Pulse Modulation

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Chapter 5
Pulse Modulation: Transition from Analog to Digital Communications

Pulse Modulation

Reference:

Sections 5.1- 5.6 of

S. Haykin and M. Moher, Introduction to Analog & Digital Communications, 2nd ed., John Wiley & Sons, Inc., 2007. ISBN-13 978-0-471-43222-7.

Sampling

From duality:

periodic in time $\rightleftharpoons$ discrete in frequency
discrete in time $\rightleftharpoons$ periodic in frequency

Sampling

We can model sampling as multiplication of an analog waveform $g(t)$ with an impulse train:

- analog waveform: $g(t)$
- impulse train:

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_S)$$

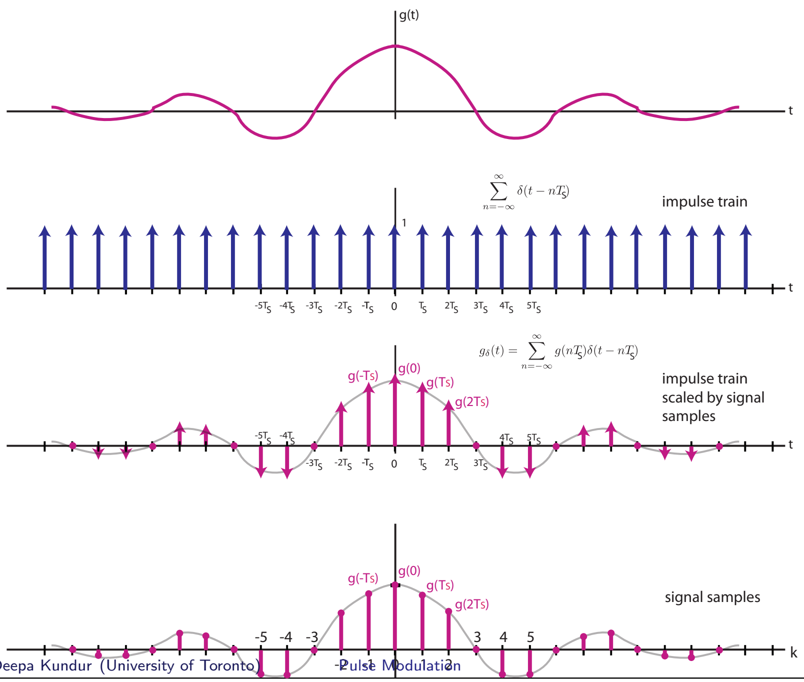
- model of sampling:

$$\begin{aligned} g_\delta(t) &= g(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_S) \\ &= \sum_{n=-\infty}^{\infty} g(nT_S)\delta(t - nT_S) \end{aligned}$$

Sampling

$$\begin{aligned} g_\delta(t) &= g(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_S) \\ &= \sum_{n=-\infty}^{\infty} g(nT_S)\delta(t - nT_S) \end{aligned}$$

Note: $g_\delta(t)$ contains the information of $g(nT_S)$ and represents a good model of sampling.



$$g_\delta(t) \overset{\mathcal{F}}{\longleftrightarrow} G_\delta(f)$$

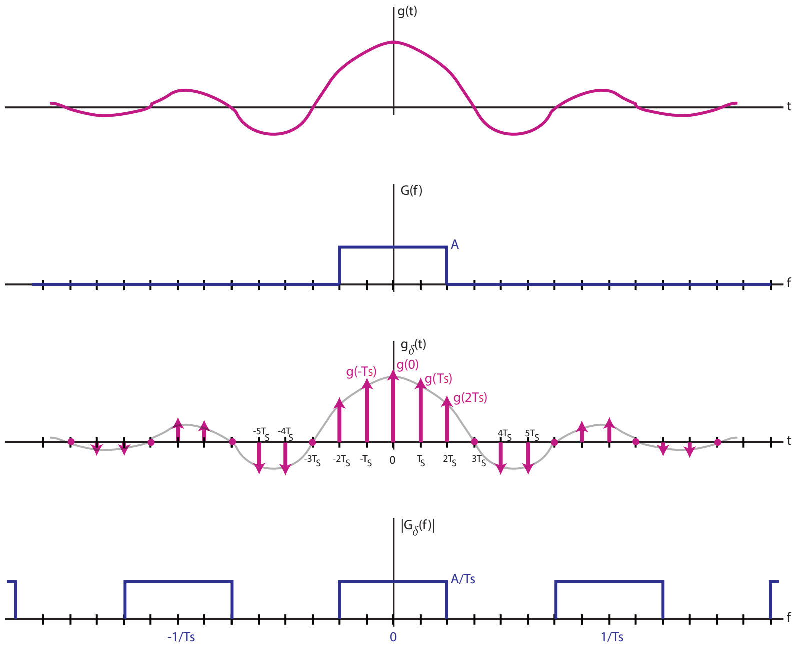
$$g(t) \overset{\mathcal{F}}{\longleftrightarrow} G(f)$$

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_S) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{T_S} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T_S}\right)$$

$$g_\delta(t) = g(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_S) \overset{\mathcal{F}}{\longleftrightarrow} G_\delta(f) = G(f) \star \frac{1}{T_S} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T_S}\right)$$

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(t)\delta(t - nT_S) \overset{\mathcal{F}}{\longleftrightarrow} G_\delta(f) = \frac{1}{T_S} \sum_{k=-\infty}^{\infty} G(f) \star \delta\left(f - \frac{k}{T_S}\right)$$

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_S)\delta(t - nT_S) \overset{\mathcal{F}}{\longleftrightarrow} G_\delta(f) = \frac{1}{T_S} \sum_{k=-\infty}^{\infty} G\left(f - \frac{k}{T_S}\right)$$



Let $f_s = \frac{1}{T_S}$:

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_S)\delta(t - nT_S) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad G_\delta(f) = \frac{1}{T_S}\sum_{k=-\infty}^{\infty} G(f - \frac{k}{T_S})$$

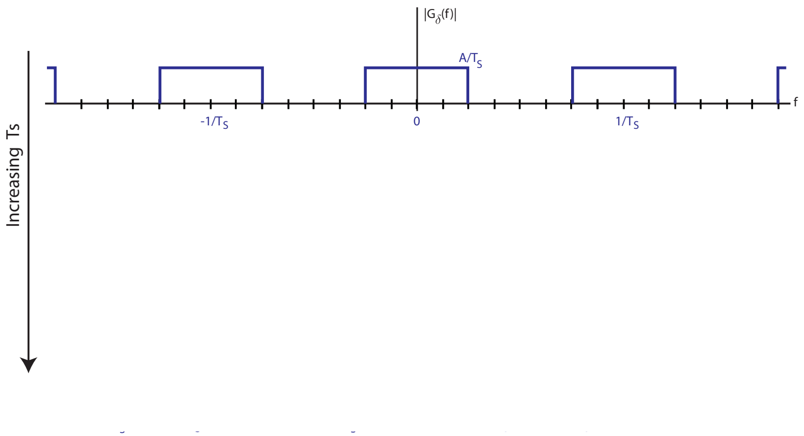
$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_S)\delta(t - nT_S) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad G_\delta(f) = f_S\sum_{k=-\infty}^{\infty} G(f - kf_S)$$

