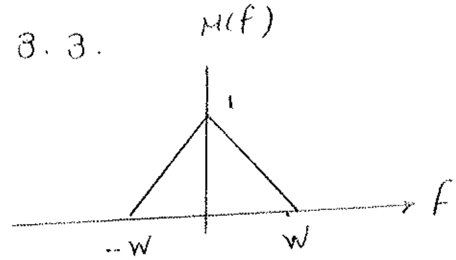
ECE 316 CHAPTER 3 PROBLEM SET SOLUTIONS

Drill. Problem 3.3.

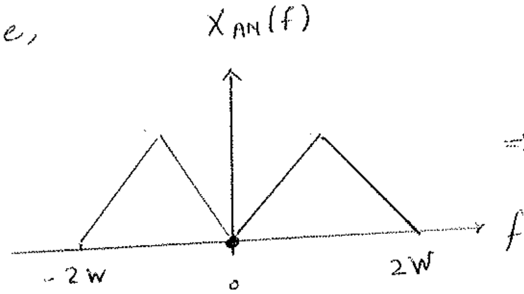
$m(t) \xrightarrow{\mathcal{F}} M(f)$



$x_{AM}(t) = A_c\,[1 + k_a m(t)]\cos 2\pi f_c t$

⇒ Assume $f_c = W$

Therefore,



⇒ if $f_c > W$: No overlap

if $f_c = W$: spectral overlap

if $f_c < W$: overlap

2. Drill problem 3.1.

$A_{min} = A_c(1-\mu) = 0$

100% modulation ; $|k_a m(t)| = 1$ ⇒ if $k_a m(t) = -1$

$A_c(1 + k_a m(t))\cos\omega_c t = 0$ ⇒ envelope : $1 + k_a m(t) = 0$

3. Drill problem 3.2.

AM with a sinusoidal modulating signal :

$s(t) = A_c(1 + k_a A_m \cos\omega_m t)\cos\omega_c t.$

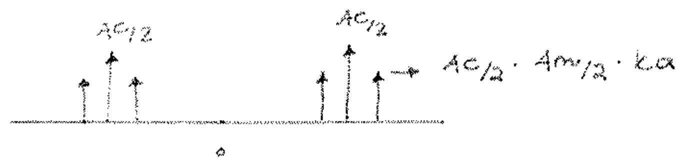
$= A_c \cos w_c t + A_c \mu \cos w_m t \cos w_c t$

$\mu = k_a A_m$ modulation factor.

Total average power components: $A_c^2/2$ (in carrier)

(upper-side freq) $2\left(A_m^2/4 \cdot A_c^2/4 \cdot k_a^2\right) = \mu^2 \frac{A_c^2}{8}$

(lower-side freq) $2\left(A_m^2/4 \cdot A_c^2/4 \cdot k_a^2\right) = \mu^2 A_c^2/8$.



$\Rightarrow$ (a) Average power in the carrier: $A_c^2/2$.

(b) each side freq: $A_c^2/8 \cdot \mu^2 = (0.2)^2 A_c^2/8$. if $\mu = 0.2$.

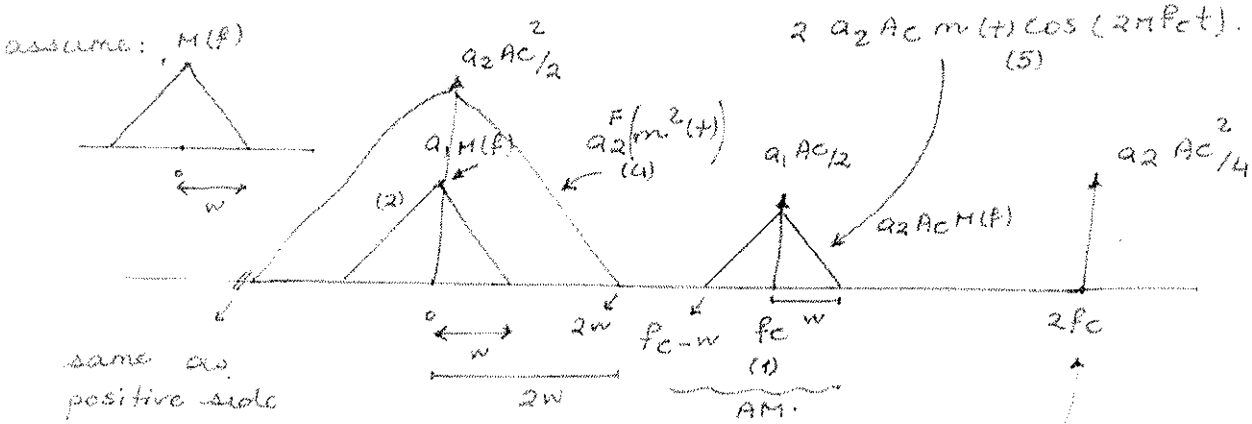
4. Drill problem 3.4. (* important).

