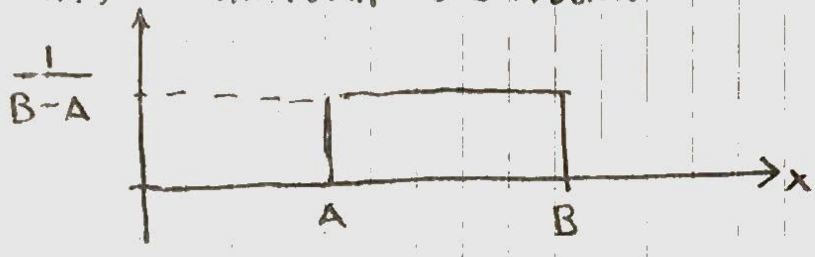
6.1) uniform distribution:



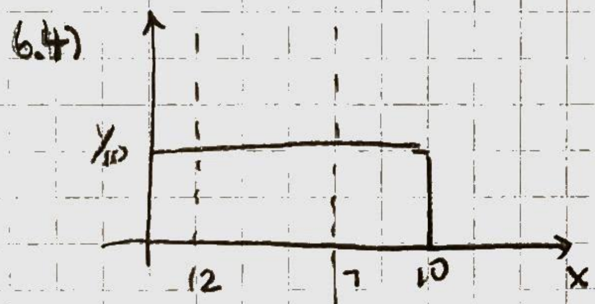
a) $\mu = \int_A^B x f(x)dx = \int_A^B \frac{x}{B-A}dx = \frac{B^2-A^2}{2(B-A)} = \frac{A+B}{2}$

b) $E(x^2) = \int_A^B \frac{x^2}{B-A}dx = \frac{B^3-A^3}{3(B-A)}$

$\sigma^2 = E(X^2) - \mu^2 = \frac{B^3-A^3}{3(B-A)} - \left(\frac{A+B}{2}\right)^2$

$= \frac{4(B^2+AB+A^2) - 3(B^2+2AB+A^2)}{12} = \frac{(B-A)^2}{12}$

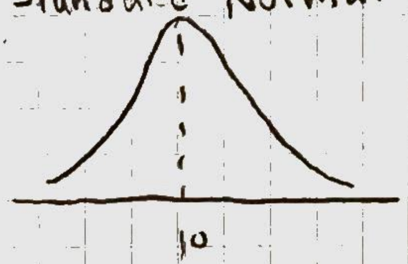
6.4)



a) $P(x>7) = \frac{10-7}{10} = 0.3$

b) $P(2<x<7) = \frac{7-2}{10} = 0.5$

6.5) Standard Normal Distribution (Table A.3, pg 735)



a) $P(z<-1.39) = 0.0823$

b) $P(z>1.96) = 1 - P(z<1.46) = 1 - 0.9750$
$= 0.0250$

c) $P(-2.16<z<-0.65) = P(z<-0.65) - P(z<-2.16)$
$= 0.2578 - 0.0154 = 0.2424$

d) $P(z<1.43) = 0.9236$

e) $P(z>-0.89) = 1 - P(z<-0.89) = 1 - 0.1867 = 0.8133$

f) $P(-0.48<z<1.74) = P(z<1.74) - P(z<-0.48)$
$= 0.9591 - 0.3156 = 0.6435$

6.7) a) $P(Z>k) = 0.2946 \Rightarrow P(z<k) = 1-0.2946$
$= 0.7054$
from Table A.3 $\Rightarrow k = 0.54$

b) $P(z<k) = 0.0427 \Rightarrow K = -1.72$

c) $P(-0.93 < z < k) = 0.7235$

$\Rightarrow P(z<-0.93) = 0.1762$

$\Rightarrow P(z<k) = 0.1762 + 0.7235 = 0.8997 \Rightarrow k = 1.28$

6.9) normal distribution $\Rightarrow$ (not standard) can't use Table A.3 directly

