1.1 c, d

HW1 solved by

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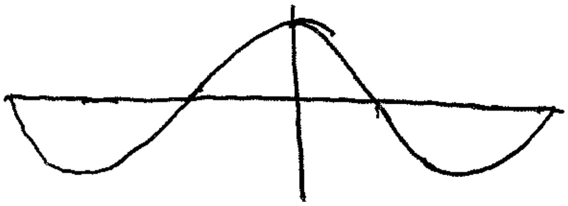
$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ .... (eqn 1.4)

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$ .... (eqn 1.5)

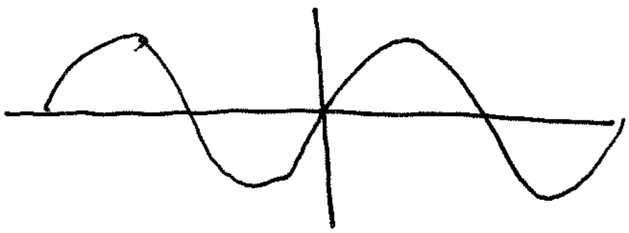
1.1 c) $x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$

$x(-t) = 1 + (-t)\cos(-t) + (-t)^2\sin(-t) + (-t)^3\sin(-t)\cos(-t)$

Remember :



$\cos(-t) = \cos(t)$



$\sin(-t) = -\sin(t)$

so

$x(-t) = 1 - t\cos(t) - t^2\sin(t) + t^3\sin(t)\cos(t)$

$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[2 + 0 + 0 + 2t^3\sin(t)\cos(t)]$

$= 1 + t^3\sin(t)\cos(t)$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[0 + 2t\cos(t) + 2t^2\sin(t)]$

$= t\cos(t) + t^2\sin(t)$

1.1

d) $x(t) = (1+t^3)\cos^3(10t)$

$x(-t) = (1-t^3)\cos^3(10(-t)) = (1-t^3)\cos^3(-10t)$

since $\cos(-t) = \cos(t)$

then $\cos^3(-t) = \cos^3(t)$

so $\cos^3(-10t) = \cos^3(10t)$

$x(-t) = (1-t^3)\cos^3(10t)$

$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}\left[((1+t^3)+(1-t^3))\cos^3(10t)\right]$

$= \frac{1}{2}\left[2\cos^3(10t)\right] = \cos^3(10t)$

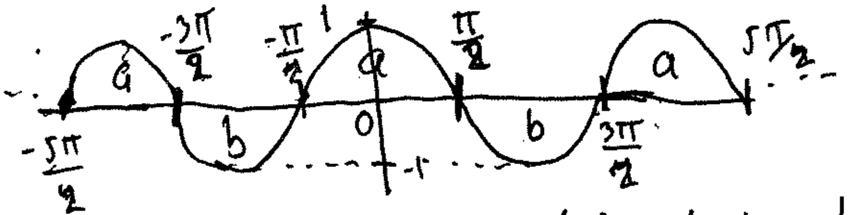
$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}\left[((1+t^3)-(1-t^3))\cos^3(10t)\right]$

$= \frac{1}{2}\left[2t^3\cos^3(10t)\right] = t^3\cos^3(10t)$

1.5 a, c, d, e, f, g

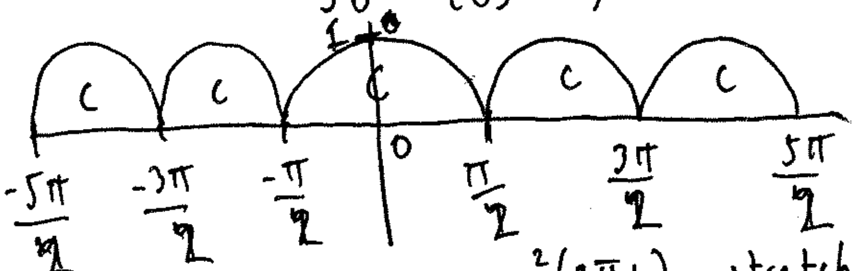
a) $\cos^2(2\pi t)$

remember $\cos(t)$




mountain a = −mountain b

so $\cos^2(t)$ looks like:

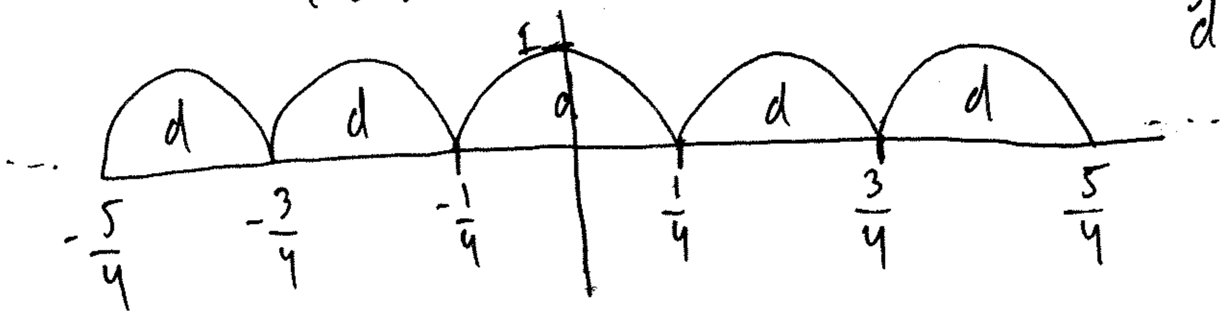



only mountain c

so $\cos^2(2\pi t)$ stretches the above plot by a factor of $1/2\pi$

$\cos^2(2\pi t) =$

only mountain d type




so $\cos^2(2\pi t)$ is periodic and it has a period of

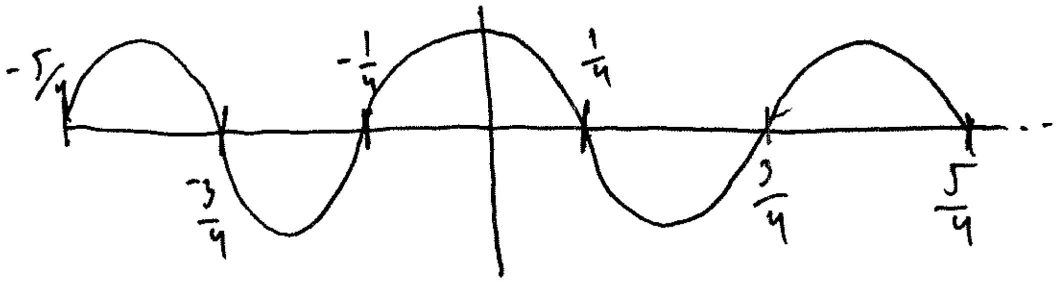
$T = \frac{1}{2}\,s = 0.5\,s$

1.5 c

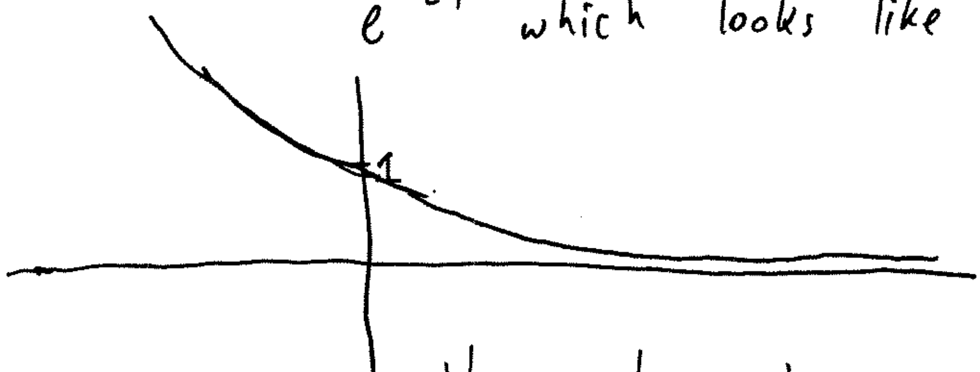
$x(t) = e^{-2t}\cos(2\pi t)$

$\cos(2\pi t)$

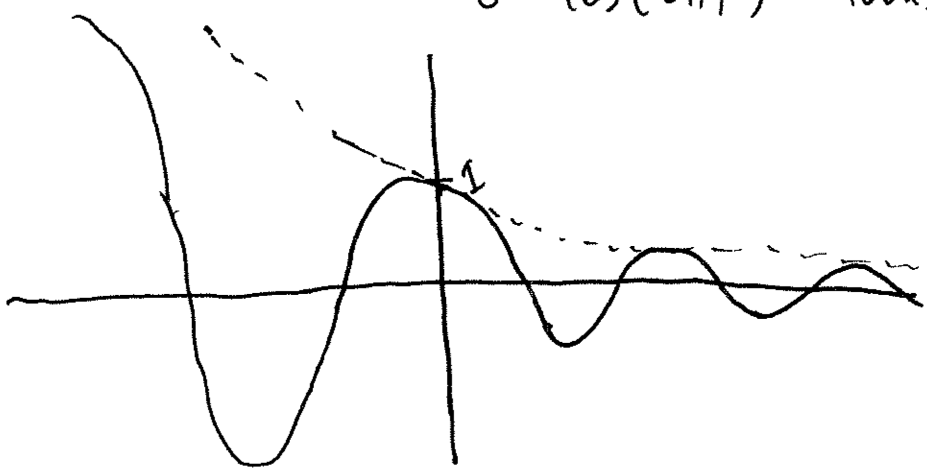
⇓




but we have to multiply this by $e^{-2t}$ which looks like




the end result $e^{-2t}\cos(2\pi t)$ looks like:



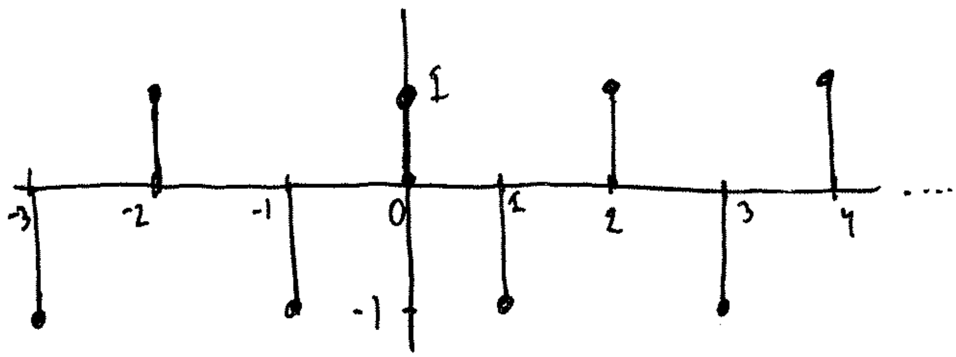

