CHAPTER 5, problem set { 5.1, 5.3, 5.4 (a,b,c,d,n), 5.9, 5.11, 5.23, 5.31, 5.65 }

problem $\boxed{5.1}$ The following input-output pairs have been observed during the operation of various systems; Determine their frequency response (LTI)

(a) $x(n) = \left(\frac{1}{2}\right)^n \xrightarrow{T_1} y(n) = \left(\frac{1}{8}\right)^n$

range of $n \in (-\infty, \infty) \Rightarrow$ fourier transform of $x(n)$ & $y(n)$ does not exist.

also $y(n) = x^3(n) \Rightarrow$ system H is non-linear

(b) $x(n) = \left(\frac{1}{2}\right)^n u(n) \xrightarrow{T_2} y(n) = \left(\frac{1}{8}\right)^n u(n)$.

$$X(\omega) = \frac{1}{1 - \frac{1}{2} e^{-j\omega}}$$

$$Y(\omega) = \frac{1}{1 - \frac{1}{8} e^{-j\omega}}$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{\frac{1}{1 - \frac{1}{8} e^{-j\omega}}}{\frac{1}{1 - \frac{1}{2} e^{-j\omega}}} = \frac{1 - \frac{1}{2} e^{-j\omega}}{1 - \frac{1}{8} e^{-j\omega}}$$

(c) $x(n) = e^{j\pi/5}$ ~~x(n)~~ $\xrightarrow{T_3} y(n) = 3 e^{+j\pi/5}$

$H(\omega) = 3$ for all ω,

(d) $x(n) = e^{j\pi/5} u(n) \xrightarrow{T_4} y(n) = 3 e^{j\pi/5} u(n)$.

~~$H(\omega) = 3$ for all ω~~. $H(\omega)$ @ $\omega = \frac{\pi}{5} = 3$ i.e $H(\frac{\pi}{5}) = 3$

(e) $x(n) = x(n + N_1) \xrightarrow{T_5} y(n) = y(n + N_2)$, $N_1 \neq N_2$, N_1, N_2 pri

for LTI systems: period of the input signal should be the same as the period of the output signal $\Rightarrow N_1 = N_2$ but since, $N_1 \neq N_2$, the system is non-linear.

$\boxed{\text{problem 5.3}}$ Consider an LTI system with impulse responese

$h(n) = \left(\frac{1}{2}\right)^n u(n)$

(b) Determine and sketch the magnitude and phase spectra for the input and output signals for the following inputs

(a) Determine and sketch the magnitude and phase response $|H(\omega)|$ and $\angle H(\omega)$, respectively.

$$H(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$H(\omega) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^n \quad , \sum_{n=0}^{\infty} a^n, \; |a|<1$$

$$= \frac{1}{1-\frac{1}{2}e^{-j\omega}} = \frac{1}{1-\frac{1}{2}\cos\omega - \frac{1}{2}j\sin\omega}.$$

$$|H(\omega)| = \frac{1}{\sqrt{(1-\frac{1}{2}\cos\omega)^2 + (\frac{1}{2}\sin\omega)^2}}$$

$$= \frac{1}{(1+\frac{1}{4}\cos^2\omega - 2\cdot\frac{1}{2}\cos\omega + \frac{1}{4}\sin^2\omega)^{\frac{1}{2}}}$$

$$= \frac{1}{(1+\frac{1}{4}(\cos^2\omega+\sin^2\omega) - \cos\omega)^{\frac{1}{2}}}$$

$$= \frac{1}{(5/4 - \cos\omega)^{\frac{1}{2}}}$$

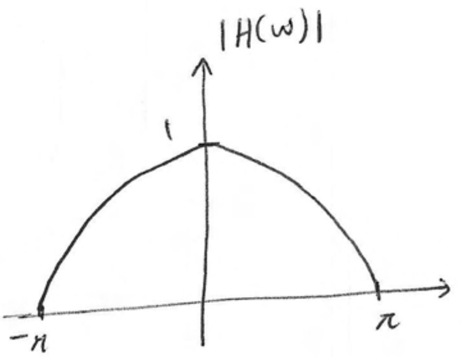
problem 5.4 Determine and sketch the magnitude and phase response of the following systems.

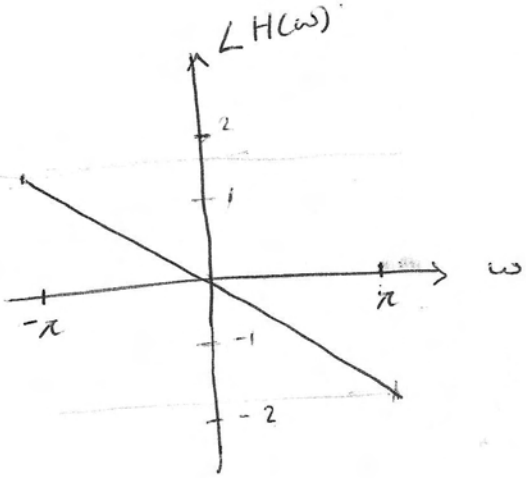
(a) $y(n) = \frac{1}{2}\left[x(n) + x(n-1)\right]$

$$Y(\omega) = \frac{1}{2}\left[1 + e^{-j\omega}\right] X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{2}(1 + e^{-j\omega}) = \frac{1}{2} \cdot e^{-j\omega/2}\left(e^{j\omega/2} + e^{-j\omega/2}\right)$$

$$= \frac{1}{2} \cdot 2 \cdot \cos\left(\frac{\omega}{2}\right) e^{-j\omega/2} = \cos\left(\frac{\omega}{2}\right) \cdot e^{-j\omega/2}$$



