5.4) $p = 3/4$ (75% of all thefts is due to drug money)

$n = 5$ (next 5 cases)

a) $P(X=2) = \binom{5}{2}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^3 = \boxed{0.0879}$

out of 5 cases, choose 2

75% probability for each of the 2 cases

25% probability of the rest 3 cases

b) $P(x<3) = \sum_{x=0}^{3} b(x;5,3/4) = 1 - \sum_{x=4}^{5} b(x;5,3/4)$

$= 1 - [P(x=4) + P(x=5)] = 1 - \left[\binom{5}{4}(3/4)^4(1/4)^1 - \binom{5}{5}(3/4)^5(1/4)^0\right]$

$= \boxed{0.3672}$

5.7) $p = 70\%$ of lung cancer are chain smokers

a) $n = 10$ patients, fewer than half $\Rightarrow P(X<5)$

$= P(x \leq 4) = \sum_{x=0}^{4} b(x;10,7/10) = \boxed{0.0473}$ (look at table A.1 in textbook)

b) $n=20$, $P(x<10) = \sum_{x=0}^{9} b(x;20,7/10) = \boxed{0.0171}$

5.11) $p = 0.9$, $n = 7$ patients

$P(x=5) = P(x \leq 5) - P(x \leq 4) = \sum_{x=0}^{5} b(x;7,0.9) - \sum_{x=0}^{4} b(x;7,0.9)$

$= 0.1497 - 0.0257 = \boxed{0.1240}$

5.15) $p = 1 - 0.6 = 0.4$, $n = 5$

a) $P(x=0) = \sum_{x=0}^{0} b(x;5,0.4) = 0.0778$

b) $P(x<2) = \sum_{x=0}^{1} b(x;5,0.4) = 0.3370$

c) $P(x>3) = 1 - P(x \leq 3) = 1 - \sum_{x=0}^{3} b(x;5,0.4)$

$= 1 - 0.9130 = 0.0870$

5.29) hypergeometric distribution:

- $N = 5$ tulip + 4 daffodil = 9 flowers total
- $n = 6$ bulbs planted
- $x = 2$ daffodils wanted
- $k = 4$ daffodils total

choose 2 tulips out of 4

choose 4 daff. out of 5

$$h(x; N, n, k) = h(2; 9, 6, 4) = \frac{\binom{4}{2}\binom{5}{4}}{\binom{9}{6}} = \boxed{\frac{5}{14}}$$

6 total bulbs out of 9

5.31)
- $N$ = total people = 4 doctors + 2 nurses = 6
- $n = 3$ committee members
- $x$ = doctors in committee
- $k = 4$ total doctors

$$h(x; 6, 3, 4) = \frac{\binom{4}{x}\binom{2}{3-x}}{\binom{6}{3}}, \quad x = 1, 2, 3$$

$$P(2 \leq x \leq 3) = h(2; 6, 3, 4) + h(3; 6, 3, 4) = \boxed{4/5}$$

5.33) $N = 52$ cards
$n = 7$ cards dealt.

a) $x$ = face cards
$K = 12$ face cards in a deck

$$P(x = 2) = h(2; 52, 7, 12) = \frac{\binom{12}{2}\binom{40}{5}}{\binom{52}{7}} = \boxed{0.3246}$$

b) prob. of at least 1 queen:
rather than solving a $\Sigma$ for $P(x \geq 1)$, we can find $1 - P(x = 0)$:

$\Rightarrow$ $x$ = Queen, $k = 4$ possible queens

$$1 - P(x = 0) = 1 - h(0; 52, 7, 4) = 1 - \frac{\binom{0}{4}\binom{48}{7}}{\binom{52}{7}}$$

$$= \boxed{0.4496}$$

5.49) use negative binomial distribution:

$$b^*(x;k,p) = \binom{x-1}{k-1} p^k q^{x-k}, \quad x = k, k+1, k+2, \ldots$$

- $x =$ 10th person
- $k =$ 5th person with dog
- $p = 0.3$ prob. of having dog
- $q = 1 - p = 0.7$

