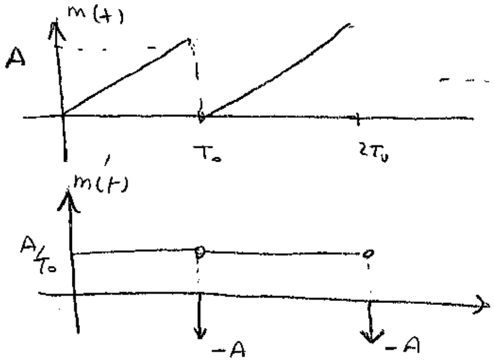
ECE 316 CHAPTER 4 PROBLEM SET SOLUTIONS

4.8) PM signal : $s_{pm}(t) = A_c \cos(2\pi f_c t + k_p m(t)) = A_c \cos(\theta_{pm}(t))$

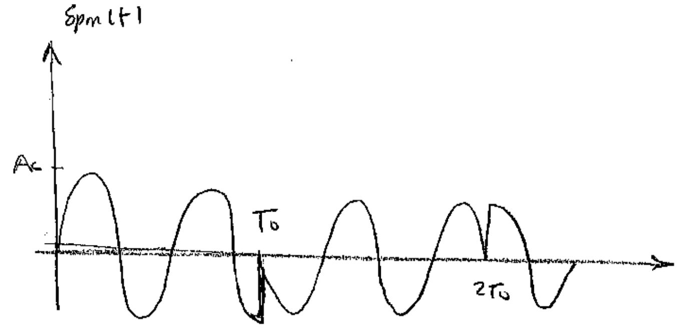
with $\theta_{pm}(t) = 2\pi f_c t + k_p m(t)$



$$\frac{1}{2\pi}\frac{d\theta_{pm}(t)}{dt} = f_c + k_p \frac{dm(t)}{dt}$$

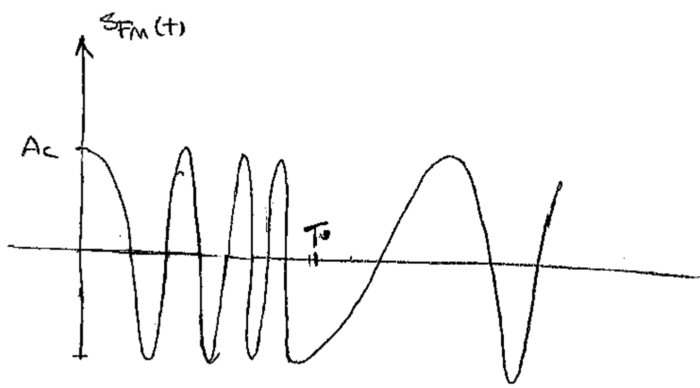
$$\rightarrow \frac{dm(t)}{dt} = A/T_0 - A\sum_{n=1}^{\infty} \delta(t - nT_0)$$

Therefore, $s_{pm}(t)$ is a cosine with a frequency $f_c + \frac{k_p}{2\pi}\frac{A}{T_0}$ when at nT_0 will have phase shifts of $-k_p A$



FM signal : $s_{FM}(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau)$

$$\theta_{FM}(t) = 2\pi f_c t + 2\pi k_f \cdot \int_0^t m(\tau)d\tau \Rightarrow \frac{1}{2\pi}\frac{d\theta_{FM}}{dt} = f_c + k_f m(t)$$



i.e., signal is a cosine with an instantaneous frequency increasing from f_c to $k_f \cdot A$ and then resetting to f_c at nT_0.

4.11) $m(t) = A_m \cos(2\pi f_m t)$, $S_{pm}(t) = A_c \cos(2\pi f_c t + k_p m(t))$

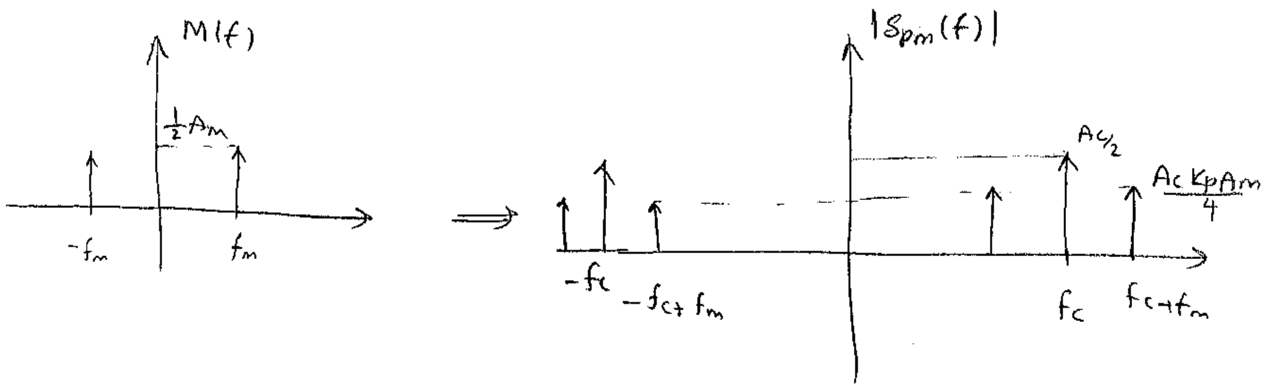
we know that $\beta = k_p A_m \leq 0.3$.