$v_1(t) = A_c \cos(2\pi f_c t) + m(t)$

square law: $v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$

$\Rightarrow v_2(t) = a_1 A_c \cos(2\pi f_c t) + a_1 m(t) + a_2\left[A_c \cos(2\pi f_c t) + m(t)\right]^2$

$= \underset{(1)}{a_1 A_c \cos(2\pi f_c t)} + \underset{(2)}{a_1 m(t)} + \underset{(3)}{a_2 A_c^2 \cos^2(2\pi f_c t)} + \underset{(4)}{a_2 m^2(t)} + \underset{(5)}{2 a_2 A_c m(t) \cos(2\pi f_c t)}$.



(3): $a_2 A_c^2 \cos^2(2\pi f_c t) = a_2 A_c^2 \, 1/2\left[1 + \cos(4\pi f_c t)\right]$

$= a_2 A_c^2/2 + a_2 A_c^2/2 \cos(2(2\pi f_c)t)$

in summary :

(1) $a_1 A_c \cos(2\pi f_c t) \xrightarrow{F} a_1 A_c/2 \left[\delta(f - f_c) + \delta(f + f_c) \right]$

(2) $a_1 m(t) \xrightarrow{F} a_1 M(f)$ (baseband $BW = w$)

(3) $a_2 A_c^2 \cos^2(2\pi f_c t) = a_2 A_c^2/2 + a_2 A_c^2/2 \cos(2(2\pi f_c t))$

DC signal (0 Hz)

$\xrightarrow{F} a_2 A_c^2/2 \, \delta(f)$

$\xrightarrow{F} a_2 A_c^2/4 \left[\delta(f - 2f_c) + \delta(f + 2f_c) \right]$

(4) $a_2 m^2(t) \xrightarrow{F} a_2 M(f) * M(f) \Rightarrow BW = 2w$ (baseband)

(5) $2 a_2 A_c m(t) \cos(2\pi f_c t) \xrightarrow{F} a_2 A_c \left[M(f - f_c) + M(f + f_c) \right]$

(b) Combining (1) and (5)

$a_1 A_c \cos(2\pi f_c t) + 2 a_2 A_c m(t) \cos(2\pi f_c t) =$

$a_1 A_c \left[1 + \frac{2 a_2}{a_1} m(t) \right] \cos(2\pi f_c t)$ AM signal.

$\Rightarrow$ BPF : f_c : centre frequency

$BW = 2w$.

(c) To have <u>No</u> frequency distortion

$f_c - w > 2w \Rightarrow \underline{\underline{f_c > 3w}}$

($w < f_c < 2w$ mentioned in the book is <u>IN</u>correct).

Drill 3.6.

DSB - SC signal : $x_{DSB} = A_c m(t) \cos 2\pi f_c t$

where $m(t) = A_m \cos 2\pi f_m t$

$\Rightarrow x_{DSB-SC} = A_c A_m \cos 2\pi f_c t \cos 2\pi f_m t$

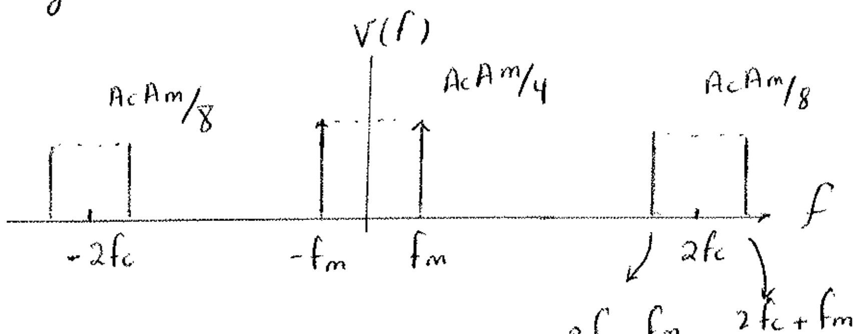
Consider the received local oscillator as : $\cos 2\pi f_c t$

$\Rightarrow$ The output $v(t) = A_c A_m \cos 2\pi f_m t \cos^2 2\pi f_c t$

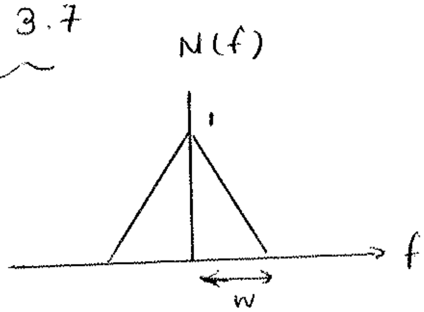
$$= \frac{A_c A_m}{2} \cos 2\pi f_m t \left[1 + \cos 2\pi (2 f_c) t \right]$$

where we use : $\cos^2 \alpha = \frac{1}{2}\left[1 + \cos 2\alpha \right]$

Now, taking the Fourier transform, we have :