$\mu = 18 \quad \sigma = 2.5$

a) $z = (15-18)/2.5 = -1.2$
$\Rightarrow P(x<15) = P(Z<-1.2) = 0.1151$

b) if $P(z<Z_0) = 0.2236$, then $Z_0 = -0.76$
$Z_0 = \frac{k-\mu}{\sigma} \Rightarrow k = (2.5)(-0.76) + 18 = 16.1$

c) $P(z>Z_0) = 0.1814 \Rightarrow Z_0 = 0.91$
$k = (2.5)(0.91) + 18 = 20.275$

d) $Z_1 = (17-18)/2.5 = -0.4 \quad Z_2 = (21-18)/2.5 = 1.2$

$P(17<x<21) = P(-0.4<Z<1.2) = 0.8849 - 0.3446 = 0.5403$

6.13) a) $\mu = 40 \quad \sigma = 6.3 \quad z = (32-40)/6.3 = -1.27$

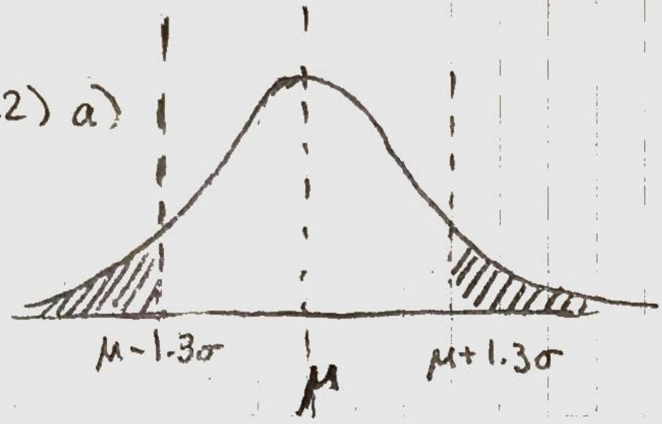
$P(x>32) = P(Z>-1.27) = 1-0.1020 = 0.8980$

b) $z = (28-40)/6.3 = -1.90$
$P(x<28) = P(z<-1.90) = 0.0287$

c) $z_1 = (37-40)/6.3 = -0.48 \quad Z_2 = (49-40)/6.3 = 1.43$

$P(37<x<49) = P(-0.48<Z<1.43) = 0.9236 - 0.3156$
$= 0.0062$

6.22) a)



$x_1 = \mu + 1.3\sigma$
$x_2 = \mu - 1.3\sigma$

$z_1 = \frac{x_1 - \mu}{\sigma} = 1.3$

$z_2 = -1.3$

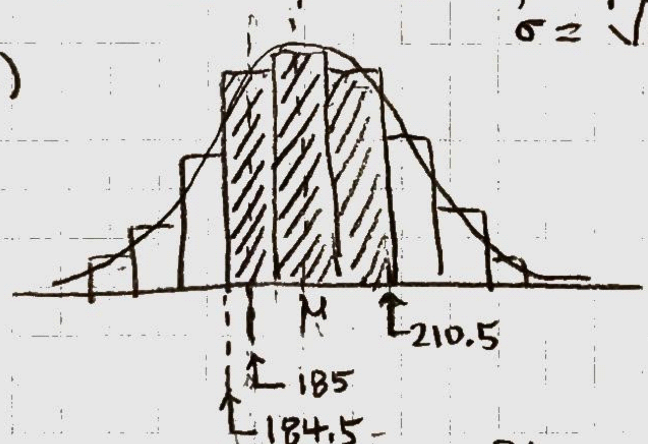
$$P(X > x_1) + P(x < x_2) = P(z > 1.3) + P(z < -1.3) = 2P(z < -1.3) = 0.1936$$

b) $x_1 = \mu + 0.52\sigma$ $\quad z_1 = 0.52$
$x_2 = \mu - 0.52\sigma$ $\quad z_2 = -0.52$

$$P(x_2 < x < x_1) = P(-0.52 < x < 0.52) = 0.6985 - 0.3015 = 0.3970$$

6.24) $n = 400$ $\quad p = 1/2 \Rightarrow \mu = np = 400(1/2) = 200$
$\sigma = \sqrt{npq} = \sqrt{400(1/2)(1/2)} = 10$

a)



$x_1 = 184.5 \Rightarrow z_1 = \frac{(184.5 - 200)}{10}$

$\Rightarrow z_1 = -1.55$

$x_2 = 210.5 \Rightarrow z_2 = \frac{(210.5 - 200)}{10}$

$\Rightarrow z_2 = 1.05$

$$P(-1.55 < z < 1.05) = 0.8531 - 0.0606$$
$$= 0.7925$$

b) $z_1 = (204.5 - 200)/10 = 0.45$
$z_2 = (205.5 - 200)/10 = 0.55$

$$P(0.45 < z < 0.55) = 0.7088 - 0.6736 = 0.0352$$

c) $z_1 = (175.5 - 200)/10 = -2.45$

$z_2 = (227.5 - 200)/10 = 2.75$

$$P(z < -2.45) + P(z > 2.75) = P(z < -2.45) + 1 - P(z < 2.75)$$
$$= 0.0071 + 1 - 0.9970 = 0.0101$$

