Properties of Angle Modulation

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Reference:

Sections 4.2 and 4.3 of

S. Haykin and M. Moher, Introduction to Analog & Digital Communications, 2nd ed., John Wiley & Sons, Inc., 2007. ISBN-13 978-0-471-43222-7.

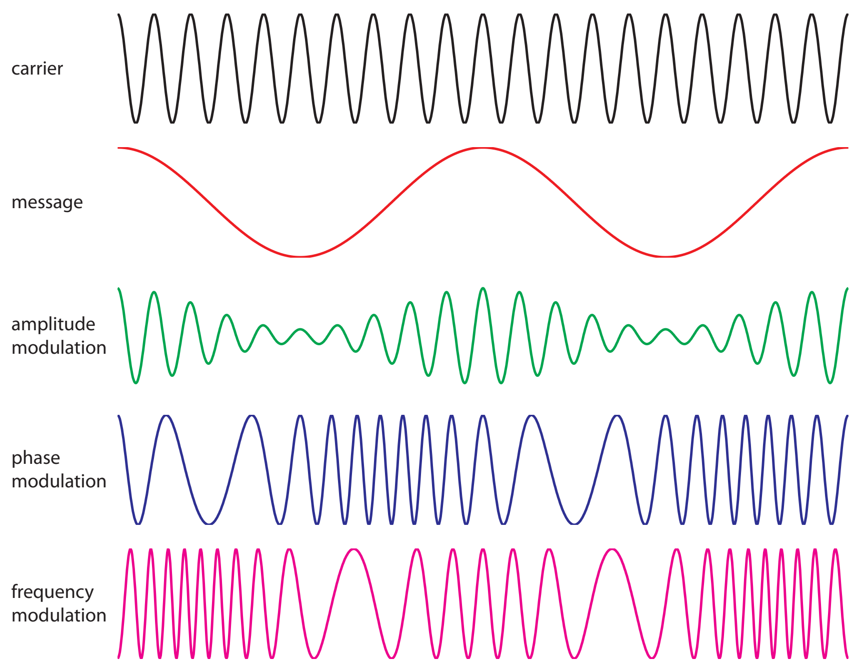
Angle Modulation

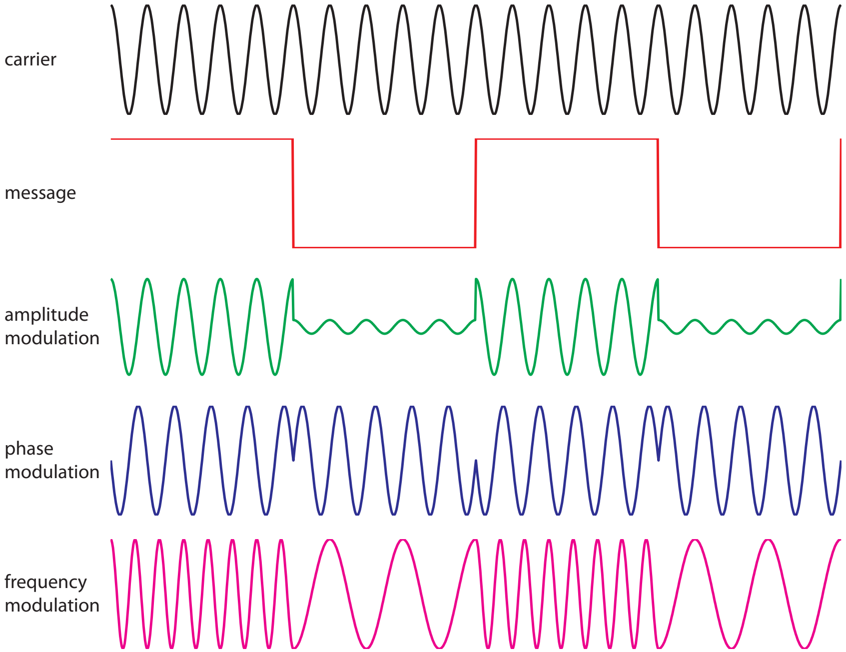
- Phase Modulation (PM):

