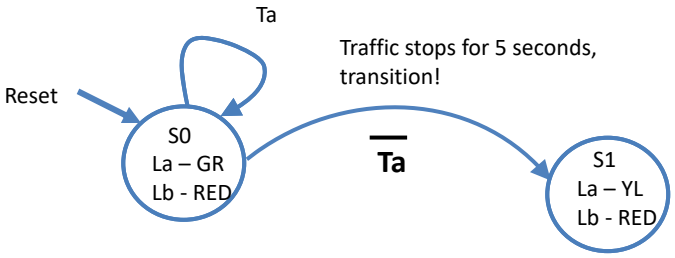
Traffic Light FSM



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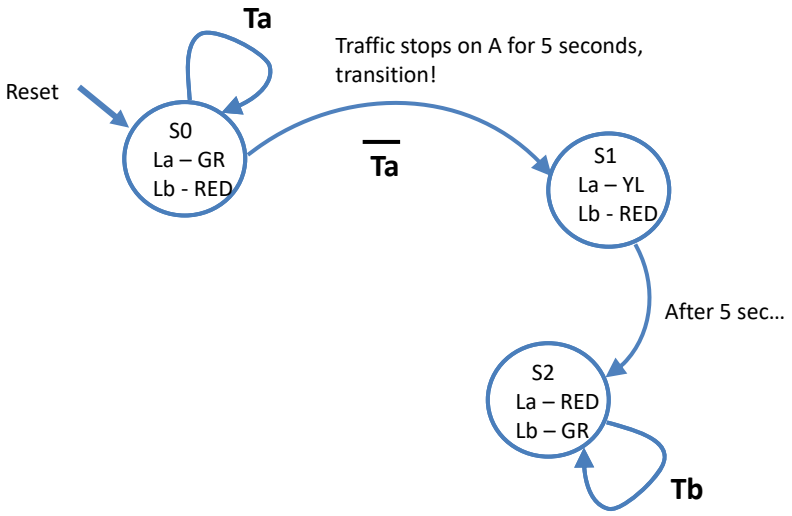
Traffic Light FSM



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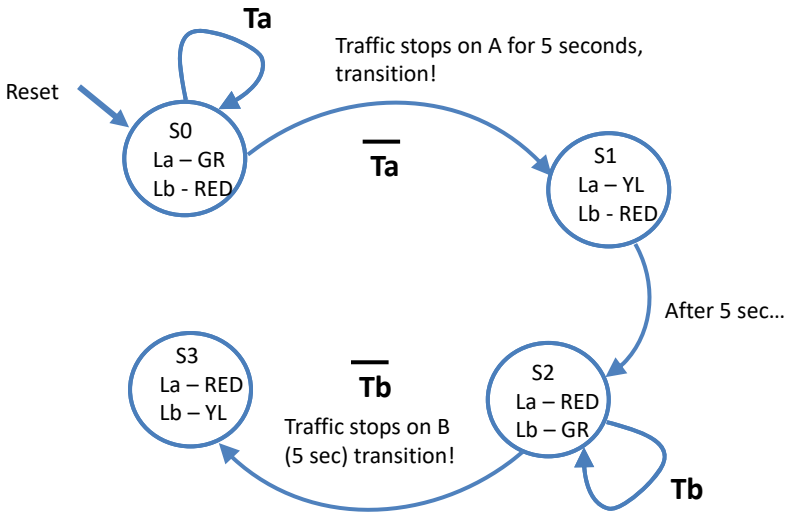
Traffic Light FSM



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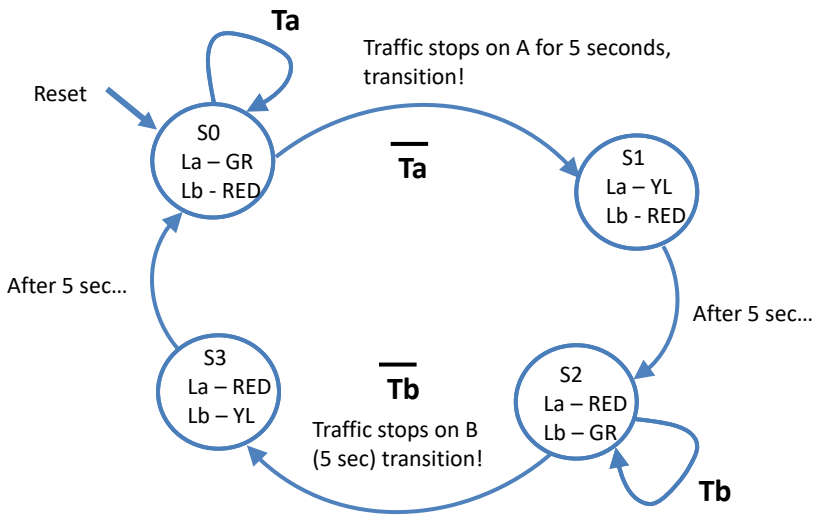
Traffic Light FSM