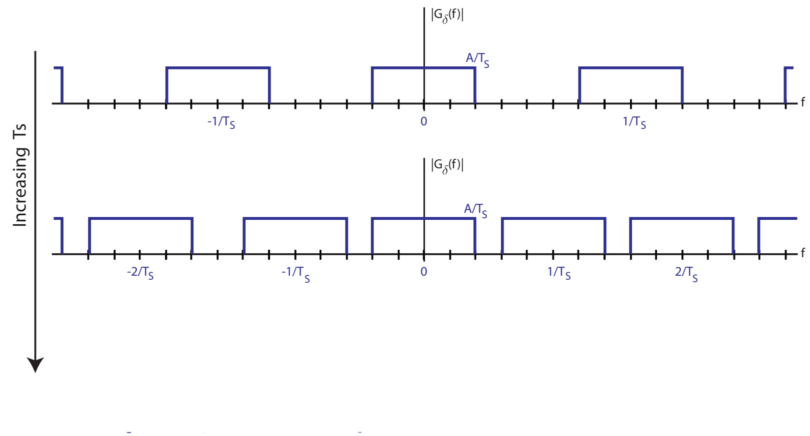
Recall, $\delta(t - t_0) \stackrel{\mathcal{F}}{\longleftrightarrow} e^{j2\pi t_0 f}$

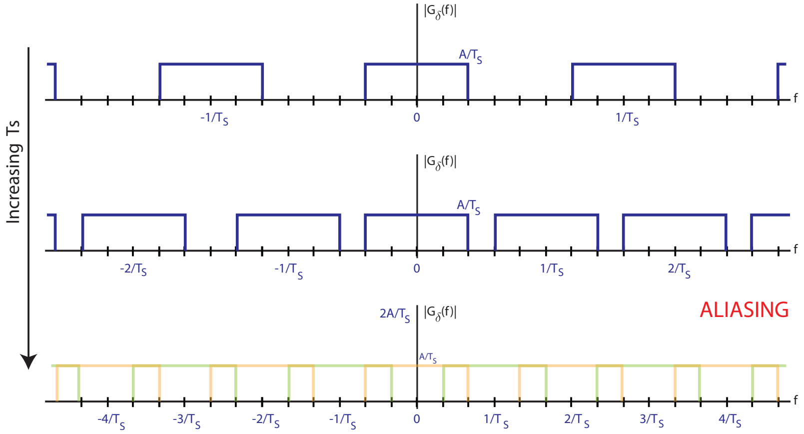
$$\begin{aligned} G_\delta(f) &= \mathcal{F}[g_\delta(t)] = \mathcal{F}\left[\sum_{n=-\infty}^{\infty} g(nT_S)\delta(t - nT_S)\right] \\ &= \sum_{n=-\infty}^{\infty} g(nT_S)\mathcal{F}[\delta(t - nT_S)] = \sum_{n=-\infty}^{\infty} g(nT_S)e^{j2\pi nT_S f} \end{aligned}$$

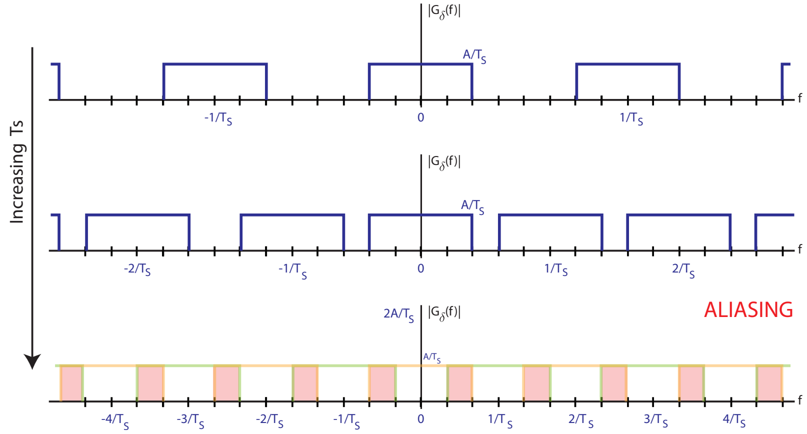
Therefore,

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_S)e^{j2\pi nT_S f} = f_S\sum_{k=-\infty}^{\infty} G(f - kf_S)$$






Increasing Ts
|Gδ(f)|
A/TS
-1/TS
0
1/TS
f
-2/TS
2/TS
2A/TS
ALIASING
-4/TS
-3/TS
3/TS
4/TS


Increasing Ts
|Gδ(f)|
A/TS
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ALIASING
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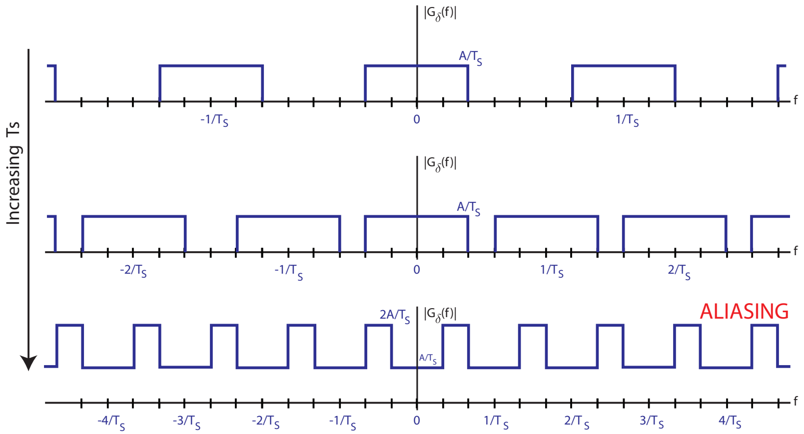
-3/TS
3/TS
4/TS


