Consider the following / Signal : periodic

$$x(n) = \{ \ldots, 1, 0, 1, 2, 3, 2, 1, 0, 1, \ldots \}$$

(a) sketch the signal $x(n)$ and its magnitude and phase spectra

$$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \, e^{-j2\pi kn/N} = \frac{1}{6} \sum_{n=0}^{5} x(n) \, e^{-j2\pi kn/6}$$

$$= \frac{1}{6} \left[3 + 2 e^{-j2\pi k/6}, + e^{-j2\pi k 2/6}, + 0 + e^{-j2\pi k 4/6} + 2 e^{-j2\pi k \frac{5}{6}} \right]$$

$$= \frac{1}{6} \left[3 + 2 \left(e^{-j\frac{\pi k}{3}} + e^{-j\frac{\pi k 5}{3}} \right) + \left(e^{-j\frac{2\pi k}{3}} + e^{-j\frac{2\pi k}{3}} \right) \right]$$

$$= \frac{1}{6} \left[3 + 4 \cos\left(\frac{\pi k}{3}\right) + 2 \cos\left(\frac{2\pi k}{3}\right) \right]$$

$$C_0 = \frac{1}{6} [3 + 4 + 2] = \frac{9}{6}$$

$$C_1 = \frac{1}{6} \left[3 + 4 \cdot \frac{1}{2} + 2 \cdot \left(\frac{-1}{2}\right) \right] = \frac{4}{6}$$
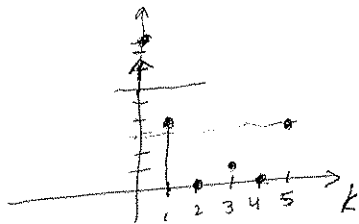
$$C_2 = \frac{1}{6} \left[4 + 4 \left(\frac{-1}{2}\right) + 2 \cdot \left(\frac{-1}{2}\right) \right] = 0$$

$$C_3 = \frac{1}{6}$$

$$C_4 = C_2 = 0$$

$$C_5 = C_1 = \frac{4}{6}$$

phase = 0

(b) using the results in part a, verify Parseval's relation by computing the power in the time and frequency domains.

$$P_{time} = \frac{1}{6} \sum_{n=0}^{5} |x(n)|^2$$

$$= \frac{1}{6}(3^2 + 2^2 + 1^2 + 0^2 + 1^2 + 2^2) = \frac{19}{6}$$

$$P_{freq} = \sum_{k=0}^{5} |C_k|^2 = \left[\left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + 0^2 + \left(\frac{1}{6}\right)^2 + 0^2 + \left(\frac{4}{6}\right)^2\right]$$

$$= \frac{19}{6}$$

$$P_{time} = P_{freq}. \quad \square$$

4.5 Consider the signal

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

(a) Determine and sketch its power density spectrum.

$$\underline{N=8}$$

$$C_k = \frac{1}{8}\sum_{n=0}^{7} x(n)\, e^{-j2\pi kn/8}$$

$$x(n) = \{5.5, 3.0607, 1, 0.9393, 0.5, 0.9393, 1, 3.0607\}$$

or

$$= \left\{\frac{11}{2}, 2+\frac{3}{4}\sqrt{2}, 1, 2-\frac{3}{4}\sqrt{2}, \frac{1}{2}, 2-\frac{3}{4}\sqrt{2}, 1, 2+\frac{3}{4}\sqrt{2}\right\}.$$

$$\Rightarrow \quad C_0 = 2, \quad C_1 = 1 = C_7, \quad C_2 = 0.5, \quad C_3 = \frac{1}{4} = C_5, \quad C_4 = 0$$

(b) power spectral density $\Rightarrow |C_k|^2$

4.6.

4.5 (6) Evaluate the power of the signal.

$$P = \sum_{i=0}^{7} |c(i)|^2 = 4 + 1 + 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} = \frac{53}{8}$$

4.5 (6) Evaluate the power of the signal.

$$P = \sum_{i=0}^{7} |c(i)|^2 = 4 + 1 + 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} = \frac{53}{8}$$

$\boxed{4.6}$ Determine and sketch the magnitude and phase spectra of the following periodic signals.

