http://www.comm.utoronto.ca/~dkundur/course/discrete-time-systems/

7.1

$x(n)$ is a real valued sequence. The first five points of its 8-point DFT are:

{0.25, 0.125 - j·0.3018, 0, 0.125 - j·0.0518, 0}

To compute the 3 remaining points, we can use the following property for real valued sequences:

$$X(N-k) = X^*(k) = X(-k) \text{ (page 468 in the book)}$$

In our case N=8 and therefore we have the equations for $X(5)$, $X(6)$ and $X(7)$:

$$\begin{aligned} X(5) &= X(8-3) = X^*(3) \\ &= (0.125 - j\cdot 0.0518)^* \\ &= 0.125 + j\cdot 0.0518 \end{aligned}$$

$$\begin{aligned} X(6) &= X(8-2) = X^*(2) \\ &= (0)^* \\ &= 0 \end{aligned}$$

$$\begin{aligned} X(7) &= X(8-1) = X^*(1) \\ &= (0.125 - j\cdot 0.3018)^* \\ &= 0.125 + j\cdot 0.3018 \end{aligned}$$

Hence the complete 8-point DFT of $x(n)$ is:

{0.25, 0.125 - j·0.3018, 0, 0.125 - j·0.0518, 0, 0.125 + j·0.0518, 0, 0.125 + j·0.3018}.

7.3

$X(k), 0 \le k \le N-1,$ is the N-point DFT of $x(n)$, $0 \le n \le N-1$. We define the DFT $\tilde{X}(k)$ as:

$$\tilde{X}(k) = \begin{cases} X(k), & 0 \le k \le k_c, \quad N-k_c \le k \le N-1, \\ 0, & k_c \le k \le N-k_c \end{cases}$$

From this definition, we can represent $\tilde{X}(k)$ as the product of $X(k)$ with the ideal lowpass filter $H(k)$ where:

$$H(k) = \begin{cases} 1, & 0 \le k \le k_c, \quad N-k_c \le k \le N-1, \\ 0, & k_c \le k \le N-k_c \end{cases}$$

Hence this leads to the conclusion that $\hat{x}(n)$, the inverse N-point DFT of $\tilde{X}(k)$, is a lowpass version of $x(n)$.

7.7

$X(k)$ is the N-point DFT of the sequence $x(n)$. We want to determine the N-point DFTs of the two sequences derived from $x(n)$:

$$x_c(n) = x(n)\cdot\cos\left(\frac{2\pi k_0 n}{N}\right),\quad 0 \le n \le N-1$$

$$x_s(n) = x(n)\cdot\sin\left(\frac{2\pi k_0 n}{N}\right),\quad 0 \le n \le N-1$$

The DFT of $x_c(n)$, $X_c(k)$, is given by:

$$X_c(k) = \sum_{n=0}^{N-1} x(n)\cdot\cos\left(\frac{2\pi k_0 n}{N}\right)\cdot\exp\left(-j\frac{2\pi kn}{N}\right)$$

Developing the cosine in the previous equality we get:

$$X_c(k) = \sum_{n=0}^{N-1} x(n)\cdot\left[\frac{1}{2}\left(\exp\left(j\frac{2\pi k_0 n}{N}\right)+\exp\left(-j\frac{2\pi k_0 n}{N}\right)\right)\right]\cdot\exp\left(-j\frac{2\pi kn}{N}\right)$$

$$=\frac{1}{2}\cdot\sum_{n=0}^{N-1} x(n)\cdot\exp\left(-j\frac{2\pi(k-k_0)n}{N}\right)+\frac{1}{2}\cdot\sum_{n=0}^{N-1} x(n)\cdot\exp\left(-j\frac{2\pi(k+k_0)n}{N}\right)$$

From the properties of the DFT, this expression simply becomes:

$$X_c(k) = \frac{1}{2}\cdot X\left(k-k_0\right)_{\mathrm{mod}\,N} + \frac{1}{2}\cdot X\left(k+k_0\right)_{\mathrm{mod}\,N}$$

Operating the same way for the sequence $x_s(n)$ we get the corresponding DFT $X_s(k)$:

$$X_s(k) = \frac{1}{2j}\cdot X\left(k-k_0\right)_{\mathrm{mod}\,N} - \frac{1}{2j}\cdot X\left(k+k_0\right)_{\mathrm{mod}\,N}$$

7.13 a)

$x_p(n)$ is a periodic sequence with fundamental period N. We have the N-point DFT of $x_p(n)$: $x_p(n) \xleftrightarrow[N]{DFT} X_1(k)$ and the 3N-point DFT of $x_p(n)$: $x_p(n) \xleftrightarrow[3N]{DFT} X_3(k)$.

