Introduction to Image Processing

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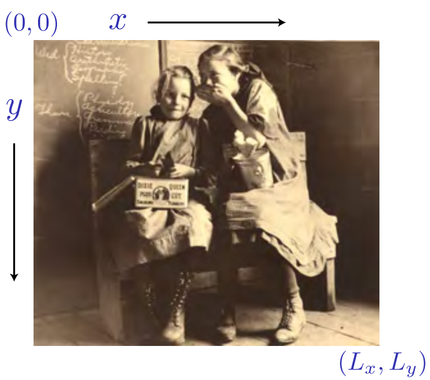
Analog Intensity Images

The image shown is "Dixie Queens" (two schoolgirls at lunch from Hadleyville, Oregon, circa 1911), Roy C. Andrews collection, PH003-P954, Special Collections and University Archives, University of Oregon, Eugene, Oregon 97403-1299.

Analog Intensity Images



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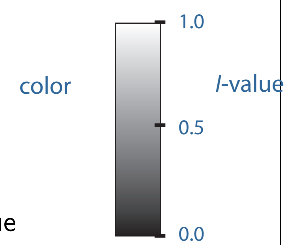
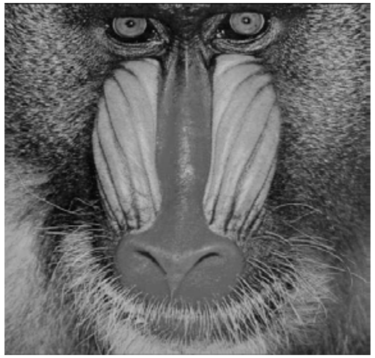
Analog Intensity Images

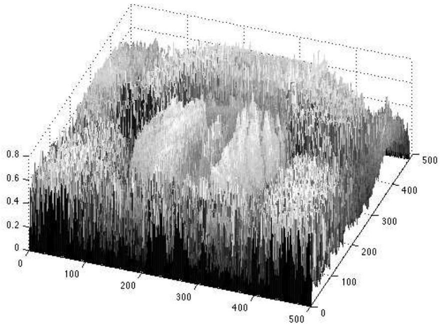
- continuous-space and continuous-amplitude image consisting of intensity (grayscale) values
- $I(x, y)$ is a two-dimensional signal representing the grayscale value at location (x, y) where:
  - $0 \leq x \leq L_x$ and $0 \leq y \leq L_y$
  - $I(x, y) = 0$ represents black
  - $I(x, y) = 1$ represents white
  - $0 < I(x, y) < 1$ represents proportional gray-value



Analog Intensity Images

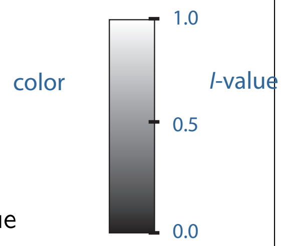
- $I(x, y)$ can be displayed as an intensity image or as a mesh graph





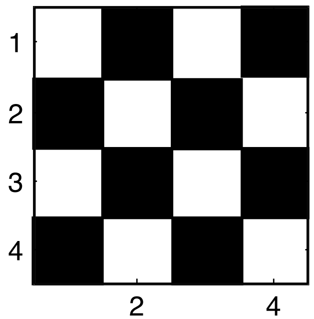
Discrete-Space Intensity Images

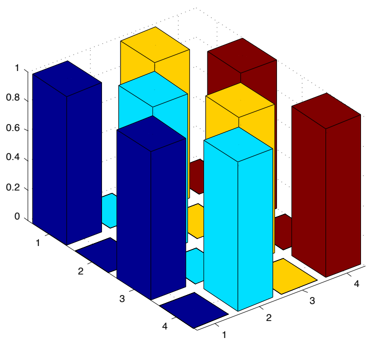
- discrete-space and continuous-amplitude image consisting of intensity (grayscale) values
- $I(m, n)$ is a two-dimensional signal representing the grayscale value at location (m, n) where:
- $m = 0, 1, \ldots, N_x - 1$ and $n = 0, 1, \ldots, N_y - 1$
- $I(m, n) = 0$ represents black
- $I(m, n) = 1$ represents white
- $0 < I(m, n) < 1$ represents proportional gray-value