(b) $x(n) = \cos \frac{2\pi}{3} n + \sin \frac{2\pi}{5} n$

$\Rightarrow N = 15$

⊛ $\cos \frac{2\pi}{3} n = \frac{1}{2} \left[e^{j\frac{2\pi}{3} n} + e^{-j\frac{2\pi}{3} n} \right]$

$$C_{k_1} = \begin{cases} \frac{1}{2} & , k = 5, 10 \\ 0 & , o/w \end{cases}$$

⊛ $\sin \frac{2\pi}{5} n = \frac{1}{2j} \left[e^{j\frac{2\pi}{5} n} - e^{-j\frac{2\pi}{5} n} \right]$

$$C_{k_2} = \begin{cases} \frac{1}{2j} & , k = 3 \\ \frac{-1}{2j} & , k = 12 \\ 0 & , o/w \end{cases}$$

$\Rightarrow C_{k_{(overall)}} = C_{k_1} + C_{k_2} = \begin{cases} \frac{1}{2j} & , k = 3 \\ \frac{1}{2} & , k = 5 \\ \frac{1}{2} & , k = 10 \\ \frac{-1}{2j} & , k = 12 \\ 0 & , o/w \end{cases}$

phase

$\begin{cases} \frac{3\pi}{2} & , k = 3 \\ 0 & , k = 5 \\ 0 & , k = 10 \\ \pi/2 & , k = 12 \\ 0 & , o/w \end{cases}$

④

(d) $x(n) = \{ \ldots, -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2 \} - - \}$

$\Rightarrow \boxed{N=5}$

$$C_k = \frac{1}{5} \sum_{n=0}^{4} x(n) \, e^{-j\frac{2\pi n k}{5}}$$

$$C_k = \frac{1}{5}\left[1 \cdot e^{-j\frac{2\pi k}{5}} + 2 \cdot e^{-j\frac{2\pi \cdot 2 \cdot k}{5}} - 2\, e^{-j\frac{2\pi}{5}\cdot 3 \cdot k} - e^{-j\frac{2\pi}{5}\cdot 4 \cdot k} \right]$$

$$= \frac{1}{5}\left[\left(e^{-j\frac{2\pi k}{5}} - e^{-j\frac{8\pi}{5}k + 2N} \right) + 2\left(e^{-j\frac{4\pi}{5}k} - e^{-j\frac{6\pi k + 10}{5}} \right) \right]$$

$$= \frac{-2j}{5}\left[\sin\left(\frac{2\pi k}{5}\right) + 2 \cdot \sin\left(\frac{4\pi k}{5}\right) \right]$$

$$\underline{phase}$$
$$0$$
$$\pi + \frac{\pi}{2} + \angle[\quad]$$

$C_0 = 0,$

$C_1 = \frac{-2j}{5}\left[+\sin\left(\frac{2\pi}{5}\right) + 2\sin\left(\frac{4\pi}{5}\right)\right]$

$C_2 = \frac{-2j}{5}\left[\sin\left(\frac{4\pi}{5}\right) + 2\sin\left(\frac{8\pi}{5}\right)\right]$

$C_3 = -C_2$

$C_4 = -C_1$

(f) $x(n) = \{ \ldots, 0, 0, \underset{\uparrow}{1}, 1, 0, 0, 0, 1, 1, 0, 0, \ldots \}$

$N = 5$

$$C_k = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{-j\frac{2\pi k n}{5}} = \frac{1}{5}\left[1 + e^{-j\frac{2\pi k}{5}}\right] \times \frac{e^{j\frac{\pi k}{5}}}{e^{j\frac{\pi k}{5}}}$$

$$= \frac{2}{5}\cos\left(\frac{\pi k}{5}\right)e^{-j\frac{\pi k}{5}}$$

$\rightarrow C_0 = \frac{2}{5}, \quad C_1 = \frac{2}{5}\cos\left(\frac{\pi}{5}\right)e^{-j\frac{\pi}{5}}$