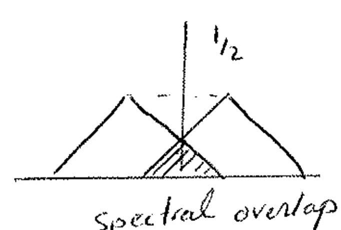


Drill 3.7



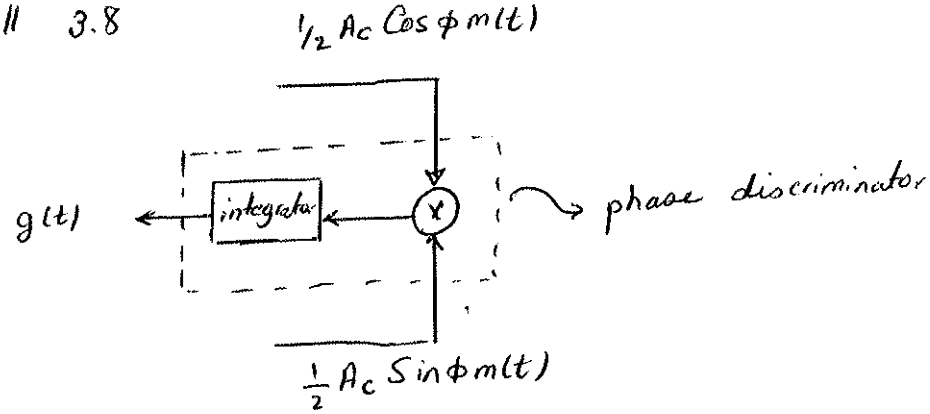
assuming that $f_c < W$

$\mathcal{F}(m(t) \cos 2\pi f_c t)$ is $\rightarrow$



The effects of the spectral overlap can not be mitigated using a coherent detector.

Drill 3.8

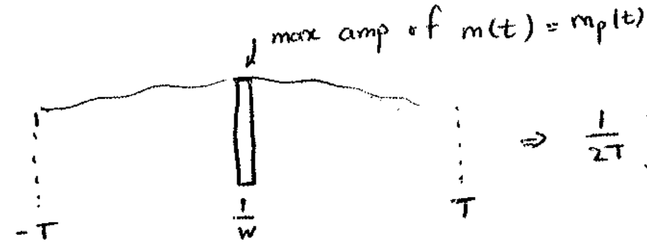


$$\Rightarrow \quad g(t) = \frac{1}{2T}\int_{-T}^{T} \frac{1}{4} A_c^2 \cos\phi \sin\phi \, m^2(t)\, dt$$

$$= \frac{1}{4} A_c^2 \underbrace{\cos\phi \sin\phi}_{\frac{1}{2}\cos 2\phi} \cdot \frac{1}{2T}\int_{-T}^{T} m^2(t)\, dt \overset{(*)}{=} \frac{1}{4} A_c^2 \phi \cdot \frac{1}{2T}\int_{-T}^{T} m^2(t)\, dt$$

ϕ is small $\Rightarrow \cos 2\phi \simeq 2\phi$ (*)

Now $2T \gg \underbrace{\frac{1}{W}}_{T_m}$



$$\Rightarrow \frac{1}{2T}\int_{-T}^{T} m^2(t)\, dt = \frac{1}{W} m_p(t)$$

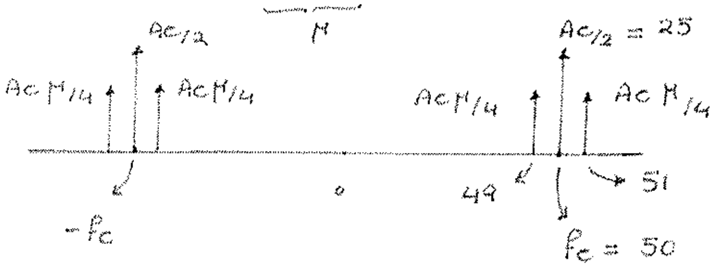
$$\Rightarrow \quad \boxed{g(t) = \frac{1}{4} A_c^2 \phi \cdot \frac{1}{W} \cdot m_p(t)}$$

5. problem 3.18.

$m(t) = 20\cos(2\pi t)$ message. (1 Hz)

$c(t) = 50\cos(100\pi t)$ carrier (50 Hz) $A_c = 50$

AM $s(t) = A_c\left(1 + \underbrace{k_a A_m}_{\mu} \cos(2\pi t)\right)\cos(100\pi t)$.