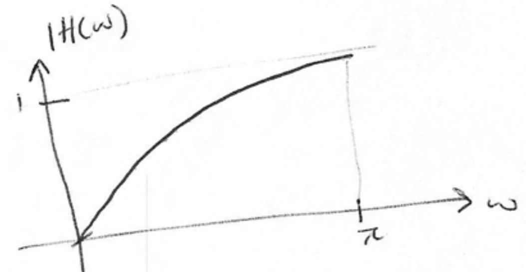


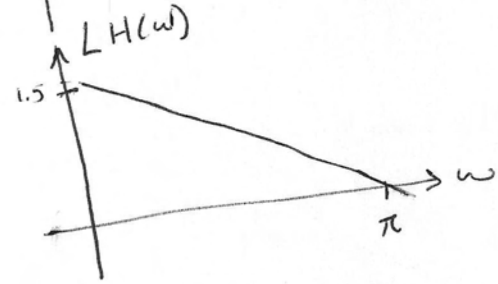
(b) $y(n) = \frac{1}{2}\left[x(n) - x(n-1)\right]$

$$Y(\omega) = \frac{1}{2}(1 - e^{-j\omega}) X(\omega)$$

$$H(\omega) = \frac{1}{2}(1 - e^{-j\omega})$$

$$= \sin\left(\frac{\omega}{2}\right) e^{-j\omega/2} e^{j\pi/2}$$



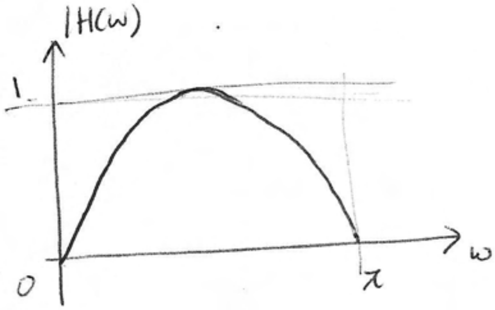


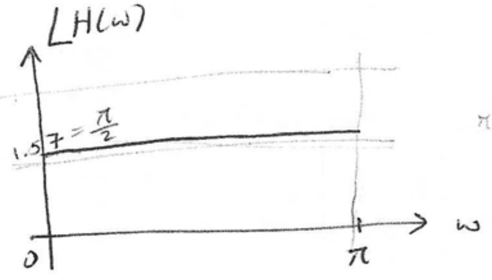
(c) $y(n) = \frac{1}{2}[x(n+1) - x(n-1)]$