$C_2 = \frac{2}{5}\cos\left(\frac{2\pi}{5}\right)e^{-j\frac{2\pi}{5}}, \quad C_3 = \frac{2}{5}\cos\left(\frac{3\pi}{5}\right)e^{-j\frac{3\pi}{5}}, \quad C_4 = \frac{2}{5}\cos\left(\frac{4\pi}{5}\right)e^{-j\frac{4\pi}{5}}$

⑤

Determine the periodic signal $x(n)$, with fundamental period $N=8$, if the Fourier coefficients are given by:

(a) $c_k = \cos \dfrac{\pi k}{4} + \sin \dfrac{3\pi k}{4}$ $\Rightarrow$ $c_k = \dfrac{1}{2}\left[e^{j\frac{2\pi k}{8}} + e^{-j\frac{2\pi k}{8}} \right] + \dfrac{1}{2j}\left[e^{j\frac{6\pi k}{8}} - e^{-j\frac{6\pi k}{8}} \right]$

$$x(n) = \sum_{k=0}^{7} c_k \, e^{j\frac{2\pi nk}{8}}$$

if $c_k = e^{j\frac{2\pi nk}{8}}$ $\Rightarrow$ $\displaystyle\sum_{k=0}^{7} e^{j\frac{2\pi pk}{8}} \, e^{j\frac{2\pi nk}{8}} = \begin{cases} 8 & , \ n = -p \\ 0 & , \ n \neq -p \end{cases}$

$$\Rightarrow x(n) = 4\,\delta(n+1) + 4\,\delta(n-1) + 4j\,\delta(n+3) + 4j\,\delta(n-5)$$

Two DT signals, $s_k(n)$ and $s_\ell(n)$, are said to be orthogonal over an interval $[N_1, N_2]$ if

$$\sum_{n=N_1}^{N_2} s_k(n)\, s_\ell^{*}(n) = \begin{cases} A_k & , \ k = \ell \\ 0 & , \ k \neq \ell \end{cases}$$

if $A_k = 1$, the signals are orthonormal.

(a) Prove the relation

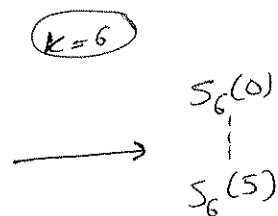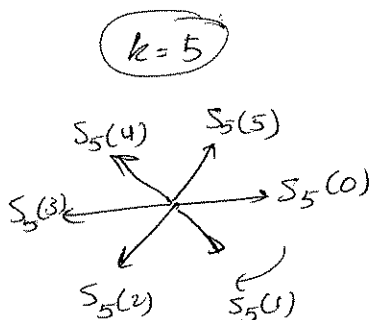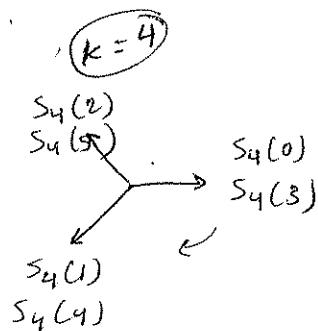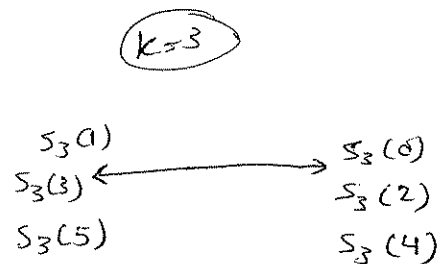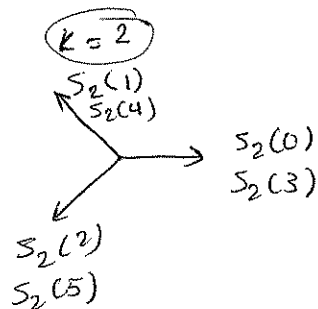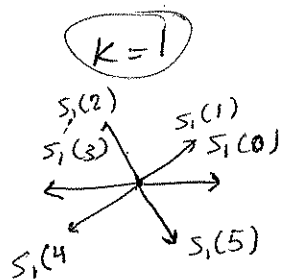$$\sum_{n=0}^{N-1} e^{j\frac{2\pi kn}{N}} = \begin{cases} N & , \ k = 0, \mp N, \mp 2N, \cdots \\ 0 & , \ o/w \end{cases}$$

if $k = 0, \mp N, \mp 2N, \cdots$

$$\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \sum_{n=0}^{N-1} \underset{1}{\cos(2\pi \tfrac{kn}{N})} - \underset{0}{\sin\left(2\pi \tfrac{kn}{N}\right)} = N$$

for $k \neq 0, \mp N, \mp 2N, \cdots$ $\Rightarrow \displaystyle\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \dfrac{1 - e^{j2\pi k}}{1 - e^{j2\pi k/N}} = 0$