We want to find an expression for $X_3(k)$ as a function of $X_1(k)$. Let's first define W_N^{kn} as $W_N^{kn} = \exp\left(-j\dfrac{2\pi kn}{N}\right)$. We can then write:

$$X_1(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$X_3(k) = \sum_{n=0}^{3N-1} x(n) W_{3N}^{kn}$$

If we develop the previous expression for $X_3(k)$ we get:

$$X_3(k) = \sum_{n=0}^{N-1} x(n) W_{3N}^{kn} + \sum_{n=N}^{2N-1} x(n) W_{3N}^{kn} + \sum_{n=2N}^{3N-1} x(n) W_{3N}^{kn}$$

$$= \sum_{n=0}^{N-1} x(n) W_N^{\frac{k}{3}n} + \sum_{n=0}^{N-1} x(n) W_{3N}^{k(n+N)} + \sum_{n=0}^{N-1} x(n) W_{3N}^{k(n+2N)}$$

$$= \sum_{n=0}^{N-1} x(n) W_N^{\frac{k}{3}n} + \sum_{n=0}^{N-1} x(n) W_3^{k} \cdot W_N^{\frac{k}{3}n} + \sum_{n=0}^{N-1} x(n) W_3^{2k} \cdot W_N^{\frac{k}{3}n}$$

$$= \sum_{n=0}^{N-1} x(n) \left[1 + W_3^k + W_3^{2k}\right] W_N^{\frac{k}{3}n}$$

$$= \left[1 + W_3^k + W_3^{2k}\right] \cdot \sum_{n=0}^{N-1} x(n) W_N^{\frac{k}{3}n}$$

Finally the desired expression is obtained:

$$\boxed{X_3(k) = \left[1 + W_3^k + W_3^{2k}\right] \cdot X_1\left(\frac{k}{3}\right)}$$

7.23

We have to compute the N-point DFT of 4 signals:

a) $x(n) = \delta(n)$

b) $x(n) = \delta(n - n_0), \quad 0 < n_0 < N$

c) $x(n) = a^n, \quad 0 \le n \le N-1$

h) $x(n) = \begin{cases} 1, & n \quad even \\ 0, & n \quad even \end{cases} \quad 0 \le n \le N-1$

a) $x(n) = \delta(n)$

The N-point DFT of $x(n) = \delta(n)$ is defined as:

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} \delta(n) \exp\left(-j\frac{2\pi kn}{N}\right) \\
&= \delta(0) \exp\left(-j\frac{2\pi k(0)}{N}\right) \\
&= 1
\end{aligned}
$$

Therefore:

$$
x(n) = \delta(n), \quad 0 \le n \le N-1 \xleftrightarrow[N]{DFT} X(k) = 1, \quad 0 \le k \le N-1
$$

b) $x(n) = \delta(n - n_0), \quad 0 < n_0 < N$

The N-point DFT of $x(n) = \delta(n - n_0), \quad 0 < n_0 < N$ is defined as:

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} \delta(n - n_0) \exp\left(-j\frac{2\pi kn}{N}\right) \\
&= \delta(n_0) \exp\left(-j\frac{2\pi k(n_0)}{N}\right) \\
&= \exp\left(-j\frac{2\pi k(n_0)}{N}\right)
\end{aligned}
$$

Therefore:

$$x(n) = \delta(n-n_0),\ \ 0 < n_0 < N \ \xleftrightarrow[N]{DFT}\ X(k) = \exp\left(-j\frac{2\pi k n_0}{N}\right),\ \ 0 \le k \le N-1$$

c) $x(n) = a^n,\ \ 0 \le n \le N-1$

The N-point DFT of $x(n) = a^n,\ \ 0 \le n \le N-1$ is defined as:

$$X(k) = \sum_{n=0}^{N-1} a^n \cdot \exp\left(-j\frac{2\pi kn}{N}\right)$$

$$= \sum_{n=0}^{N-1}\left[a \cdot \exp\left(-j\frac{2\pi k}{N}\right)\right]^n$$

$$= \frac{1-\left[a \cdot \exp\left(-j\frac{2\pi k}{N}\right)\right]^N}{1-a\cdot\exp\left(-j\frac{2\pi k}{N}\right)}$$

$$= \frac{1-a^N}{1-a\cdot\exp\left(-j\frac{2\pi k}{N}\right)}$$

Therefore:

$$x(n) = a^n,\ \ 0 \le n \le N-1 \ \xleftrightarrow[N]{DFT}\ X(k) = \frac{1-a^N}{1-a\cdot\exp\left(-j\frac{2\pi k}{N}\right)},\ \ 0 \le k \le N-1$$

h) $x(n) = \begin{cases} 1, & n \ \ even \\ 0, & n \ \ even \end{cases} \ \ 0 \le n \le N-1$

The N-point DFT of $x(n) = \begin{cases} 1, & n \ \ even \\ 0, & n \ \ even \end{cases} \ \ 0 \le n \le N-1$ is defined as:

$$X(k) = \sum_{n=0}^{N-1} x(n)\cdot\exp\left(-j\frac{2\pi kn}{N}\right)$$

If we assume N odd, then N-1 is even and we have:

$$X(k) = \underbrace{1 + \exp\left(-j\frac{2\pi k(2n)}{N}\right) + \exp\left(-j\frac{2\pi k(4n)}{N}\right) + \ldots + \exp\left(-j\frac{2\pi k(N-1)}{N}\right)}_{\frac{N-1}{2}\ terms}$$

i.e.,

$$X(k) = \frac{1 - \left[\exp\left(-j\frac{2\pi(2k)}{N}\right)\right]^{\frac{N+1}{2}}}{1 - \cdot\exp\left(-j\frac{2\pi(2k)}{N}\right)}$$

$$= \frac{1 - \exp\left(-j\frac{2\pi k}{N}\right)}{1 - \cdot\exp\left(-j\frac{4\pi k}{N}\right)}$$

$$= \frac{1 - \exp\left(-j\frac{2\pi k}{N}\right)}{\left[1 - \exp\left(-j\frac{2\pi k}{N}\right)\right] \cdot \left[1 + \exp\left(-j\frac{2\pi k}{N}\right)\right]}$$

$$= \frac{1}{1 + \exp\left(-j\frac{2\pi k}{N}\right)}$$

Therefore:

$$x(n) = \begin{cases} 1, & n \quad even \\ 0, & n \quad even \end{cases} \quad 0 \le n \le N-1 \quad \xleftrightarrow[N]{DFT} \quad X(k) = \frac{1}{1 + \exp\left(-j\frac{2\pi k}{N}\right)}, \quad 0 \le k \le N-1$$

