STA286 Problem Set 2 Solutions

Chapter 2 Questions

Question 2.51

Sample space for different amounts of money:

S = {$10, $25, $100}

Probabilities to each sample point:

$10: $p = \frac{275}{500} = 0.55$

$25: $p = \frac{150}{500} = 0.30$

$100: $p = \frac{75}{500} = 0.15$

Probability of first purchase containing less than $100:

P = p($10) + p($25) = 0.55 + 0.30 = 0.85

Question 2.55

There are 5 vowels out of 26 letters; and 4 even digits out of 9 non-zero digits.

Amount of codes that satisfies the requirement:

5 X (26-1) X (26-2) X (9-1) X (9-2) X (9-3) X 4

= 5 X 25 X 24 X 8 X 7 X 6 X 4

= 4032000

Note: in the expression of the second half, we started by having 9-1 numbers since we have "reserved" one even number for the last digit.

Amount of codes in total:

26 X (26-1) X (26-2) X 9 X (9-1) X (9-2) X (9-3)

= 26 X 25 X 24 X 9 X 8 X 7 X 6

= 47174400

The probability:

$$P = \frac{4032000}{47174400} = 0.085$$

Question 2.58

Sample space size: 6 X 6 = 36

With both dices being fair, each one of the 36 sample point is with the same probability: 1/36.

Part (a)

The following sample points correspond to a total of 8:

(2,6); (3,5); (4,4); (5,3); (6,2)

Total of 5 sample points, thus the probability is: 5/36

Part (b)

The following sample points correspond to the condition:

Total of 2: (1,1)

Total of 3: (1,2); (2,1)

Total of 4: (1,3); (2,2); (3,1)

Total of 5: (1,4); (2,3); (3,2); (4,1)

There are a total of 10 sample points, corresponding to the probability: 10/36 = 5/18

Question 2.61

Note that we can assume the probability of selecting every individual is uniform across all 100 students.

Part (a)

Amount of students took math or history:

$$54 + 69 - 35 = 88$$

Probability:

$$P = 88/100 = 22/25$$

Part (b)

Amount of students didn’t take either:

$$100 - 88 = 12$$

Probability:

$$P = 12/100 = 3/25$$

Part (c)

Amount of students took history but not math:

$$69 - 35 = 34$$

Probability:

$$P = 34/100 = 17/50$$

Question 2.67

Part (a)

P = p(3 cars served) + p(4 cars served) = 0.12 + 0.19 = 0.31

Part (b)

P = 1 – p(8 or more cars served) = 1 – 0.07 = 0.93

Part (c)

Note that as two events here are mutually exclusive, we can add them up together:

P = p(3 cars served) + p(4 cars served) = 0.12 + 0.19 = 0.31

Question 2.73

Part (a)

The probability that the convict committed armed robbery given that this convict pushed dope.

Part (b)

The probability that the convict did not push the dope given that this convict committed armed robbery.

Part (c)

The probability that the convict did not commit armed robbery given that this convict did not push dope.

Question 2.85

First of all, there is a common sense implied: if the doctor diagnosis correctly, then the patient won't sue.

P(incorrect diagnosis ∩ sue) = p(sue | incorrect diagnosis) X p(incorrect diagnosis)

= 0.9 X (1-0.7)

= 0.27

Question 2.93

Part (a)

In this case, computing the probability through compliment is easier.

Think its compliment: the system doesn't work. For this to be true, both lines have to break.

It's probability is: p' = (1-0.7X0.7) X (1-0.8X0.8X0.8) = 0.51 X 0.488 = 0.249

Thus, the probability of the system working:

P = 1 - p' = 1 – 0.249 = 0.751

Part (b)

P(A not work | system works) = p(A not work ∩ system works) / p(system works)

P(A not work ∩ system works): if A doesn't work, then C, D, E all have to work, thus we can compute the probability as:

P(A not work ∩ system works) = (1-p(A))P(C)P(D)P(E) = (1-0.7)X0.8X0.8X0.8 = 0.1536

Thus:

P(A not work | system works) = 0.1536 / 0.751 = 0.205

Question 2.94

P(A not work | system not work) = p(A not work ∩ system not work) / p(system not work)

P(system not work) = 1 – p(system works) = 1- 0.751 = 0.249 (From question 2.93)

P(A not work ∩ system not work): if A doesn't work, then as long as one of the C,D,E doesn't work, the system doesn't work. Can compute this probability by taking the compliment:

P(A not work ∩ system not work)

= p(system not work | A not work) X P(A not work)