First not that for $-0.3 \leq x \leq 0.3$, $\begin{cases} \cos(x) \simeq 1 \\ \sin(x) \simeq x \end{cases}$

For: $S_{pm}(t)$ we have:

$$S_{pm}(t) = A_c \left[\cos(2\pi f_c t) \cos(k_p m(t)) - \sin(2\pi f_c t) \sin(k_p m(t)) \right]$$

$$\simeq A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t) . k_p . m(t)$$



more specifically:

$$S_{pm}(f) = \frac{1}{2} A_c \left[\delta(f - f_c) + \delta(f + f_c) \right]$$

$$- \frac{1}{4j} \beta_p A_c \left[\delta(f - f_c - f_m) - \delta(f + f_c + f_m) \right]$$

$$- \frac{1}{4j} \beta_p A_c \left[\delta(f - f_c + f_m) - \delta(f + f_c - f_m) \right]$$

4.15) if the VCO Frequency is different than that of the crystall then after mixer and frequency discriminator there is a term $x(t) + \Delta$ where $x(t)$ is due to the message and Δ is because of the error in the VCO. After LPF, $x(t)$ is canceled and Δ is used to adjust the error in the VCO.

4.16) $v(t) = s(t) - s(t-T) = A_c \cos(2\pi f_c t + \phi(t)) - A_c \cos(2\pi f_c (t-T) + \phi(t-T))$

$$= 2A_c \sin\left(\frac{2\pi f_c(2t-T) + \phi(t) + \phi(t-T)}{2}\right) \sin\left(\frac{2\pi f_c T + \phi(t) - \phi(t-T)}{2}\right)$$

where $\phi(t) = \beta \sin(2\pi f_m t)$

then $\phi(t) - \phi(t-T) = \beta\Big(\sin(2\pi f_m t) - \sin(2\pi f_m (t-T))\Big)$

$$= \beta\left(\sin(2\pi f_m t) - \sin(2\pi f_m t)\cos(2\pi f_m T) + \cos(2\pi f_c t)\sin(2\pi f_m T)\right)$$

we have $\cos(2\pi f_m T) \simeq 1$, $\sin(2\pi f_m T) \simeq 2\pi f_m T$ (true if $f_m \ll f_c$)

$\Longrightarrow \phi(t) - \phi(t-T) \simeq 2\pi \Delta f\, T \cos(2\pi f_m t)$, $\Delta f = \beta f_m$

we have $2\pi f_c T = \frac{\pi}{2}$, then

$$\sin\left(\frac{2\pi f_c T + \phi(t) - \phi(t-T)}{2}\right) \simeq \sin(\pi f_c T + \pi \Delta f\, T \cos(2\pi f_m t)) = \sin\left(\frac{\pi}{4} + \pi \Delta f\, T \cos(2\pi f_m t)\right)$$

$$\simeq \frac{\sqrt{2}}{2}\left(\cos(\pi \Delta f\, T \cos(2\pi f_m t)) + \sin(\pi \Delta f\, T \cos(2\pi f_m t))\right)$$

$$\simeq \frac{\sqrt{2}}{2}\left(1 + \pi \Delta f\, T \cos(2\pi f_m t)\right)$$

Finally, it follows for $v(t)$ that

$$v(t) \simeq -\sqrt{2}\,A_c\,(1+\pi D_f T\cos(2\pi f_m t))\sin\left(\pi f_c (2t-T) + \frac{\phi(t)-\phi(t-T)}{2}\right)$$

then, the output of the envelope detector is

$$a(t) = \sqrt{2}\,A_c\,(1+\pi D_f T\cos(2\pi f_m t))$$

4.17) $S_{PM} = A\cos(2\pi f_c t + k_p m_2(t))$, $S_{FM} = A\cos(2\pi f_c t + 2\pi k_f \int_0^t m_1(\tau))$

$\Longrightarrow$ $S_{PM} = A\cos(2\pi f_c t + k_p(b_2 t^2 + b_1 t + b_0))$

$S_{FM} = A\cos(2\pi f_c t + 2\pi k_f(\frac{a_1}{2}t^2 + a_0 t))$

$\Longrightarrow$ we need to have 1) $k_p b_0 = 0 \Longrightarrow b_0 = 0$

2) $k_p b_1 = 2\pi k_f a_0$

3) $k_p b_2 = \pi k_f a_1$