Final Exam Review

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Final Exam Review

Reference:

Sections:
2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7
3.1, 3.2, 3.3, 3.4, 3.5, 3.6
4.1, 4.2, 4.3, 4.4, 4.6, 4.7, 4.8
5.1, 5.2, 5.3, 5.4, 5.5
6.1, 6.2, 6.3, 6.4, 6.5, 6.6
7.1, 7.2, 7.3, 7.4, 7.5, 7.6

S. Haykin and M. Moher, Introduction to Analog & Digital Communications, 2nd ed., John Wiley & Sons, Inc., 2007. ISBN-13 978-0-471-43222-7.

Chapter 7: Digital Band-Pass Modulation Techniques

Binary Modulation Schemes

$$c(t) = A_c \cos(2\pi f_c t + \phi_c)$$

- Binary amplitude-shift keying (BASK): carrier amplitude is keyed between two possible values (typically $\sqrt{E_b}$ and 0 to represent 1 and 0, respectively); carrier phase and frequency are held constant.
- Binary phase-shift keying (BPSK):carrier phase is keyed between two possible values (typically 0 and π to represent 1 and 0, respectively); carrier amplitude and frequency are held constant.
- Binary frequency-shift keying (BFSK): carrier frequency is keyed between two possible values (typically f_1 and f_2 to represent 1 and 0, respectively); carrier amplitude and phase are held constant.

Preliminaries

$$c(t) = A_c \cos(2\pi f_c t + \phi_c)$$

- T_b represents the bit duration
- E_b represents the energy of the transmitted signal per bit
- In digital communications the carrier amplitude is normalized to have unit energy in one bit duration; thus we set

$$A_c = \sqrt{\frac{2}{T_b}}$$

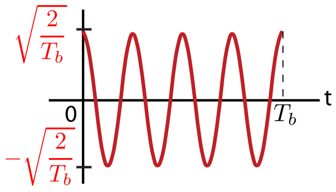
- The carrier frequency $f_c = \frac{k}{T_b}$ for $k \in \mathbb{Z}$ to ensure an integer number of carrier cycles in a bit duration.

Carrier for Digital Communications

Therefore

$$c(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t + \phi_c).$$

For $k = 4$



Binary Amplitude-Shift Keying (BASK)

Let $\phi_c = 0$ and the carrier frequency is f_c.

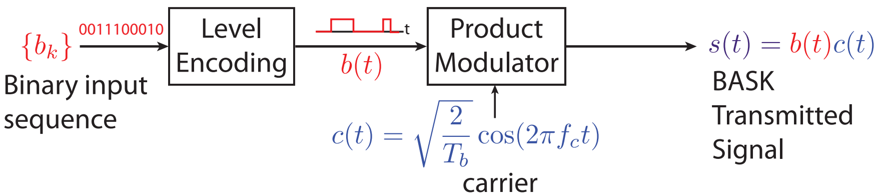
$$b(t) = \begin{cases} \sqrt{E_b} & \text{for binary symbol 1} \\ 0 & \text{for binary symbol 0} \end{cases}$$

$$c(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t + \phi_c) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

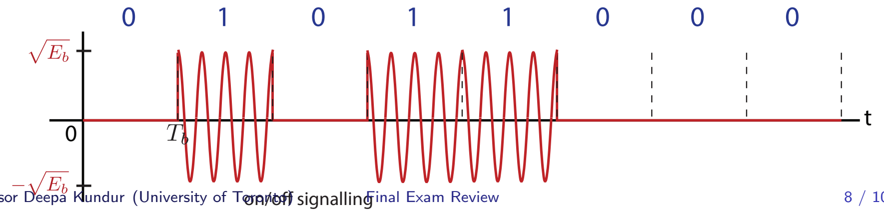
$$\begin{aligned} s(t) &= b(t) \cdot c(t) \\ &= \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{for symbol 1} \\ 0 & \text{for symbol 0} \end{cases} \end{aligned}$$

BASK Transmitter and Receiver

Transmitter

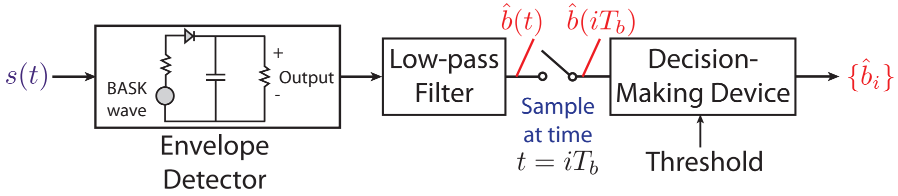


Transmitted BASK Signal

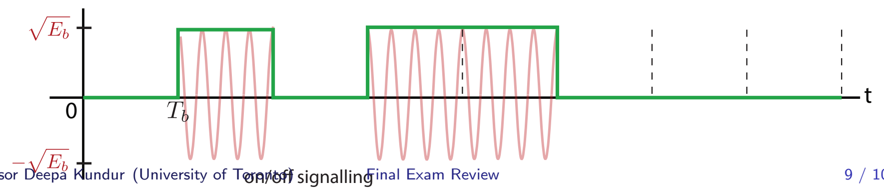


BASK Transmitter and Receiver

(Non-coherent) Receiver



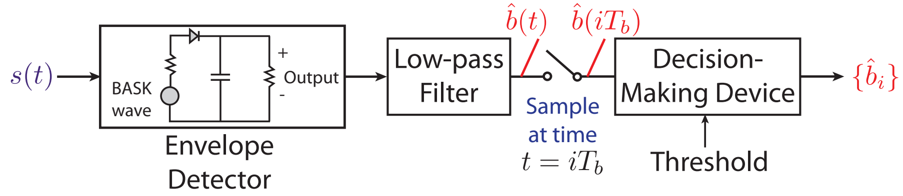
Output of Ideal Envelope Detector



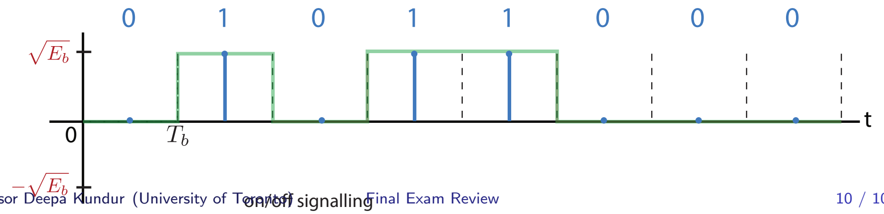
on/off signalling

BASK Transmitter and Receiver

(Non-coherent) Receiver



Output of Sampler and Decision-Making Device



on/off signalling

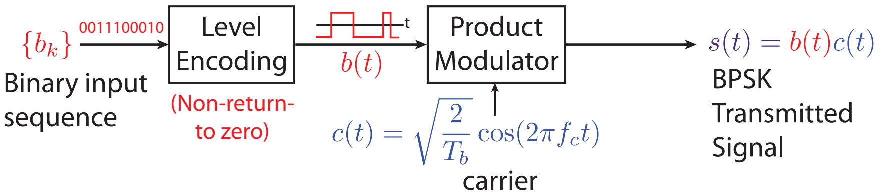
Binary Phase-Shift Keying (BPSK)

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{for symbol 1 } (i = 1) \\ \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) & \text{for symbol 0 } (i = 2) \end{cases}$$

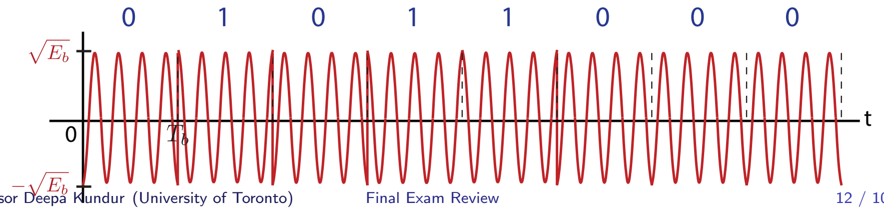
$$= \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{for symbol 1 } (i = 1) \\ -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{for symbol 0 } (i = 2) \end{cases}$$

BPSK Transmitter and Receiver

Transmitter

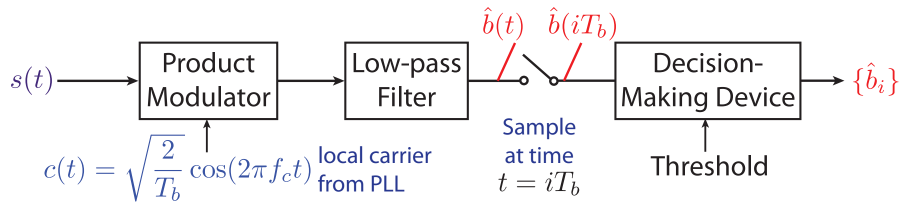


Transmitted BPSK Signal

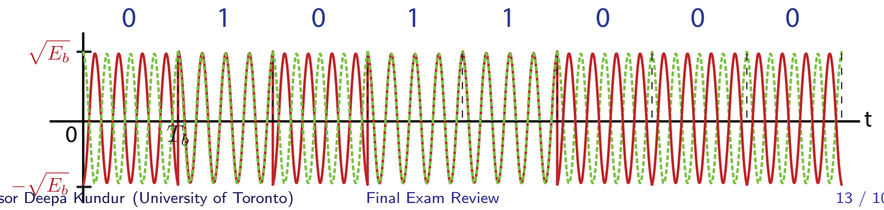


BPSK Transmitter and Receiver

(Coherent) Receiver

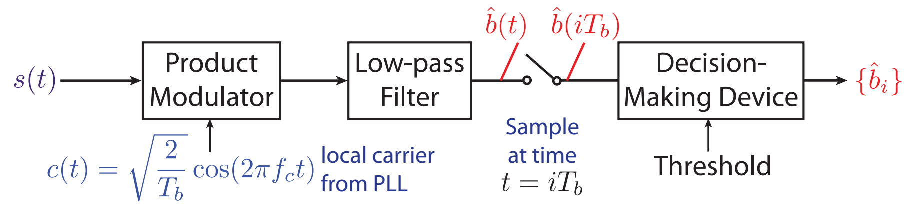


Inputs to Product Modulator

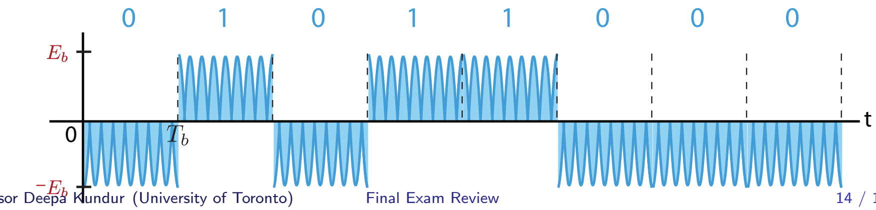


BPSK Transmitter and Receiver

(Coherent) Receiver

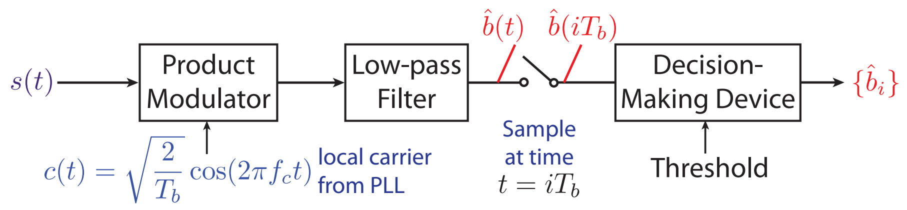


Output of Product Modulator

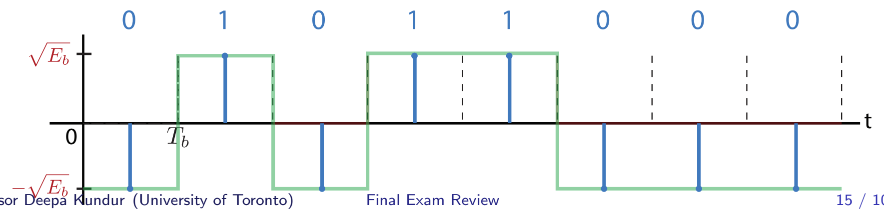


BPSK Transmitter and Receiver

(Coherent) Receiver



Output of Lowpass Filter and Sampler



DPSK

DPSK = Differential Phase Shift Keying

Differential Phase Shift Keying = Differential Encoding + PSK

- To send "0", we advance the carrier phase by π
- To send a "1", we leave the carrier phase unchanged