Discrete-Space Intensity Images

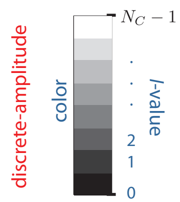
Example: 4×4 Checkerboard



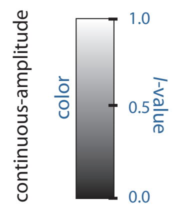


Digital Images

- discrete-space and discrete-amplitude
- $m = 0, 1, \ldots, N_x - 1$ and $n = 0, 1, \ldots, N_y - 1$
- image consisting of grayscale colors from a finite set $\mathcal{C}$ and indexed via the set: $\{0, 1, 2, \ldots, N_C - 1\}$
- Example: $N_C = 8$ and grayscale values linearly distributed in intensity between black (0) and white $(N_C - 1)$



. . . vs.

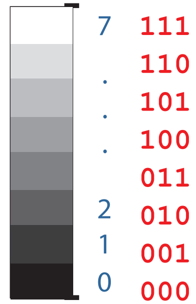


Digital Images: Common Format

- $I(m, n)$ is a two-dimensional signal representing the grayscale value at location (m, n) where:
  - $I(m, n) \in \{0, 1, 2, \ldots, N_C - 1\}$; N_C = no. of colors
  - $I(m, n) = 0$ represents black
  - $I(m, n) = N_C - 1$ represents white
  - $I(m, n) \in \{1, 2, \ldots, N_C - 2\}$ represents proportional gray-value

Digital Images: Common Format

- N_C is usually of the form 2^N, so that the 2^N different colors are efficiently represented with N-bit binary notation; Example: $N = 3$

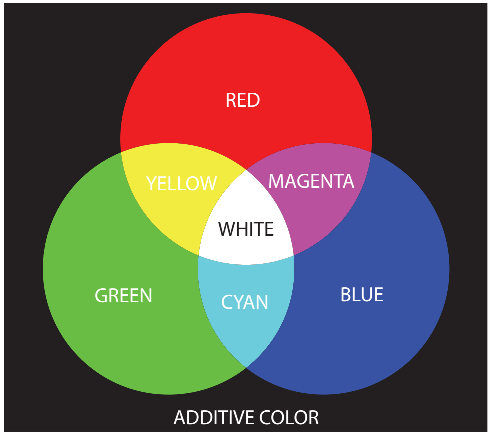


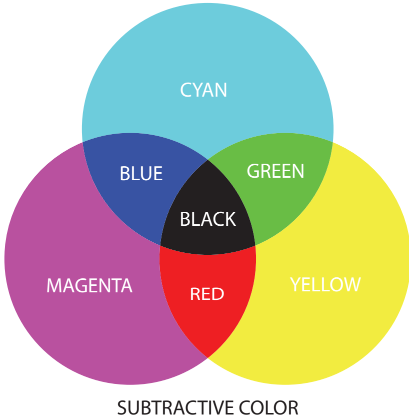
Digital Images: 8-Bit Grayscale Images

- Standard 8-bit images use color indices from 0 through 255 to cover shades of gray ranging from black to white (inclusive).
  - convenient for programming: color representation occupies a single byte
  - perceptually acceptable: barely sufficient precision to avoid visible banding