$= 50 \cdot \frac{0.75}{4} = \frac{150}{16}$

(a) $\mu = 0.75$ 75 percent modulation

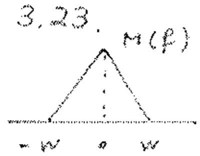
(b) power across $100\,\Omega$: $P = \frac{1}{100}\left(A_c^2/2\right) + \frac{1}{100}\left(A_c^2\mu^2/8 \times 2\right)$

$= \frac{25 \times 100}{200} + \frac{25 \times 100 \times 0.75^2}{400}$

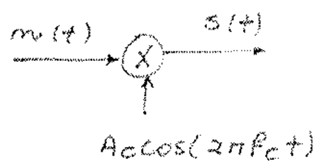
$= 12.5 + 3.5156 = \underline{15.02}$

6. problem 3.23.

$m(t)$:



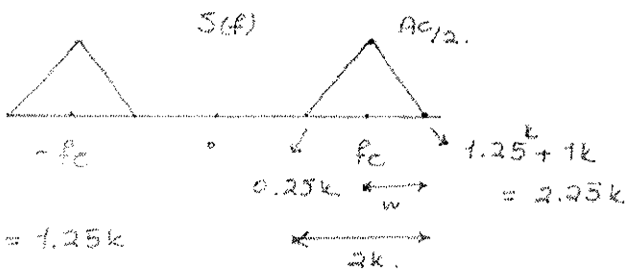
$W = 1\,kHz$.

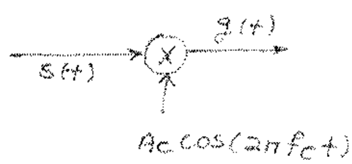


$A_c\cos(2\pi f_c t)$: carrier

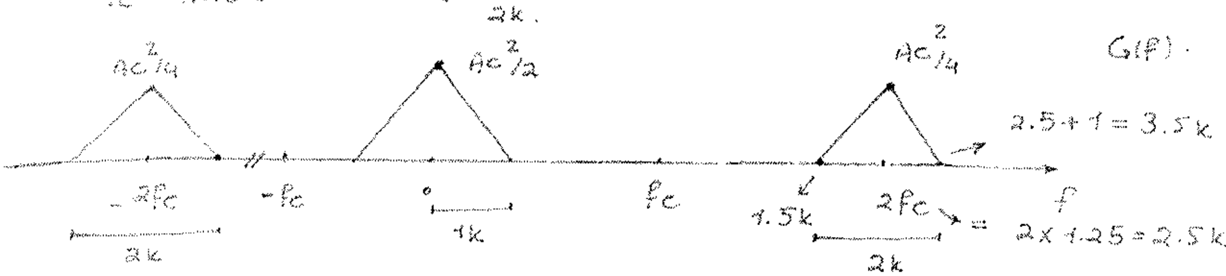
Detector :

(a)



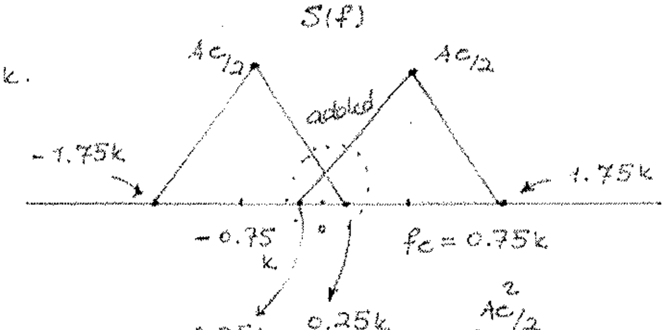


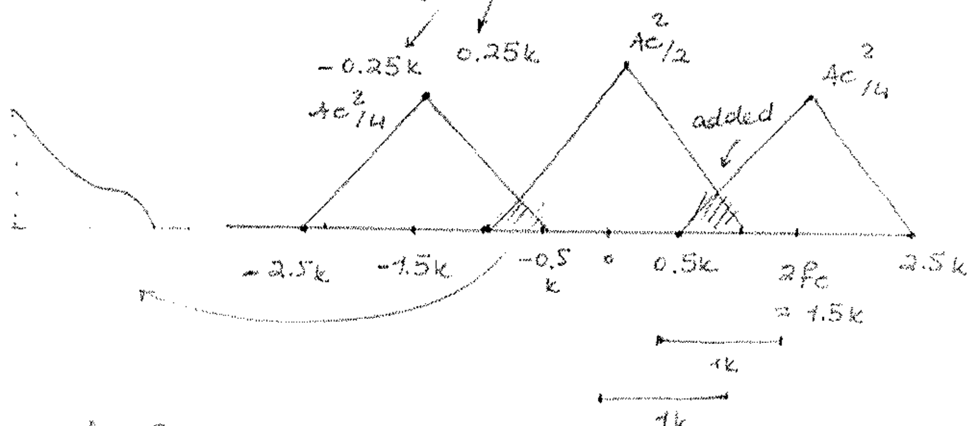
$f_c = 1.25k$



4,

(b) if $f_c = 0.75k$.