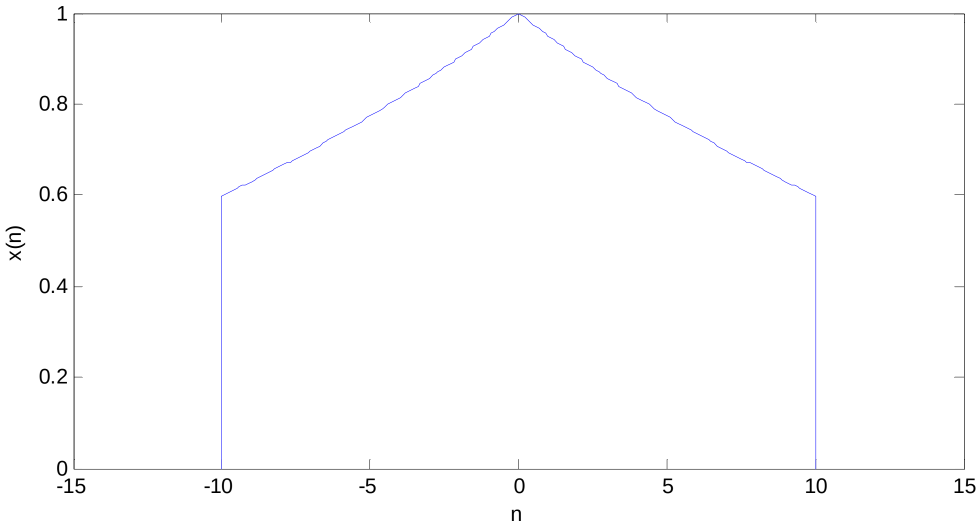
7.28

We are given a discrete-time signal $x(n)=\begin{cases} a^{|n|}, & |n|\le L \\ 0, & |n|>L \end{cases}$ where $a=0.95$ and $L=10$.

(a) Here we need to compute and plot $x(n)$. Obviously from the given values of a and L, we have:

$$x(n)=\begin{cases} 0.95^{|n|}, & |n|\le 10 \\ 0, & |n|>10 \end{cases}$$

The corresponding plot can be found below.

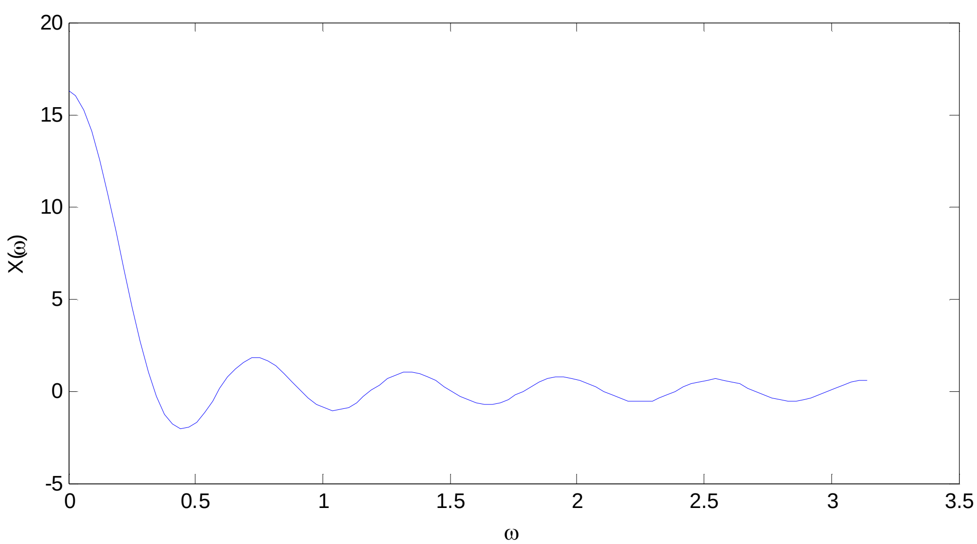


(b) We need to show that $X(\omega)=x(0)+2\cdot\sum_{n=1}^{L}x(n)\cdot\cos(\omega n)$. By definition, we have

$X(\omega)=\sum_{n=-\infty}^{\infty}x(n)\cdot\exp(-j\omega n)$ which in our case becomes:

$$
\begin{aligned}
X(\omega) &= \sum_{n=-\infty}^{\infty} a^{|n|} \cdot \exp(-j\omega n) \\
&= \sum_{n=-L}^{L} a^{|n|} \cdot \exp(-j\omega n) \\
&= \sum_{n=-L}^{-1} a^{-n} \cdot \exp(-j\omega n) + a^{0} \cdot \exp(-j\omega(0)) + \sum_{n=1}^{L} a^{n} \cdot \exp(-j\omega n) \\
&= \sum_{n=1}^{L} a^{n} \cdot \exp(j\omega n) + 1 + \sum_{n=1}^{L} a^{n} \cdot \exp(-j\omega n) \quad when \quad n \to -n \quad in \quad 1st \quad sum \\
&= 1 + \sum_{n=1}^{L} a^{n} \cdot \left[\exp(j\omega n) + \exp(-j\omega n)\right] \\
&= 1 + \sum_{n=1}^{L} a^{n} \cdot \left[2 \cdot \cos(\omega n)\right] \\
&= x(0) + 2 \cdot \sum_{n=1}^{L} x(n) \cdot \cos(\omega n)
\end{aligned}
$$

The corresponding plot at $\omega = \frac{\pi k}{100}, \quad k = 0,1,\ldots,N-1,$ can be found below.



