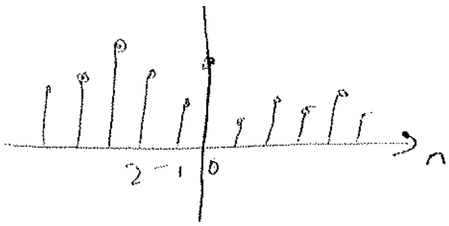
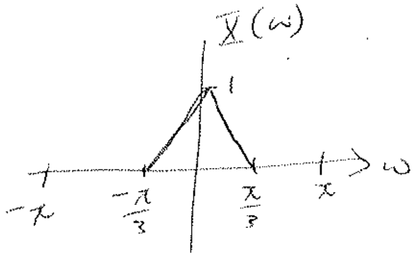
CH. 11 problems {11.5, 11.9, 11.12, 11.13}

11.5 In this problem we illustrate the concepts of sampling and decimation for discrete-time signals. To this end consider a signal $x(n)$ with Fourier transform $X(\omega)$ as in





(a) Sampling $x(n)$ with a sampling period $D=2$, results in the signal

$$X_s(n) = \begin{cases} x(n), & n = 0, \mp 2, \mp 4, \ldots \\ 0, & n = \mp 1, \mp 3, \mp 5, \ldots \end{cases}$$

compute and sketch the signal $X_s(n)$ and its Fourier transform $X_s(\omega)$. Can we reconstruct $x(n)$ from $X_s(n)$? How?

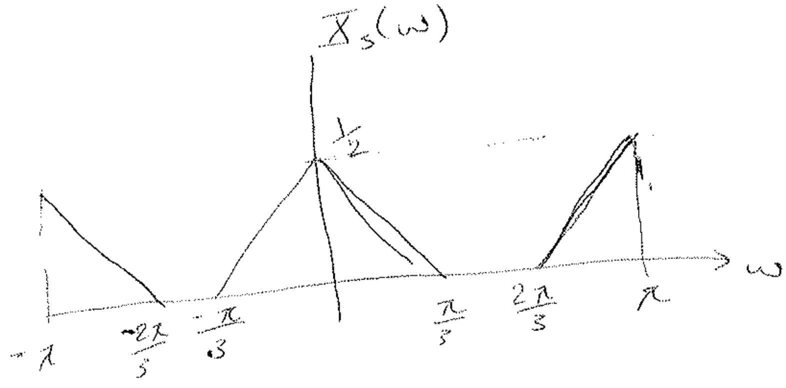
let $w = \frac{\Omega}{F_x}$, $w' = \frac{2\Omega}{F_x}$

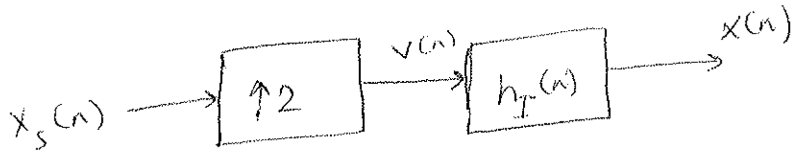
$$X_s(w) = \sum_n x_s(n) e^{-jwn} = \sum_n x(2m) e^{-jw2m}$$

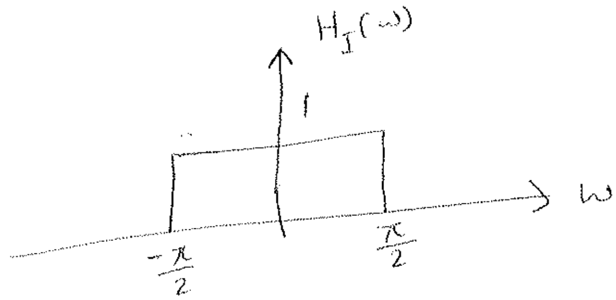
$$= \frac{1}{2} \sum_q X\left(w - \frac{2\pi q}{2}\right)$$

$$= \frac{1}{2} \sum_q X(w - \pi q)$$

to recover $X(n)$ from $X_s(n)$







b) Decimating $x(n)$ by a factor of $D=2$, produces the signal

$$x_d(n) = x(2n), \quad \text{all } n$$

and its transform $X_d(\omega)$. Show that $X_d(\omega) = X_s\left(\frac{\omega}{2}\right)$. Plot the signal $x_d(n)$ and its transform $X_d(\omega)$. Do we lose any information when we decimate the sampled signal $x_s(n)$?