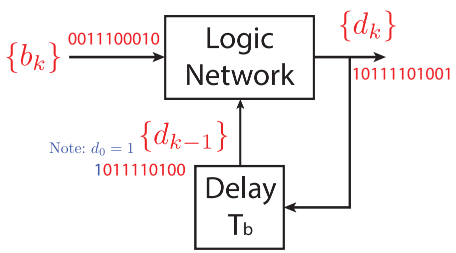
Consequence: DPSK detector must measure the relative phase difference between waveforms received in two consecutive intervals.

Differential Encoding

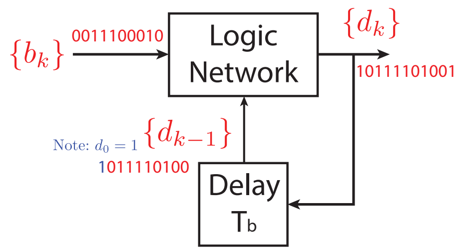
$$d_k = d_{k-1} \oplus \overline{b_k}$$

To produce d_k, need:

1. d_{k-1} (previous differentially encoded bit)
2. b_k (current input bit)



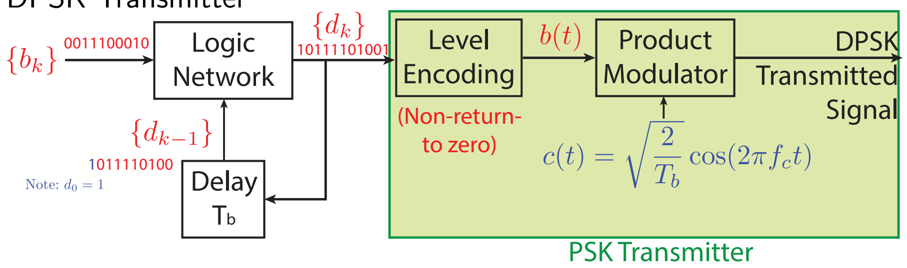
Differential Encoding



b_k		0	0	1	1	1	0	0	0	1	0
d_{k-1}		1	0	1	1	1	1	0	1	0	0
$d_k = d_{k-1} \oplus \overline{b_k}$	1	0	1	1	1	1	0	1	0	0	1

DPSK Transmitter and Receiver

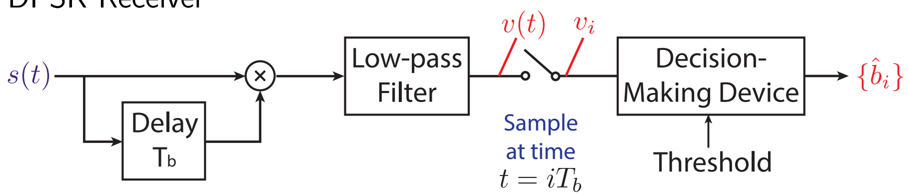
DPSK Transmitter



b_k		0	0	1	1	1	0	0	0	1	0
d_{k-1}		1	0	1	1	1	1	0	1	0	0
$d_k = d_{k-1} \oplus \overline{b_k}$	1	0	1	1	1	1	0	1	0	0	1
ϕ_c	0	π	0	0	0	0	π	0	π	π	0

DPSK Transmitter and Receiver

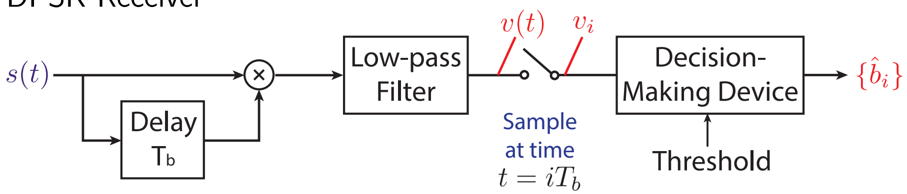
DPSK Receiver



b_k		0	0	1	1	1	0	0	0	1	0
d_{k-1}		1	0	1	1	1	1	0	1	0	0
$d_k = d_{k-1} \oplus \overline{b_k}$	1	0	1	1	1	1	0	1	0	0	1
ϕ_c	0	π	0	0	0	0	π	0	π	π	0

DPSK Transmitter and Receiver

DPSK Receiver



Case 1: No phase difference (note: $\gamma \in \{0, 1\}$)

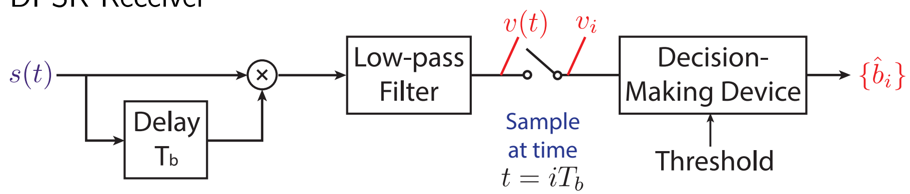
$$\begin{aligned} v(t) &= LPF\left[\frac{2}{T_b}\cos(2\pi f_c t + \gamma\pi)\cdot\cos(2\pi f_c t + \gamma\pi)\right] \\ &= LPF\left[\frac{2}{T_b}\cos^2(2\pi f_c t + \gamma\pi)\right] = LPF\left[\frac{1}{T_b}[1 + \cos(4\pi f_c t + \gamma\pi)]\right] = +\frac{1}{T_b} > 0 \end{aligned}$$

Case 2: Phase difference of π

$$\begin{aligned} v(t) &= LPF\left[\frac{2}{T_b}\cos(2\pi f_c t)\cdot\cos(2\pi f_c t + \pi)\right] \\ &= LPF\left[[-\frac{2}{T_b}\cos^2(2\pi f_c t)\right] = LPF\left[-\frac{1}{T_b}[1 + \cos(4\pi f_c t)]\right] = -\frac{1}{T_b} < 0 \end{aligned}$$

DPSK Transmitter and Receiver

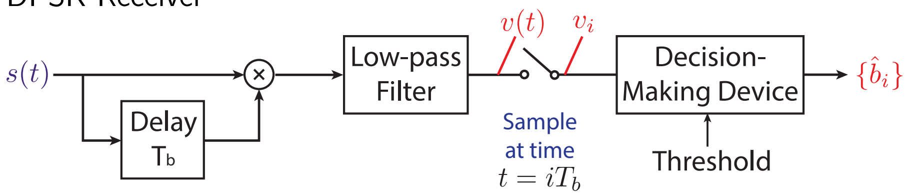
DPSK Receiver



b_k		0	0	1	1	1	0	0	0	1	0
d_{k-1}		1	0	1	1	1	1	0	1	0	0
$d_k = d_{k-1} \oplus \overline{b_k}$	1	0	1	1	1	1	0	1	0	0	1
ϕ_c	0	π	0	0	0	0	π	0	π	π	0
v_k		$-$	$-$	$+$	$+$	$+$	$-$	$-$	$-$	$+$	$-$

DPSK Transmitter and Receiver

DPSK Receiver

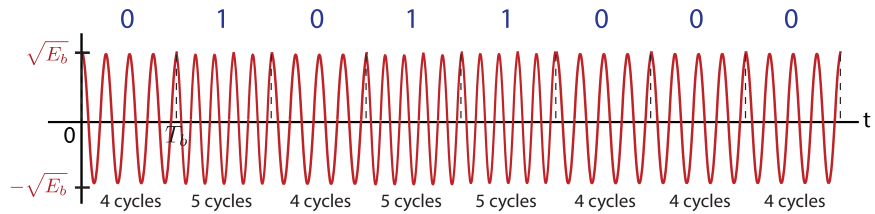


b_k		0	0	1	1	1	0	0	0	1	0
d_{k-1}		1	0	1	1	1	1	0	1	0	0
$d_k = d_{k-1} \oplus \overline{b_k}$	1	0	1	1	1	1	0	1	0	0	1
ϕ_c	0	π	0	0	0	0	π	0	π	π	0
v_k		$-$	$-$	$+$	$+$	$+$	$-$	$-$	$-$	$+$	$-$
$\hat{b}_k$		0	0	1	1	1	0	0	0	1	0

Binary Frequency-Shift Keying (BFSK)

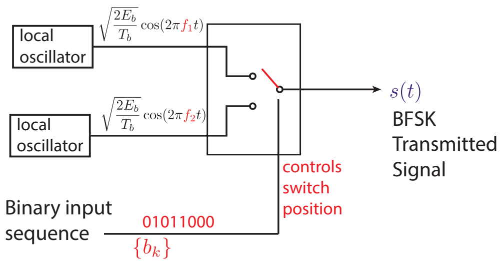
Let $\phi_c = 0$, $|f_1 - f_2| = \frac{1}{T_b}$ and $f_i = \frac{k_i}{T_b}$ (integer number of cycles in a bit duration).