Increasing Ts
|Gδ(f)|
A/TS
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ALIASING
-4/TS
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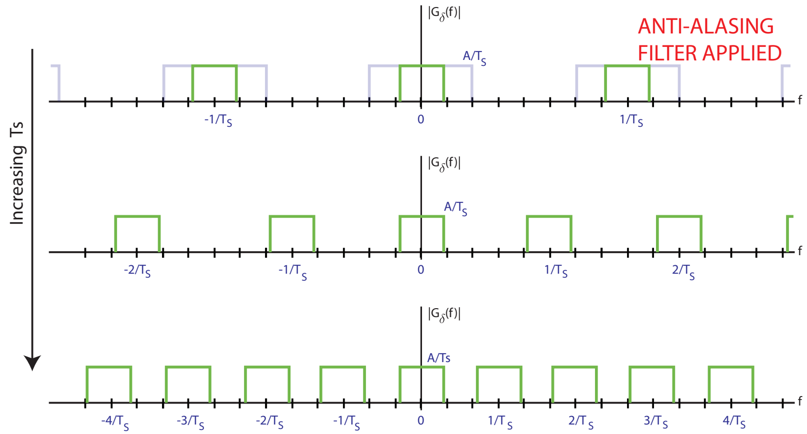
ANTI-ALASING
FILTER APPLIED
Increasing Ts
|Gδ(f)|
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1/TS
f
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2/TS
A/Ts
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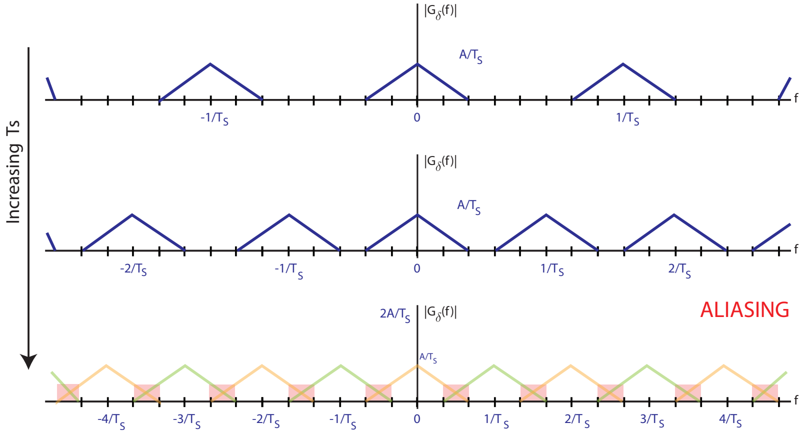
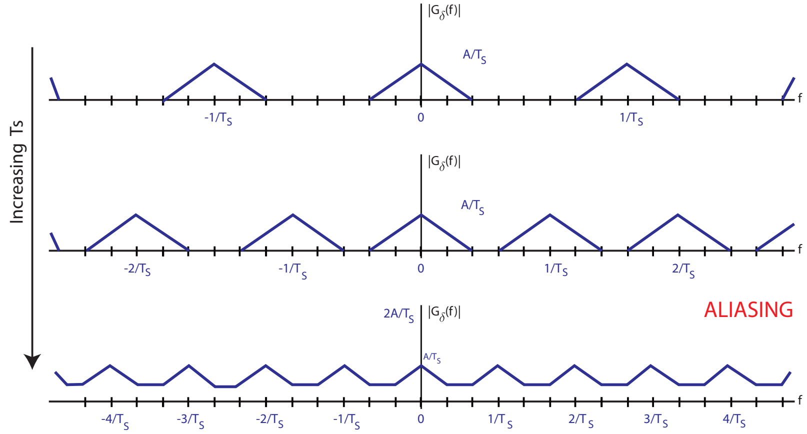
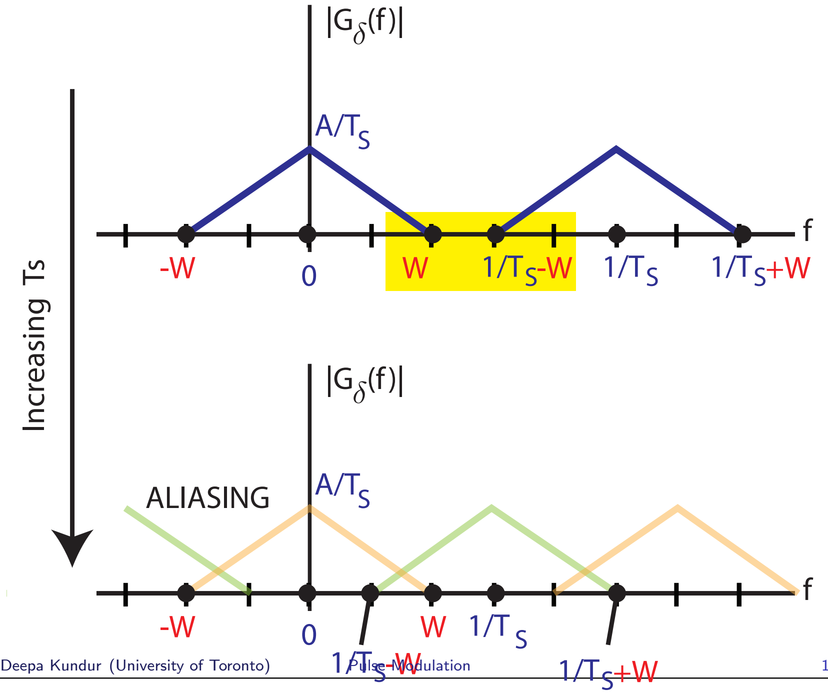
-3/TS
3/TS
4/TS







Sampling Theorem

$$W < \frac{1}{T_S} - W$$
$$W < f_S - W$$
$$f_S > 2W = \text{Nyquist Rate}$$

OR

$$f_S > 2W = \text{Nyquist Rate}$$
$$T_S < \frac{1}{2W} = \text{Maximum Sampling Period}$$

Sampling Theorem

Suppose that a signal $g(t)$ is *strictly band-limited* with no frequency components higher than W Hz. That is, $G(f)$ is zero for $|f| \geq W$.

Then $g(t)$ can be exactly recovered from its sample values $g(nT_S)$ for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$ through *band-limited interpolation* if:

$$f_S = \frac{1}{T_S} \quad > \quad 2W$$

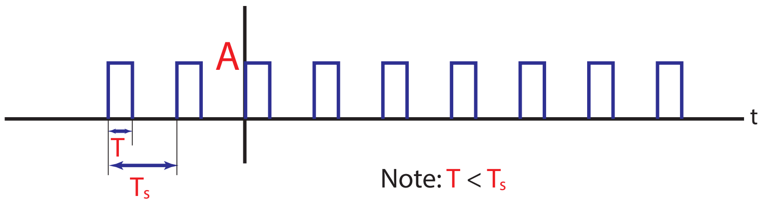
where $2W$ is called the Nyquist Rate.

We will assume that all message signals $m(t)$ from now on are sampled above the Nquist Rate.