$$Y(w) = \frac{1}{2}\left[e^{jw} - e^{-jw}\right] X(w)$$

$$H(w) = \frac{1}{2}\left(e^{jw} - e^{-jw}\right) = \sin(w)\, e^{j\pi/2}$$

$= j$

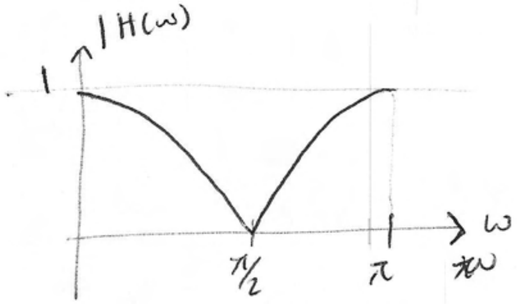


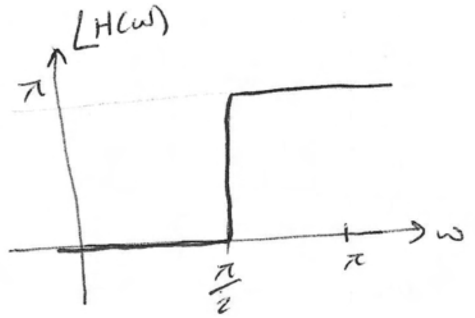


(d) $y(n) = \frac{1}{2}[x(n+1) + x(n-1)]$

$$Y(w) = \frac{1}{2}\left[e^{jw} + e^{-jw}\right] X(w)$$

$$H(w) = \frac{1}{2}\left(e^{jw} + e^{-jw}\right) = \cos(w)$$



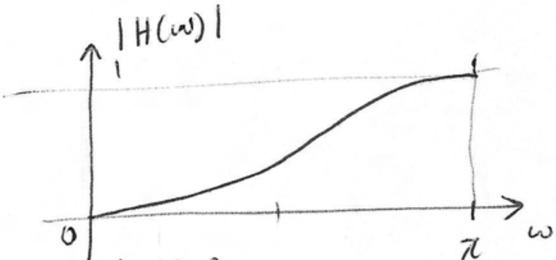


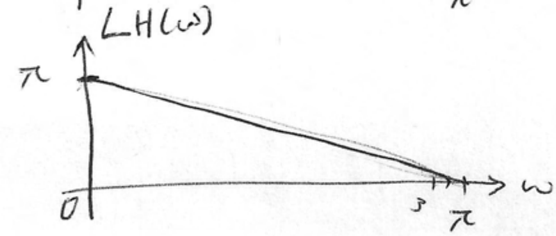
(n) $y(n) = \frac{1}{4}[x(n) - 2x(n-1) + x(n-2)]$

$$Y(w) = \frac{1}{4}\left[1 - 2e^{-jw} + e^{-j2w}\right] X(w)$$

$$H(w) = \frac{1}{4}\left(1 - e^{jw}\right)^2$$

$$= \sin^2\left(\frac{w}{2}\right) e^{-j(w-\pi)}$$





problem 5.9 Determine the frequency content ~~response~~ of the outputs of the following systems to the input signal.

$$x(n) = A\cos\frac{\pi}{4}n$$

(a) $y(n) = x(2n) = A\cos\left(\frac{\pi}{2}n\right) \Rightarrow \omega = \frac{\pi}{2}$

(b) $y(n) = x^2(n) = A^2\cos^2\left(\frac{\pi}{4}n\right) = A^2\left(\frac{\cos\left(2\cdot\frac{\pi}{4}n\right)+1}{2}\right)$

$= \frac{1}{2}A^2 + \frac{1}{2}A^2\cos\left(\frac{\pi}{2}n\right) \Rightarrow \quad \omega = 0,\ \omega = \frac{\pi}{2}$

(c) $y(n) = (\cos\pi n)\,x(n)$.

$= A.\cos\left(\frac{\pi}{4}n\right).\cos(\pi n) = \frac{A}{2}.\cos\left(\frac{5\pi}{4}n\right) + \frac{A}{2}\cos\left(\frac{3\pi}{4}n\right)$

$\Rightarrow \omega = \frac{5\pi}{4},\ \frac{3\pi}{4}$

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problem 5.11 Determine the magnitude and phase response of the multipath channel $y(n) = x(n) + x(n-M)$

at what frequencies does $H(\omega) = 0$

$y(n) = x(n) + x(n-M)$

$Y(\omega) = \left(1 + e^{-j\omega M}\right) X(\omega)$

$H(\omega) = 1 + e^{-j\omega M}$

$H(\omega) = 0$ @ $\omega = \frac{(2k+1)\pi}{M}$, $k = 0, 1, \ldots$

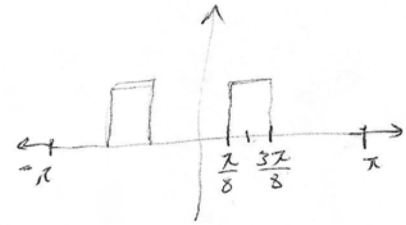
odd.

$|H(\omega)| = \left|2\cos\frac{\omega M}{2}\right| \Rightarrow H(\omega) = 0$ @ $\frac{\omega M}{2} = \frac{(2k+1)\pi}{2}$.

$\omega = \frac{(2k+1)\pi}{M}$, $k = 0, 1, \ldots$

problem 5.23 The frequency response of an ideal bandpass filter is given by

$$H(\omega) = \begin{cases} 0, & |\omega| \leq \frac{\pi}{8} \\ 1, & \frac{\pi}{8} < |\omega| < \frac{3\pi}{8} \\ 0, & \frac{3\pi}{8} \leq |\omega| \leq \pi \end{cases}$$



a) determine the impulse response.

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\frac{3\pi}{8}}^{\frac{3\pi}{8}} e^{j\omega n}\, d\omega - \int_{-\frac{\pi}{8}}^{\pi/8} e^{-j\omega n}\, d\omega \right]$$

$$= \frac{1}{\pi n} \left[\sin \frac{3\pi}{8} n - \sin \frac{\pi}{8} n \right]$$, using $\cos(u)\sin(v) = \frac{1}{2}[\sin(u+v) - \sin(u-$

$$= \frac{2}{\pi n} \sin\left(\frac{\pi}{8} n\right) \cos\left(\frac{\pi}{4} n\right)$$

(b) Show that the impulse response can be expressed as the product of $\cos\left(\frac{n\pi}{4}\right)$ and the impulse response of a lowpass filter.

if $h_1(n) = \dfrac{2 \sin\left(\frac{\pi}{8} n\right)}{n\pi}$

then, $H_1(\omega) = \begin{cases} 2, & |\omega| \leq \frac{\pi}{8} \\ 0, & \frac{\pi}{8} < |\omega| < \pi \end{cases}$

$$h(n) = h_1(n) \cos \frac{\pi}{4} n$$