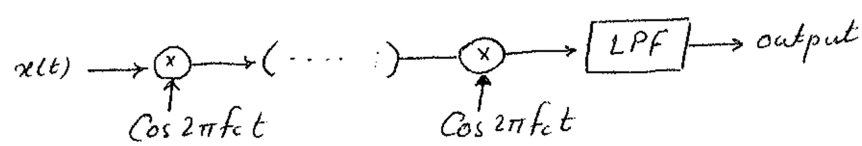


Therefore : if $f_c > w$

$\Rightarrow$ $2f_c > 2w$ $\Rightarrow$ the high freq. component centered at $2f_c$ will not ~~overlap~~ overlap with the base-band component.

$\Rightarrow$ Both components of the signal are uniquely determined. Hence $f_c > 1kHz$

Problem 3.25. Coherent detector



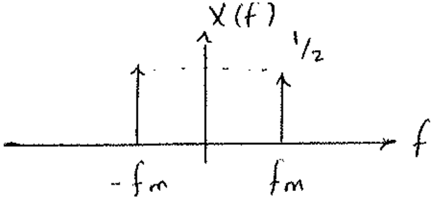
incoming DSB signal : $x(t) \cos 2\pi f_c t$

if there is an error $\Rightarrow$ $\cos 2\pi (f_c + \Delta f)t$ at the receiver side.

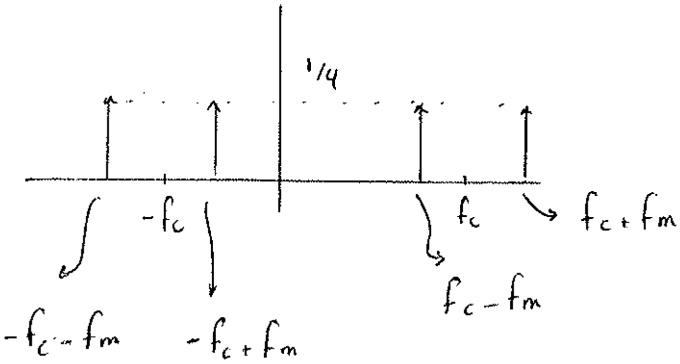
$\Rightarrow$ $x(t) \cos 2\pi f_c t . \cos 2\pi (f_c + \Delta f)t . = \frac{1}{2} x(t) \left[\cos 2\pi \Delta f t + \cos 2\pi (2f_c + \Delta f)t \right]$

Therefore, without any error, i.e., $\Delta f = 0$, the msg signal can be recovered using a LPF. However, with $\Delta f \neq 0$ (but Δf small), $\cos 2\pi \Delta f t$ appears as an extra term.

(b) if $x(t) = \cos 2\pi f_m t \xRightarrow{F}$ The FT of the output is :

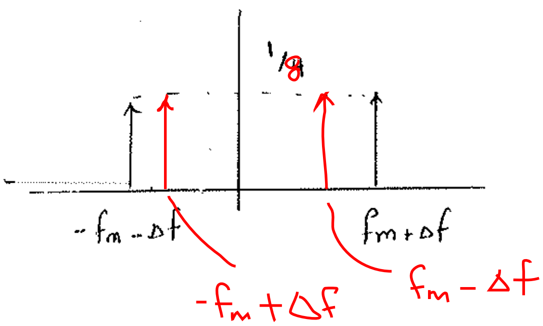


$\xrightarrow{\times \cos 2\pi f_c t}$



$\xrightarrow{\times \cos 2\pi (f_c+\Delta f) t}$

and LPF



Drill 3.9

$$s(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t$$

Consider the upper branch.

$$x(t) = s(t) \cdot A_c' \cos 2\pi f_c t = A_c A_c' m_1(t) \cos^2 2\pi f_c t + A_c A_c' m_2(t) \cos 2\pi f_c t \times \sin 2\pi f_c t$$

$$= \frac{A_c A_c' m_1(t)}{2} + \underbrace{\frac{1}{2} A_c A_c' \cos 2\pi(2f_c)t}_{\text{high freq.}} + \underbrace{\frac{1}{2} A_c A_c' m_2(t) \sin 2\pi(2f_c)t}_{\text{high freq}}.$$

$\Rightarrow$ output of the LPF is $\frac{A_c A_c'}{2} m_1(t)$.

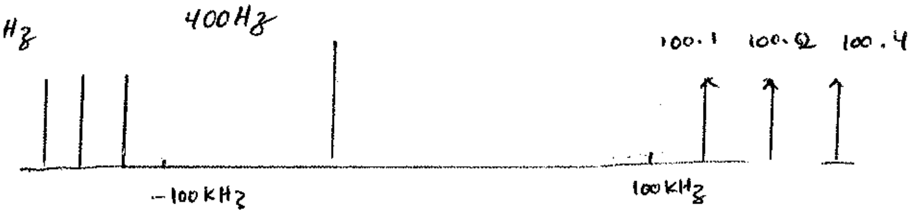
In a similar way, the output of the lower branch can be derived.