Pulse Modulation

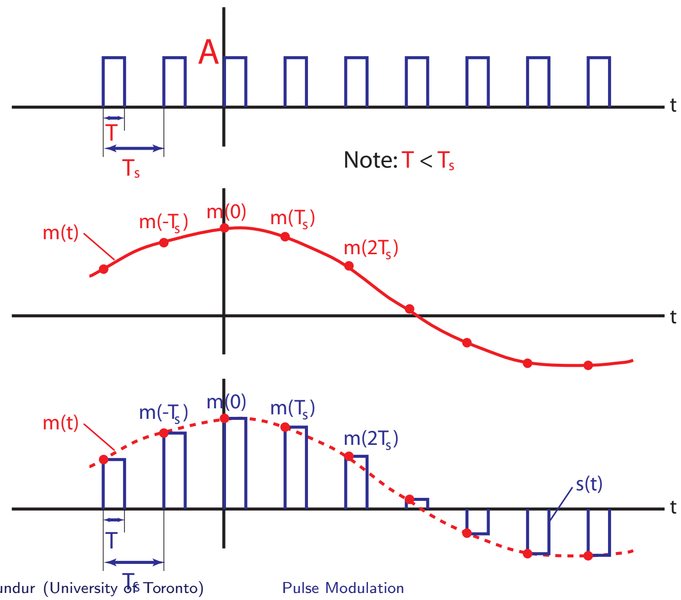
Pulse Modulation

- the variation of a regularly spaced constant amplitude pulse stream to superimpose information contained in a message signal.



- Three types:
  1. pulse amplitude modulation (PAM)
  2. pulse duration modulation (PDM)
  3. pulse position modulation (PPM)

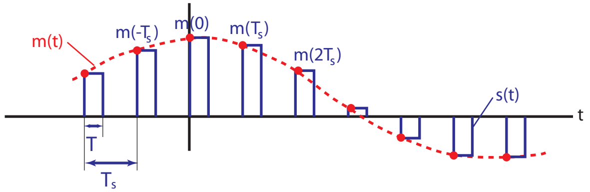
Pulse Amplitude Modulation (PAM)



Pulse Amplitude Modulation

Two steps: Sampling and Hold

1. Instantaneous sampling: the message $m(t)$ is sampled every T_s seconds where $f_s = \frac{1}{T_s}$ obeys the sampling theorem.
2. Lengthening: extending the duration of each sample so that it occupies T seconds.



Sample-and-Hold Analysis

Suppose

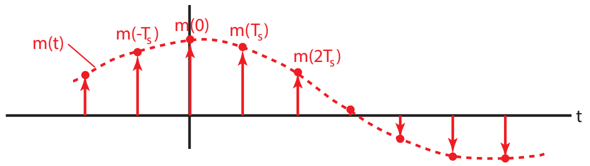
$$h(t) = \text{rect}\left(\frac{t-\frac{T}{2}}{T}\right) = \begin{cases} 1 & 0 < t < T \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} m_\delta(t) &= \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t-nT_S) \\ m_\delta(t) \star h(t) &= \left[\sum_{n=-\infty}^{\infty} m(nT_s)\delta(t-nT_S)\right] \star h(t) \\ &= \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t-nT_S) \star h(t) \\ &= \sum_{n=-\infty}^{\infty} m(nT_s)h(t-nT_S) \end{aligned}$$

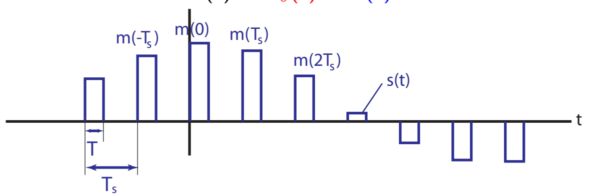
Sample-and-Hold Analysis

$$s(t) = m_\delta(t) \star h(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t-nT_S)$$

$m_\delta(t)$



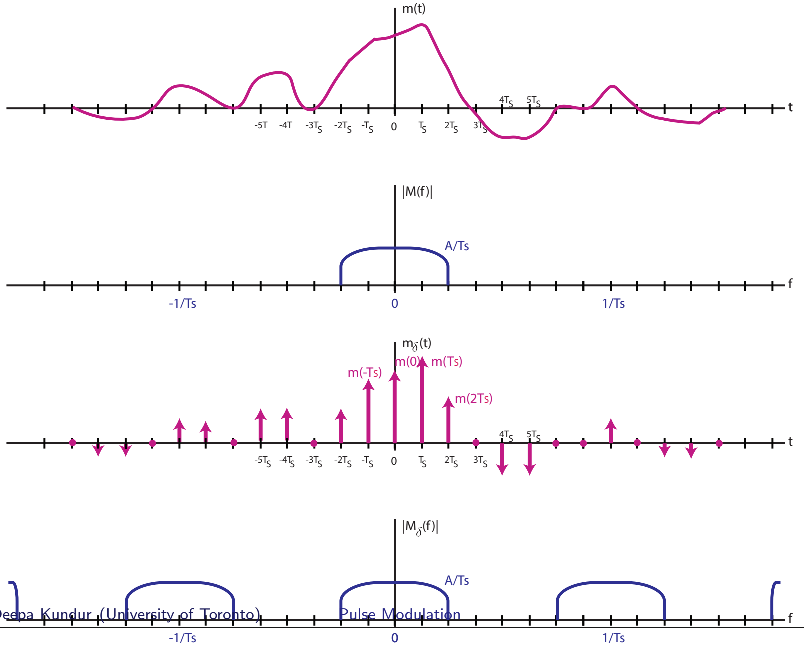
$s(t) = m_\delta(t) \star h(t)$



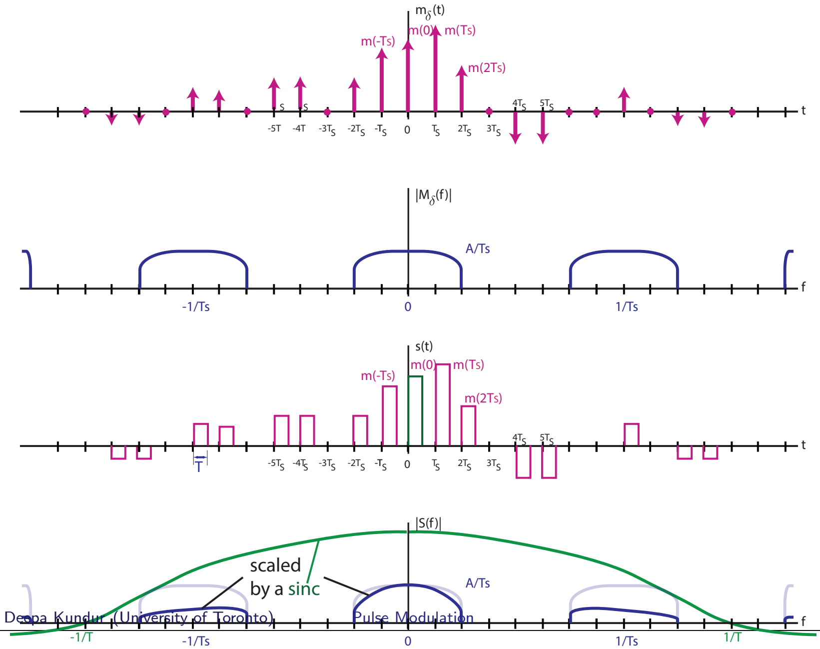
Sample-and-Hold Analysis

$$\begin{aligned} s(t) &= m_\delta(t) \star h(t) \\ S(f) &= M_\delta(f) \cdot H(f) \\ h(t) &\overset{\mathcal{F}}{\longleftrightarrow} H(f) \\ \text{rectangle} &\overset{\mathcal{F}}{\longleftrightarrow} \text{sinc} \end{aligned}$$

Step 1: Instantaneous sampling.