6.27) $n = 100 \quad p = 0.9 \quad \mu = np = 100(0.9) = 90$

$\sigma = \sqrt{npq} = \sqrt{100(0.9)(0.1)} = 3$

a) $z_1 = (83.5 - 90)/3 = -2.17$

$z_2 = (95.5 - 90)/3 = 1.83$

$P(83.5 < x < 95.5) = P(-2.17 < z < 1.83) = 0.9664 - 0.0150$
$= 0.9514$

b) $z = (85.5 - 90)/3 = -1.50 \quad P(x < 85.5) = P(z < -1.50) = 0.0668$

6.33) $\mu = np = 180(1/6) = 30 \quad \sigma = \sqrt{npq} = \sqrt{(180)(1/6)(5/6)} = 5$

a) $z = (31.5 - 40)/6 = -1.42 \quad P(z < -1.42) = 0.0778$

b) $z = (49.5 - 40)/6 = 1.58 \quad P(z > 1.58) = 1 - 0.9429 = 0.0571$

c) $z_1 = (34.5 - 40)/6 = -0.92 \quad z_2 = (46.5 - 40)/6 = 1.08$

$P(-0.92 < x < 1.08) = 0.8599 - 0.1788 = 0.6811$

6.36) $n = 200 \quad X =$ number of no shows , $p = 0.02$

$$z = \frac{(3 - 0.5) - (200)(0.02)}{\sqrt{(200)(0.02)(0.98)}} = -0.76$$

$P(\text{airline overbooks}) = P(X \leq (3 - 0.5)) = P(z < -0.76) = 0.2236$

6.41) $\alpha = 2 \quad \beta = 1 \quad , \Gamma(n) = (n-1)(n-2)\cdots(1)\Gamma(1)$

$\Gamma(\alpha) = \Gamma(2) = (1)\Gamma(1) = 1$

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha - 1} e^{-x/\beta} = xe^{-x}$$

$$P(1.8 < x < 2.4) = \int_{1.8}^{2.4} xe^{-x}\,dx = \left[-xe^{-x} - e^{-x}\right]_{1.8}^{2.4} = 0.1545$$

6.44) $\mu = 6 = \alpha\beta$ $\quad \sigma^2 = 12 = \alpha\beta^2$

$\beta = \frac{\sigma}{\mu} = 2 \quad \alpha = \frac{\mu}{\beta} = 3$

$\Gamma(3) = 2(1)\Gamma(1) = 2 \quad f(x) = \frac{1}{2^3\Gamma(3)} x^2 e^{-x/2} = \frac{1}{16} x^2 e^{-x/2}$

$P(X>12) = \int_{12}^{\infty} \frac{1}{16} x^2 e^{-x/2}\, dx = \frac{1}{16}\left[-2x^2e^{-x/2} - 8xe^{-x/2} - 16e^{-x/2}\right]_{12}^{\infty}$

$= 25e^{-6} = 0.0620$

6.47) $\alpha = 1/2 \quad \beta = 2 \quad f(x;\alpha,\beta) = \alpha\beta x^{\beta-1} e^{-\alpha x^\beta} = xe^{-x^2/2}$

a) $E(X) = \int_0^\infty x(xe^{-x^2/2})\,dx = \int_0^\infty x^2 e^{-x^2/2}\,dx$

$= -xe^{-x^2/2}\Big|_0^\infty + \int_0^\infty e^{-x^2/2}\,dx = 0 + \sqrt{2\pi}\cdot\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-x^2/2}\,dx$

$= \frac{\sqrt{2\pi}}{2} = \sqrt{\frac{\pi}{2}} = 1.2533$

b) Weibull cdf $= 1 - e^{-\alpha x^\beta}$

$P(X>2) = 1 - P(X<2) = 1 - (1 - e^{-x^2/2})\Big|_{x=2} = e^{-x^2/2}\Big|_{x=2}$

$= e^{-2} = 0.1353$