Digital Images: Color

Digital Images: Color

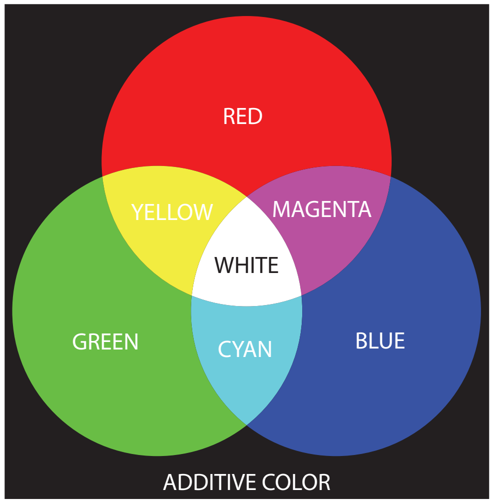


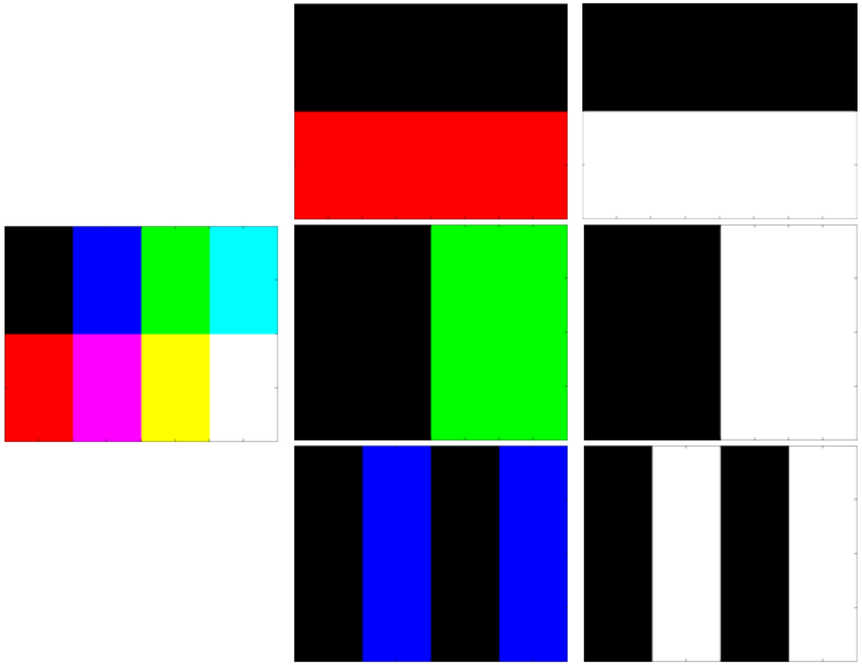


Color Spaces

- Color space: model describing a way to represent colors as mathematical vectors
- usually three or four numbers are needed to represent any color; common color spaces include:
  - red (R), green (G), blue (B) popular for LCD displays
  - cyan (C), magenta (M), yellow (Y), key (K) popular for print
  - YCbCr, HSV, . . .

Digital Images: Additive Color Theory





Digital Images: Truecolor Images

- From Wiki (March 18, 2013): method of representing and storing graphical image information (especially in computer processing) in an RGB color space such that a very large number of colors, shades, and hues can be displayed in an image, such as in high quality photographic images or complex graphics
- usually at least 256 shades of each red, green and blue are employed resulting in at least $256^3 = 16,777,216$ (16 million) color variations
- human eye can discern as many as ten million colors, so representation should exceed human visual system (HVS) capabilities!

RGB versus Grayscale

- RGB to grayscale conversion:

$$I(m,n) = 0.299R(m,n) + 0.587G(m,n) + 0.114B(m,n)$$

RGB versus Grayscale

- RGB to grayscale conversion:

$$I(m,n) = 0.299R(m,n) + 0.587G(m,n) + 0.114B(m,n)$$

  - Note: $0.299 + 0.587 + 0.114 = 1$.
  - The luminance compensates for the eye's distinct sensitivity to different colors.
  - The human eye is most sensitive to green, then red, and last blue.
    - There are evolutionary justifications for this difference.
    - A color with more green is brighter to the eye than a color with more blue.

RGB versus Grayscale

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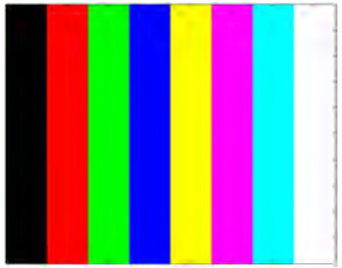
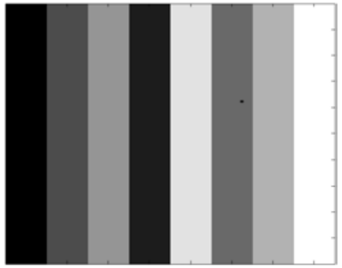
$$I(m,n) = 0.299R(m,n) + 0.587G(m,n) + 0.114B(m,n)$$