Step 2: Lengthening.



Pulse Amplitude Modulation

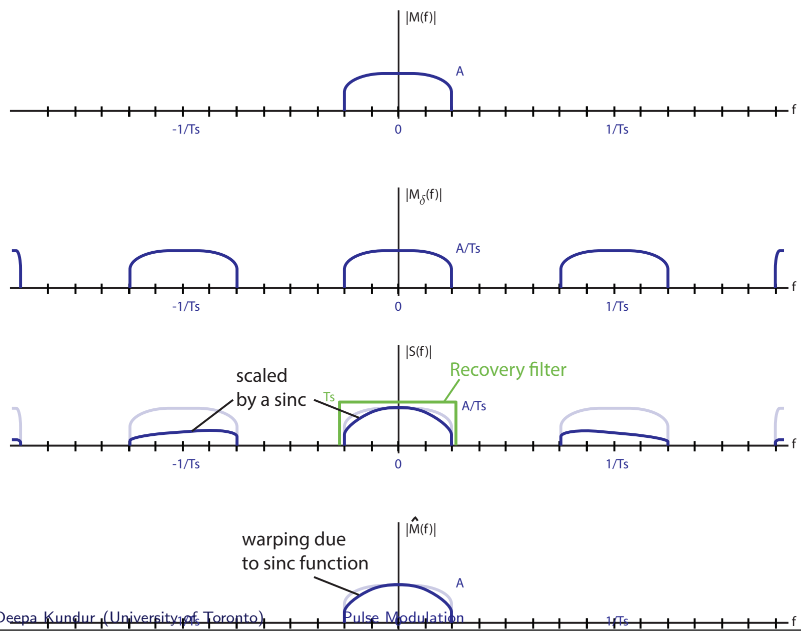
Recovery:

- Pass the samples $s(t)$ through a lowpass filter.

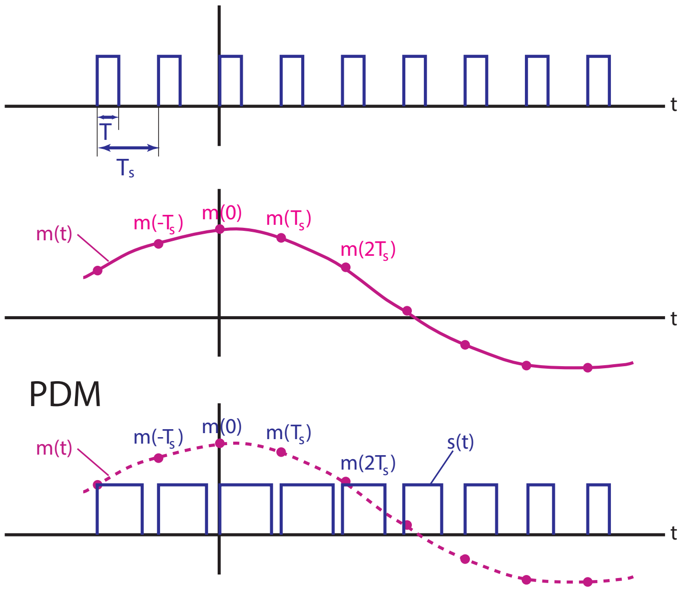
There is a trade-off to the pulse width T:

- The signal lengthening stage reduces the bandwidth of the overall pulse making it more efficient for communications.
- However, this is some distortion when recovering the signal as the sinc function in the frequency domain warps the frequency domain of the information signal.

Recovery.

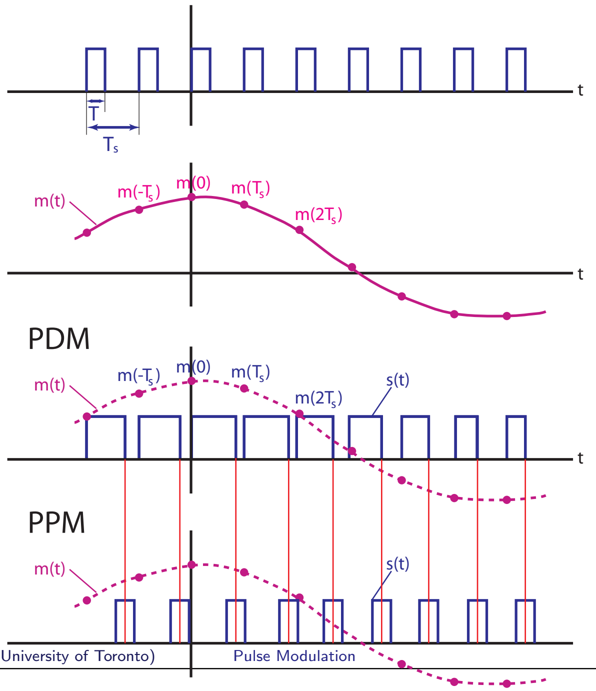


Pulse Duration Modulation (PDM)



Pulse Duration Modulation

- The width of the pulse reflects the sampled signal amplitude.
  - the position of the leading edge, trailing edge or both may be modified to reflect the changing duration of the pulse
  - also known as: pulse width modulation or pulse length modulation
- PDM is wasteful of energy when the pulses are long, but the information is only in the pulse transitions



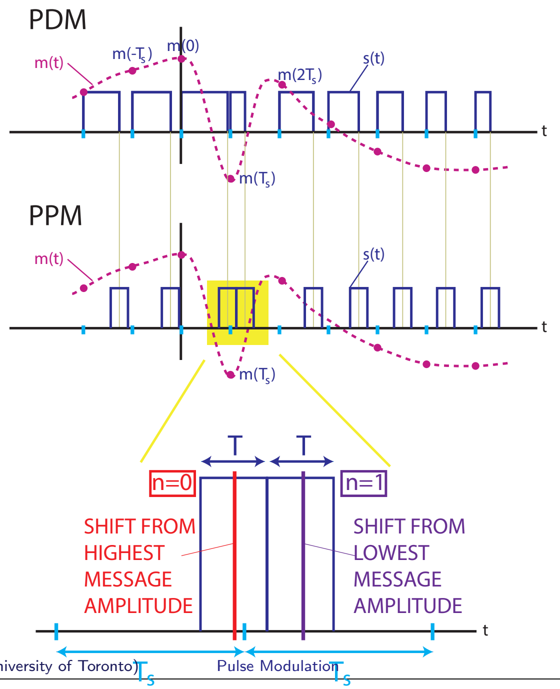
Pulse Position Modulation

- The position of the pulse reflects the sampled signal amplitude.
- PPM can be represented as:

$$s(t) = \sum_{n=-\infty}^{\infty} g(t - nT_s - k_p m(nT_s))$$

where

- k_p is the sensitivity factor
- the adjacent pulses must be strictly non-overlapping



Pulse Position Modulation

$$s(t) = \sum_{n=-\infty}^{\infty} g(t - nT_s - k_p m(nT_s))$$

where

$$k_p \, |m(t)|_{\max} < (T_s/2)$$

for strictly non-overlapping pulses.