So the function is clearly not periodic

1.5 d

$x[n] = (-1)^n$ ← the function (signal) is discrete!

remember $(-1)^n = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$

so the plot of the function is:




So the signal $x[n] = (-1)^n$ is periodic with period $T = 2$ samples

1.5e

$x[n] = (-1)^{n^2}$ ← discrete signal

remember that

$$(-1)^n = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

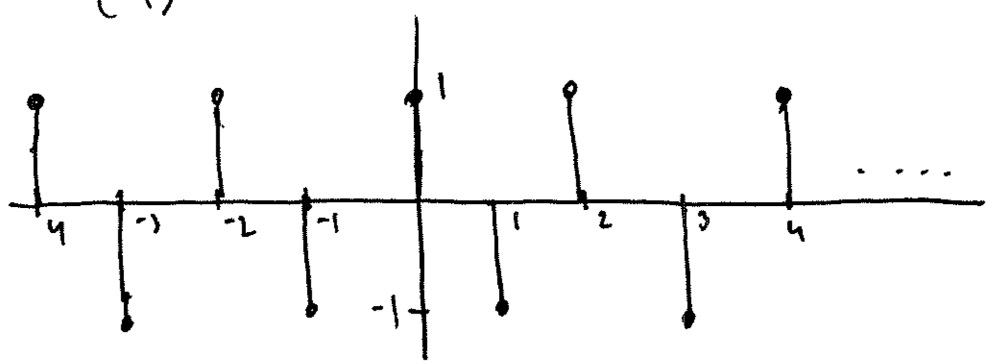
let's look at $n^2$ in terms of $n$

$n$ being even means that $n$ is divisible by 2 (i.e 2 is a factor of $n$)
therefore $n^2$ must also be divisible by 2