RGB versus Grayscale

- RGB to grayscale conversion:

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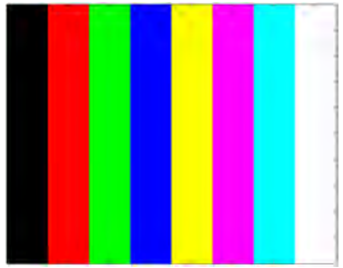


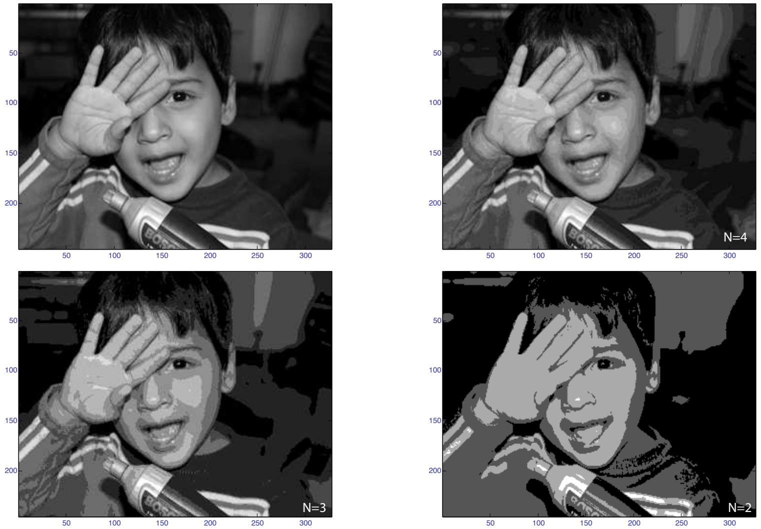
$$\begin{aligned} \text{black} &= [0\ 0\ 0] = [R\ G\ B] \\ \text{red} &= [1\ 0\ 0] \\ \text{green} &= [0\ 1\ 0] \\ \text{blue} &= [0\ 0\ 1] \\ \text{yellow} &= [1\ 1\ 0] \\ \text{magenta} &= [1\ 0\ 1] \\ \text{cyan} &= [0\ 1\ 1] \\ \text{white} &= [1\ 1\ 1] \end{aligned}$$

Image Parameters

- The following parameters have an effect on the image quality:
  - sampling rate: spatial resolution or dimension of image
  - color depth: number of colors or number of bits to represent colors

Sampling Rate and Subsampling

Color Depth and Amplitude Quantization



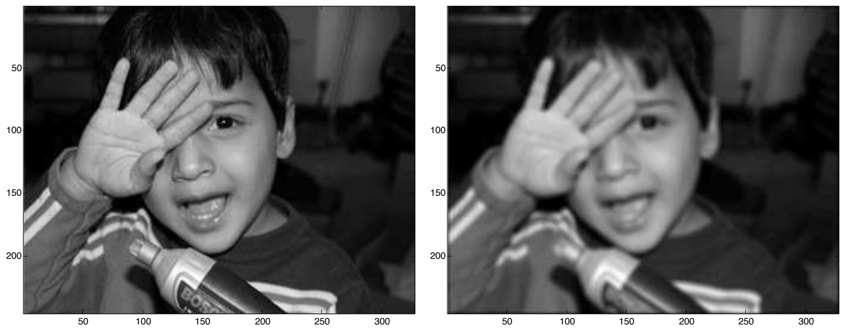
Lowpass Filtering

$$H = \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$I_H(m,n) = I(m,n) * H(m,n)$$

Lowpass Filtering

$I(m,n)$ and $I_H(m,n)$:



Highpass Filtering

$$H = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$I_H(m,n) = I(m,n) * H(m,n)$$

Highpass Filtering

$I(m,n)$ and $I_H(m,n)$:

Edge Enhancement

$$H = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$I_H(m,n) = I(m,n) * H(m,n)$$

$$I_E(m,n) = I_H(m,n) + I(m,n)$$

Edge Enhancement

$I(m,n)$ and $I_E(m,n)$:

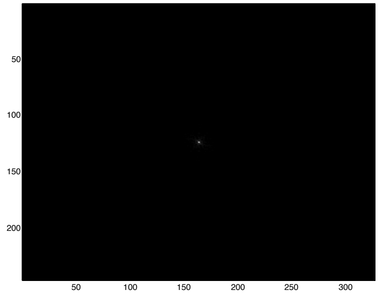
2-D Discrete Fourier Transform

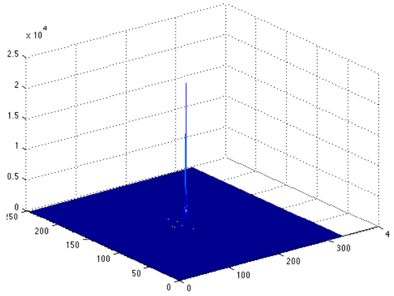
$$\mathcal{I}_F(U,V) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} I(m,n) e^{-j2\pi(Um+Vn)}$$

$I(m,n)$:

2-D Discrete Fourier Transform

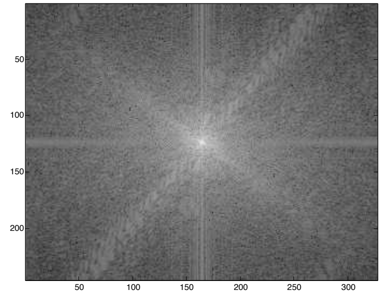
$\mathcal{I}_F(U, V)$:

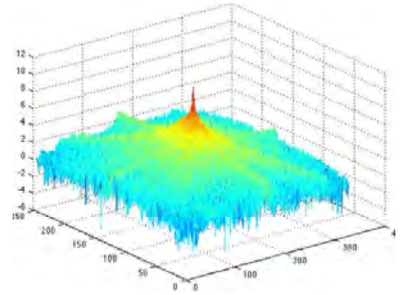




2-D Discrete Fourier Transform

$\mathcal{I}_F(U, V)$ on log-scale:





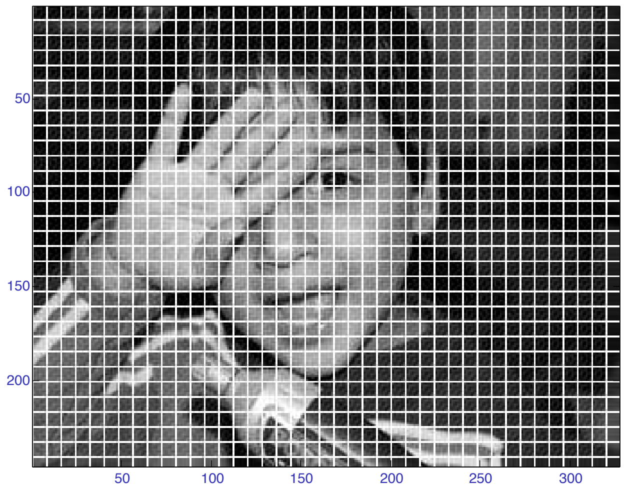
2-D Discrete Cosine Transform

Consider an $N_x \times N_y$-dimensional digital image $I(m, n)$:

$$\mathcal{I}_{DCT}(k, l) = \sum_{m=0}^{N_x-1} \sum_{n=0}^{N_y-1} I(m, n) \cos\left[\frac{\pi}{N}\left(n + \frac{1}{2}\right)k\right] \cos\left[\frac{\pi}{M}\left(m + \frac{1}{2}\right)l\right]$$

2-D Discrete Cosine Transform

2-D Discrete 8 × 8 Cosine Transform



2-D Discrete 8 × 8 Cosine Transform

$$\mathcal{I}^B_{DCT}(k,l) = \sum_{m=0}^{7}\sum_{n=0}^{7} I^B(m,n)\cos\left[\frac{\pi}{8}\left(n+\frac{1}{2}\right)k\right]\cos\left[\frac{\pi}{8}\left(m+\frac{1}{2}\right)l\right]$$

$$I^B(m,n) = \sum_{k=0}^{7}\sum_{l=0}^{7} \alpha(k)\alpha(l)\mathcal{I}^B_{DCT}(k,l)\cos\left[\frac{\pi}{8}\left(n+\frac{1}{2}\right)k\right]\cos\left[\frac{\pi}{8}\left(m+\frac{1}{2}\right)l\right]$$