Pulse Position Modulation

Recovery:

1. Determine pulse centers to decode values of $\{m(nT_s)\}$.
2. Use bandlimited interpolation to obtain $\hat{m}(t)$.

Assuming that the samples $\{m(nT_s)\}$ obey the Sampling Theorem, then $\hat{m}(t) = m(t)$ leading to ideal communication reconstruction.

Analog vs. Digital Communications

- Channel noise and signal distortion on analog communication system is cumulative.
- Regenerative repeaters in digital communication system can practical eliminate degrading effects of channel noise and signal distortion.
- Coding can be used in digital communication systems for greater reliability and security.
- Digital communications requires that we not just sample in time, but quantize in amplitude.

Amplitude Quantization

Amplitude quantization: the process of transforming the sample amplitude $m(nT_s)$ of a baseband signal $m(t)$ at time $t = nT_s$ into a discrete amplitude $v(nT_s)$ taken from a finite set of possible levels.

- non-reversible process

- Let us denote $m(nT_s)$ simply as m and $v(nT_s)$ simply as v.

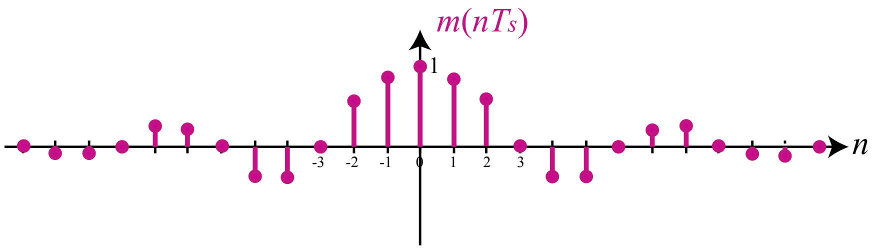
Amplitude Quantization

Let:

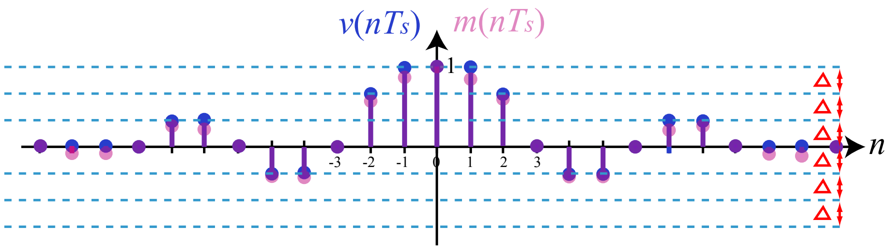
- m = original discrete-time signal sample
- v = quantized digital signal sample
- $g(\cdot)$ = quantization operator
- e_m = quantization error

$$\begin{aligned} v &= g(m) \\ e_m &= v - m \end{aligned}$$

Uniform Quantization Example

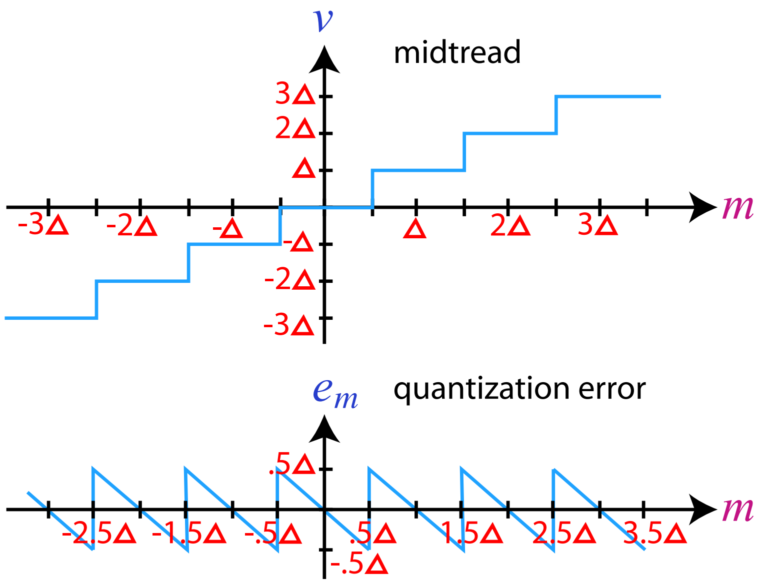


Uniform Quantization Example

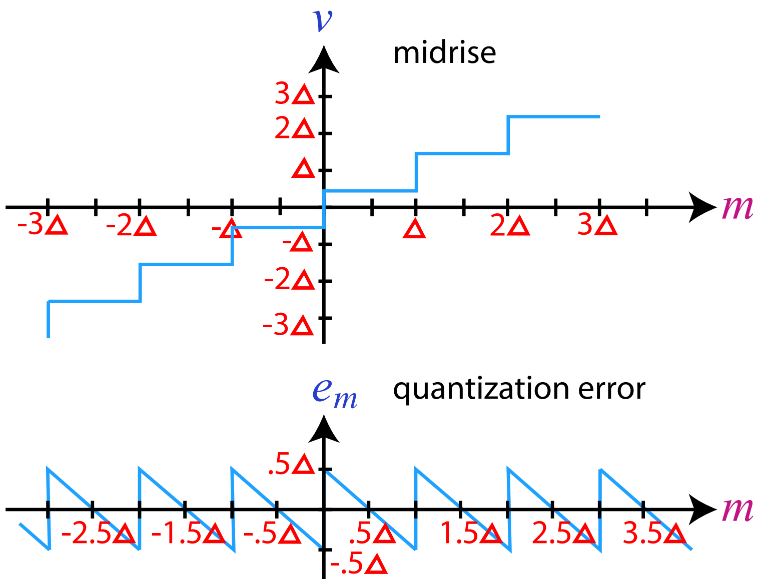


... result of rounding to the nearest quantization level.