(c) We need to compute c_k for $N = 30$ with c_k defined as:

$$
c_k = \frac{1}{N} X\left(\frac{2\pi}{N} k\right), \quad k = 0,1,\ldots,N-1
$$

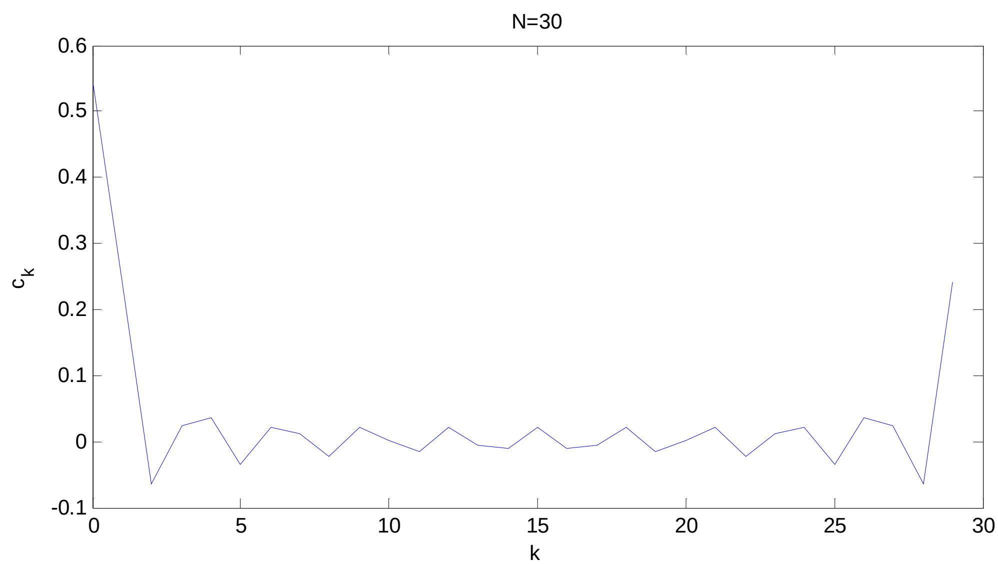
For $N = 30$, c_k becomes:

$$c_k = \frac{1}{30} X\left(\frac{2\pi}{30}k\right), \quad k = 0,1,\ldots,29$$

Using (b), we can derive the desired expression for c_k for $N = 30$:

$$\begin{aligned} c_k &= \frac{1}{30}\left[x(0) + 2\cdot\sum_{n=1}^{L} x(n)\cdot\cos\left(\frac{2\pi}{30}kn\right)\right], \quad k = 0,1,\ldots,29 \\ &= \frac{1}{30}\left[1 + 2\cdot\sum_{n=1}^{L}(0.95)^n\cdot\cos\left(\frac{2\pi}{30}kn\right)\right], \quad k = 0,1,\ldots,29 \end{aligned}$$

The corresponding plot can be found below.



(d) We need to compute $\tilde{x}(n)$ with $\tilde{x}(n)$ defined as:

$$\tilde{x}(n) = \sum_{k=0}^{N-1} c_k \exp\left(j\frac{2\pi kn}{N}\right)$$

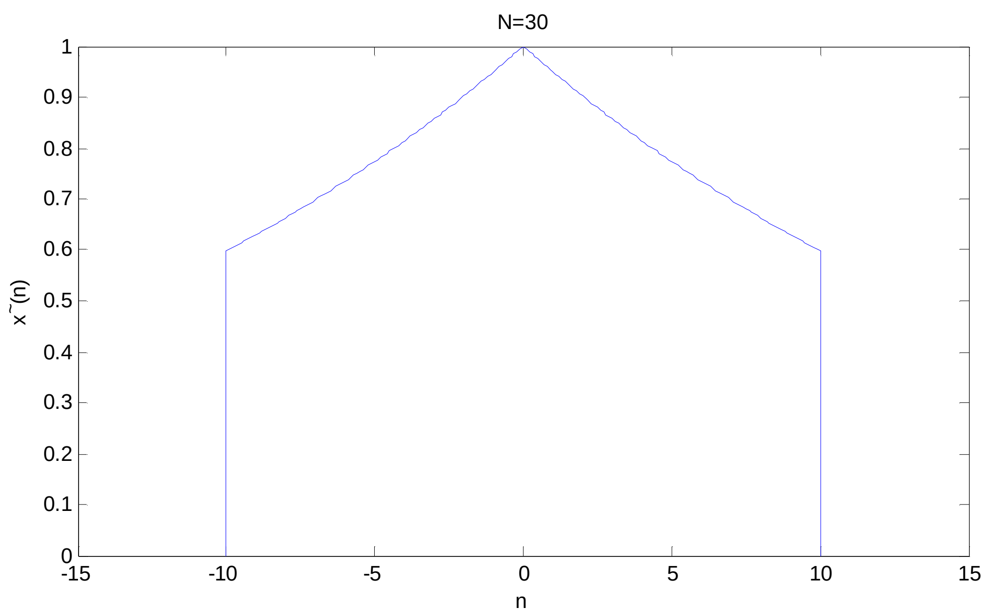
Replacing c_k by its expanded expression in the previous equality we get:

$$\tilde{x}(n)=\sum_{k=0}^{29}\frac{1}{30}X\left(\frac{2\pi}{30}k\right)\cdot\exp\left(j\frac{2\pi kn}{N}\right)$$

$$=\frac{1}{30}\cdot\sum_{k=0}^{29}X\left(\frac{2\pi}{30}k\right)\cdot\exp\left(j\frac{2\pi kn}{N}\right)$$

$$=\frac{1}{30}\cdot\sum_{k=0}^{29}X(w)\cdot\exp(jwn)$$

Therefore $\tilde{x}(n)$ is the inverse 30-point DFT of the DFT of $x(n)$. The corresponding plot can be found below.



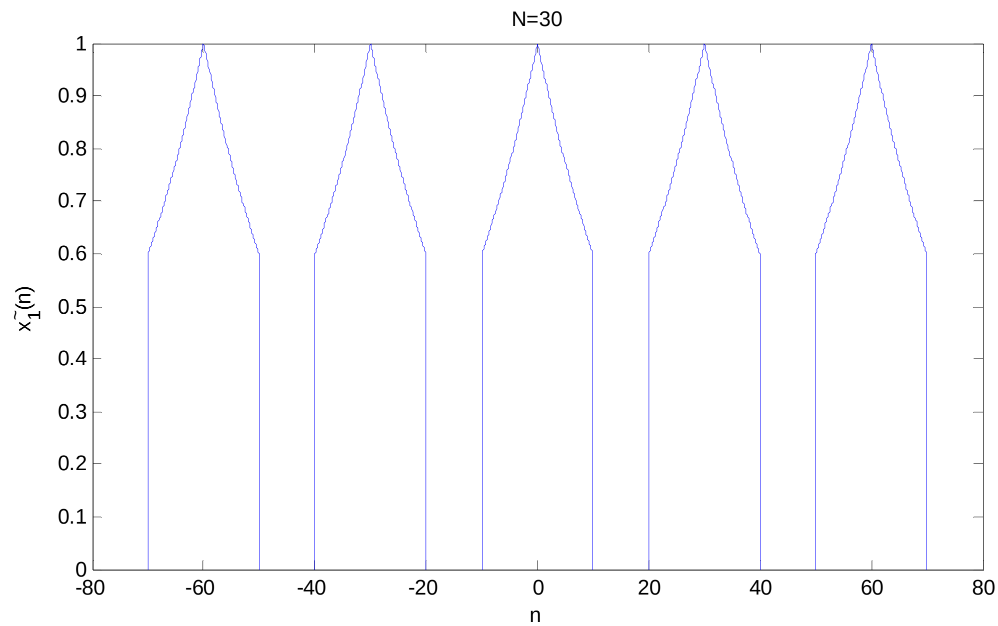
(e) We need to compute $\tilde{x}_1(n)$ for $N=30$ with $\tilde{x}_1(n)$ defined as:

$$\tilde{x}_1(n)=\sum_{l=-\infty}^{\infty}x(n-lN),\quad -L\le n\le L$$

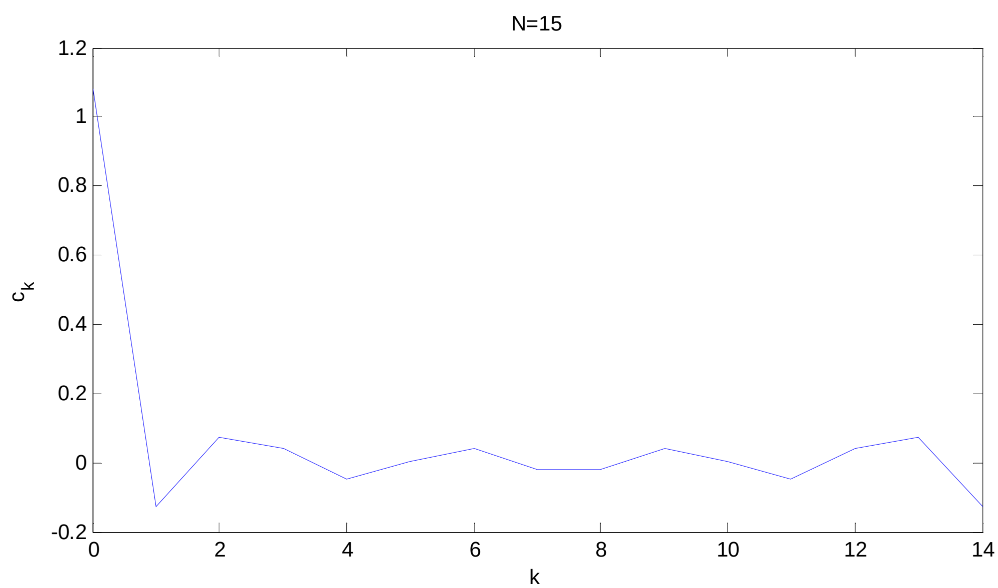
For $N=30$, $\tilde{x}_1(n)$ becomes:

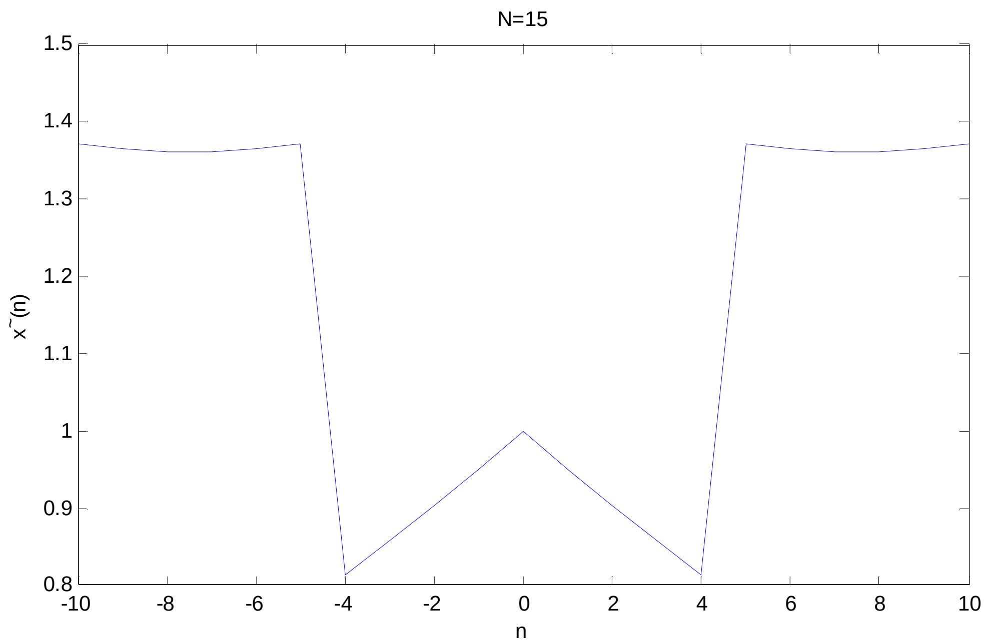
$$\tilde{x}_1(n)=\sum_{l=-\infty}^{\infty}x(n-30l),\quad -L\le n\le L$$

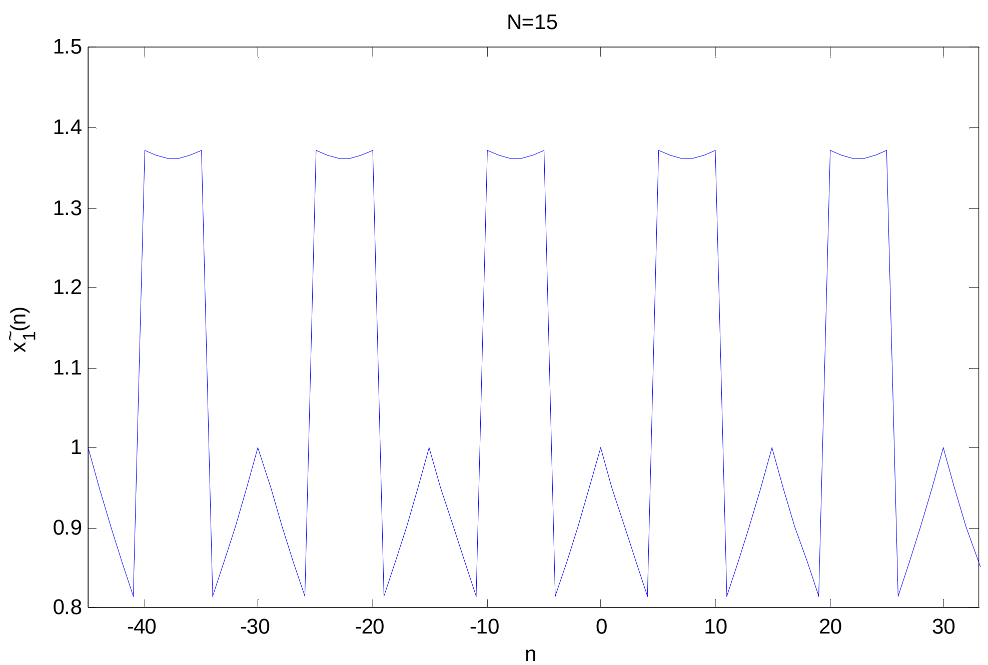
From the corresponding plot below, we can see that $\tilde{x}_1(n)$ is a periodic/repeated version of $\tilde{x}(n)$.



(f) Here we just have to replace N by 15 instead of 30 in the previously obtained equation. This is trivial so just the new plots are being shown.




N=15
x~(n)
n


N=15
x~1(n)
n

8.1

To show that $\exp\left(j\frac{2\pi k}{N}\right),\quad 0 \le k \le N-1$ is an Nth root of unity we just have to show that $X^N = 1$ for $X = \exp\left(j\frac{2\pi k}{N}\right),\quad 0 \le k \le N-1$. This is fairly obvious since:

$$\left[\exp\left(j\frac{2\pi k}{N}\right)\right]^N = \exp\left(j2\pi k\right) = 1.$$

Hence, $\exp\left(j\frac{2\pi k}{N}\right),\quad 0 \le k \le N-1$ is an Nth root of unity.