(b) Illustrate the validity of the relation in part (a) by plotting for every value of $k=1,2,\cdots,6$, the signals $S_k = e^{j(\frac{2\pi}{6})kn}$

$n = 0,1,\cdots,5$. [Note: for a given k, n the signal $S_k(n)$ can be represented as a vector in the complex plane.]

(c) Show that the harmonically related signals

$$S_k(n) = e^{j(2\pi/N)kn}$$

are orthogonal over any interval of length N.

$$\sum_{n=0}^{N-1} S_k(n)\, S_i^*(n) = \sum_{n=0}^{N-1} e^{j2\pi kn/N}\, e^{-j2\pi in/N}$$

$$= \sum_{n=0}^{N-1} e^{j\frac{2\pi(k-i)n}{N}} = \begin{cases} N, & k=i \\ 0, & k \neq i \end{cases}$$

4.10 Determine the signals having the following Fourier Transform

(b) $\underline{X}(\omega) = \cos^2\omega = \left(\frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega}\right)^2$

$$= \frac{1}{4}\left(e^{j2\omega} + 2 + e^{-j2\omega}\right)$$

$$X(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \underline{X}(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{8\pi}\left[2\pi\delta(n+2) + 4\pi\delta(n) + 2\pi\delta(n-2)\right]$$

$$= \frac{1}{4}\left[\delta(n+2) + 2\delta(n) + \delta(n-2)\right]$$

(c) $\underline{X}(\omega) = \begin{cases} 1, & \omega_0 - \delta\omega/2 \leq |\omega| \leq \omega_0 + \delta\omega/2 \\ 0, & \text{elsewhere} \end{cases}$
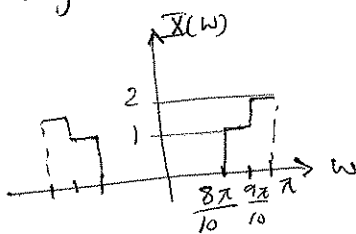
$$X(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\underline{X}(\omega)e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{\omega_0-\frac{\delta\omega}{2}}^{\omega_0+\frac{\delta\omega}{2}} e^{j\omega n}\,d\omega$$

$$= \frac{2}{\pi}\,\delta\omega\left(\frac{\sin(n\delta\omega/2)}{n\delta\omega/2}\right)e^{jn\omega_0}$$

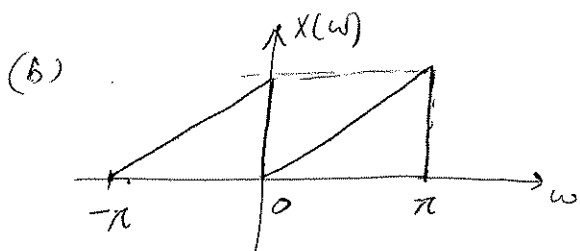**4.12** Determine the signal $X(n)$ if its Fourier transform is as given in Fig.

(a)

$$X(n) = \frac{1}{2\pi}\left[\int_{8\pi/10}^{9\pi/10}e^{j\omega n}\,d\omega + \int_{-9\pi/10}^{-8\pi/10}e^{j\omega n}\,d\omega + 2\int_{9\pi/10}^{\pi}e^{j\omega n}\,d\omega + 2\int_{-\pi}^{-9\pi/10}e^{j\omega n}\,d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{jn}\left(e^{j\frac{9\pi n}{10}} - e^{-j\frac{9\pi n}{10}} - e^{j\frac{8\pi n}{10}} + e^{-j\frac{8\pi n}{10}}\right)\right.$$