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t) & \text{for symbol 1 } (i = 1) \\ \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_2 t) & \text{for symbol 0 } (i = 2) \end{cases}$$

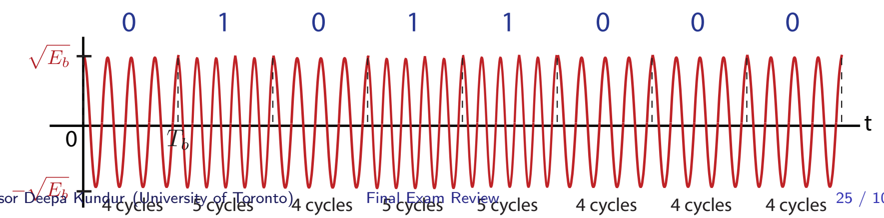


BFSK Transmitter and Receiver

Transmitter

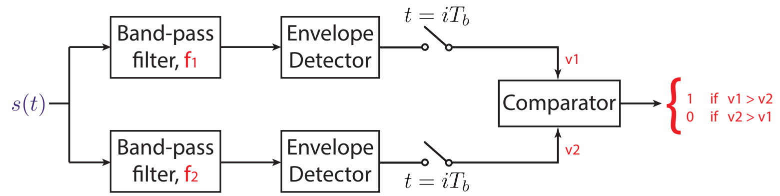


Transmitted BFSK Signal

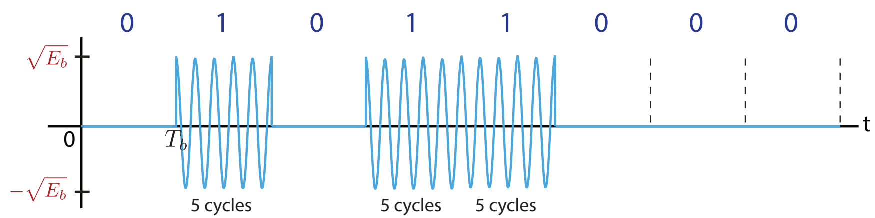


BFSK Transmitter and Receiver

Receiver

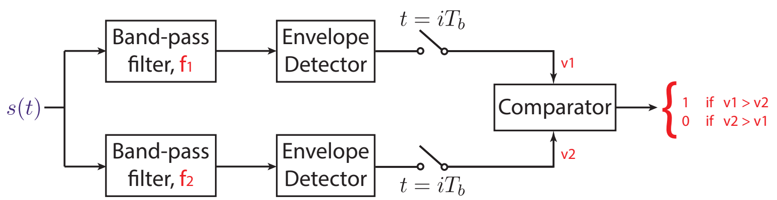


Output of Upper Band-pass Filter

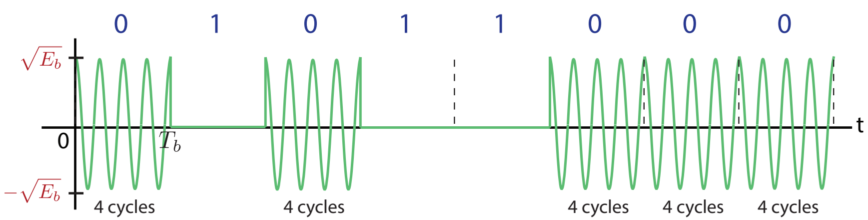


BFSK Transmitter and Receiver

Receiver

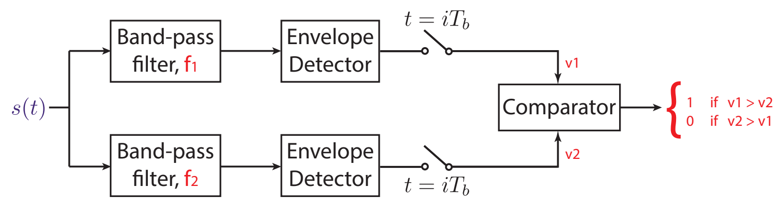


Output of Lower Band-pass Filter

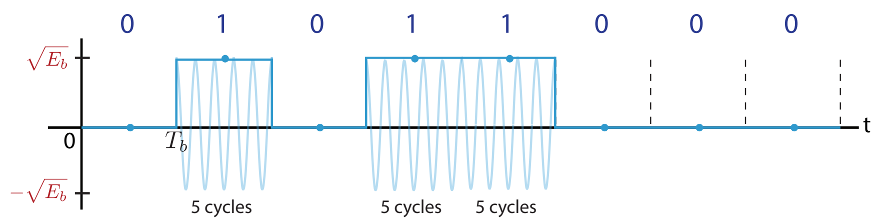


BFSK Transmitter and Receiver

Receiver

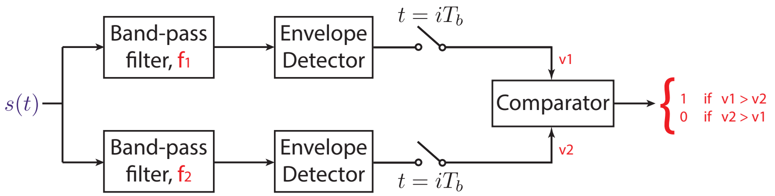


Output of Upper Envelope Detector and Sampler (v_1)

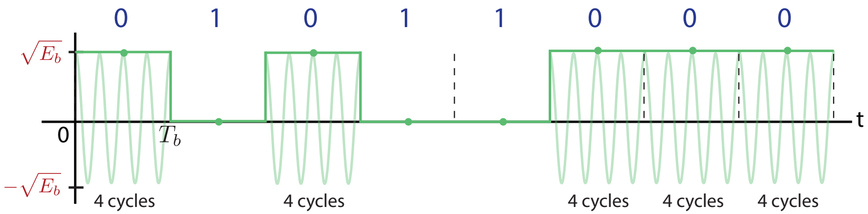


BFSK Transmitter and Receiver

Receiver

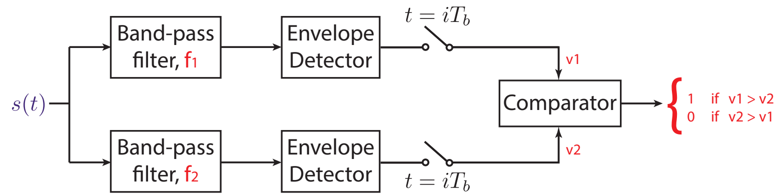


Output of Lower Envelope Detector and Sampler (v_2)

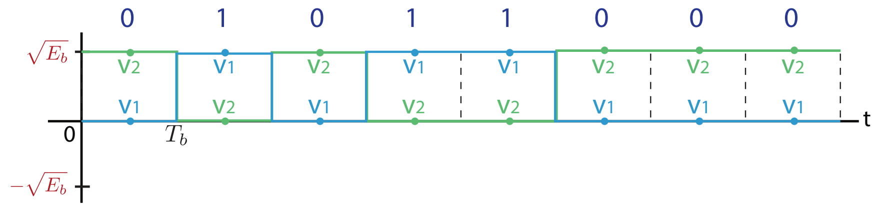


BFSK Transmitter and Receiver

Receiver



Inputs to Comparator



Summary

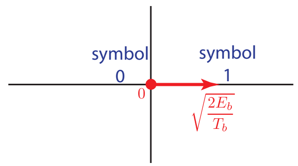
BASK
$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \quad \text{for symbol 1}$$
$$s_2(t) = 0 \quad \text{for symbol 0}$$

BPSK
$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + 0) \quad \text{for symbol 1}$$
$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi) \quad \text{for symbol 0}$$

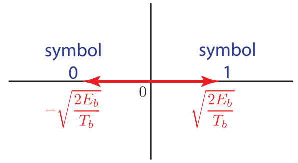
BFSK
$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t) \quad \text{for symbol 1}$$
$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_2 t) \quad \text{for symbol 0}$$

Summary: Phasor Diagrams

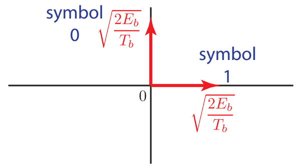
BASK



BPSK

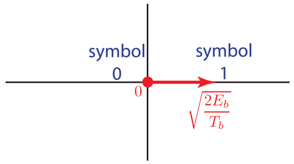


BFSK

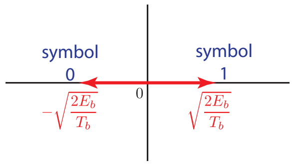


Summary: Phasor Diagrams

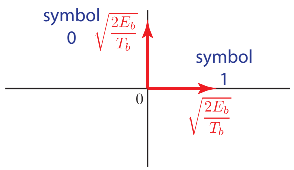
BASK



BPSK (antipodal)



BFSK (orthogonal)



M-ary Digital Modulation Schemes

For $M = 2^m$ and $T = mT_b$,

- M-ary Phase-Shift Keying

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M} i\right)$$

$i = 0, 1, \ldots, M-1$, $0 \leq t \leq T$.

- M-ary Quadrature Amplitude Modulation

$$s_i(t) = \sqrt{\frac{2E_0}{T}} a_i \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}} b_i \sin(2\pi f_c t)$$

$i = 0, 1, \ldots, M-1$, $0 \leq t \leq T$.

- M-ary Frequency-Shift Keying

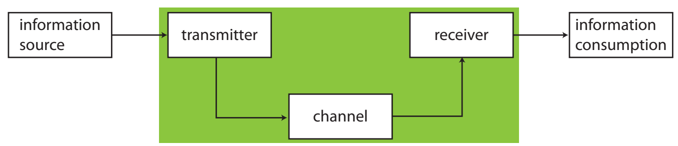
$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(\frac{\pi}{T}(n+i)t\right)$$

$i = 0, 1, \ldots, M-1$, $0 \leq t \leq T$.

Chapter 2: Fourier Representation of Signals and Systems

Communication Systems: Foundational Theories

- Modulation Theory: piggy-back information-bearing signal on a carrier signal
- Detection Theory: estimating or detecting the information-bearing signal in a reliable manner
- Probability and Random Processes: model channel noise and uncertainty at receiver
- Fourier Analysis: view signal and system in another domain to gain new insights



The Fourier Transform (FT)

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft}$$
$$g(t) = \int_{-\infty}^{\infty} G(f)e^{+j2\pi ft}$$

Notation:

$$g(t) \rightleftharpoons G(f)$$
$$G(f) = \mathbf{F}[g(t)]$$
$$g(t) = \mathbf{F}^{-1}[G(f)]$$

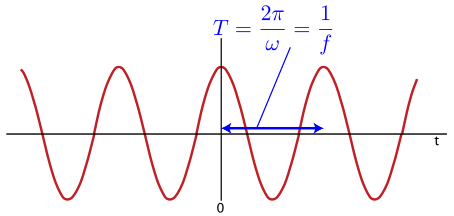
FT Synthesis Equation

$$g(t) = \int_{-\infty}^{\infty} G(f)e^{j2\pi ft}dt$$

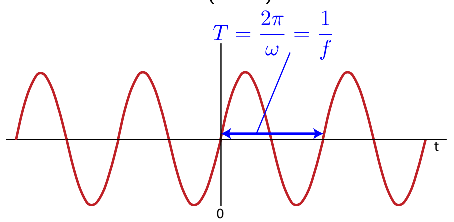
- $g(t)$ is the sum of scaled complex sinusoids
- $e^{j2\pi ft} = \cos(2\pi ft) + j\sin(2\pi ft) \quad \equiv \quad$ complex sinusoid

$$e^{j2\pi ft} = \cos(2\pi ft) + j\sin(2\pi ft)$$

$\cos(2\pi ft)$



$\sin(2\pi ft)$



FT Analysis Equation

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft}dt$$

- The analysis equation represents the <u>inner product</u> between $g(t)$ and $e^{j2\pi ft}$.
- The analysis equation states that $G(f)$ is a measure of <u>similarity</u> between $g(t)$ and $e^{j2\pi ft}$, the complex sinusoid at frequency f Hz.

$|G(f)|$ and $\angle G(f)$

$$g(t) = \int_{\infty}^{\infty} G(f) e^{j2\pi f t} df = \int_{\infty}^{\infty} |G(f)| e^{j(2\pi f t + \angle G(f))} df$$

- $|G(f)|$ dictates the relative presence of the sinusoid of frequency f in $g(t)$.
- $\angle G(f)$ dictates the relative alignment of the sinusoid of frequency f in $g(t)$.

Low, Mid and High Frequency Signals

Q: Which of the following signals appears higher in frequency?

