1.1 c, d
\[ X_e(t) = \frac{1}{2} [x(t) + x(-t)] \quad \text{(eqn 1.4)} \]
\[ X_0(t) = \frac{1}{2} [x(t) - x(-t)] \quad \text{(eqn 1.5)} \]

1.1 c)
\[ x(t) = 1 + t \cos(t) + t^2 \sin(t) + t^3 \sin(t) \cos(t) \]
\[ x(-t) = 1 + (-t) \cos(-t) + (-t)^2 \sin(-t) + (-t)^3 \sin(-t) \cos(-t) \]

**Remember:**
\[ \cos(-t) = \cos(t) \]
\[ \sin(-t) = -\sin(t) \]

So,
\[ x(-t) = 1 - t \cos(t) - t^2 \sin(t) + t^3 \sin(t) \cos(t) \]
\[ X_e(t) = \frac{1}{2} [x(t) + x(-t)] = \frac{1}{2} [2 + 0 + 0 + 2t^3 \sin(t) \cos(t)] \]
\[ = 1 + t^3 \sin(t) \cos(t) \]
\[ X_0(t) = \frac{1}{2} [x(t) - x(-t)] = \frac{1}{2} [0 + 2t \cos(t) + 2t^2 \sin(t)] \]
\[ = t \cos(t) + t^2 \sin(t) \]
\[ d) \quad x(t) = (1+t^3) \cos^3(10t) \]
\[ x(-t) = (1-t^3) \cos^3(10(-t)) = (1-t^3)\cos^3(-10t) \]

Since \( \cos(-t) = \cos(t) \)

Then \( \cos^3(-t) = \cos^3(t) \)

So \( \cos^3(-10t) = \cos^3(10t) \)

\[ x(-t) = (1-t^3) \cos^3(10t) \]

\[ x_e(t) = \frac{1}{2} [x(t) + x(-t)] = \frac{1}{2} \left[ \left( (1+t^3) + (1-t^3) \right) \cos^3(10t) \right] \]

\[ = \frac{1}{2} \left[ 2 \cos^3(10t) \right] = \cos^3(10t) \]

\[ x_o(t) = \frac{1}{2} [x(t) - x(-t)] = \frac{1}{2} \left[ \left( (1+t^3) - (1-t^3) \right) \cos^3(10t) \right] \]

\[ = \frac{1}{2} \left[ 2t^3\cos^3(10t) \right] = t^3\cos^3(10t) \]
\[ \cos^2(2\pi t) \]

1.5 a, c, d, e, f, g

a) \[ \cos^2(2\pi t) \]

Remember \( \cos(t) \)

\[ \cos^2(t) \text{ looks like:} \]

\[ \cos^2(t) = \frac{1}{2} \]

so \( \cos^2(2\pi t) \) stretches the above plot by a factor of \( \frac{1}{2\pi} \)

only mountain \( C \)

\[ \cos^2(2\pi t) \text{ is periodic and it has a period of} \]

\[ \tau = \frac{1}{2} s = 0.5 s \]
\[ x(t) = e^{-2t} \cos(2\pi t) \]

but we have to multiply this by \( e^{-2t} \) which looks like

the end result \( e^{-2t} \cos(2\pi t) \) looks like:

so the function is clearly not periodic
\[ x[n] = (-1)^n \]  

The function (signal) is discrete!  

Remember \((-1)^n = \begin{cases} 
1 & \text{if } n \text{ is even} \\
0 & \text{if } n \text{ is odd} 
\end{cases}\)

So the plot of the function is:

\[ \text{So the signal } x[n] = (-1)^n \text{ is periodic} \]

\[ \text{with period } T = 2 \text{ samples} \]
$X[n] = (-1)^n$, a discrete signal

Remember that $(-1)^n = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$

Let's look at $n^2$ in terms of $n$; $n$ being even means that $n$ is divisible by 2 (i.e., 2 is a factor of $n$); therefore, $n^2$ must also be divisible by 2.