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Traffic Light FSM

- **A systematic approach to the design**
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     - On “old-school” hardware, in code, on HDL

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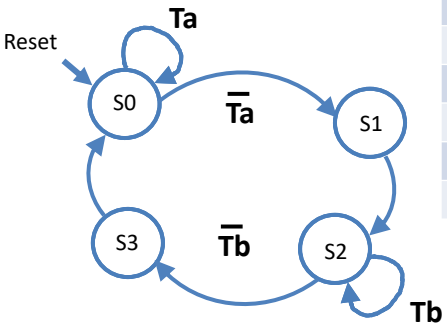
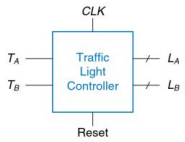
Traffic Light FSM

- State Transition Table

Current	Ta	Tb	Next
S0	0	X	S1
S0	1	X	S0
S1	X	X	S2
S2	X	0	S3
S2	X	1	S2
S3	X	X	S0





(review the examples given before)

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Traffic Light FSM

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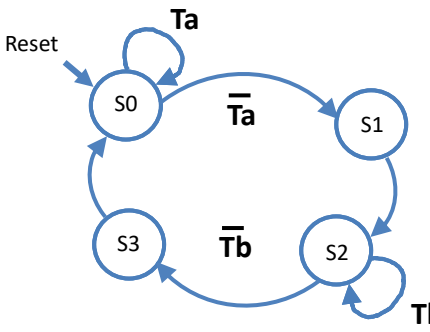
Traffic Light FSM

- State Encoding



Current	Ta	Tb	Next
0 0	0	X	0 1
0 0	1	X	0 0
0 1	X	X	1 0
1 0	X	0	1 1
1 0	X	1	1 0
1 1	X	X	0 0

Sb1 Sb0 S'b1 S'b0

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Traffic Light FSM

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Traffic Light FSM

- Logic Expressions

Current	Ta	Tb	Next
0 0	0	X	0 1
0 0	1	X	0 0
0 1	X	X	1 0
1 0	X	0	1 1
1 0	X	1	1 0
1 1	X	X	0 0

Sb1 Sb0 S'b1 S'b0

$$S'b_1 = \overline{Sb_1}Sb_0 + Sb_1\overline{Sb_0}\,\overline{T_b} + Sb_1\overline{Sb_0}T_b$$
$$S'b_0 = \overline{Sb_1}\,\overline{Sb_0}\,\overline{T_a} + Sb_1\overline{Sb_0}\,\overline{T_b}$$

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Traffic Light FSM

- Logic Expressions

Current	Ta	Tb	Next
0 0	0	X	0 1
0 0	1	X	0 0
0 1	X	X	1 0
1 0	X	0	1 1
1 0	X	1	1 0
1 1	X	X	0 0

Sb1 Sb0 S'b1 S'b0

$$S'b_1 = \overline{Sb_1}Sb_0 + Sb_1\overline{Sb_0}\,\overline{T_b} + Sb_1\overline{Sb_0}T_b$$
$$S'b_0 = \overline{Sb_1}\,\overline{Sb_0}\,\overline{T_a} + Sb_1\overline{Sb_0}\,\overline{T_b}$$

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Traffic Light FSM

- **State Transition Table** – next states from current states

Current	Ta	Tb	Next
0 0	0	X	0 1
0 0	1	X	0 0
0 1	X	X	1 0
1 0	X	0	1 1
1 0	X	1	1 0
1 1	X	X	0 0

Sb1 Sb0 S'b1 S'b0

$$S'b_1 = \overline{Sb_1}Sb_0 + Sb_1\overline{Sb_0}$$

$$S'b_0 = \overline{Sb_1}\,\overline{Sb_0}\,\overline{T_a} + Sb_1\overline{Sb_0}\,\overline{T_b}$$

$$S'b_1 = Sb_1 \oplus Sb_0$$

$$S'b_0 = \overline{Sb_1}\,\overline{Sb_0}\,\overline{T_a} + Sb_1\overline{Sb_0}\,\overline{T_b}$$

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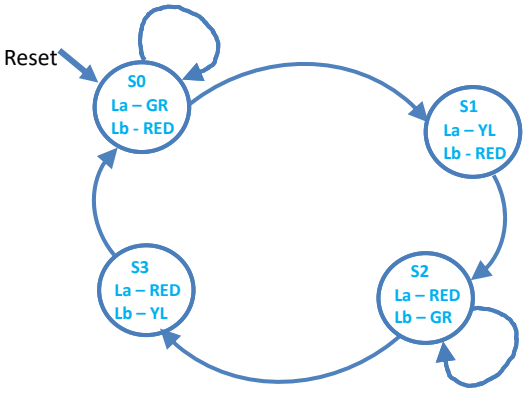
Traffic Light FSM

- **Output table** – we look at La and Lb, codified



Each output, 2 bits

Red – 0 0
Yellow – 0 1
Green – 1 0

What happens to 1 1?