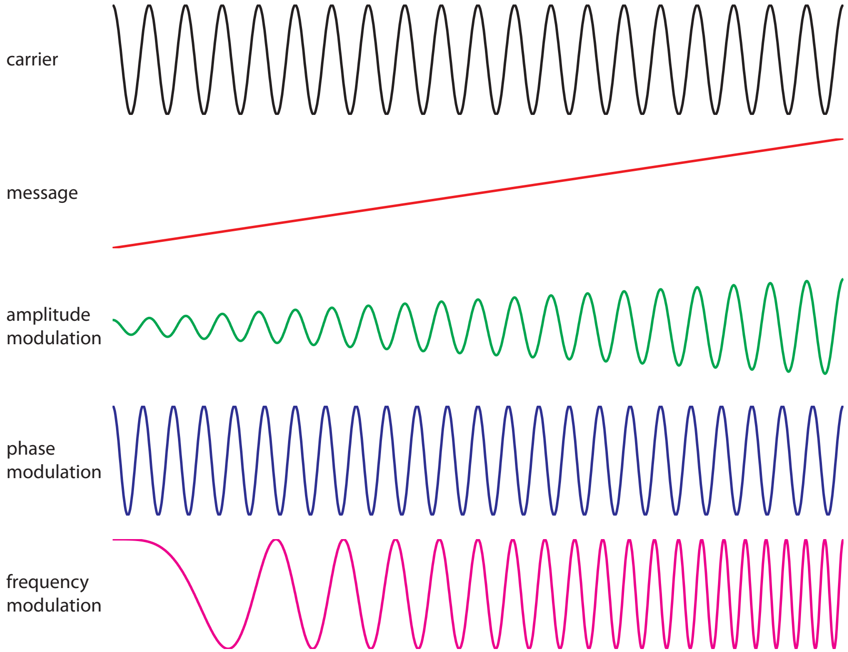
$$\begin{aligned}\theta_i(t) &= 2\pi f_c t + k_p m(t)\\ f_i(t) &= \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}\\ s_{PM}(t) &= A_c \cos[2\pi f_c t + k_p m(t)]\end{aligned}$$

- Frequency Modulation (FM):

$$\begin{aligned}\theta_i(t) &= 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\\ f_i(t) &= \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + k_f m(t)\\ s_{FM}(t) &= A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]\end{aligned}$$