Now if we consider the sum used in the orthogonality property, we can rewrite that sum as:

$$\sum_{n=0}^{N-1}\exp\left(j\frac{2\pi kn}{N}\right)\cdot\exp\left(-j\frac{2\pi ln}{N}\right) = \sum_{n=0}^{N-1}\exp\left(j\frac{2\pi\left(k-l\right)n}{N}\right)$$

If $k \ne l$, the terms in the sum represent the N equally spaced unity roots on the unit circle which add to zero.

<u>Mathematical proof if $k \ne l$:</u>

$$\sum_{n=0}^{N-1}\exp\left(j\frac{2\pi\left(k-l\right)n}{N}\right) = \frac{1-\left[\exp\left(j\frac{2\pi\left(k-l\right)}{N}\right)\right]^N}{1-\exp\left(j\frac{2\pi\left(k-l\right)}{N}\right)}$$

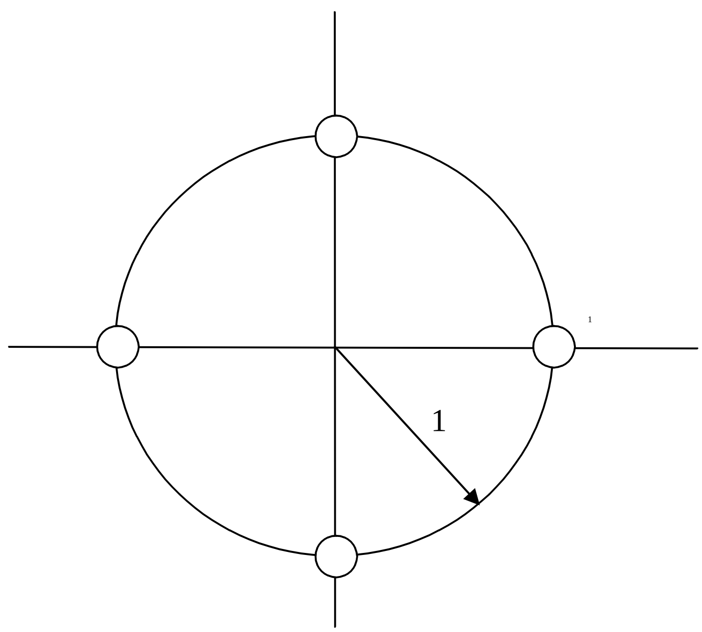
$$= \frac{1-1}{1-\exp\left(j\frac{2\pi\left(k-l\right)}{N}\right)}$$

$$= 0$$

If $k = l$, the sum adds up to N:

$$\begin{aligned}\sum_{n=0}^{N-1} \exp\left(j \frac{2\pi (k-l) n}{N} \right) &= \sum_{n=0}^{N-1} \exp\left(j \frac{2\pi (0) n}{N} \right) \\ &= \sum_{n=0}^{N-1} 1 \\ &= N\end{aligned}$$

A plot of the unitary roots for N=4 is shown below.



8.3

$x(n)$ is a real valued N-point sequence with $N = 2^{\nu}$. The N-point DFT of $x(n)$ is $X(k) = \sum_{n=0}^{N-1} x(n)\cdot \exp\left(-j\frac{2\pi kn}{N}\right)$ which , N being even, can be rewritten as:

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n)W_N^{kn} + \sum_{n=\frac{N}{2}}^{N-1} x(n)W_N^{kn}, \quad where \quad W_N^{kn} = \exp\left(-j\frac{2\pi kn}{N}\right)$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n)W_N^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right)W_N^{k\left(n+\frac{N}{2}\right)}$$

$X^{'}(k)$ corresponds to the odd harmonics of $X(k)$, i.e., $X^{'}(k) = X(2k+1)$ and therefore:

$$X^{'}(k) = \sum_{n=0}^{N-1}\left[x(n)W_N^{(2k+1)n} + x\left(n+\frac{N}{2}\right)W_N^{(2k+1)\left(n+\frac{N}{2}\right)}\right]$$

$$= \sum_{n=0}^{N-1}\left[x(n)\cdot W_N^{n}\cdot W_{\frac{N}{2}}^{kn} + x\left(n+\frac{N}{2}\right)\cdot W_N^{n}\cdot W_{\frac{N}{2}}^{kn}\cdot W_N^{\frac{N}{2}}\right]$$

because $W_N^{2kn} = W_{\frac{N}{2}}^{kn}$.

We can simplify further this expression using the fact that $W_N^{\frac{N}{2}} = -1$:

$$X^{'}(k) = \sum_{n=0}^{N-1}\left[x(n)\cdot W_N^{n}\cdot W_{\frac{N}{2}}^{kn} - x\left(n+\frac{N}{2}\right)\cdot W_N^{n}\cdot W_{\frac{N}{2}}^{kn}\right]$$

Finally we get the odd harmonics of $X(k)$ using the following formulae:

$$X^{'}(k) = X(2k+1) = \sum_{n=0}^{N-1}\left[x(n) - x\left(n+\frac{N}{2}\right)\right]\cdot W_N^{n}\cdot W_{\frac{N}{2}}^{kn}$$

8.4

We want to develop a method to compute a 24-point DFT from three 8-point DFTs.