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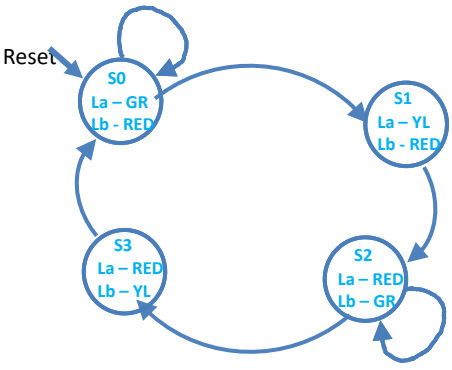
Traffic Light FSM

- Output table – we look at La and Lb, codified



Current		La		Lb	
0	0	0	0 (G)	1	0 (R)
0	1	0	1 (Y)	1	0 (R)
1	0	1	0 (R)	0	0 (G)
1	1	1	0 (R)	0	1 (Y)
Sb1	Sb0	La1	La0	Lb1	Lb0

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Traffic Light FSM

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Traffic Light FSM

- **Output table** – we look at La and Lb, codified

Current		La		Lb	
0	0	0	0 (G)	1	0 (R)
0	1	0	1 (Y)	1	0 (R)
1	0	1	0 (R)	0	0 (G)
1	1	1	0 (R)	0	1 (Y)
Sb1	Sb0	La1	La0	Lb1	Lb0

$$La_1 = Sb_1$$

$$La_0 = \overline{Sb_1}\, Sb_0$$

$$Lb_1 = \overline{Sb_1}$$

$$Lb_0 = S_1 S_0$$

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Traffic Light FSM

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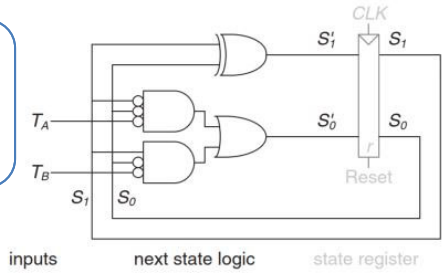
Traffic Light FSM

- We’ve got equations for next states from current states

$$S'b_1 = Sb_1 \oplus Sb_0$$

$$S'b_0 = \overline{Sb_1}\,\overline{Sb_0}\,\overline{T_a} + Sb_1\overline{Sb_0}\,\overline{T_b}$$

A 2 bit state register…



- But that does not say anything about the output…

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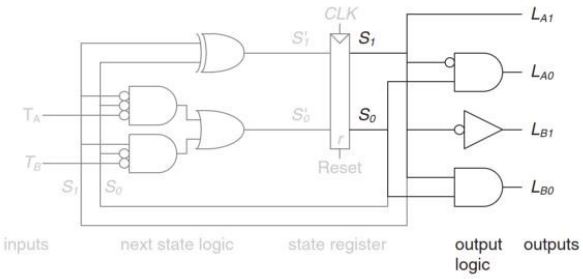
Traffic Light FSM

- We've ALSO got equations for outputs, which result in…

$$La_1 = Sb_1$$
$$La_0 = \overline{Sb_1}\, Sb$$
$$Lb_1 = \overline{Sb_1}$$
$$Lb_0 = S_1 S_0$$



Note: Outputs depend only on the present state ➔ MOORE MACHINE

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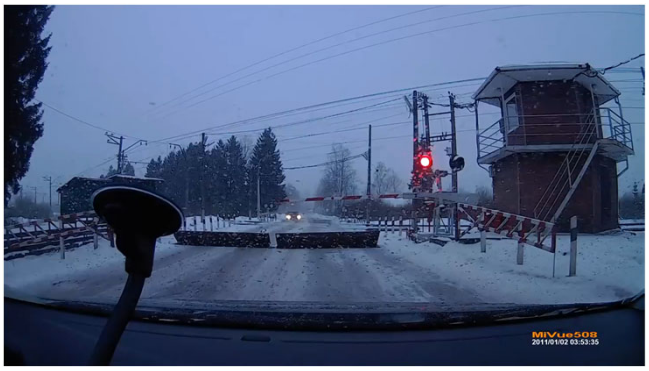
Traffic Light FSM