$$\left. + \frac{2}{jn}\left(-e^{j\frac{9\pi n}{10}} + e^{-j\frac{9\pi n}{10}} + e^{j\pi n} - e^{-j\pi n}\right)\right]$$
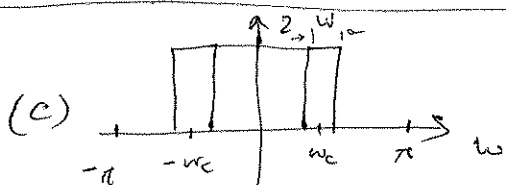
$$= \frac{1}{n\pi}\left[\sin\pi n - \sin\frac{8\pi n}{10} - \sin\frac{9\pi n}{10}\right]$$

$$=$$

(b)

$$X(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{0} X(\omega)e^{j\omega n}d\omega + \int_{0}^{\pi} X(\omega)e^{j\omega n}d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right)e^{j\omega n}d\omega + \int_{0}^{\pi}\frac{\omega}{\pi}e^{j\omega n}d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{\omega}{jn\pi}e^{j\omega n}\Big|_{-\pi}^{\pi} + \frac{e^{j\omega n}}{jn}\Big|_{-\pi}^{0}\right]$$

$$= \frac{1}{\pi n}\sin\frac{\pi n}{2}\,e^{-jn\pi/2}$$

(c)

$$X(n) = \frac{1}{2\pi}\int_{\omega_c-\frac{\omega}{2}}^{\omega_c+\frac{\omega}{2}}2e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{-\omega_c-\frac{\omega}{2}}^{-\omega_c+\frac{\omega}{2}}2e^{j\omega n}d\omega$$

$$= \frac{1}{\pi}\left[\frac{1}{jn\pi}e^{j\omega n}\Big|_{\omega_c-\frac{\omega}{2}}^{\omega_c+\frac{\omega}{2}} + \frac{e^{j\omega n}}{jn}\Big|_{-\omega_c-\frac{\omega}{2}}^{-\omega_c+\frac{\omega}{2}}\right]$$

$$X(n) = \frac{2}{\pi n}\left[\frac{e^{j(\omega_c+\frac{\omega}{2})n} - e^{j(\omega_c-\frac{\omega}{2})n} + e^{-j(\omega_c-\frac{\omega}{2})n} - e^{-j(\omega_c+\frac{\omega}{2})n}}{2j}\right]$$

$$= \frac{2}{\pi n}\left[\sin\left(\omega_c+\frac{\omega}{2}\right)n - \sin\left(\omega_c-\frac{\omega}{2}\right)n\right]$$

4.14 Consider the signal

$$X(n) = \{-1, 2, \overset{\uparrow}{-3}, 2, -1\}$$

with Fourier transform $X(\omega)$, compute the following quantities, without explicitly computing $X(\omega)$.

a) $X(0) = \sum_{n} x(n)e^{j\omega n}\Big|^{0} = -1$

b) $\angle X(\omega) = \tan^{-1}\frac{X_I(\omega)}{X_R(\omega)} = \tan^{-1}0$ (real, even)

$$= \pi \quad \text{for all } \omega.$$

hint $\left(\tan^{-1}\frac{0}{-1}\right) \rightarrow \pi$

⑨

(c) $\int_{-\pi}^{\pi} X(\omega) d\omega$

$x(0) = \dfrac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) d\omega$, $\int_{-\pi}^{\pi} X(\omega) d\omega = 2\pi x(0) = -6$

(d) $X(\pi) = \sum\limits_{n=-\infty}^{\infty} x(n) \left(\boxed{e^{-j n \pi}}\right) = \sum\limits_{n} (-1)^n x(n) = \cancel{-4} \; -3 - 2 - 2 - 1 - 1 = -9$

(e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega = 2\pi \sum\limits_{n} |x(n)|^2 = 2\pi(9 + 8 + 2) = 38\pi$

$\boxed{4.17}$ Let $x(n)$ be an arbitrary signal, not necessarily real valued
with Fourier transform $X(\omega)$. Express the Fourier transforms of the
following signals in terms of $X(\omega)$.

a) $x^*(n) \Rightarrow \sum\limits_{n} x^*(+n) e^{-j\omega n} = \left(\sum\limits_{n} x(n) e^{-j(-\omega)n} \right)^* = X^*(-\omega)$

b) $x^*(-n) \Rightarrow \sum\limits_{n} x^*(-n) e^{-j\omega n} = \sum\limits_{n=-\infty}^{\infty} x(n)^* e^{+j\omega n} = X^*(\omega)$

c) $y(n) = x(n) - x(n-1)$.