1. $\cos(4 \times 10^6 \pi t + \pi/3)$
2. $\sin(2\pi t + 10\pi) + 17\cos^2(10\pi t)$

A: $\cos(4 \times 10^6 \pi t + \pi/3)$.

Importance of FT Theorems and Properties

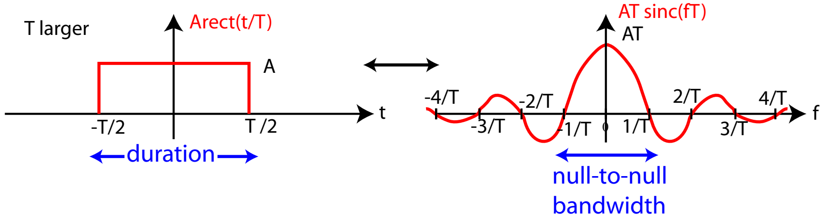
- The Fourier transform converts a signal or system representation to the frequency-domain, which provides another way to *visualize* a signal or system convenient for analysis and design.
- The properties of the Fourier transform provide valuable insight into how signal operations in the time-domain are described in the frequency-domain.

FT Theorems and Properties

Property/Theorem	Time Domain		Frequency Domain
Notation:	$g(t)$	$\rightleftharpoons$	$G(f)$
	$g_1(t)$	$\rightleftharpoons$	$G_1(f)$
	$g_2(t)$	$\rightleftharpoons$	$G_2(f)$
Linearity:	$c_1 g_1(t) + c_2 g_2(t)$	$\rightleftharpoons$	$c_1 G_1(f) + c_2 G_2(f)$
Dilation:	$g(at)$	$\rightleftharpoons$	$\frac{1}{\lvert a \rvert} G\left(\frac{f}{a}\right)$
Conjugation:	$g^*(t)$	$\rightleftharpoons$	$G^*(-f)$
Duality:	$G(t)$	$\rightleftharpoons$	$g(-f)$
Time Shifting:	$g(t - t_0)$	$\rightleftharpoons$	$G(f)e^{-j2\pi f t_0}$
Frequency Shifting:	$e^{j2\pi f_c t} g(t)$	$\rightleftharpoons$	$G(f - f_c)$
Area Under $G(f)$:	$g(0)$	$=$	$\int_{-\infty}^{\infty} G(f) df$
Area Under $g(t)$:	$\int_{-\infty}^{\infty} g(t) dt$	$=$	$G(0)$
Time Differentiation:	$\frac{d}{dt} g(t)$	$\rightleftharpoons$	$j2\pi f G(f)$
Time Integration :	$\int_{-\infty}^{t} g(\tau) d\tau$	$\rightleftharpoons$	$\frac{1}{j2\pi f} G(f)$
Modulation Theorem:	$g_1(t) g_2(t)$	$\rightleftharpoons$	$\int_{-\infty}^{\infty} G_1(\lambda) G_2(f - \lambda) d\lambda$
Convolution Theorem:	$\int_{-\infty}^{\infty} g_1(\tau) g_2(t - \tau)$	$\rightleftharpoons$	$G_1(f) G_2(f)$
Correlation Theorem:	$\int_{-\infty}^{\infty} g_1(t) g_2^*(t - \tau) dt$	$\rightleftharpoons$	$G_1(f) G_2^*(f)$
Rayleigh's Energy Theorem:	$\int_{\infty}^{\infty} \lvert g(t) \rvert^2 dt$	$=$	$\int_{\infty}^{\infty} \lvert G(f) \rvert^2 df$

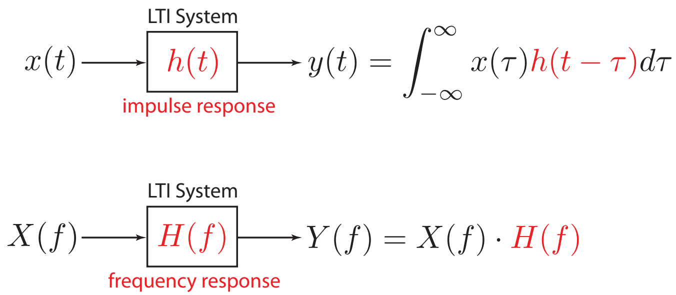
Time-Bandwidth Product

$\boxed{\text{time-duration of a signal} \times \text{frequency bandwidth} = \text{constant}}$



Note: the constant depends on the definitions of duration and bandwidth and can change with the shape of signals being considered

LTI Systems and Filtering



- For systems that are linear time-invariant (LTI), the Fourier transform provides a decoupled description of the system operation on the input signal much like when we diagonalize a matrix.
- This provides a filtering perspective to how a linear time-invariant system operates on an input signal.
- The LTI system scales the sinusoidal component corresponding to frequency f by $H(f)$ providing frequency selectivity.

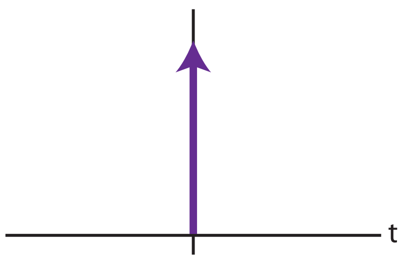
Dirac Delta Function

Definition:

1. $\delta(t) = 0, \ t \neq 0$
2. The area under $\delta(t)$ is unity:

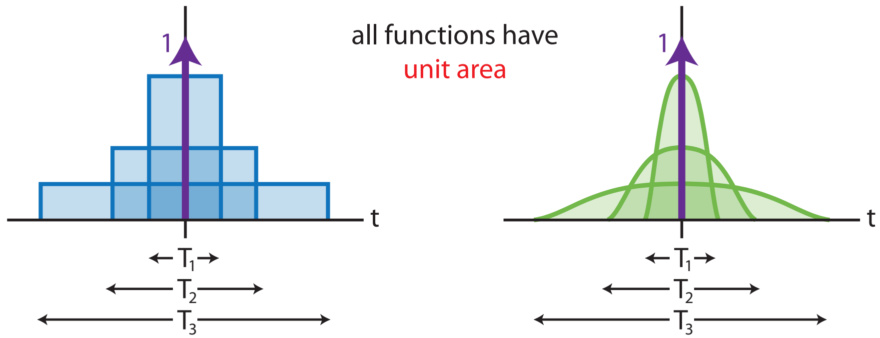
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

Note: $\delta(0) =$ undefined



Dirac Delta Function

- can be interpreted as the limiting case of a family of functions of unit area but that become narrower and higher



Dirac Delta Function

- Sifting Property:

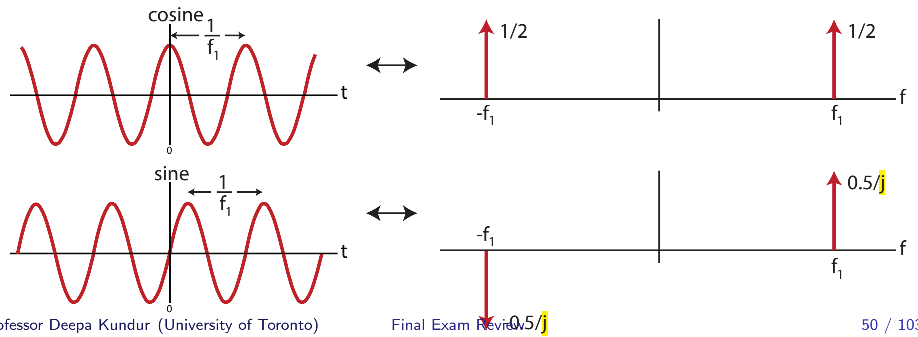
$$\int_{-\infty}^{\infty} g(t)\delta(t - t_0)dt = g(t_0)$$

- Convolution with $\delta(t)$:

$$g(t) \star \delta(t - t_0) = g(t - t_0)$$

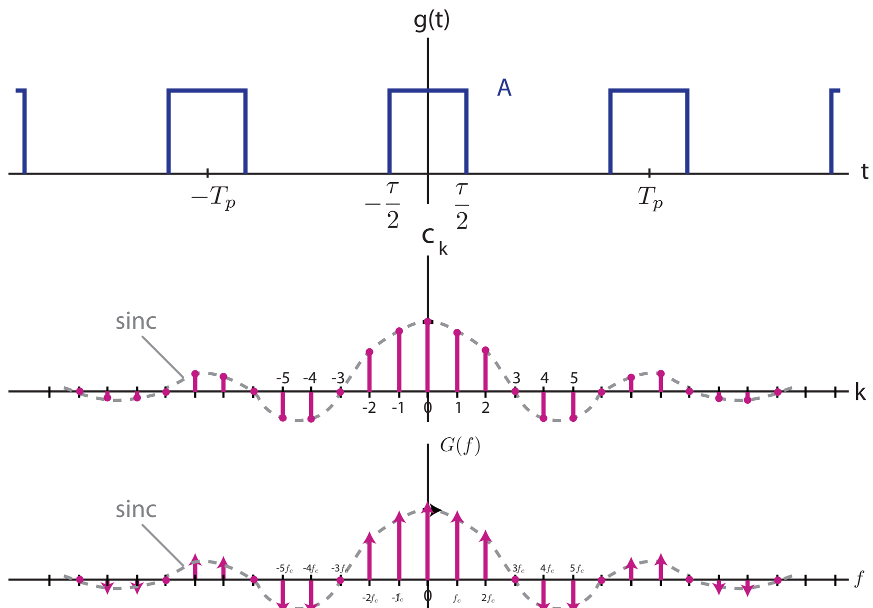
The Fourier Transform and the Dirac Delta

$$\begin{aligned}
\delta(t) &\rightleftharpoons 1 \\
1 &\rightleftharpoons \delta(f) \\
e^{j2\pi f_0 t} &\rightleftharpoons \delta(f - f_0) \\
\cos(2\pi f_1 t) = \frac{e^{j2\pi f_1 t}}{2} + \frac{e^{-j2\pi f_1 t}}{2} &\rightleftharpoons \frac{1}{2}\delta(f - f_1) + \frac{1}{2}\delta(f + f_1) \\
\sin(2\pi f_1 t) = \frac{e^{j2\pi f_1 t}}{2j} - \frac{e^{-j2\pi f_1 t}}{2j} &\rightleftharpoons \frac{1}{2j}\delta(f - f_1) - \frac{1}{2j}\delta(f + f_1)
\end{aligned}$$



Fourier Transforms of Periodic Signals

$$g(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t} \quad \rightleftharpoons \quad G(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - n f_0)$$



Chapter 3: Amplitude Modulation

Amplitude Modulation

- In modulation need two things:
  1. a modulated signal: carrier signal: $c(t)$
  2. a modulating signal: message signal: $m(t)$
- carrier:
  - $c(t) = A_c \cos(2\pi f_c t)$; phase $\phi_c = 0$ is assumed.
- message:
  - $m(t)$ (information-bearing signal)
  - assume bandwidth/max freq of $m(t)$ is W