3.27.

$$m(t) = \underbrace{\cos 2\pi f_1 t}_{100\,Hz} + \underbrace{\cos 2\pi f_2 t}_{200\,Hz} + \underbrace{\cos 2\pi f_3 t}_{400\,Hz}$$

$\Rightarrow$ $m(t) \cos 2\pi f_{c_1} t \xrightarrow{f}$

$f_{c_1} = 100\,KHz$

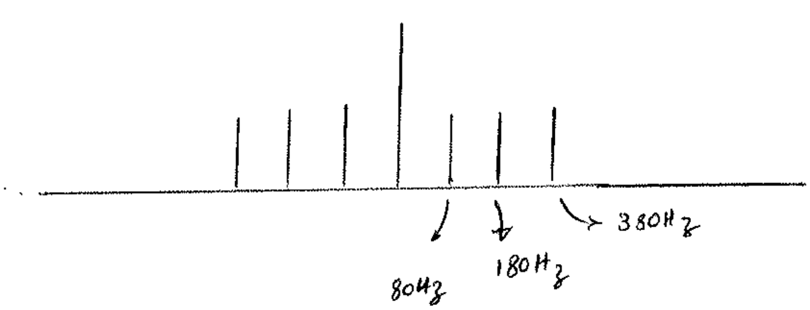


Spectrum of the SSB signal

$\Rightarrow$ At the receiver side:

$\times \cos 2\pi f_{c_2} t$

$f_{c_2} = 100.02\,kHz$

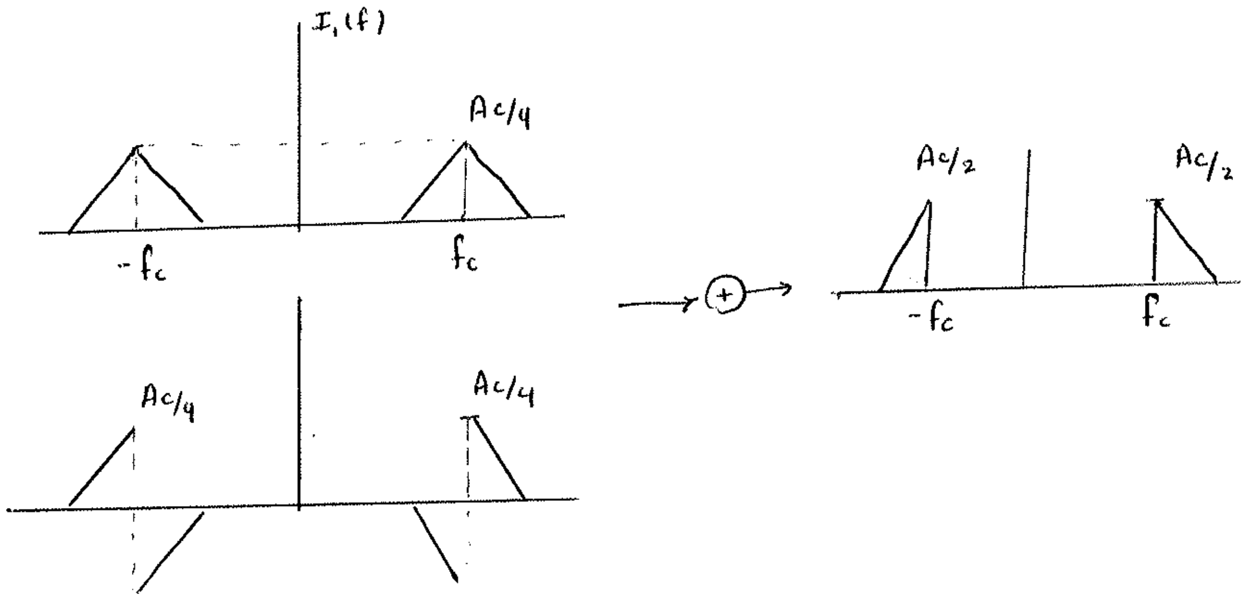


Drill 3.10.

$$s(t) = \frac{A_c}{2}\, m(t) \cos 2\pi f_c t \;-\; \frac{A_c}{2}\, \hat{m}(t) \sin 2\pi f_c t$$