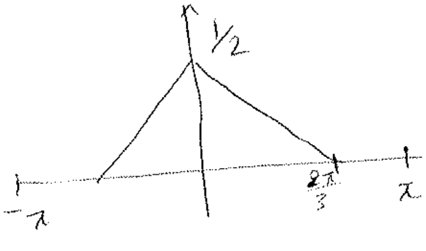
recall $\omega' = 2\omega$

$$X_d(\omega') = \sum_n x_d(n) e^{-j\omega' n}$$

$$= \sum_{n \text{ even}} x_s(n) e^{-j\omega' \frac{n}{2}}$$

$$= \sum_n x_s(n) e^{-j\omega' \frac{n}{2}} \quad \text{since } x_s(n) = 0 \text{ for odd } n.$$

$$= X_s\left(\frac{\omega'}{2}\right) \quad \square$$

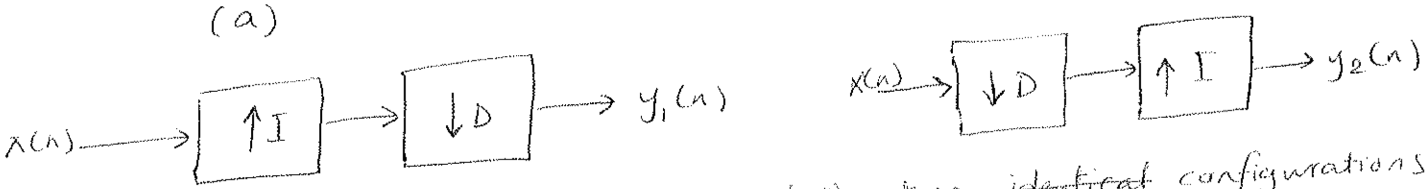


No information is lost, since the decimated sample rate still exceeds twice the band limit of the original sig.

11.9 Consider the two different ways of cascading a decimator with an interpolator shown in

(b)

(a)



(a) if $D = I$, show that the outputs of the two ~~identical~~ configurations are different. Hence, in general, the two systems are not identical.

let $x(n) = \{x_0, x_1, \dots\}$

$D = I = 2$, decimation first (fig-b)

$$Z_2(n) = \{x_0, x_2, x_4, \dots\}$$

$$y_2(n) = \{x_0, 0, x_2, 0, x_4, 0, \dots\}$$

Interpolation first (fig-a)

$$Z_1(n) = \{x_0, 0, x_1, 0, x_2, 0, \dots\}$$

$$y_1(n) = \{x_0, x_1, x_2, \dots\}$$

$$\Rightarrow y_2(n) \neq y_1(n)$$

(b) show that the two systems are identical if and only if D & I are relatively prime.

let $D = dk$ and $I = ik$ and d, i are relatively prime.

$$x(n) = \{x_0, x_1, x_2, \dots\}$$

decimation first

$$Z_2(n) = \{x_0, x_{dk}, x_{2dk}, \dots\}$$

$$y_2(n) = \{x_0, \underbrace{0, 0, 0, \dots, 0}_{ik-1}, x_{dk}, \underbrace{0, \dots, 0}_{ik-1}, x_{2dk}, \dots\}$$

interpolation first

$$z_1(n) = \{x_0, \underbrace{0, 0, \cdots, 0}_{ik-1}, x_1, \underbrace{0, 0, \cdots, 0}_{ik-1}, x_2, 0, \cdots, 0, \cdots\}$$

$$y_1(n) = \{x_0, \underbrace{0, 0, \cdots 0}_{d-1}, x_d, \underbrace{0, \cdots, 0}_{d-1}, \cdots\}$$

$\Rightarrow y_1 = y_2$ iff $d = dk$ ok $k = 1$, $\Rightarrow$ D and I are relatively prime

example let $x_n = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$.