$$\Rightarrow b^*(10;5,0.3) = \binom{9}{4}(0.3)^5(0.7)^5 = \boxed{0.0515}$$

5.50) $p = 0.5$ probability of a head

a) $x =$ 7th flip
$k =$ 3rd head

$$b^*(7;3,0.5) = \binom{6}{2}(0.5)^3(0.5)^4 = \boxed{0.1172}$$

b) $b^*(4;1,0.5) = g(4;0.5) = (0.5)(0.5)^3 = \boxed{1/16}$

geometric distribution

5.54) a) $p = 2/3$ $\quad x = 5$

"first" prescribing Valium $\Rightarrow$ geometric distribution

$$\Rightarrow g(5;2/3) = (2/3)(1/3)^4 = \boxed{2/243}$$

b) $k = 3$rd prescribing valium $\Rightarrow$ negative binomial.

$$\Rightarrow b^*(5;3,2/3) = \binom{4}{2}(2/3)^3(1/3)^2 = \boxed{\frac{16}{81}}$$

5.56) " probability ... at this section" ⇒ Poisson Distribution

$$p(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!} \quad x = 0,1,2,\ldots$$

a) $x = 5$ accidents occur
$\mu = \lambda t = 3$ accidents per month

$$\Rightarrow P(x=5) = p(5;3) = \sum_{x=0}^{5} p(x;3) - \sum_{x=0}^{4} p(x;3) \Rightarrow$$ look at table A.2 in textbook

$$= \boxed{0.1008}$$

b) $P(x<3) = P(x \le 2) = \sum_{x=0}^{2} p(x;3) = \boxed{0.4232}$

c) $P(x \ge 2) = 1 - P(x \le 1) = 1 - \sum_{x=0}^{1} p(x;3) = \boxed{0.8009}$

5.61) " 10,000 returns selected" ⇒ specific set ⇒ Poisson dist.

$$\mu = \lambda t = np = (10{,}000)(1/1000) = 10$$

$$P(x=\{6,7,8\}) = P(6 \le x \le 8) = P(x \le 8) - P(x \le 5)$$

$$= \sum_{x=0}^{8} p(x;10) - \sum_{x=0}^{5} p(x;10) = \boxed{0.2657}$$

5.66) $\mu = 6t$

a) $t = 1$ hour period $\rightarrow \mu = 6$
$x = 4$

$$P(x=4) = P(x \le 4) - P(x \le 3) = \sum_{x=0}^{4} p(x;6) - \sum_{x=0}^{3} p(x;6)$$

$$= 0.2851 - 0.1512 = \boxed{0.1339}$$

b) $t=1 \Rightarrow \mu = 6.$ $P(x \ge 4) = 1 - P(x \le 3) = 1 - \sum_{x=0}^{3} p(x;6)$

$$= 1 - 0.1512 = \boxed{0.8488}$$

c) $t = 12 \Rightarrow \mu = 72$ $\quad P(x \ge 75) = 1 - \sum_{x=0}^{74} p(x;72) = \boxed{0.3773}$

5.94) $n = 500$ parts sampled
$p = 0.01$ defective

$x = 15$ defectives observed

a) $P(X \geq 15) = 1 - P(X \leq 14) = 1 - \sum_{x=0}^{14} b(x; 500, 0.01)$

$= 0.00021$ ⇒ having at least 15 defectives observed is very rare. Since it did happen in this test, the assumption that $p = 0.01$ may not be true

b) $P(X=3) = b(3; 500, 0.01) = \binom{500}{3}(0.01)^3(0.99)^{497}$

$= \boxed{0.1402}$

c) i) if $p = 0.01$, then $\mu = (500)(0.01) = 5$

"in this sample of 500, 5 were defective"

$P(X \geq 15) = 1 - P(X \leq 14) = 1 - \sum_{x=0}^{14} p(x; 5) = 1 - 0.9998$

$\boxed{= 0.0002}$

ii) $P(x=3) = P(x \leq 3) - P(x \leq 2) = \sum_{x=0}^{3} p(x;5) - \sum_{x=0}^{2} p(x;5)$

$\Rightarrow 0.2650 - 0.1247 = \boxed{0.1403}$