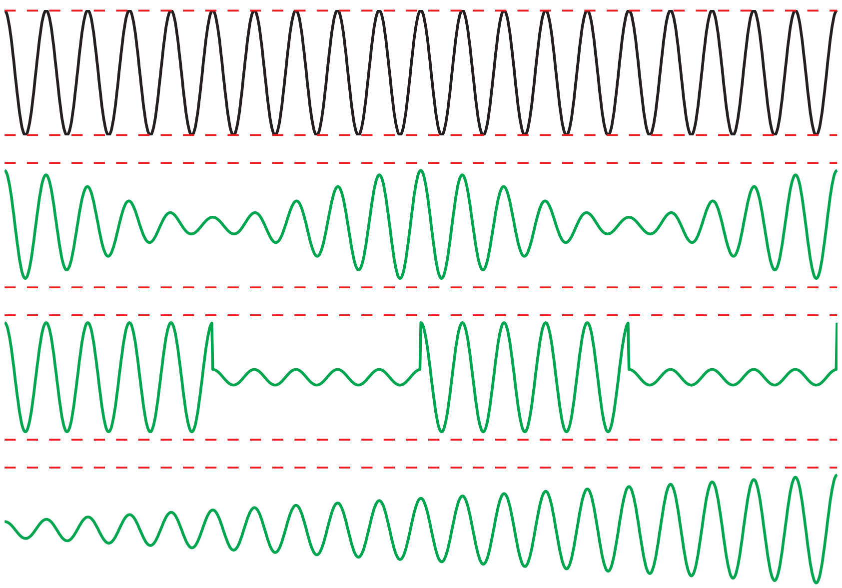
Constancy of Transmitted Power

- Consider a sinusoid $g(t) = A_c \cos(2\pi f_0 t + \phi)$ where $T_0 = \frac{1}{f_0}$ or $T_0 f_0 = 1$.
- The power of $g(t)$ (over a 1 ohm resister) is defined as:

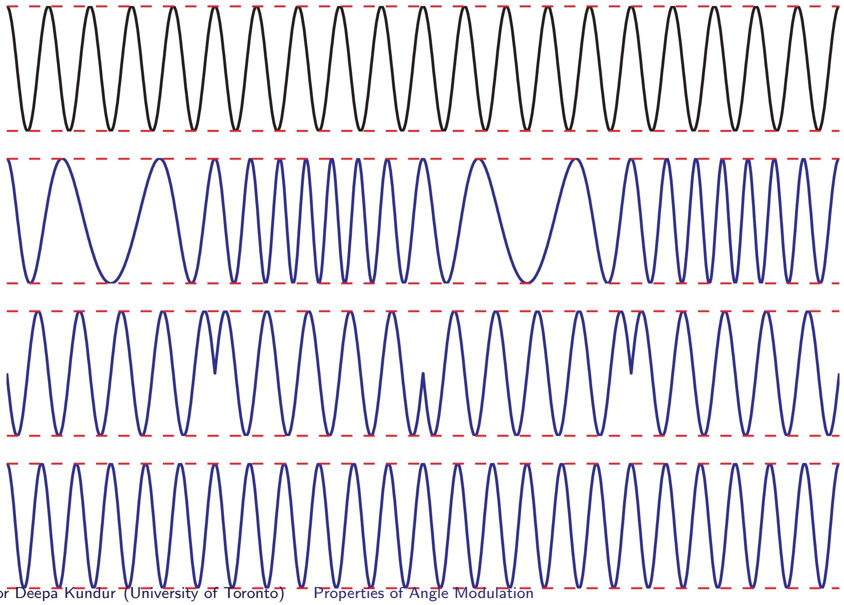
$$
\begin{aligned}
P &= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g^2(t)dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A_c^2 \cos^2(2\pi f_0 t + \phi)dt \\
&= \frac{A_c^2}{T_0}\int_{-T_0/2}^{T_0/2} \frac{1}{2}\left[1 + \cos(4\pi f_0 t + 2\phi)\right] dt \\
&= \frac{A_c^2}{2T_0}\left[t + \frac{\sin(4\pi f_0 t + 2\phi)}{4 f_0}\right]\Bigg|_{-T_0/2}^{T_0/2} \\
&= \frac{A_c^2}{2T_0}\left[\left(\frac{T_0}{2} - \frac{-T_0}{2}\right) + \left(\frac{\sin(4\pi f_0 T_0/2 + 2\phi) - \sin(-4\pi f_0 T_0/2 + 2\phi)}{4\pi f_0}\right)\right] \\
&= \frac{A_c^2}{2T_0}\left[T_0 + \frac{\sin(2\pi + 2\phi) - \sin(-2\pi + 2\phi)}{4\pi f_0}\right] = \frac{A_c^2}{2}
\end{aligned}
$$

- Therefore, the power of a sinusoid is NOT dependent on f_0, just its envelop A_c.