$\sum\limits_{n} y(n) e^{-j\omega n} = \sum\limits_{n} x(n) e^{-j\omega n} - \sum\limits_{n} x(n-1) e^{-j\omega n}$

$\quad Y(\omega) = X(\omega) + X(\omega) e^{-j\omega}$

$\qquad = (1 - e^{-j\omega}) X(\omega)$.

$\overset{O}{\cancel{\;\nearrow\;}}$ (d) $y(n) = \sum\limits_{k=-\infty}^{n} x(k) -$

$\qquad y(n) = \cancel{y(n) - y(n-1)} \underset{\underbrace{\;\;}_{y(n-1)}}{\overset{n-1}{\sum\limits_{-\infty}}} x(k) + x(n)$.

$\qquad x(n) = y(n) - y(n-1)$
$\qquad X(\omega) = Y(\omega) - Y(\omega) e^{-j\omega}$
$\qquad X(\omega) = Y(\omega)(1 - e^{-j\omega})$
$\qquad \Rightarrow Y(\omega) = \dfrac{X(\omega)}{1 - e^{-j\omega}}$

(e) $y(n) = x(2n)$

$$Y(\omega) = \sum_n x(2n) e^{-j\omega n}$$

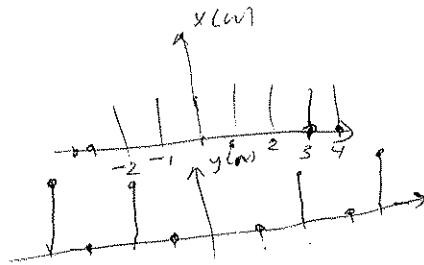$$= \sum_n x(n) e^{-j\frac{\omega}{2} n}$$

$$= X\left(\frac{\omega}{2}\right)$$

(f) $y(n) = \begin{cases} x(n/2) & , n \text{ even} \\ 0 & , n \text{ odd} \end{cases}$

$$Y(\omega) = \sum_n x\left(\frac{n}{2}\right) e^{-j\omega n}$$

$$= \sum_n x(n) e^{-j(2\omega)n}$$
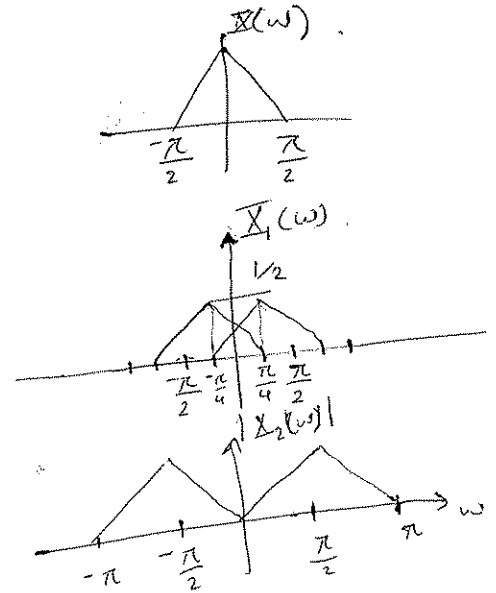
$$= X(2\omega)$$

~~down sampling~~.

4.19 let $x(n)$ be a signal with Fourier transform as in Fig. P4.19. Determine and sketch the Fourier transformation of the following signals.