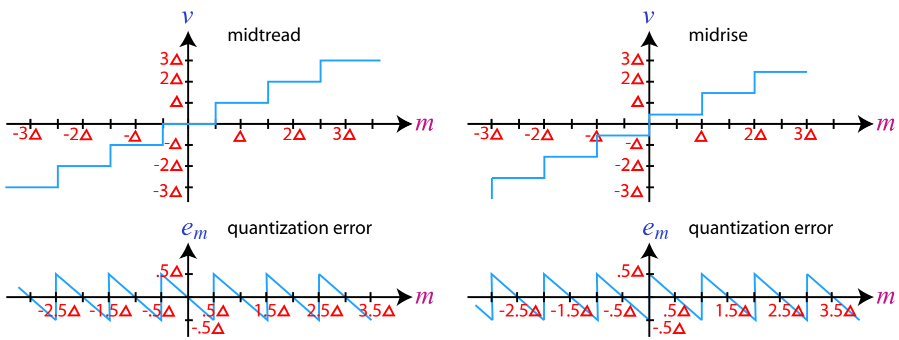
Uniform Quantization: Midtread



Uniform Quantization: Midrise

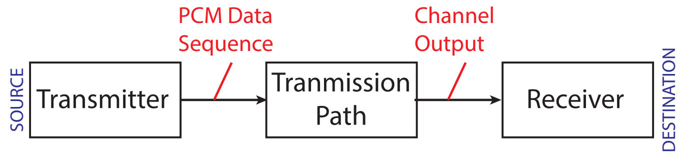


Uniform Quantization: Midtread vs. Midrise

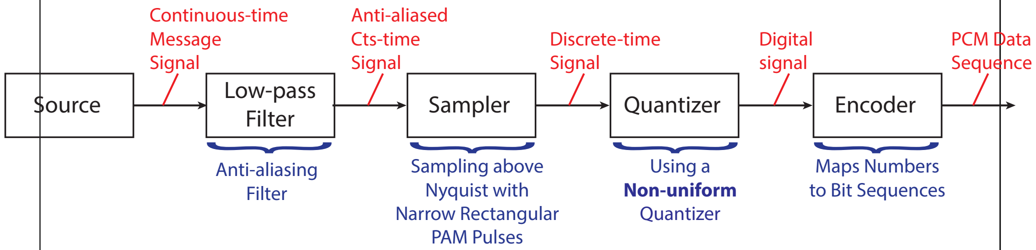


Pulse-Code Modulation

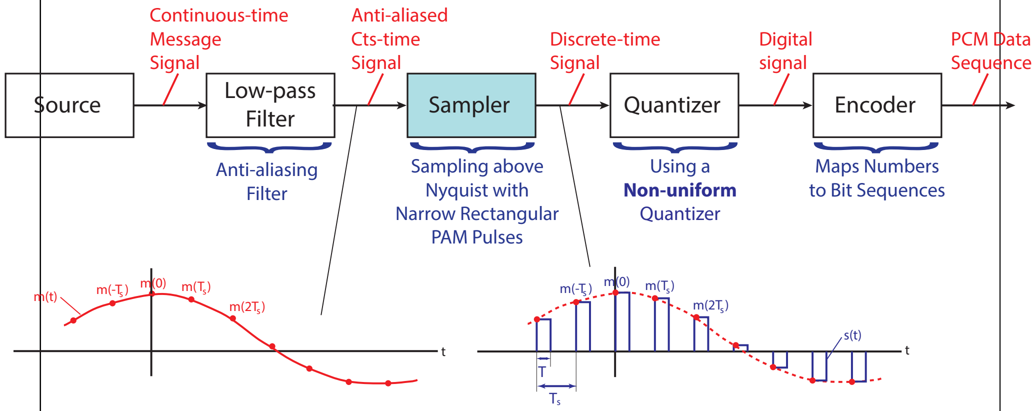
- Most basic form of digital pulse modulation
- Elements of pulse-code modulation (PCM):
  1. Transmitter
  2. Transmission Path
  3. Receiver



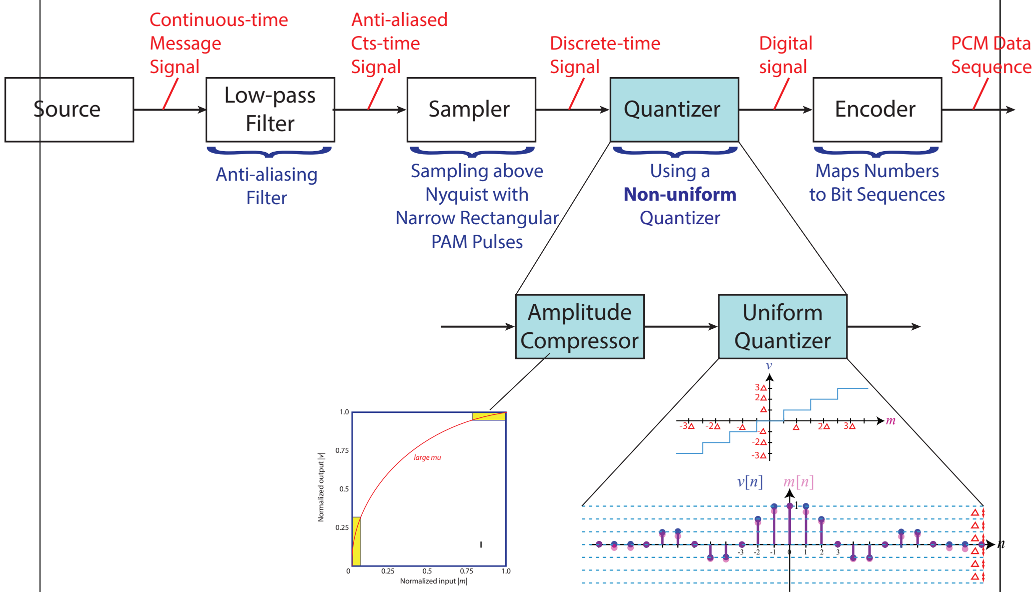
PCM Transmitter



PCM Transmitter: Sampler



PCM Transmitter: Non-Uniform Quantizer



PCM Transmitter: Non-Uniform Quantizer

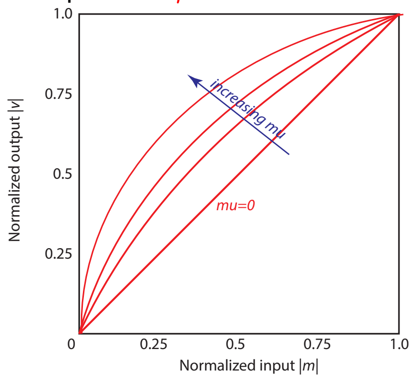
Compressor: μ-law:

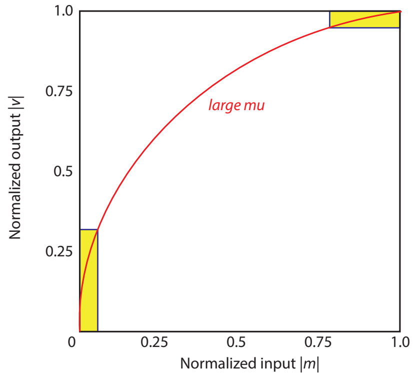
$$
\begin{aligned}
m &\equiv \text{message sample}\\
v &\equiv \text{quantized value}\\
\log &\equiv \text{natural logarithm}\\
|v| &= \frac{\log(1+\mu|m|)}{\log(1+\mu)}
\end{aligned}
$$

Note: $\mu = 0$ corresponds to a linear quantizer.
Typically $\mu \approx 255$ used in practice.