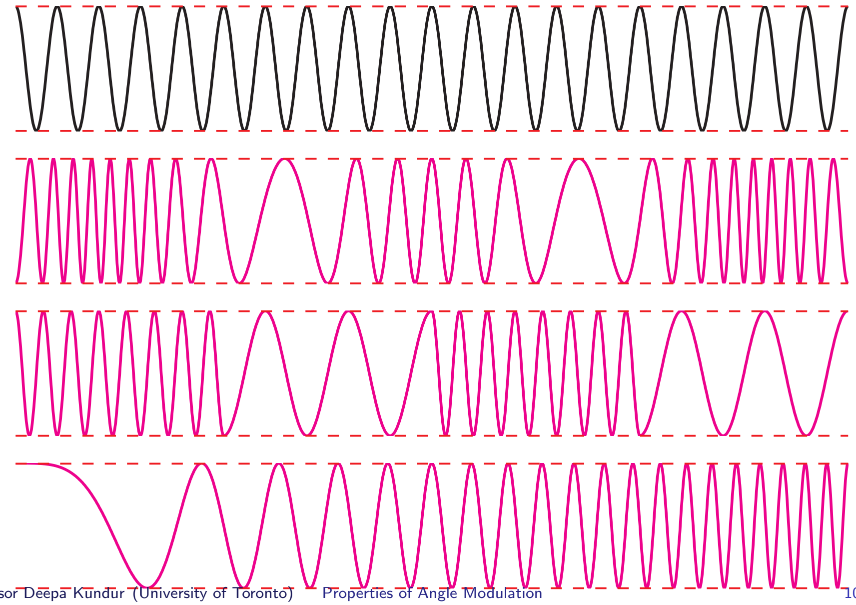
Constancy of Transmitted Power: AM



Constancy of Transmitted Power: PM



Constancy of Transmitted Power: FM



Constancy of Transmitted Power

Therefore, angle modulated signals exhibit constancy of transmitted power.

Nonlinearity of Angle Modulation

Consider PM (proof also holds for FM).

- Suppose

$$\begin{aligned} s_1(t) &= A_c \cos\left[2\pi f_c t + k_p m_1(t)\right] \\ s_2(t) &= A_c \cos\left[2\pi f_c t + k_p m_2(t)\right] \end{aligned}$$

- Let $m_3(t) = m_1(t) + m_2(t)$.

$$\begin{aligned} s_3(t) &= A_c \cos\left[2\pi f_c t + k_p(m_1(t) + m_2(t))\right] \\ &\neq s_1(t) + s_2(t) \\ & \quad \because \cos(2\pi f_c t + A + B) \neq \cos(2\pi f_c t + A) + \cos(2\pi f_c t + B) \end{aligned}$$

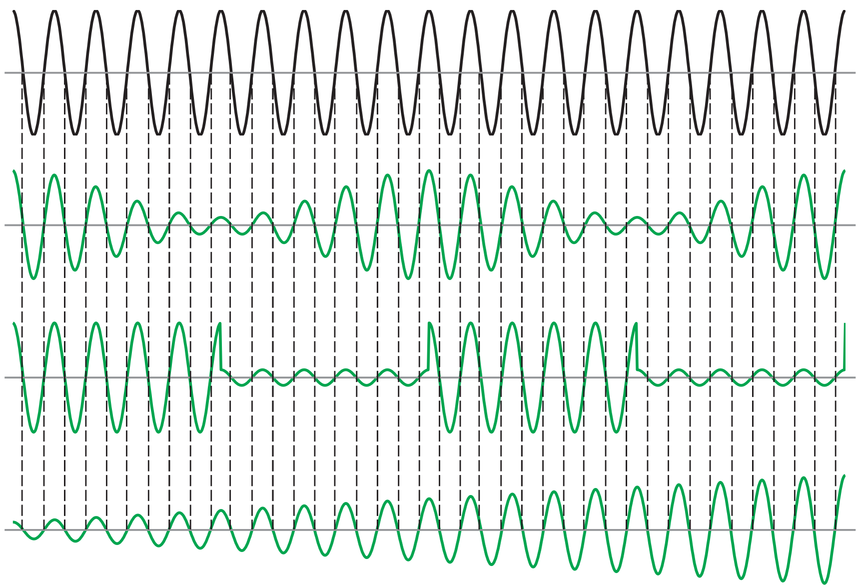
Nonlinearity of Angle Modulation

Therefore, angle modulation is nonlinear.