= (1-p(B)XP(C)XP(D)) X (1-0.7) = (1- 0.8X0.8X0.8) X 0.3 = 0.1464

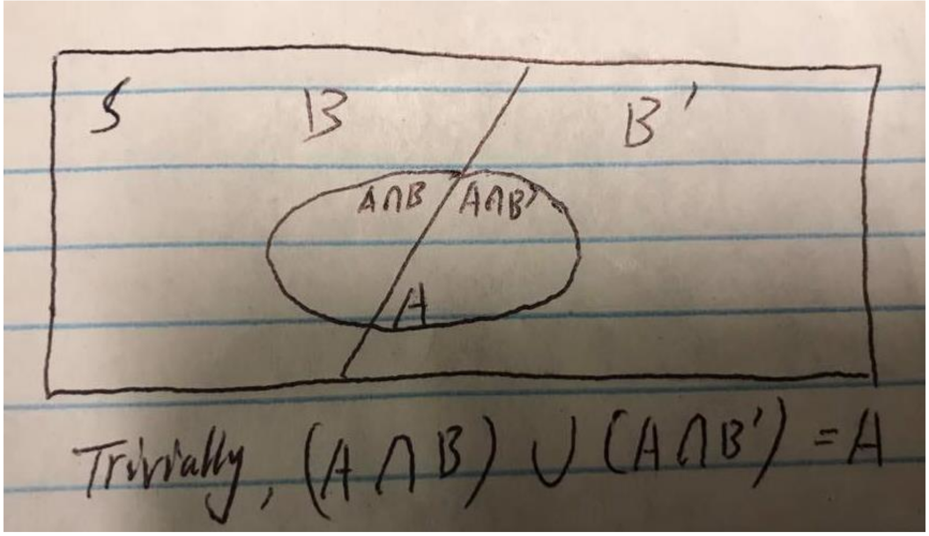
Thus:

P(A not work | system not work) = 0.1464 / 0.249 = 0.588

Question 2.105

Part (a).

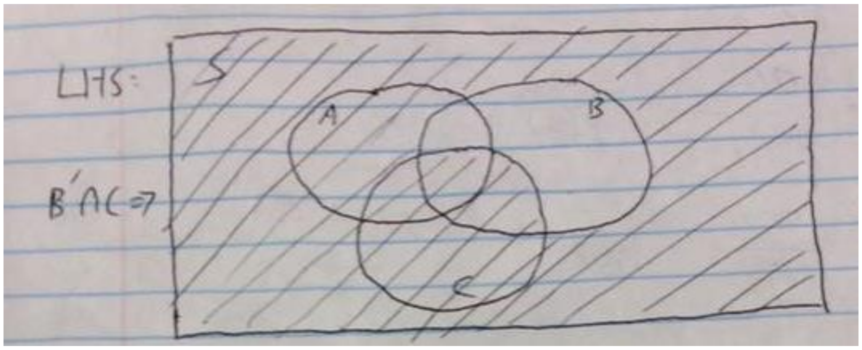
Note that essentially B and B' form a partition of S. The Venn diagram is shown as following:

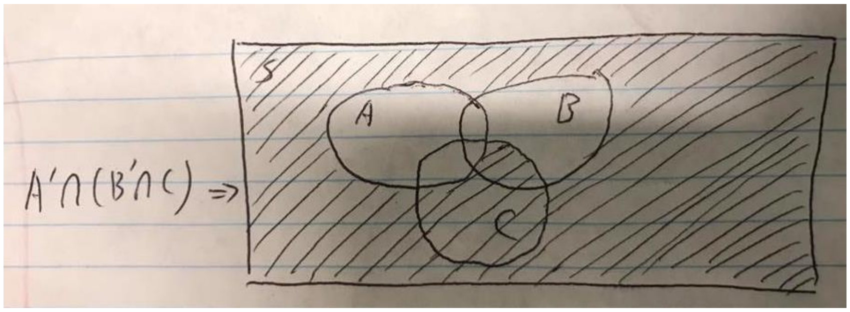


Part (b)