PCM Transmitter: Non-Uniform Quantizer

Compressor: μ-law:





PCM Transmitter: Non-Uniform Quantizer

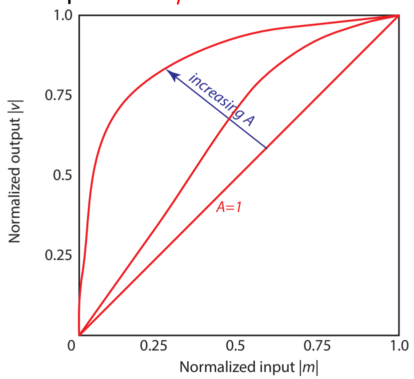
Compressor: A-law:

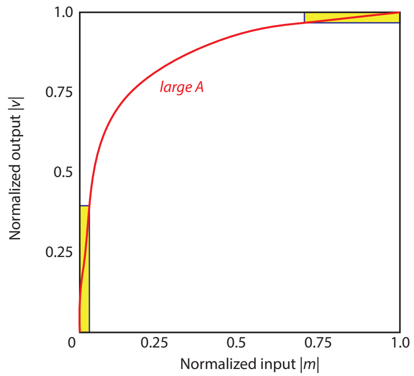
$$|v| = \begin{cases} \frac{A|m|}{1+\log(A)} & 0 \le |m| \le \frac{1}{A} \\ \frac{1+\log(A|m|)}{1+\log(A)} & \frac{1}{A} \le |m| \le 1 \end{cases}$$

Note: $A = 1$ corresponds to a linear quantizer.
Typically $A \approx 100$ used in practice.

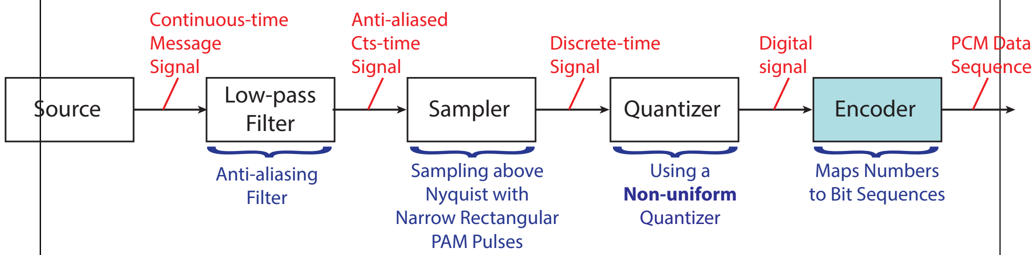
PCM Transmitter: Non-Uniform Quantizer

Compressor: μ-law:





PCM Transmitter: Encoder



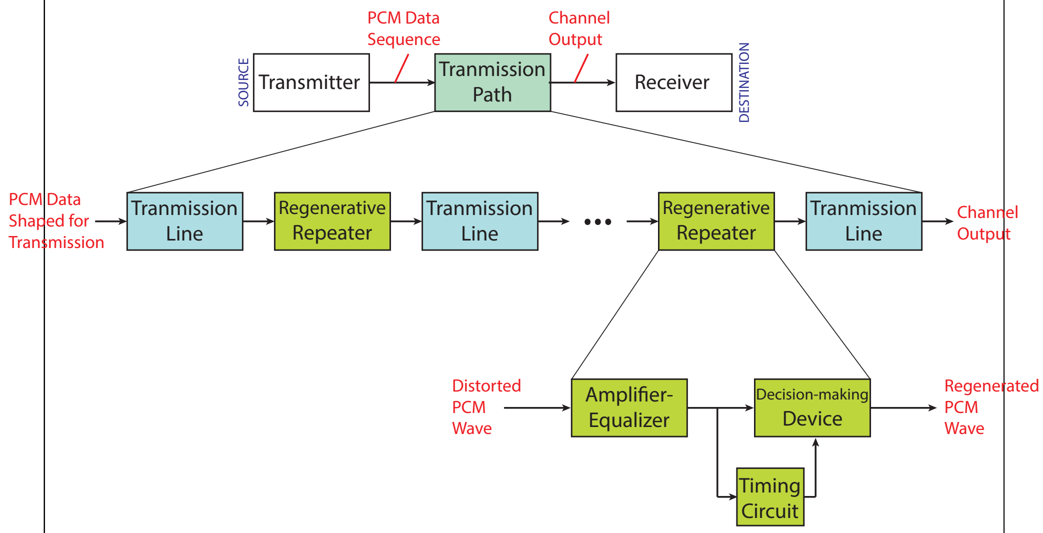
- maps quantization-level output to a code word
- typically binary code words are employed

Encoder: Example

8 Quantization-levels, or $R = 3$-bit code words:

Quantization-Level Index	Binary Codeword
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

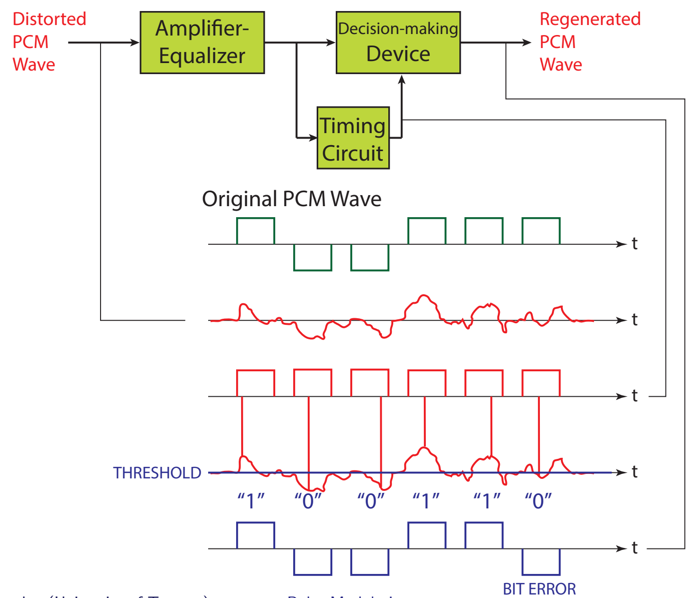
PCM: Transmission Path



PCM: Regenerative Repeater

- Amplifier-Equalizer: shapes the received pulses to compensate for amplitude and phase distortions produced by transmission line
- Timing Circuit: produces periodic pulse train derived from received pulses to sample the pulses where signal-to-noise ratio is highest
- Decision-making Device: sample of pulse is compared to a pre-determined threshold
  - if threshold exceed a clean new pulse representing 1 is transmitted
  - otherwise a clean new pulse representing 0 is transmitted

PCM: Regenerative Repeater



PCM: Receiver

Two Stages:

1. Decoding and Expanding:
   1.1 regenerate the pulse one last time and interpret bit sequence
   1.2 group bits into code words
   1.3 interpret code words as quantization level
   1.4 pass level through expander (opposite of compressor)
2. Reconstruction:
   2.1 pass expander output through low-pass reconstruction filter (cutoff is equal to message bandwidth) to estimate original message $m(t)$

■