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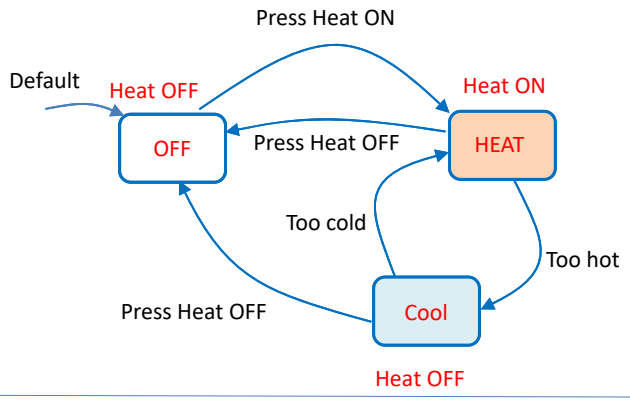
Moore Machine

- A machine whose present outputs are a function of the present state ONLY



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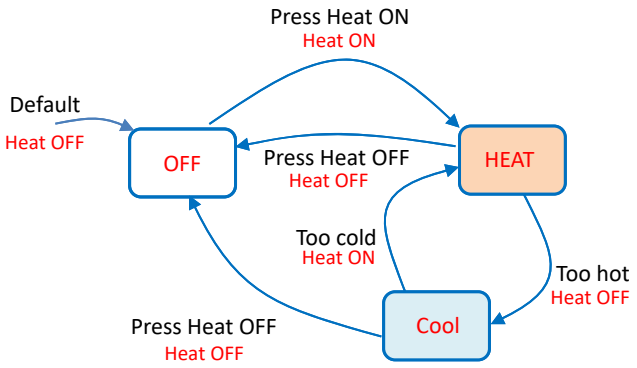
Mealy Machine

- A machine whose present outputs are a function of the present state and the present input.



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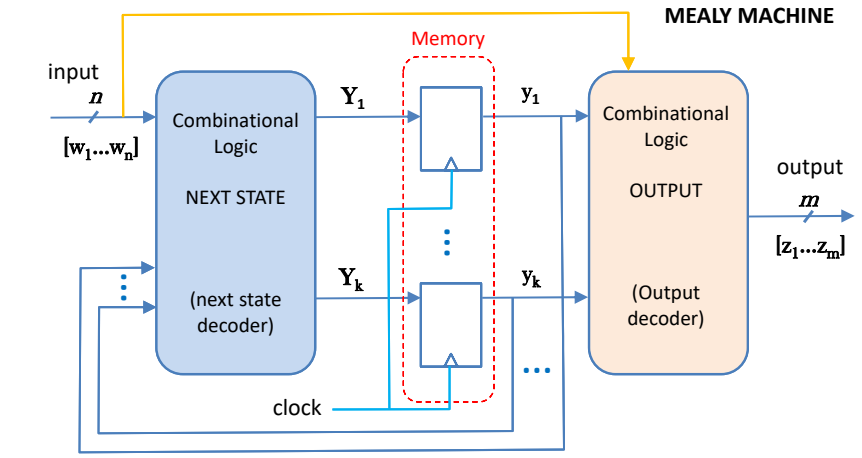
Definitions



- On clock edge, Y_i becomes y_i

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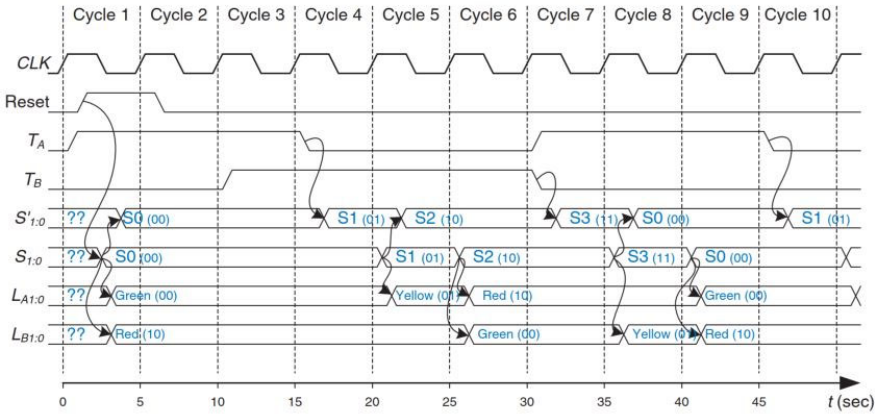
Traffic Light FSM

- As for the timing of all this…



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