If $n$ is odd, then 2 is not a factor of $n$, and so squaring it means that $n^2$ will still not be divisible by 2. Therefore, we can conclude $(-1)^{n^2} = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$

So $X[n] = (-1)^n$ is periodic with period $P = 2$ samples.
\[ x[n] = \cos(2\pi n) \]

We know \( \cos(2\pi t) \) is periodic with period \( \Pi \), \( \cos(2\pi n) \) is simply \( \cos(2\pi t) \) sampled at \( n \) such that \( t \) is an integer. So for \( \cos(2\pi n) \) to be periodic there has to be \( n_0 \) such that the distance between \( n_0 \) and \( n_\Pi \) is a multiple of \( \Pi \). Since \( \Pi \) is irrational (it cannot be expressed as a fraction \( \frac{a}{b} \), \( a, b \in \mathbb{N} \)) then there is no \( n_0 \) and \( n_\Pi \) whose distance is a multiple of \( \Pi \).

The signal

\[ x[n] = \cos(2\pi n) \]

is NOT periodic.

This example is important.
$x[n] = \cos(2\pi n)$

We know
$\cos(2\pi n) = 1$ for all $n$

so
$\cos(2\pi n)$

so
$x[n] = \cos(2\pi n)$ is periodic with
period $\frac{\pi}{n} = 1$ sample
a) What is the total energy of the rectangular pulse shown in Fig 1.14 (b)?

![Fig 1.14b](image)

Total energy \( E = \int_{-T_1/2}^{T_1/2} x(t)^2 dt \) (eqn. 1.15)

so \( E = \int_{-T_1/2}^{T_1/2} A^2 dt = \left[ T_1 \right]_{-T_1/2}^{T_1/2} = A^2 \left( \frac{T_1}{2} \right) - A^2 \left( -\frac{T_1}{2} \right) \)

\[ \frac{E}{T} = A^2 \]

b) What is the average power of the square wave shown in Fig 1.14 (a)?

![Fig 1.14a](image)

For a periodic signal

\[ P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt \]

the period of the signal is 0.2 and \( x(t) \) is:

![Area](image)

\[ P = \frac{0.2}{0.2} = 0.2 = 1 \]
1.7 Determine the average power of the triangular wave shown in Fig. 1.15.

Fig. 1.15

-1
-0.5
0
0.5
1

0.1 0.2 0.3 0.4 0.5 0.6

so \( T = 0.2 \)

look at -0.1 to 0.1 interval

\[ x(t) = 20t - 1 \]

from \([-0.1, 0]\] \( x(t) = -20t - 1 \)

\[
P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) \, dt
\]

\[ x^2(t) \] looks like:

due to symmetrical symmetry:

\[
P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) \, dt = \frac{2}{T} \int_{0}^{T/2} x^2(t) \, dt
\]

for \((0,0.1)\) \( x^2(t) = 400t^2 - 40t + 1 \)

continued on next page
\[ p = \frac{2}{0.2} \int_0^{0.1} (400 + t - 40 + 1) \, dt = 10 \left( \frac{400}{3} \left[ \frac{t^3}{0.1} \right]_0^{0.1} - \frac{40}{2} \left[ \frac{t^2}{0.1} \right]_0^{0.1} + \left[ t \right]_0^{0.1} \right) \]

\[ = 10 \left( \frac{400}{3} \left( \frac{1}{10} \right)^3 - \frac{40}{2} \left( \frac{1}{40} \right)^2 + 0.1 \right) \]

\[ = 10 \left( \frac{400}{3000} - \frac{40}{200} + 0.1 \right) = \frac{40}{30} - \frac{40}{20} + 1 = \frac{4}{3} - 2 + 1 = \frac{4}{3} - 1 \]

\[ = \frac{1}{3} \]
1.8: Determine the total energy of the discrete-time signal shown in Fig. 1.17

Fig. 1.17:

\[ E = \sum_{n=-\infty}^{\infty} x^2[n] \quad \text{(eqn. 1.18)} \]

\[ E = \sum_{n=-1}^{1} 1^2 = 1^2 + 1^2 + 1^2 = 3 \]
Problem 1.9 \( a, b, c, f, h \)

a) \( x(t) = \begin{cases} 
2-t & 0 \leq t \leq 1 \\
2 & 1 \leq t \leq 2 \\
0 & \text{otherwise}
\end{cases} \)