(a) $x_1(n) = x(n) \cos(\pi n /4)$

$$x_1(n) = x(n) \cdot \frac{1}{2}\left(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}\right)$$

$$X_1(\omega) = \frac{1}{2}\left[X\left(\omega - \frac{\pi}{4}\right) + X\left(\omega + \frac{\pi}{4}\right)\right]$$

(b) $x_2(n) = x(n) \sin\left(\frac{\pi n}{2}\right)$

$$X_2(\omega) = \frac{1}{2j}\left[X\left(\omega - \frac{\pi}{2}\right) + X\left(\omega + \frac{\pi}{2}\right)\right]$$

(c) $x_3(n) = x(n) \cos\left(\frac{\pi n}{2}\right)$

$$X_3(\omega) = \frac{1}{2}\left[X\left(\omega - \frac{\pi}{2}\right) + X\left(\omega + \frac{\pi}{2}\right)\right]$$

(d) $x_4(n) = x(n) \cos \pi n$

$$X_4(\omega) = \frac{1}{2}\left[X(\omega - \pi) + X(\omega + \pi)\right]$$

$$= X(\omega - \pi)$$

⑪

4.18

a) $X_1(\omega) = \sum_n x_1(n) e^{-j\omega n}$

$= e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$

$= 1 + 2\cos\omega + 2\cos 2\omega$

b) $X_2(\omega) = \sum_n x_2(n) e^{-j\omega n}$

$= e^{j4\omega} + e^{j2\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$

$= 1 + 2\cos 2\omega + 2\cos 4\omega$

c) $X_3(\omega) = \sum_n x_3(n) e^{-j\omega n}$

$= e^{j6\omega} + e^{j3\omega} + 1 + e^{-j3\omega} + e^{-j6\omega}$

$= 1 + 2\cos 3\omega + 2\cos 6\omega$

d) $X_2(\omega) = X_1(\omega)$, and $X_3(\omega) = X_1(3\omega)$

e) if
$$x_k(n) \begin{cases} x(n/k) & n/k \text{ an integer} \\ 0 & ow \end{cases}$$

Then, $X_k(\omega) = \sum_{n, \frac{n}{k}} x_k(n) e^{-j\omega n}$

$= \sum_n x(n) e^{-jk\omega n}$

$= X(k\omega)$

$\boxed{4.22}$ A signal $x(n)$ has the following Fourier transform

$$X(w) = \frac{1}{1-ae^{-jw}}$$

Determine the Fourier transforms of the following signals.

(a) $x(2n+1)$

$$X_1(w) = \sum_n x(2n+1)e^{-jwn}$$

$$= \sum_k x(k)e^{-j\frac{w k}{2}}e^{j\frac{w}{2}}$$

$$= \overline{X}\left(\frac{w}{2}\right)e^{jw/2}$$

$$= \frac{e^{j\frac{w}{2}}}{1-ae^{j\frac{w}{2}}}$$

(b) $e^{\pi n/2}\, x(n+2)$

$$\overline{X}_2(w) = \sum_n x(n+2)e^{\pi n/2}e^{-jwn} =$$

$$= -\sum_k x(k)e^{-jk(w+j\pi/2)}e^{j2w}$$

$$= -\overline{X}\left(w+\frac{j\pi}{2}\right)e^{j2w}$$

(c) $x(-2n)$

$K = -2n$
$n = -\frac{k}{2}$

$$\overline{X}_3(w) = \sum_n x(-2n)e^{-jwn} = +\sum_{,k} x(k)e^{-jk(w/2)}$$

$$= \overline{X}\left(-\frac{w}{2}\right)$$

(d) $x(n)\cos(0.3\,\pi n)$

$$\overline{X}_4(w) = \sum_n \frac{1}{2}\left(e^{j0.3\pi n} + e^{-j0.3\pi n}\right)x(n)e^{-jwn}$$

$$= \frac{1}{2}\sum_n x(n)\left[e^{-j(w-0.3\pi)n} + e^{-j(w+0.3\pi)n}\right]$$

$$= \frac{1}{2}\left[\overline{X}(w-0.3\pi) + \overline{X}(w+0.3\pi)\right]$$

⑫

(e) $x(n) \circledast x(n-1)$

$$X_5(\omega) = X(\omega)\left[X(\omega)e^{-j\omega}\right] = X^2(\omega)e^{-j\omega}$$

(f) $x(n) \circledast x(-n)$

$$X_6(\omega) = X(\omega)\, X(-\omega)$$

$$= \frac{1}{(1-ae^{-j\omega})(1-ae^{j\omega})}$$

$$= \frac{1}{(1-2a\cos\omega+a^2)}$$