Let $Y(k)$ denote the 24-point DFT and $Y_1(k),\ Y_2(k),\ Y_3(k)$ denote the three 8-point DFTs. We then have:

$$Y(k)=\sum_{n=0}^{N-1} y(n)W_N^{kn}=\sum_{n=0}^{23} y(n)W_N^{kn}$$

We can rewrite this sum as three sums that would take values n among the sets {0, 3, 6, …, 21}, {1, 4, 7, …, 22} and {2, 5, 8, …, 23} respectively.

$$Y(k)=\sum_{n=0,3,6,...}^{21} y(n)W_N^{kn}+\sum_{n=1,4,7,...}^{22} y(n)W_N^{kn}+\sum_{n=2,5,8,...}^{23} y(n)W_N^{kn}$$

$$=\sum_{n=0.}^{7} y(3n)W_{\frac{N}{3}}^{kn}+\sum_{n=0.}^{7} y(3n+1)W_{\frac{N}{3}}^{kn}\cdot W_N^{k}+\sum_{n=0.}^{7} y(3n+2)W_{\frac{N}{3}}^{kn}\cdot W_N^{2k}$$

$$=\underbrace{\sum_{n=0.}^{7} y(3n)W_{\frac{N}{3}}^{kn}}_{8-pt\ \ DFT}+\underbrace{\left[\sum_{n=0.}^{7} y(3n+1)W_{\frac{N}{3}}^{kn}\right]}_{8-pt\ \ DFT}\cdot W_N^{k}+\underbrace{\left[\sum_{n=0.}^{7} y(3n+2)W_{\frac{N}{3}}^{kn}\right]}_{8-pt\ \ DFT}\cdot W_N^{2k}$$

$$=Y_1(k)+Y_2(k)\cdot W_N^{k}+Y_3(k)\cdot W_N^{2k}$$

With three 8-point DFTs, $Y_1(k),\ Y_2(k),\ Y_3(k)$, we can create a 24-point DFT $Y(k)$ using the following formulae:

$$Y(k)=Y_1(k)+Y_2(k)\cdot W_N^{k}+Y_3(k)\cdot W_N^{2k}$$

8.7

We want to derive the radix-2 decimation in time using the steps 8.1.16 to 8.1.18 in the book.

Page 519 in the book already gives some guidelines on how to proceed such as selecting $M = \frac{N}{2}$ and $L = 2$.

1) The first step to follow (8.1.16) makes us compute the M-point DFTs $F(l,q)$ defined as:

$$F(l,q) = \sum_{m=0}^{M-1} x(l,m) W_M^{mq}, \quad 0 \le l \le L-1; \quad 0 \le q \le M-1$$

Therefore we have two $\frac{N}{2}$-point DFTs to compute for $l = 0$ and $l = 1$.

$$F(0,q) = \sum_{m=0}^{M-1} x(0,m) W_M^{mq} = \sum_{m=0}^{\frac{N}{2}-1} x(0,m) W_{\frac{N}{2}}^{mq}$$

$$F(1,q) = \sum_{m=0}^{\frac{N}{2}-1} x(1,m) W_{\frac{N}{2}}^{mq}$$

2) The second step (8.1.17) consists in computing a new rectangular array $G(l,q)$ defined as:

$$G(l,q) = W_N^{lq} \cdot F(l,q), \quad 0 \le l \le L-1; \quad 0 \le q \le M-1$$

Therefore we have two rectangular arrays to compute for $l = 0$ and $l = 1$.

$$G(0,q) = W_M^{(0)q} \cdot F(0,q) = F(0,q)$$

$$G(1,q) = W_M^{(1)q} \cdot F(1,q) = W_M^{(1)q} \cdot \sum_{m=0}^{\frac{N}{2}-1} x(1,m) W_{\frac{N}{2}}^{mq}$$

3) The third and last step (8.1.18) consists in computing the L-point DFT $X(p,q)$ defined as:

$$X(p,q)=\sum_{l=0}^{L-1}G(l,q)\cdot W_L^{lp},\quad 0\le p\le L-1;\quad 0\le q\le M-1$$

Therefore we have two L-point DFTs to compute for $p=0$ and $p=1$.

$$X(0,q)=\sum_{l=0}^{L-1}G(l,q)\cdot W_L^{(0)l}=\sum_{l=0}^{L-1}G(l,q)$$

$$=G(0,q)+G(1,q)$$

$$=F(0,q)+W_N^q\cdot F(1,q)$$

$$X(1,q)=\sum_{l=0}^{L-1}G(l,q)\cdot W_L^{(1)l}=\sum_{l=0}^{L-1}G(l,q)\cdot W_L^l$$

$$=G(0,q)\cdot W_2^0+G(1,q)\cdot W_2^1$$

$$=G(0,q)-G(1,q)\quad as\quad W_2^1=-1$$

$$=F(0,q)-W_N^q\cdot F(1,q)$$

$F(0,q)$ and $F(1,q)$ here are the same as $F_1(k)$ and $F_2(k)$ in equation 8.1.26 of the book and therefore we get the desired radix-2 decimation in time:

$$\begin{cases} X(0,q)=X(k)=F_1(k)+W_N^k\cdot F_2(k) \\ \\ X(1,q)=X\left(k+\dfrac{N}{2}\right)=F_1(k)-W_N^k\cdot F_2(k) \end{cases},0\le k\le\frac{N}{2}-1$$