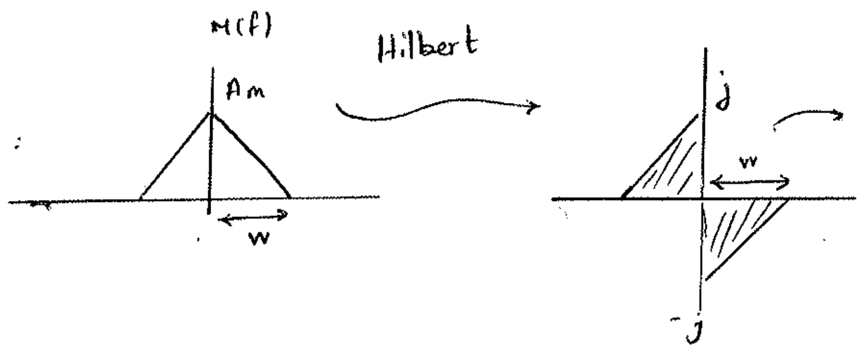
$$\hat{m}(t) \xrightarrow{\mathcal{F}} -j\,\mathrm{Sgn}(f)\, M(f)$$

$$\Rightarrow \quad S(f) = \underbrace{\frac{A_c}{4} M(f) * \left[\delta(f-f_c) + \delta(f+f_c)\right]}_{I_1(f)} \;-\; \underbrace{\frac{A_c}{4}\left(-j\,\mathrm{Sgn}(f) M(f)\right) \cdot \frac{1}{2j}\left(\delta(f-f_c) - \delta(f+f_c)\right)}_{I_2(f)}$$



Drill . 3.11

Suppose that :



still it is lowpass
and BW = W

Drill 3.12.

Same as Drill. 3.7

Drill 3.13.

$$s(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t - \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t \longrightarrow$$ Coherent detector $\times A_c' \cos 2\pi f_c t$

$$s(t) \times A_c' \cos 2\pi f_c t = \frac{A_c A_c'}{2} m(t) \cos^2 2\pi f_c t - \frac{A_c A_c'}{2} \hat{m}(t) \underbrace{\cos 2\pi f_c t \sin 2\pi f_c t}_{\text{high freq.}}$$

$$= \frac{A_c A_c'}{4} m(t) [1 + \underbrace{\cos 2\pi (2 f_c) t}_{\text{high freq.}}]$$

$\Rightarrow$ Output of the LPF is

$$\frac{A_c A_c'}{4} m(t)$$

(11) $s(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t$

$\rightarrow g_1(t)$ before LPF $= A_c m_1(t) \cos 2\pi f_c t \cos(2\pi f_c t + \phi)$

$+ A_c m_2(t) \sin 2\pi f_c t \cos(2\pi f_c t + \phi)$

high freq.

$$= \frac{A_c m_1(t)}{2} \cos(4\pi f_c t + \phi) + \frac{A_c m_1(t)}{2} \cos\phi$$

$$+ \frac{A_c m_2(t)}{2} \sin(4\pi f_c t + \phi) - \frac{A_c m_2(t)}{2} \sin\phi$$

high freq

$$\Rightarrow g_1(t) = \frac{A_c m_1(t)}{2} \cos\phi - \frac{A_c m_2(t)}{2} \sin\phi.$$

$g_2(t)$ can be derived in a same way.

(12) $g(t) = \cos 2\pi f_c t \Rightarrow G(f) = \frac{1}{2}\delta(f - f_c) + \frac{1}{2}\delta(f + f_c)$

$$\Rightarrow \hat{g}(t) : \quad \hat{G}(f) = -j\,\mathrm{Sgn}(f) \cdot \left[\frac{1}{2}\delta(f - f_c) + \frac{1}{2}\delta(f + f_c)\right]$$

$$= -j/2\,\delta(f - f_c) + j/2\,\delta(f + f_c)$$

$$= \underbrace{\frac{1}{2j}\left[\delta(f - f_c) - \delta(f + f_c)\right]}$$

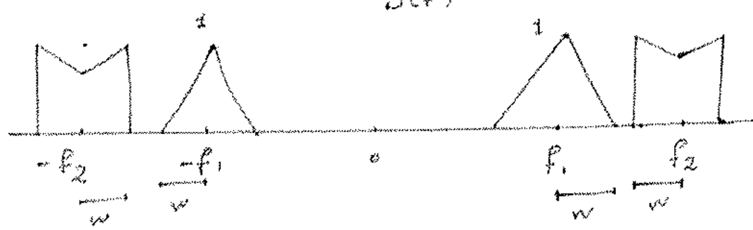
$$\downarrow \mathcal{F}^{-1}$$

$$\sin 2\pi f_c t.$$

(13) Same as Drill 3.11.

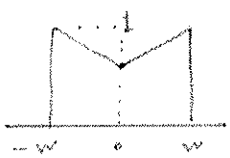
6. $s(t) = A_c m_1(t) \cos(2\pi f_1 t) + A_c m_2(t) \cos(2\pi f_2 t)$.

(a) $f_2 - f_1 > 2W$.