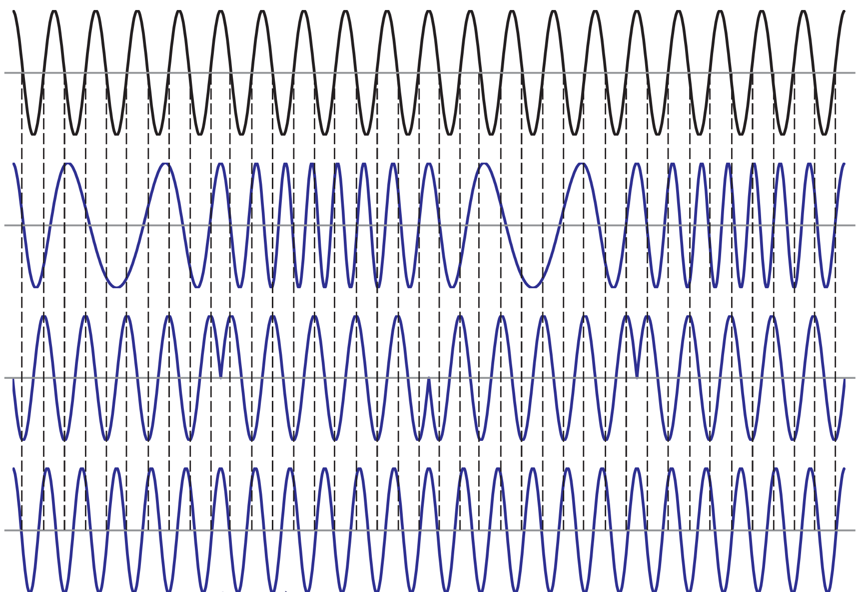
Irregularity of Zero-Crossings

- Zero-crossing: instants of time at which waveform changes amplitude from positive to negative or vice versa.