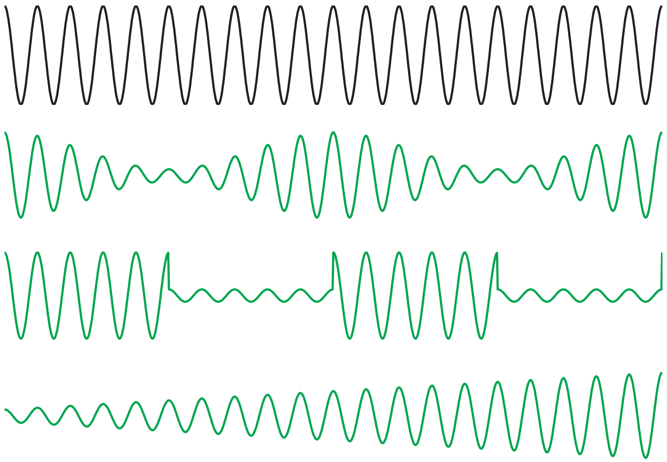
Amplitude Modulation

Three types studied:

1. Amplitude Modulation (AM)
   (yes, it has the same name as the class of modulation techniques)
2. Double Sideband-Suppressed Carrier (DSB-SC)
3. Single Sideband (SSB)

Amplitude Modulation

$$s_{AM}(t) = A_c \left[1 + k_a m(t)\right] \cos(2\pi f_c t)$$

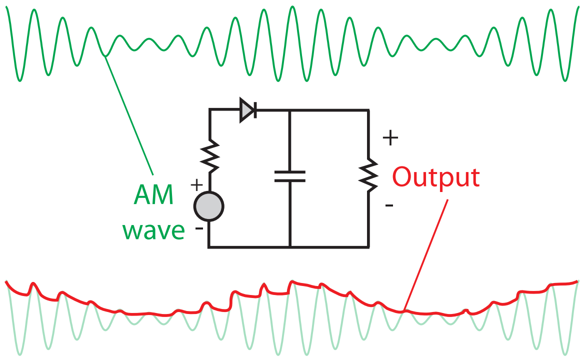


Amplitude Modulation Techniques

AM: For:

1. $1 + k_a m(t) > 0$ (envelope is always positive); and
2. $f_c \gg W$ (message moves slowly compared to carrier)

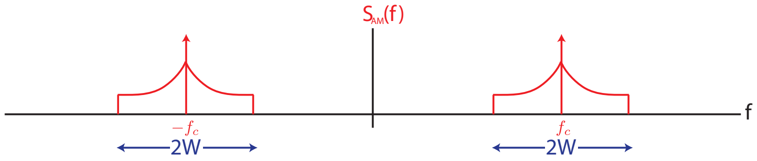
$m(t)$ can be recovered with an envelope detector.



Amplitude Modulation Techniques

AM:

$$s_{AM}(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$
$$S_{AM}(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

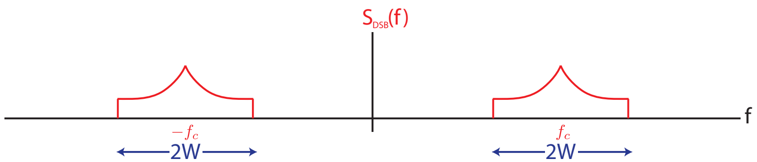


- highest power
- $B_T = 2W$
- lowest complexity

Amplitude Modulation Techniques

DSB-SC:

$$s_{DSB}(t) = A_c \cos(2\pi f_c t) m(t)$$
$$S_{DSB}(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

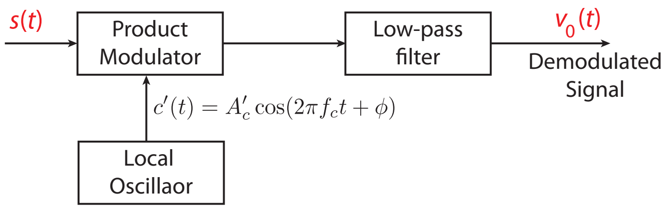


- lower power
- $B_T = 2W$
- higher complexity

Amplitude Modulation Techniques

DSB-SC:

- An envelope detector will not be able to recover $m(t)$; it will instead recover $|m(t)|$.
- Coherent demodulation is required.

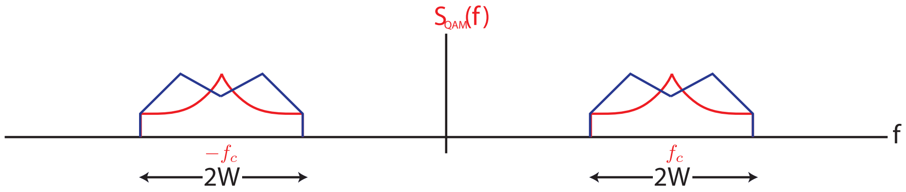


Amplitude Modulation Techniques

QAM:

$$s_{QAM}(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

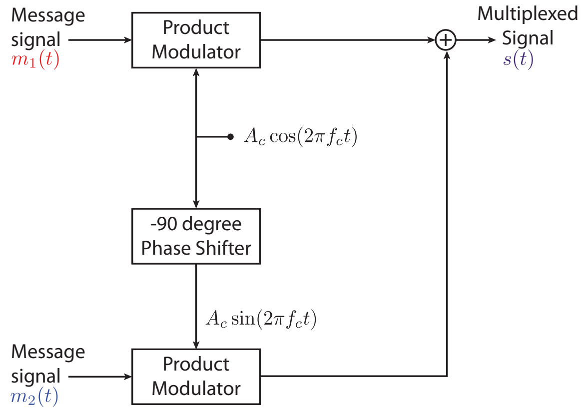
$$S_{QAM}(f) = \frac{A_c}{2}[M_1(f-f_c) + M_1(f+f_c)] + \frac{A_c}{2}[jM_2(f-f_c) - jM_2(f+f_c)]$$



- lower power (no carrier)
- $B_T = 2W/2$ messages $= W$ per message.
- higher complexity

Amplitude Modulation Techniques

QAM Transmitter:

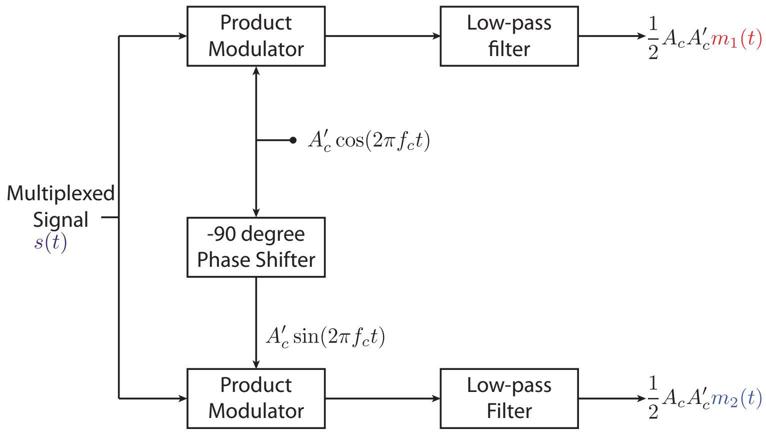


$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

Amplitude Modulation Techniques

QAM Receiver:

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$



Amplitude Modulation Techniques

SSB:

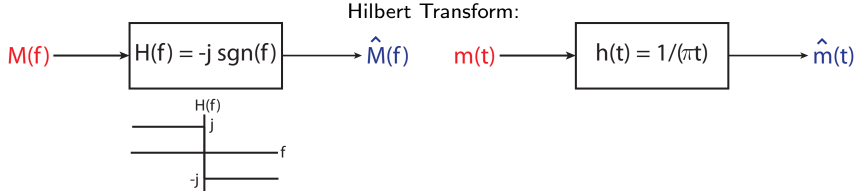
$$s_{USSB}(t) = \frac{A_c}{2}m(t)\cos(2\pi f_c t) - \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

$$S_{USSB}(f) = \begin{cases} \frac{A_c}{2}[M(f-f_c)+M(f+f_c)] & |f| \geq f_c \\ 0 & |f| < f_c \end{cases}$$

$$s_{LSSB}(t) = \frac{A_c}{2}m(t)\cos(2\pi f_c t) + \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

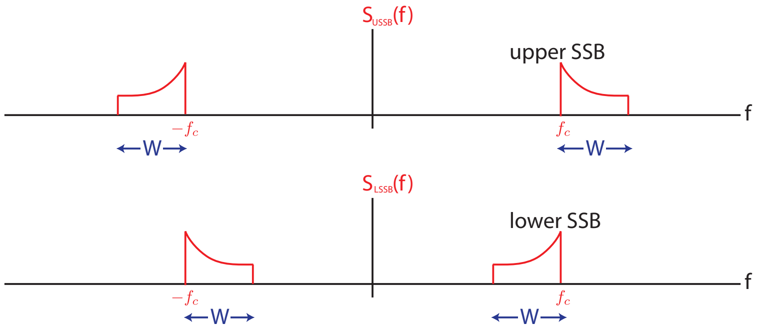
$$S_{LSSB}(f) = \begin{cases} 0 & |f| > f_c \\ \frac{A_c}{2}[M(f-f_c)+M(f+f_c)] & |f| \leq f_c \end{cases}$$

Hilbert Transform:



Amplitude Modulation Techniques

SSB:

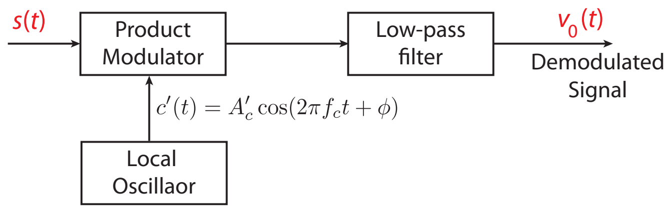


- lowest power
- $B_T = W$
- highest complexity

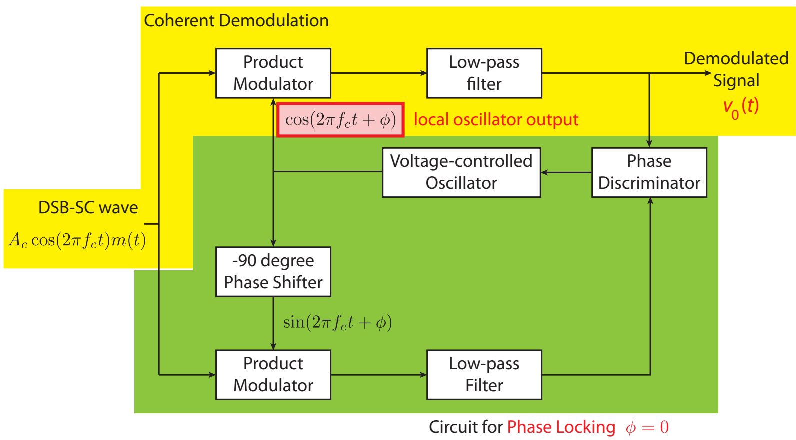
Amplitude Modulation Techniques

SSB:

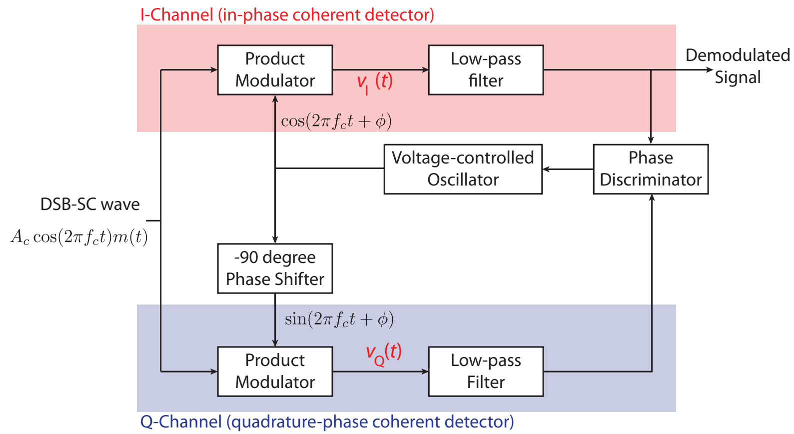
- Coherent demodulation works here as well.