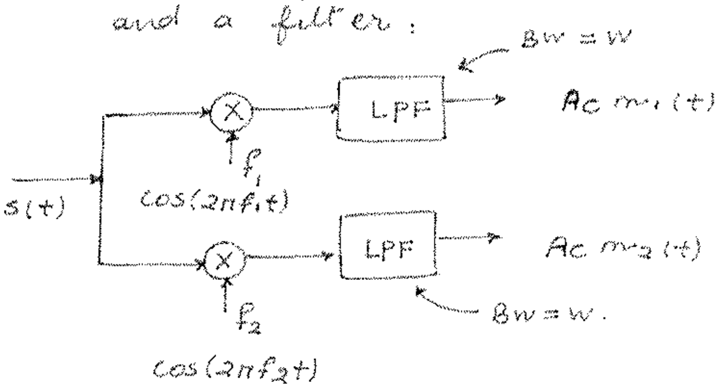


$M_1(f)$:

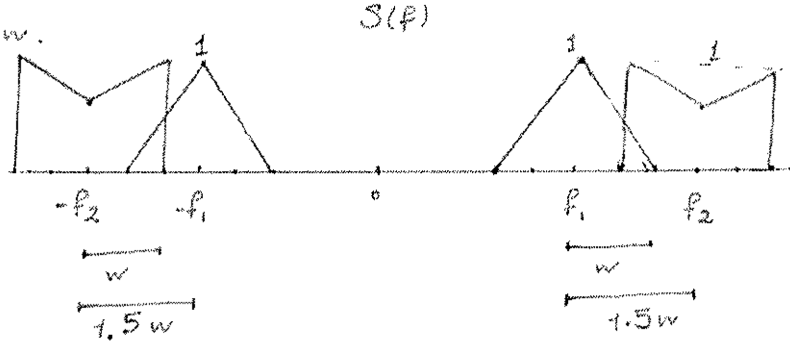
$M_2(f)$:



since $f_2 - f_1 > 2W \Rightarrow$ each signal can be recovered separately with a down-convert. and a filter:

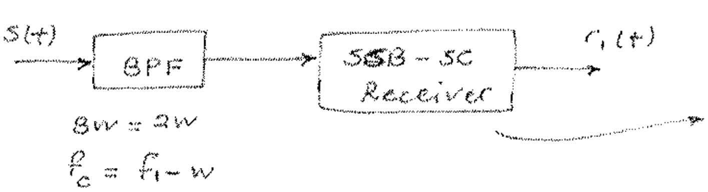


(b) $f_2 - f_1 = 1.5W$.

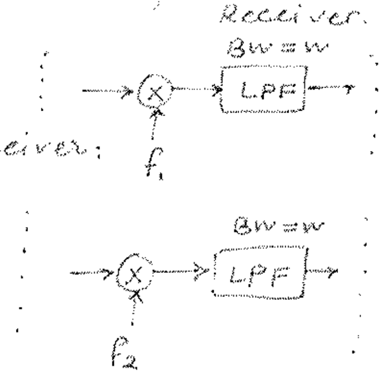


Block Diagra for SSB-SC Receiver.

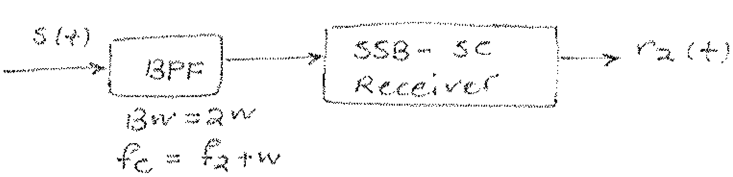
for m_1 :



Receiver:



for m_2 :



Supplement Q2.

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

$$s(t)\cos(2\pi f_c t + \phi)$$

$$= A_c m_1(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \phi) + A_c m_2(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \phi)$$

$$= \frac{1}{2} A_c m_1(t)\left[\cos\phi + \cos(2\pi 2 f_c t + \phi)\right] + \frac{1}{2} A_c m_2(t)\left[\sin(2\pi 2 f_c t + \phi) + \sin(-\phi)\right]$$

$$\xrightarrow{LPF} g_1(t) = \frac{1}{2} A_c m_1(t)\cos\phi + \frac{1}{2} A_c m_2(t)\sin(-\phi)$$

$$= \frac{1}{2} A_c\left[\cos\phi\, m_1(t) - \sin\phi\, m_2(t)\right]$$

$$s(t)\sin(2\pi f_c t + \phi)$$

$$= A_c m_1(t)\cos(2\pi f_c t)\sin(2\pi f_c t + \phi) + A_c m_2(t)\sin(2\pi f_c t)\sin(2\pi f_c t + \phi)$$

$$= \frac{1}{2} A_c m_1(t)\left[\sin(2\pi 2 f_c t + \phi) + \sin\phi\right] + \frac{1}{2} A_c m_2(t)\left[\cos\phi - \cos(2\pi 2 f_c t + \phi)\right]$$

$$\xrightarrow{LPF} g_2(t) = \frac{1}{2} A_c\left[\sin\phi\, m_1(t) + \cos\phi\, m_2(t)\right]$$