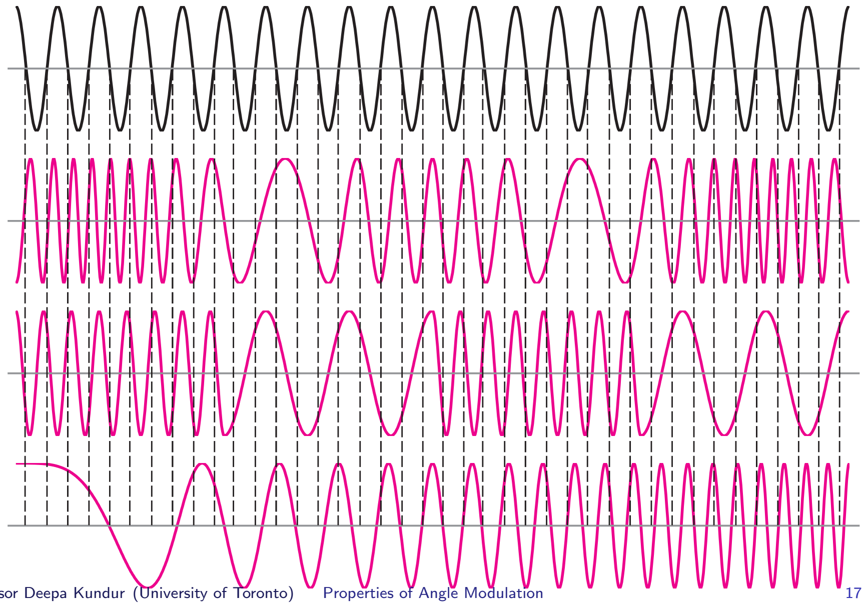
Zero-Crossings: AM



Zero-Crossings: PM



Zero-Crossings: FM



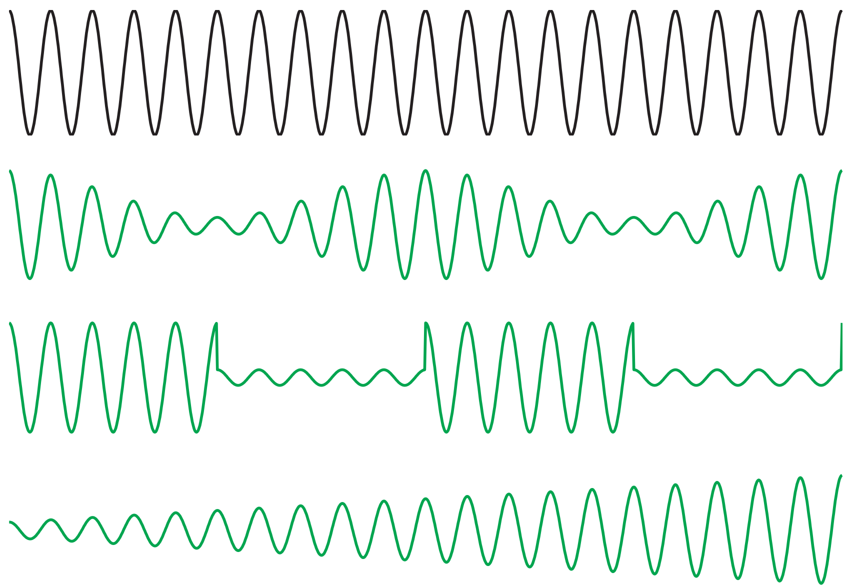
Irregularity of Zero-Crossings

Therefore angle modulated signals exhibit irregular zero-crossings because they contain information about the message (which is irregular in general).

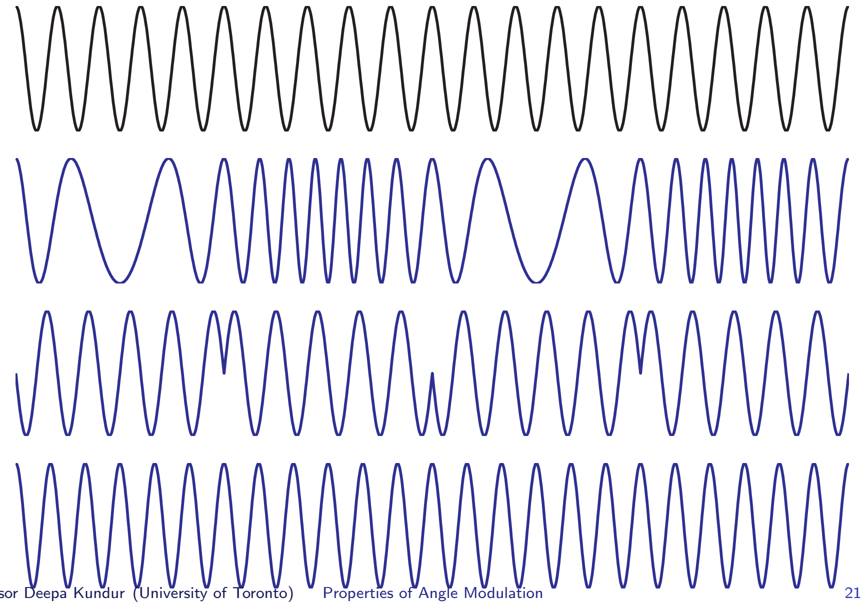
Visualization Difficulty of Message

- Visualization of a message refers to the ability to glean insights about the shape of $m(t)$ from the modulated signal $s(t)$.