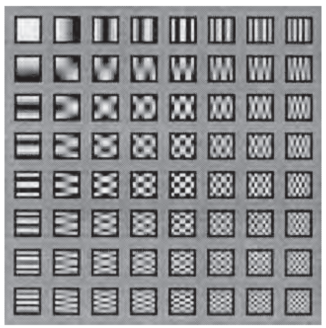
where

$$\alpha(k) = \begin{cases} \sqrt{\frac{1}{8}} & \text{for } k=0 \\ \sqrt{\frac{2}{8}} & \text{for } k=1,2,\ldots,7 \end{cases}$$

2-D Discrete 8 × 8 Cosine Transform

For $k,l \in \{0,1,2,\ldots,7\}$,

$$\cos\left[\frac{\pi}{8}\left(n+\frac{1}{2}\right)k\right]\cos\left[\frac{\pi}{8}\left(m+\frac{1}{2}\right)l\right]:$$



2-D Discrete 8 × 8 Cosine Transform

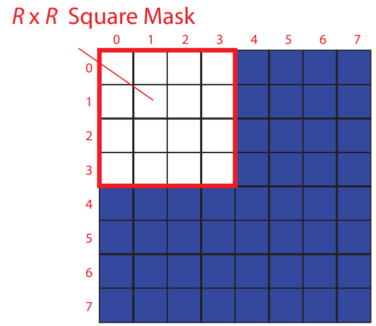
$$I^B(m,n) = \sum_{k=0}^{7}\sum_{l=0}^{7} \alpha(k)\alpha(l)\mathcal{I}^B_{DCT}(k,l)\cos\left[\frac{\pi}{8}\left(n+\frac{1}{2}\right)k\right]\cos\left[\frac{\pi}{8}\left(m+\frac{1}{2}\right)l\right]$$

Lossy versus Non-lossy Compression for Digital Images

- Lossy compression: remove signal components to reduce storage requirements
  - often exploits perceptual irrelevancy to shape the signal in order to reduce storage size
  - process is not reversible
- Non-lossy compression: exploit statistical redundancy to employ efficient codes (on average) to reduce storage requirements
  - process is reversible

Lossy Compression via the DCT

Consider removing (i.e., zeroing) signal components from 8×8-DCT domain outside a pre-defined mask.



Note: this is only an instructive example and there are multitudes of other ways to achieve this.

Lossy Compression via the DCT

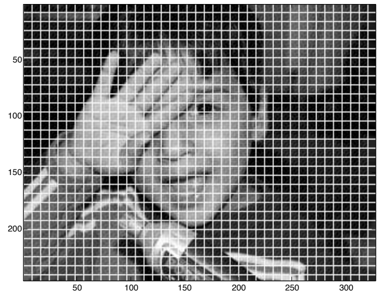
Step 1: Compute the 8×8-block DCT on $I(m, n)$.

$$\mathcal{I}^B_{DCT}(k,l) = \sum_{m=0}^{7} \sum_{n=0}^{7} I^B(m,n) \cos\left[\frac{\pi}{8}\left(n + \frac{1}{2}\right) k\right] \cos\left[\frac{\pi}{8}\left(m + \frac{1}{2}\right) l\right]$$

Lossy Compression via the DCT

Step 1: Compute the 8×8-block DCT on $I(m, n)$.

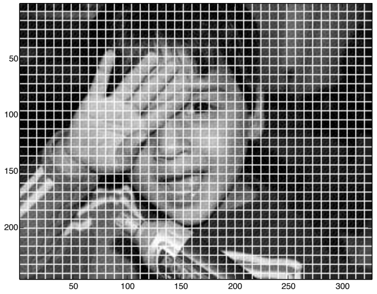
$I(m, n)$ and $I^B(m, n)$:

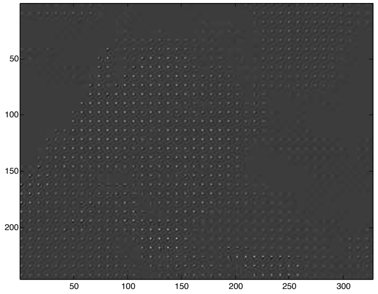


Lossy Compression via the DCT

Step 1: Compute the 8×8-block DCT on $I(m,n)$.