Costas Receiver



Costas Receiver



Chapter 4: Angle Modulation

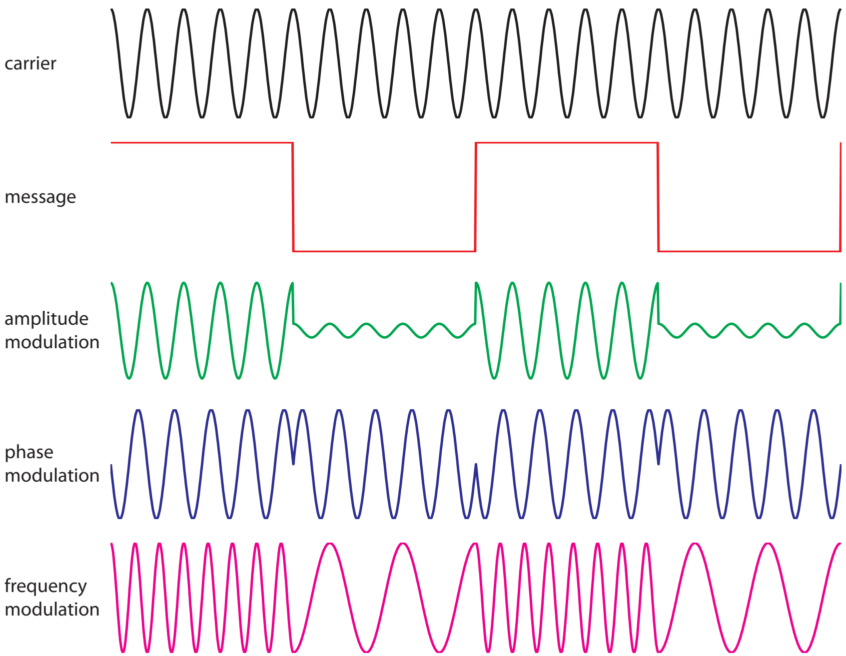
Angle Modulation

- Phase Modulation (PM):

$$\begin{aligned}
\theta_i(t) &= 2\pi f_c t + k_p m(t) \\
f_i(t) &= \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt} \\
s_{PM}(t) &= A_c \cos[2\pi f_c t + k_p m(t)]
\end{aligned}$$

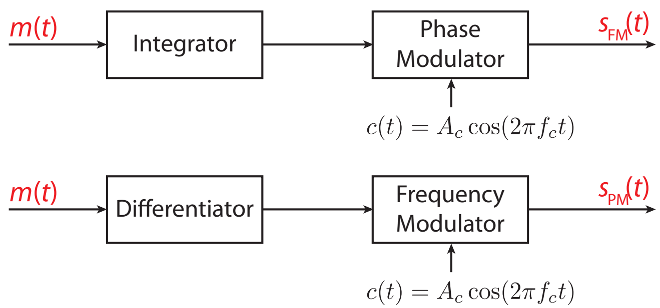
- Frequency Modulation (FM):

$$\begin{aligned}
\theta_i(t) &= 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \\
f_i(t) &= \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + k_f m(t) \\
s_{FM}(t) &= A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right]
\end{aligned}$$



Angle Modulation

PM and FM:



Properties of Angle Modulation

1. Constancy of transmitted power
2. Nonlinearity of angle modulation
3. Irregularity of zero-crossings
4. Difficulty in visualizing message
5. Bandwidth versus noise trade-off

Narrowband FM

- Suppose $m(t) = A_m \cos(2\pi f_m t)$.

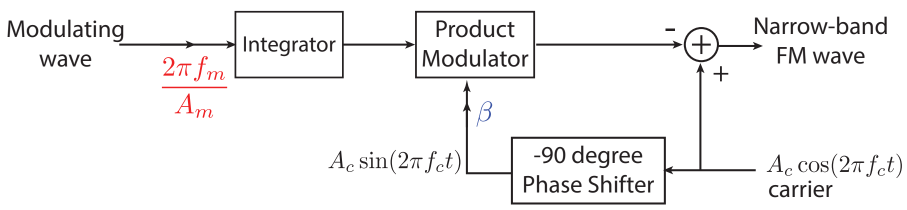
$$\begin{aligned} f_i(t) &= f_c + k_f A_m \cos(2\pi f_m t) = f_c + \Delta f \cos(2\pi f_m t) \\ \Delta f &= k_f A_m \equiv \text{frequency deviation} \\ \theta_i(t) &= 2\pi \int_0^t f_i(\tau) d\tau \\ &= 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) = 2\pi f_c t + \beta \sin(2\pi f_m t) \\ \beta &= \frac{\Delta f}{f_m} \\ s_{FM}(t) &= A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \end{aligned}$$

For narrow band FM, $\beta \ll 1$.

Narrowband FM

Modulation:

$$s_{FM}(t) \approx \underbrace{A_c \cos(2\pi f_c t)}_{\text{carrier}} - \beta \underbrace{\underbrace{A_c \sin(2\pi f_c t)}_{-90^o \text{shift of carrier}} \underbrace{\sin(2\pi f_m t)}_{\frac{2\pi f_m}{A_m} \int_0^t m(\tau) d\tau}}_{\text{DSB-SC signal}}$$



Carson's Rule

A significant component of the FM signal is within the following bandwidth:

$$B_T \approx 2\Delta f + 2f_m = 2\Delta f \left(1 + \frac{1}{\beta}\right)$$

where Δf is the maximum frequency deviation and f_m is the highest frequency in the modulating signal.

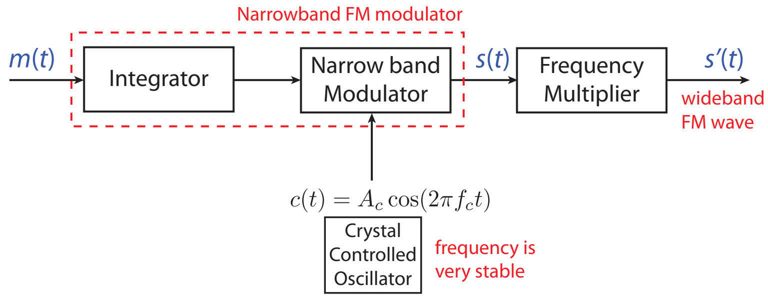
- For $\beta \gg 1$, $B_T \approx 2\Delta f = 2k_f A_m$
- For $\beta \ll 1$, $B_T \approx 2\Delta f \frac{1}{\beta} = \frac{2\Delta f}{\Delta f / f_m} = 2f_m$

Carson's Rule

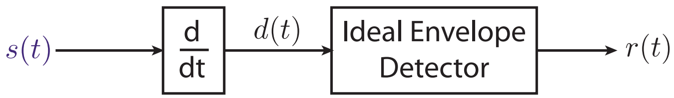
Example: Find the bandwidth of the following signal:

$$
\begin{aligned}
s(t) &= 17\cos\left[4\times 10^6\pi t - 4\pi\cos(25t - \frac{3\pi}{8}) + 2\pi\sin(5000\pi t - \frac{\pi}{7})\right] \\
&= \underbrace{17}_{A_c}\cos\left[\underbrace{4\times 10^6\pi t}_{2\pi f_c t}\underbrace{-4\pi\cos(25t - \frac{3\pi}{8}) + 2\pi\sin(5000\pi t - \frac{\pi}{7})}_{+k_p m(t)}\right] \\
\theta_i(t) &= 4\times 10^6\pi t - 4\pi\cos(25t - \frac{3\pi}{8}) + 2\pi\sin(5000\pi t - \frac{\pi}{7}) \\
f_i(t) &= \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = \underbrace{2\times 10^6}_{f_c} + \underbrace{50\sin(25t - \frac{3\pi}{8}) + 5000\pi\cos(5000\pi t - \frac{\pi}{7})}_{\text{related to frequency deviation } \Delta f} \\
\Delta f &\approx 5000\pi \quad \text{and} \quad f_m = \frac{5000\pi}{2\pi} = 2500 \\
B_T &\approx = 2\Delta f + 2f_m = 2\cdot 5000\pi + 2\cdot 2500 \approx 41415 \text{ Hz.}
\end{aligned}
$$

Generation of FM Waves: Armstrong Modulator



Demodulation of FM Waves

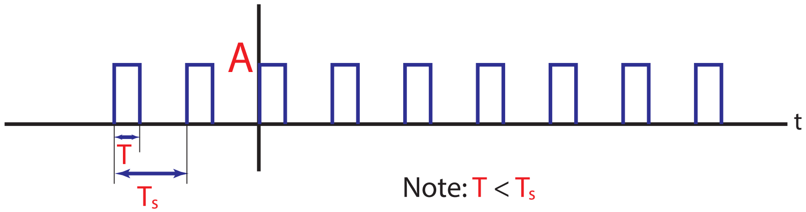


- Frequency Discriminator: uses positive and negative slope circuits in place of a differentiator, which is hard to implement across a wide bandwidth
- Phase Lock Loop: tracks the angle of the in-coming FM wave which allows tracking of the embedded message

Chapter 5: Pulse Modulation

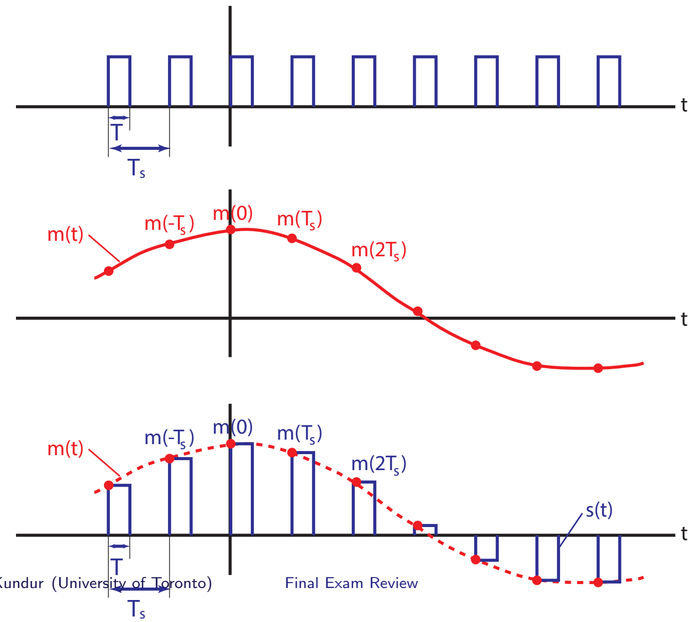
Pulse Modulation

- the variation of a regularly spaced constant amplitude pulse stream to superimpose information contained in a message signal

