Chapter 3 Problem Set Supplement

1. Consider the situation where we must transmit *two* DSB-SC signals with carrier frequencies f_1 and f_2. The overall modulated signal is given by:

$$s(t) = A_c m_1(t)\cos(2\pi f_1 t) + A_c m_2(t)\cos(2\pi f_2 t).$$

The two message signals $m_1(t)$ and $m_2(t)$ have Fourier transforms $M_1(f)$ and $M_2(f)$, respectively, that are shown in Figure 1; note that both $m_1(t)$ and $m_2(t)$ have bandwidth W. Assume that $f_1 \gg W$ and $f_2 \gg W$.

Figure 1: Fourier transforms of message signals $m_1(t)$ and $m_2(t)$ denoted $M_1(f)$ and $M_2(f)$, respectively.

   (a) Assuming $f_2 - f_1 > 2W$, plot the Fourier transform of $s(t)$ denoted $S(f)$. Give a block diagram of a system that recovers $m_1(t)$ and $m_2(t)$ separately from $s(t)$.

   (b) Now, assuming $f_2 - f_1 = \frac{3}{2}W$, plot $S(f)$ and give the block diagram of a system that recovers $m_1(t)$ and $m_2(t)$ separately from $s(t)$. It can be done!

2. Consider a QAM signal of the form

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t).$$

The QAM receiver has a phase error as illustrated in Figure 2.

Similar to the discussion in class, obtain the output signals $g_1(t)$ and $g_2(t)$. Note that

$$\begin{aligned} 2\cos A\cos B &= \cos(A-B)+\cos(A+B) \\ 2\sin A\sin B &= \cos(A-B)-\cos(A+B) \\ 2\sin A\cos B &= \sin(A+B)+\sin(A-B) \end{aligned}$$

3. Recall that the frequency response of the Hilbert transform is given by $\mathcal{H}(f) = -j\text{sgn}(f)$; that is positive frequency components of the signal that pass through the Hilbert transform are phase shifted by $-90°$ and the negative frequency components are phase shifted by $+90°$. The magnitude of the frequency components are unaffected. Show that the Hilbert transform of $g(t) = \cos(2\pi f_c t)$ is given by $\hat{g}(t) = \sin(2\pi f_c t)$.

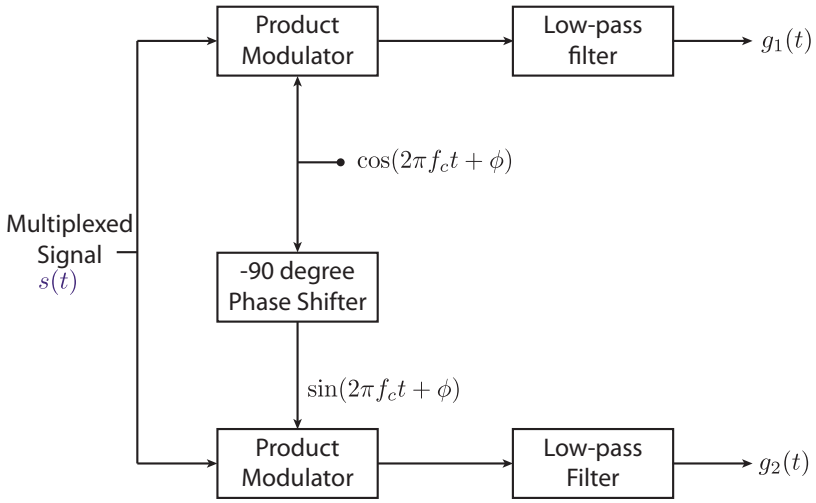

Figure 2: QAM Receiver.

4. The bandwidth of a signal $g(t)$ is W. What is the bandwidth of $\hat{g}(t)$, the Hilbert transform of $g(t)$?