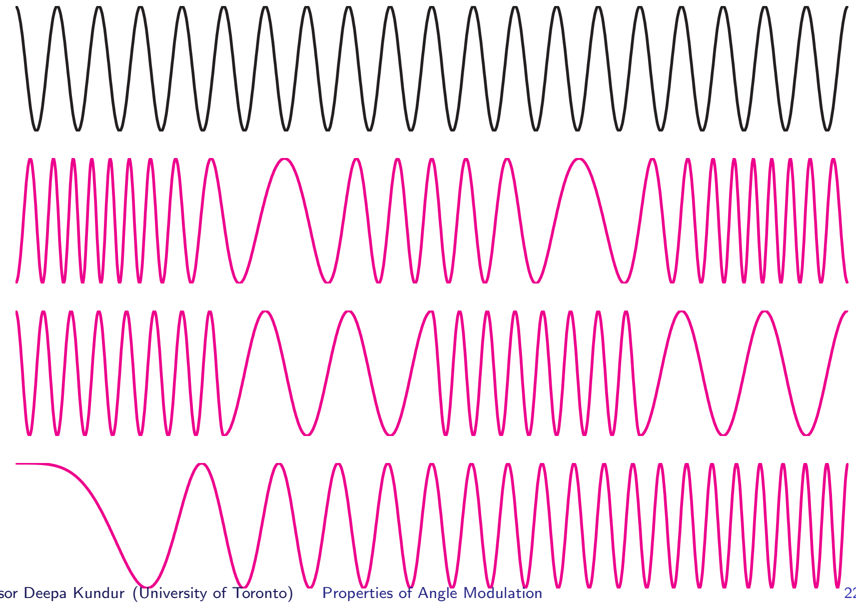
Visualization: AM



Visualization: PM



Visualization: FM

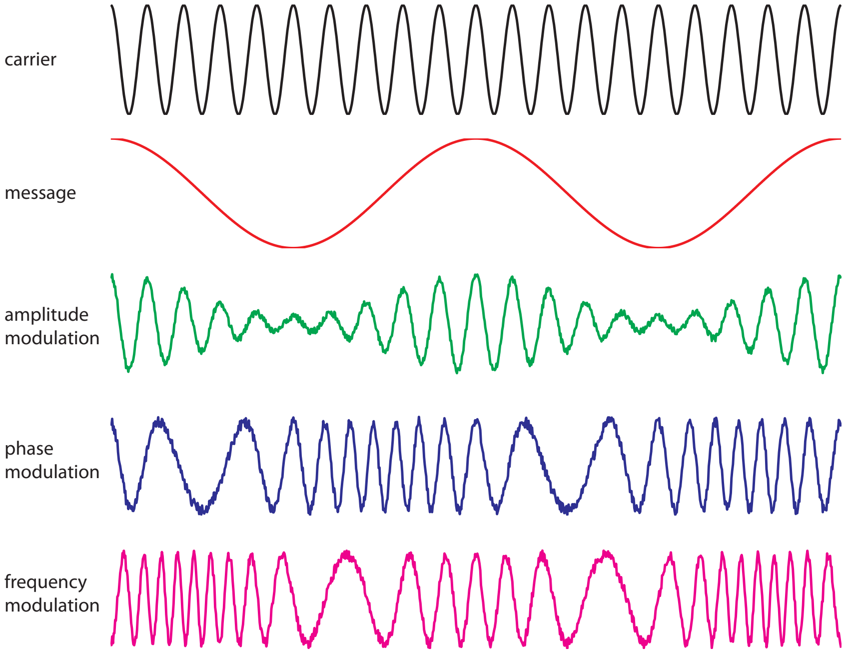


Visualization Difficulty of Message

Therefore it is difficult to visualize the message in angle modulated signals due to the nonlinear nature of the modulation process.

Bandwidth vs. Noise Trade-Off

- Noise affects the message signal piggy-backed as amplitude modulation more than it does when piggy-backed as angle modulation.
- The more bandwidth that the angle modulated signal takes, typically the more robust it is to noise.


carrier
message
amplitude modulation
phase modulation
frequency modulation