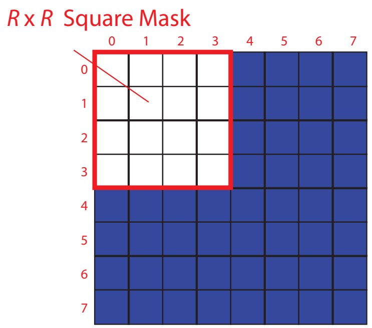
$I^B(m,n)$ and $\mathcal{I}^B_{DCT}(k,l)$:





Lossy Compression via the DCT

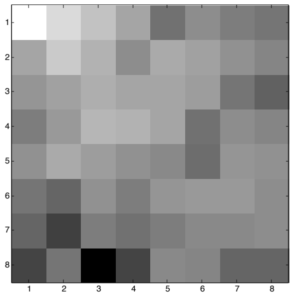
Step 2: Remove high-frequency components via $R \times R$ mask.

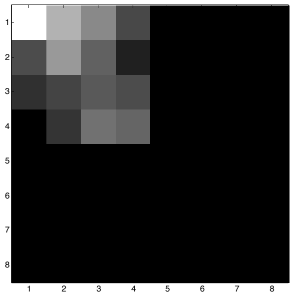


Lossy Compression via the DCT

Step 2: Remove high-frequency components via $R \times R$ mask.

$\mathcal{I}^B_{DCT}(k,l)$ and compressed version $\tilde{\mathcal{I}}^B_{DCT}(k,l)$ for $R = 4$:





Note: images displayed on log-amplitude scale.

Lossy Compression via the DCT

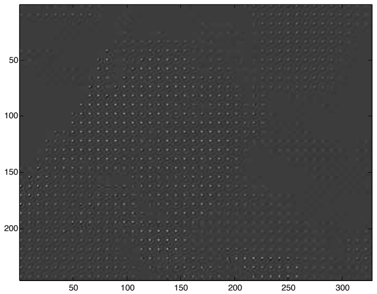
Step 3: Compute the 8×8-block IDCT on compressed DCT coefficients.

$$\tilde{I}^B(m,n) = \sum_{k=0}^{7}\sum_{l=0}^{7} \alpha(k)\alpha(l)\tilde{\mathcal{I}}^B_{DCT}(k,l) \cos\left[\frac{\pi}{8}\left(n+\frac{1}{2}\right)k\right] \cos\left[\frac{\pi}{8}\left(m+\frac{1}{2}\right)l\right]$$

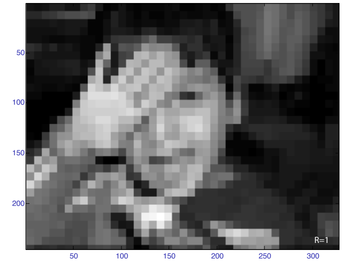
Lossy Compression via the DCT

Step 3: Compute the 8 × 8-block IDCT on compressed DCT coefficients.

$\tilde{\mathcal{I}}^{B}_{DCT}(k,l)$ and $\tilde{I}(m,n)$ for $R = 4$:

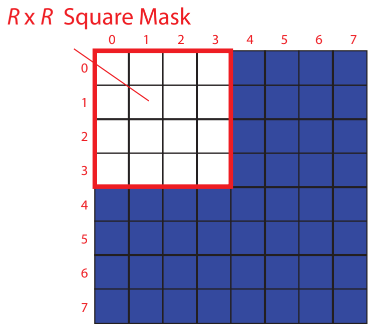


Lossy Compression Results



Further Compression Gains

- coefficients within the mask can be quantized with a factor determined by tests on human perception



- compressed coefficients are passed through a non-lossy arithmetic coder for additional compression efficiency