$D = 2, I = 3$

Decimation First:

$$z_1(n) = \{x_1, x_3, x_5, \cdots\}$$

$$y_1(n) = \{x_1, \underbrace{0, 0}_{I-1}, x_3, 0, 0, x_5, \cdots\}$$

Interpolation first:

$$z_2(n) = \{x_1, 0, 0, \cancel{x_2}, 0, 0, x_3, 0, 0, \cancel{x_4}, 0, 0, x_5, \cdots\}$$

$$y_2(n) = \{x_1, 0, 0, x_3, 0, 0, x_5, \cdots\}.$$

$\Rightarrow y_1(n) = y_2(n)$.

11.12 A sequence $x(n)$ is upsampled by $I=2$, it passes through an LTI system $H_1(z)$, and then it is down sampled by $D=2$. Can we replace this process with a single LTI system $H_2(z)$? if the answer is positive, determine the system function of this system.

output of upsampler is $X(z^2)$

$$Y_1(z) = \frac{1}{2}\left[X(z) H_1(z^{1/2}) + X(z) H_1(z^{1/2} W^{1/2})\right]$$

$$= \frac{1}{2}\left[H_1(z^{1/2}) + H_1(z^{1/2} W^{1/2})\right] X(z)$$

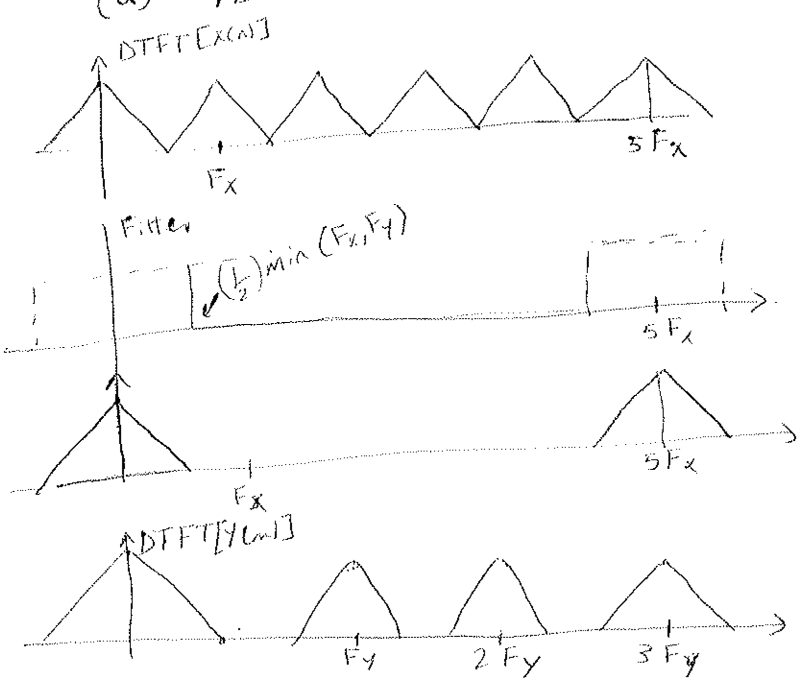
$$= H_2(z) \; X(z)$$

11.13 Plot the signals and their corresponding spectra for rational sampling rate conversion by (a) $I/D = 5/3$

(b) $I/D = 3/5$.

Assume that the spectrum of the input signal $x(n)$ occupies the entire range $-\pi \le \omega_x \le \pi$

(a) $I/D = 5/3$



$\frac{F_x}{I} \to 5$

(b) $I/D = 3/5$