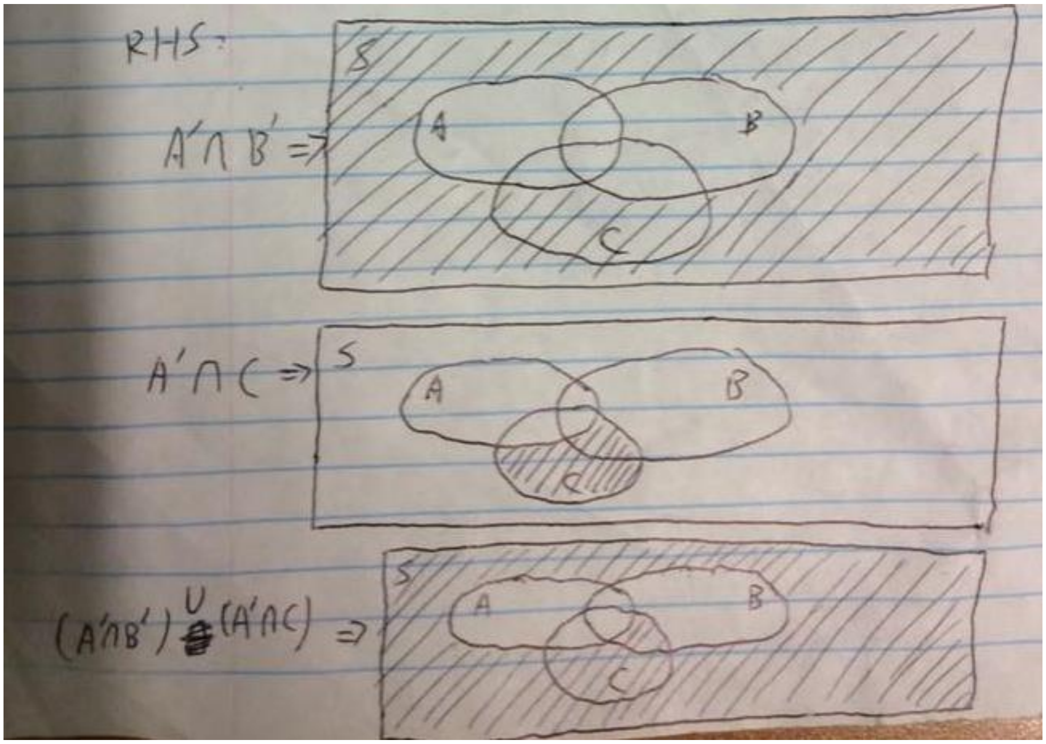
The two Venn diagrams are shown below, with each one represent the region specified by one side of the equation:

Left Hand Side:





Right Hand Side:



As shown on the final diagrams of two hand sides, the two resulting set denoted by the shaded areas are equal for two sides, thus the set expression equality is verified through Venn diagram.

Question 2.110

Part (a).

Not considering ordering, exactly 2 out of 3 patients survived:

$$P^* = 0.8 \times 0.8 \times (1-0.8) = 0.128$$

Now consider the amount of ordering: the patient that didn't survive could be the first, the second, or the third patient (the remaining two patients are the ones survived), thus leading to a total of 3 orderings. They are mutually disjoint, with same probability P*, thus their probabilities could be added up, leading to the result:

$$P = 3 \times P^* = 3 \times 0.128 = 0.384$$

Part (b).

There is only one possible case as there is no distinct ordering that could lead to 3 patients surviving, the probability is thus naturally computed as:

$$P = 0.8 \times 0.8 \times 0.8 = 0.512$$

Question 2.113

As all balls are replaced into the box, thus the probability of each draw is identically distributed.

Part (a).

Two cases:

Case 1: all balls are with color black, then probability for three balls all being black is:

P1 = 6/(6+4) X 6/(6+4) X 6/(6+4) = 0.216

Case 2: all balls are with color green, then probability for three balls all being green is:

P2 = 4/(6+4) X 4/(6+4) X 4/(6+4) = 0.064

Thus, the total probability is:

P = p1 + p2 = 0.28

Part (b).

This is exactly the compliment of part (a), thus the probability is trivially computed:

P = 1 – 0.28 = 0.72

Question 2.127

Use Baye's rule:

$$P(\text{queen carrier} \mid \text{three princes no disease}) = \frac{p(\textit{queen carrier} \cap \textit{three princes no disease})}{p(\textit{three princes no disease})}$$

$$= \frac{p(\textit{three princes no disease} \mid \textit{queen carrier})p(\textit{queen carrier})}{p(\textit{three princes no disease} \mid \textit{queen carrier})p(\textit{queen carrier}) + p(\textit{three princes no disease} \mid \textit{queen not carrier})p(\textit{queen not carrier})}$$

As each prince's situation is independently distributed, we can have the following decomposition:

$$= \frac{p(\textit{prince no disease} \mid \textit{queen carrier})^3 p(\textit{queen carrier})}{p(\textit{prince no disease} \mid \textit{queen carrier})^3 p(\textit{queen carrier}) + p(\textit{prince no disease} \mid \textit{queen not carrier})^3 p(\textit{queen not carrier})}$$

Plug in the numbers, we have:

$$= \frac{0.5^3 \times 0.5}{0.5^3 \times 0.5 + 1^3 \times 0.5}$$

= 1/9