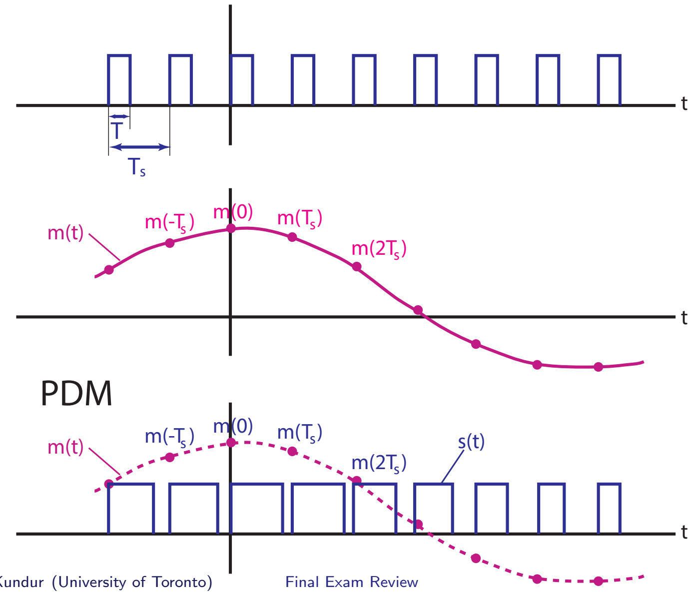


- Three types:
  1. pulse amplitude modulation (PAM)
  2. pulse duration modulation (PDM)
  3. pulse position modulation (PPM)

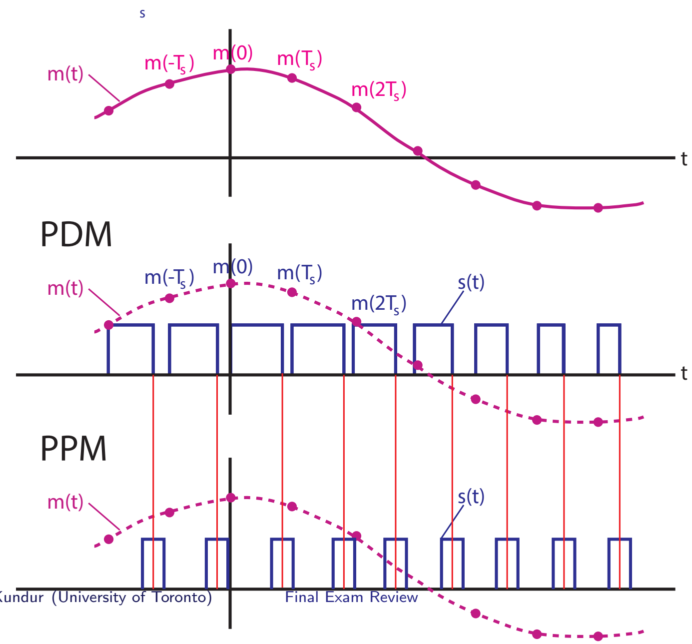
Pulse Amplitude Modulation (PAM)



Pulse Duration Modulation (PDM)



Pulse Position Modulation (PPM)



Summary of Pulse Modulation

Let $g(t)$ be the pulse shape.

- PAM:

$$s_{PAM}(t) = \sum_{n=-\infty}^{\infty} k_a m(nT_s) g(t - nT_S)$$

where k_a is an amplitude sensitivity factor; $k_a > 0$.

- PDM:

$$s_{PDM}(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{t - nT_s}{k_d m(nT_s) + M_d}\right)$$

where k_d is a duration sensitivity factor; $k_d\,|m(t)|_{\max} < M_d$.

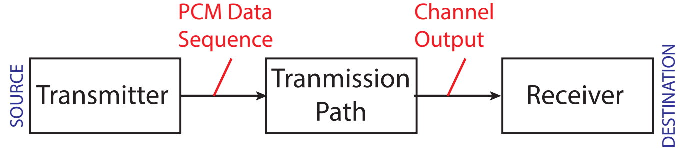
- PPM:

$$s_{PPM}(t) = \sum_{n=-\infty}^{\infty} g(t - nT_s - k_p m(nT_s))$$

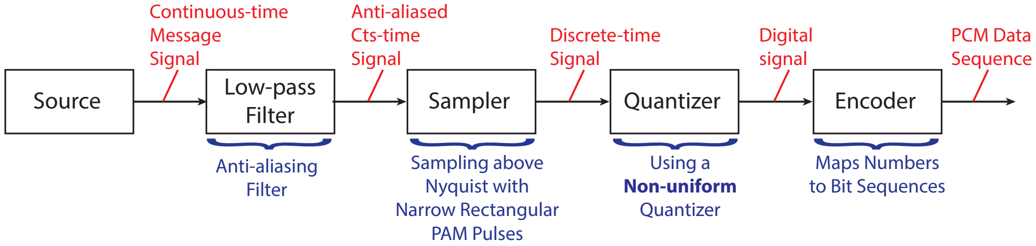
where k_p is a position sensitivity factor; $k_p\,|m(t)|_{\max} < (T_s/2)$.

Pulse-Code Modulation

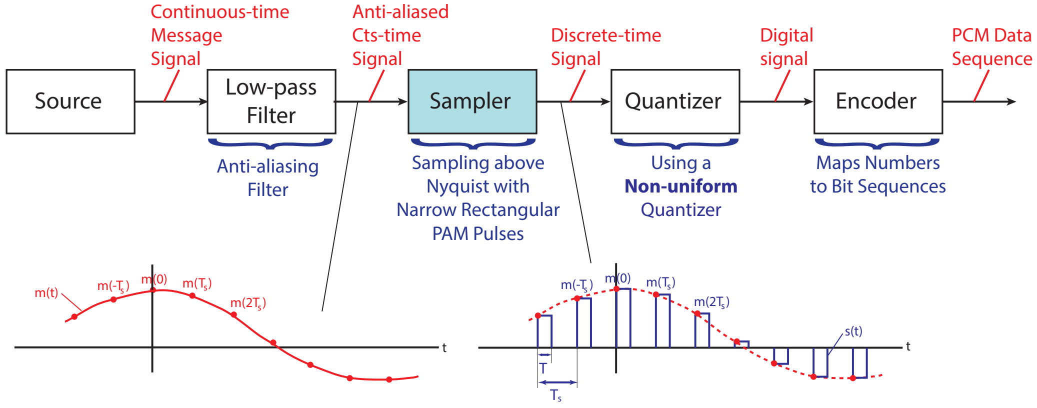
- Most basic form of digital pulse modulation



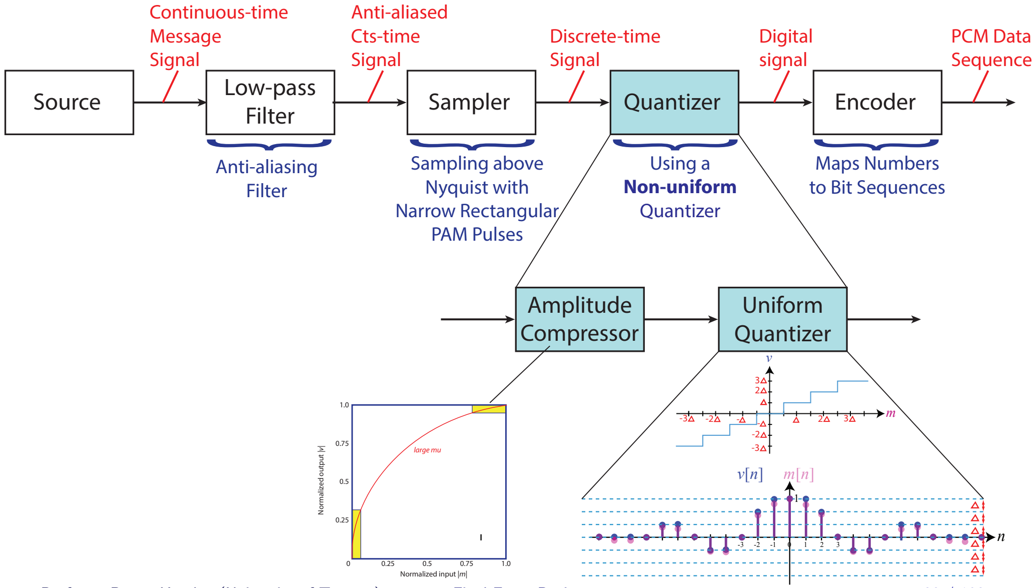
PCM Transmitter



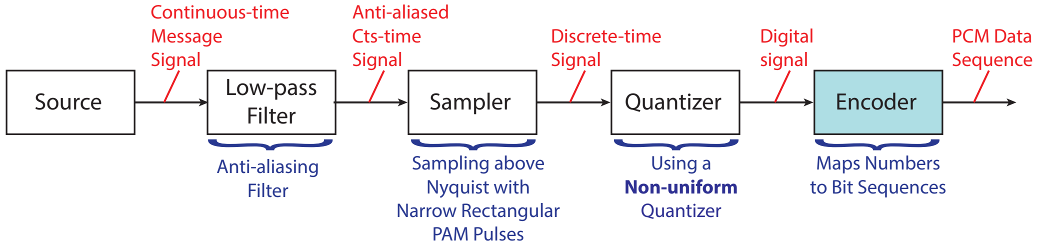
PCM Transmitter: Sampler



PCM Transmitter: Non-Uniform Quantizer

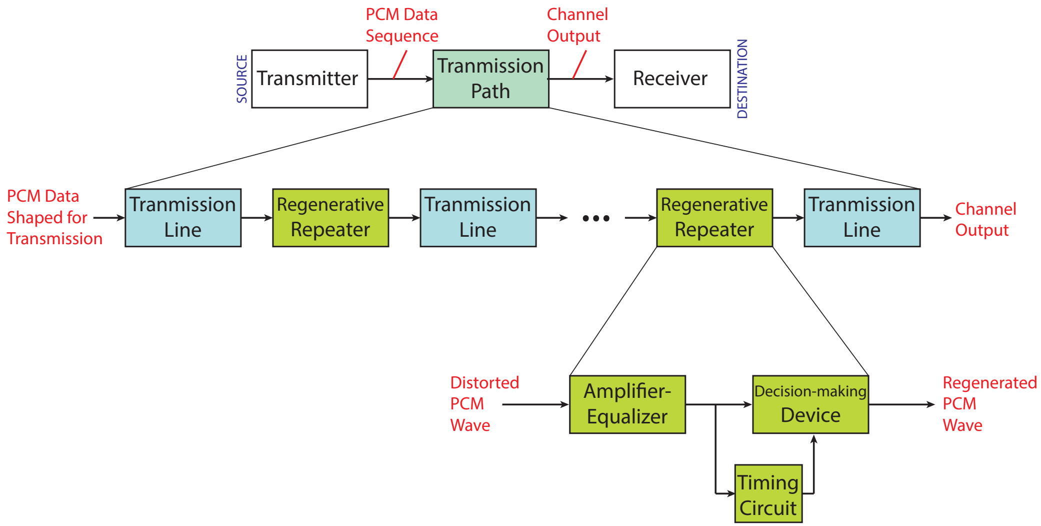


PCM Transmitter: Encoder

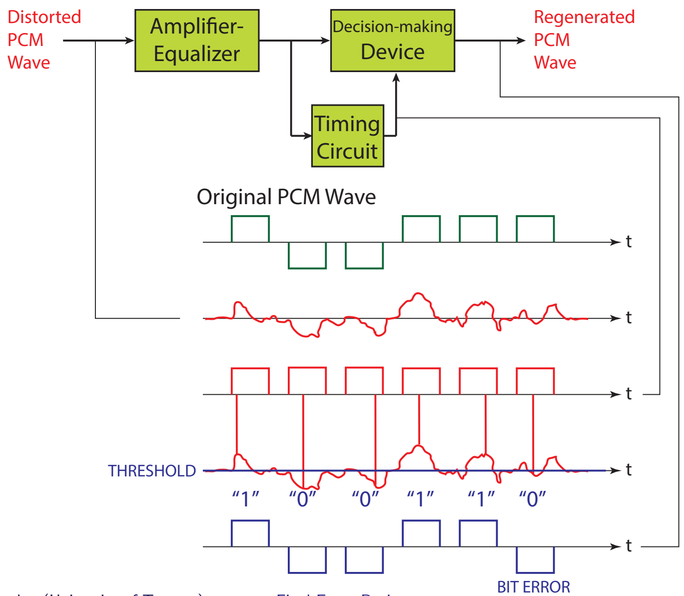


Quantization-Level Index	Binary Codeword ($R = 3$)
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