This is an energy signal, the energy is

\[
E = \int_0^\infty x^2(t)dt = \int_0^1 t^2dt + \int_1^2 (2-4t+t^2)dt
\]

\[
= \frac{t^3}{3} \Big|_0^1 + (2t^2 - \frac{4t^3}{3} + \frac{t^4}{2}) \Big|_1^2
\]

\[
= \frac{1}{3} + 4(2-1) - \frac{4}{2}(4-1) + \frac{1}{3}(8-1)
\]

\[
= \frac{1}{3} + 4 - 2(3) + \frac{1}{3}(7) = \frac{1}{3} + 4 - 6 + \frac{7}{3}
\]

\[
= \frac{8}{3} - 2 = \frac{2}{3}
\]
1.9 b \[ x[n] = \begin{cases} 
  n & 0 \leq n < 5 \\
  10-n & 5 \leq n \leq 10 \\
  0 & \text{otherwise} 
\end{cases} \]

This is an energy signal with energy

\[ E = \sum_{n=-8}^{10} x[n]^2 = \sum_{n=0}^{4} n^2 + \sum_{n=5}^{10} 100 - 20n + n^2 \]

\[
\begin{align*}
  n = 0 & \quad n^2 = 0 \\
  n = 1 & \quad n^2 = 1 \\
  n = 2 & \quad n^2 = 4 \\
  n = 3 & \quad n^2 = 9 \\
  n = 4 & \quad n^2 = 16 \\
\end{align*}
\]

\[
\begin{align*}
  n = 5 & \quad 100 - 20n + n^2 = 100 - 100 + 25 = 25 \\
  n = 6 & \quad 100 - 20n + n^2 = 100 - 120 + 36 = 16 \\
  n = 7 & \quad 100 - 20n + n^2 = 100 - 140 + 49 = 9 \\
  n = 8 & \quad 100 - 20n + n^2 = 100 - 160 + 64 = 4 \\
  n = 9 & \quad 100 - 20n + n^2 = 100 - 180 + 81 = 1 \\
  n = 10 & \quad 100 - 20n + n^2 = 100 - 200 + 100 = 0 \\
\end{align*}
\]

\[ 30 + 55 = 85 \]
c) 

\[ x(t) = 5 \cos(\pi t) + \sin(5\pi t) \]

\[ \uparrow \quad \uparrow \]

\[ \frac{\pi}{2} \quad \omega = 5\pi = 2\pi f \Rightarrow f = \frac{5}{2} \Rightarrow T = \frac{2}{5} \]

Since 2 is an integer multiple of \( \frac{2}{5} \), then the period of \( x(t) \) is \( T = 2 \).

So,

\[ P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt = \frac{1}{2} \int_{-1}^{1} (5\cos(\pi t) + \sin(5\pi t))^2 dt \]

This integral can get pretty ugly so it's ok to solve it numerically with Matlab or a calculator.

\[ P = \frac{1}{2} - 26 = 13 \]
1.9 f

\[ x[n] = \begin{cases} 
\sin(\pi n) & -y \leq n \leq y \\
0 & \text{otherwise} 
\end{cases} \]

for integer \( n \) \( \sin(\pi n) = 0 \)

so this is a zero signal and its energy
and power are therefore zero as well
$x[n] = \begin{cases} 
\cos(\pi n), & n \geq 0 \\
0, & \text{otherwise}
\end{cases}$

$\cos(\pi n) = \begin{cases} 
1, & \text{if } n \text{ is even} \\
-1, & \text{if } n \text{ is odd}
\end{cases}$

This is a power signal with period $N = 2$ samples:

$$p = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n] = \frac{1}{2} \sum_{n=0}^{4} (\cos^2(\pi n)) = \frac{1}{2} (-1)^2 + \frac{1}{2}(1)^2 = 1$$