PCM: Transmission Path



PCM: Regenerative Repeater



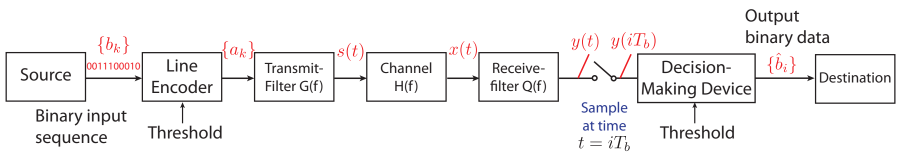
PCM: Receiver

Two Stages:

1. Decoding and Expanding:
   1.1 regenerate the pulse one last time
   1.2 group into code words
   1.3 interpret as quantization level
   1.4 pass through expander (opposite of compressor)
2. Reconstruction:
   2.1 pass expander output through low-pass reconstruction filter (cutoff is equal to message bandwidth) to estimate original message $m(t)$

Chapter 6: Baseband Data Transmission

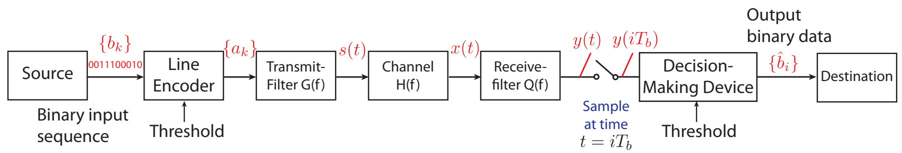
Baseband Transmission of Digital Data



$$b_k = \{0,1\} \quad \text{and} \quad a_k = \begin{cases} +1 & \text{if } b_k \text{ is symbol 1} \\ -1 & \text{if } b_k \text{ is symbol 0} \end{cases}$$

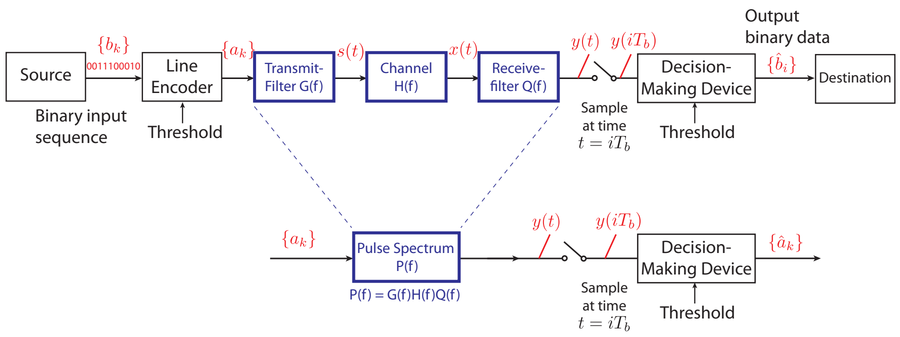
$$\begin{aligned} s(t) &= \sum_{k=-\infty}^{\infty} a_k g(t - kT_b) \\ x(t) &= s(t) \star h(t) \\ y(t) &= x(t) \star q(t) = s(t) \star h(t) \star q(t) \\ &= \sum_{k=-\infty}^{\infty} a_k g(t - kT_b) \star h(t) \star q(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT_b) \end{aligned}$$

Baseband Transmission of Digital Data

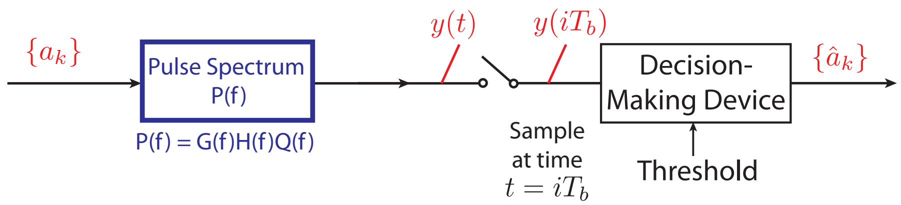


$$\begin{aligned} \therefore y(t) &= \sum_{k=-\infty}^{\infty} a_k p(t - kT_b) \\ \text{where } p(t) &= g(t) \star h(t) \star q(t) \\ P(f) &= G(f) \cdot H(f) \cdot Q(f). \end{aligned}$$

Baseband Transmission of Digital Data



Baseband Transmission of Digital Data



$$y_i = y(iT_b) \quad \text{and} \quad p_i = p(iT_b)$$

$$y_i = \underbrace{\sqrt{E}a_i}_{\text{signal to detect}} + \underbrace{\sum_{k=-\infty, k\neq i}^{\infty} a_k p_{i-k}}_{\text{intersymbol interference}} \qquad \text{for } i \in \mathbb{Z}$$

To avoid intersymbol interference (ISI), we need $p_i = 0$ for $i \neq 0$.

The Nyquist Channel

- Minimum bandwidth channel
- Optimum pulse shape:

$$p_{opt}(t) = \sqrt{E}\,\mathrm{sinc}(2B_0 t)$$

$$P_{opt}(f) = \begin{cases} \frac{\sqrt{E}}{2B_0} & -B_0 < f < B_0 \\ 0 & \text{otherwise} \end{cases}, \qquad B_0 = \frac{1}{2T_b}$$

Note: No ISI.
$p_i = p(iT_b) = \sqrt{E}\,\mathrm{sinc}(2B_0 i T_b)\sqrt{E}\,\mathrm{sinc}\left(2 \cdot \frac{1}{2T_b} i T_b\right) = \sqrt{E}\,\mathrm{sinc}(i) = 0$ for $i \neq 0$.
Disadvantages: (1) physically unrealizable (sharp transition in freq domain); (2) slow rate of decay leaving no margin of error for sampling times.

Raised-Cosine Pulse Spectrum

- has a more graceful transition in the frequency domain
- more practical pulse shape:

$$p(t) = \sqrt{E}\,\mathrm{sinc}(2B_0 t)\left(\frac{\cos(2\pi\alpha B_0 t)}{1 - 16\alpha^2 {B_0}^2 t^2}\right)$$

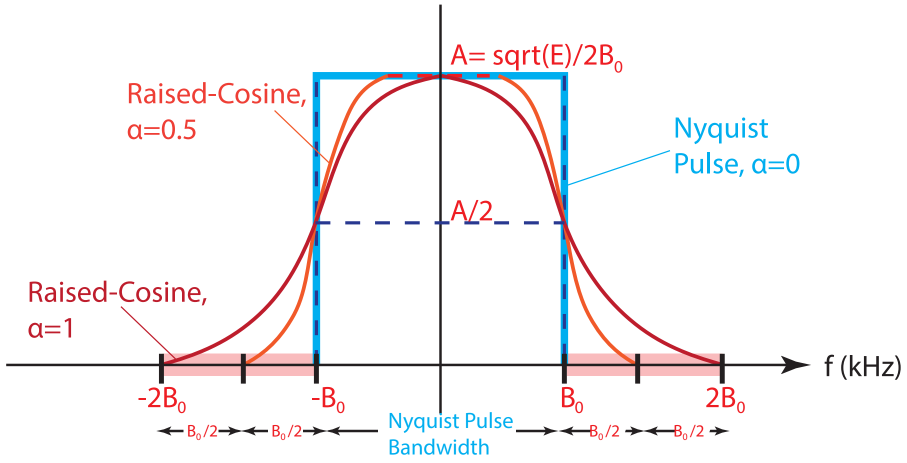
$$P(f) = \begin{cases} \frac{\sqrt{E}}{2B_0} & 0 \leq |f| < f_1 \\ \frac{\sqrt{E}}{4B_0}\left\{1 + \cos\left[\frac{\pi(|f| - f_1)}{(B_0 - f_1)}\right]\right\} & f_1 < f < 2B_0 - f_1 \\ 0 & 2B_0 - f_1 \leq |f| \end{cases}$$

$$\alpha = 1 - \frac{f_1}{B_0}$$

$$B_T = B_0(1 + \alpha) \quad \text{where } B_0 = \frac{1}{2T_b} \text{ and } f_v = \alpha B_0$$

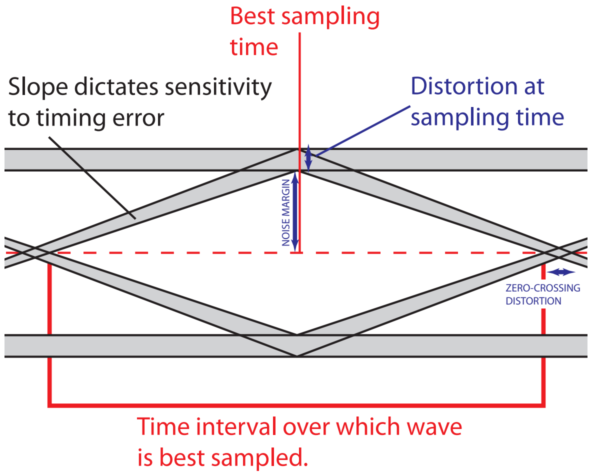
Note: No ISI. $\because p_i = 0$ for $i \neq 0$.

Raised-Cosine Pulse Spectrum



Trade-off: larger bandwidth than Nyquist pulse.

The Eye Pattern



Note: an "open" eye denotes a larger noise margin, lower zero-crossing distortion and greater robustness to timing error.

Important Identities

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$
$$\cos(A)\sin(B) = \frac{1}{2}\sin(A+B) - \frac{1}{2}\sin(A-B)$$
$$\cos(A) = \sin\left(A+\frac{\pi}{2}\right) \qquad \cos(A+\pi) = -\cos(A)$$
$$\cos(A) = \cos(-A) \qquad \sin(A) = -\sin(-A)$$
$$\cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A)$$
$$\cos^2(A) + \sin^2(A) = 1$$
$$\cos(A) \approx 1 \quad \text{for } |A| \ll 1$$
$$\sin(A) \approx A \quad \text{for } |A| \ll 1$$