Students should check this result as it does not match with the book.
The sinusoidal signal
\[ x(t) = 3\cos(200t + \pi/6) \]
is passed through a square-law device defined by the input-output relationship
\[ y(t) = x^2(t) \]
Using the trig. identity
\[ \cos^2 \theta = \frac{1}{2} (\cos 2\theta + 1) \]
Show that the output \( y(t) \) consists of a dc component and a sinusoidal component.
a) Specify the dc component
b) Specify amplitude and frequency of the sinusoidal component in output \( y(t) \)
\[ y(t) = x^2(t) = 9\cos^2(200t + \pi/6) = \frac{9}{2} \left( \cos(400t + \frac{\pi}{3}) + 1 \right) \]
\[ = \frac{9}{2} \cos(400t + \frac{\pi}{3}) + \frac{9}{2} \]
Sinusoidal component
Amplitude is \( \frac{9}{2} \)
Frequency is \( 2\pi f = 400 \Rightarrow f = \frac{400}{2\pi} \)

D. C. component = \( \frac{9}{2} \)
1.44 Consider the signal
\[ x(t) = A \cos (\omega t + \phi) \]

Determine the average power

Since we are only looking to get the average power of \( x(t) \), we can get away with only considering

\[ x'(t) = A \cos (\omega t) \]

\[ \omega = 2\pi f \implies f = \frac{\omega}{2\pi} \implies T = \frac{2\pi}{\omega} \]

\[ P = \frac{1}{T} \int_{-T/2}^{T/2} x'^2(t) dt = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} A^2 \cos^2 (\omega t) dt \]

\[ = \frac{\omega A^2}{2\pi} \left[ \int_{-\pi/\omega}^{\pi/\omega} \cos^2 (\omega t) dt \right] \]

\[ = \frac{\omega A^2}{2\pi} \left[ \frac{\pi}{2} + \frac{1}{4\omega} \sin(2\omega t) \right]_{-\pi/\omega}^{\pi/\omega} \]

\[ = \frac{\omega A^2}{2\pi} \left[ \left( \frac{\pi}{2\omega} - \frac{-\pi}{2\omega} \right) + \frac{1}{4\omega} \left( \sin(2\pi) - \sin(-2\pi) \right) \right] \]

\[ = \frac{\omega A^2}{2\pi} \left[ \frac{\pi}{\omega} + \frac{1}{4\omega} \cdot 0 \right] = \frac{\omega A^2}{2\pi} \cdot \frac{\pi}{\omega} = \frac{A^2}{2} \]
The angular frequency \( \Omega \) of the sinusoidal signal \( x[n] = A \cos(\Omega n + \phi) \) satisfies the condition for \( x[n] \) to be periodic.

determine the average power of \( x[n] \)

Again because we care for average power it's equivalent to finding the average power of

\[ x'[n] = A \cos(\Omega n) \]

\[ P = \frac{1}{N} \sum_{n=0}^{N-1} x'[n]^2 = \frac{1}{N} \sum_{n=0}^{N-1} A^2 \cos^2(\Omega n) \]

\[ = \frac{A^2}{N} \sum_{n=0}^{N-1} \cos^2(\Omega n) = \frac{A^2}{N} \sum_{n=0}^{N-1} \frac{1}{2} (\cos(2\Omega n) + 1) \]

\[ = \frac{A^2}{2N} \left( \sum_{n=0}^{N-1} \cos(2\Omega n) + \sum_{n=0}^{N-1} 1 \right) \]

\[ = \frac{A^2}{2N} \cdot N = \frac{A^2}{2} \]
A rectangular pulse $x(t)$ is defined by
\[
x(t) = \begin{cases} A, & 0 \leq t \leq T \\ 0, & \text{Otherwise} \end{cases}
\]

The pulse $x(t)$ is applied to an integrator defined by
\[
y(t) = \int_0^t x(\tau) \, d\tau
\]

Find the total energy of the output $y(t)$

First let's draw $x(t)$

$y(t)$ will be the area under $x(t)$ from 0 to $t$ so it should look like this:

Since $E = \int_0^\infty y^2(t) \, dt$ then looking at the above plot of $y(t)$ $E$ is clearly infinite.