if $n$ is odd then 2 is not a factor of $n$ and so squaring it means that $n^2$ will still not be divisible by 2. therefore we can conclude

$$(-1)^{n^2} = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

$(-1)^{n^2}$




so $x[n] = (-1)^{n^2}$ is peiodica with period
$T = 2$ samples

1.5 f

$x[n] = \cos[2n]$

we know $\cos(2t)$ is periodic with period $\pi$, $\cos[2n]$ is simply $\cos[2t]$ sampled at ~~in~~ all $t$ such that $t$ is an integer. So for $\cos[2n]$ to be periodic ~~a~~ there has to be $n_0, n_T$ such that the distance between $n_0$ and $n_T$ is a multiple of $\pi$. Since $\pi$ is irrational (it cannot be expressed as a fraction $\frac{a}{b}$ $a, b \in \mathbb{N}$) then there is no $n_0$ and $n_T$ whose distance is a multiltjiple of $\pi$.

The signal

$x[n] = \cos[2n]$ is NOT periodic

this example is important

1.5 g)

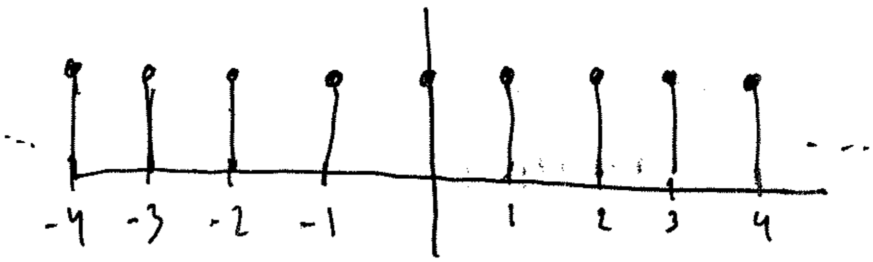
$x[n] = \cos(2\pi n)$

we know

$\cos(2\pi n) = 1$ for all $n$

so

$\cos(2\pi n)$



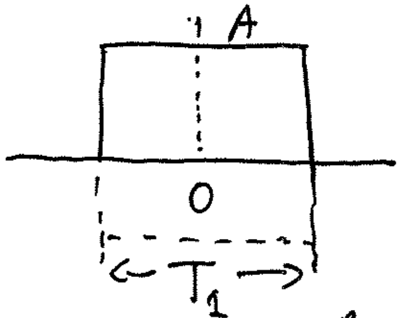

so

$x[n] = \cos(2\pi n)$ is periodic with period $T = 1$ sample

1.6

a) What is the total energy of the rectangular pulse shown in Fig 1.14 (b)?

fig 1.14b



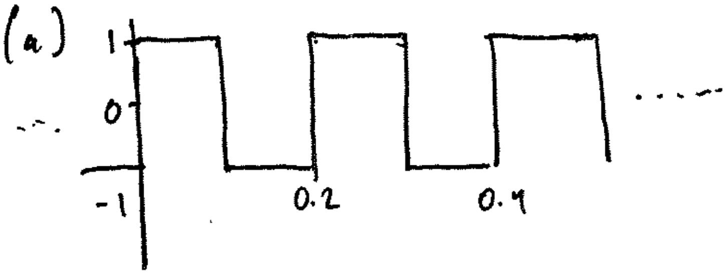

total energy $E = \int_{-\infty}^{\infty} x^2(t)\,dt$ (eqn. 1.15)

so $E = \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} A^2\,dt = A^2 t \Big|_{-\frac{T_1}{2}}^{T_1/2} = A^2\left(\frac{T_1}{2}\right) - A^2\left(-\frac{T_1}{2}\right)$

$= A^2 T_1$

b) What is the average power of the square wave shown in Fig 1.14 (a)

fig. 1.14a:

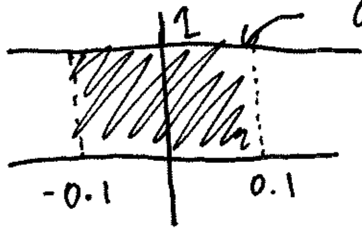



for a periodic signal

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} x^2(t)\,dt$$

the period of this signal is 0.2 and $x^2(t)$ is:

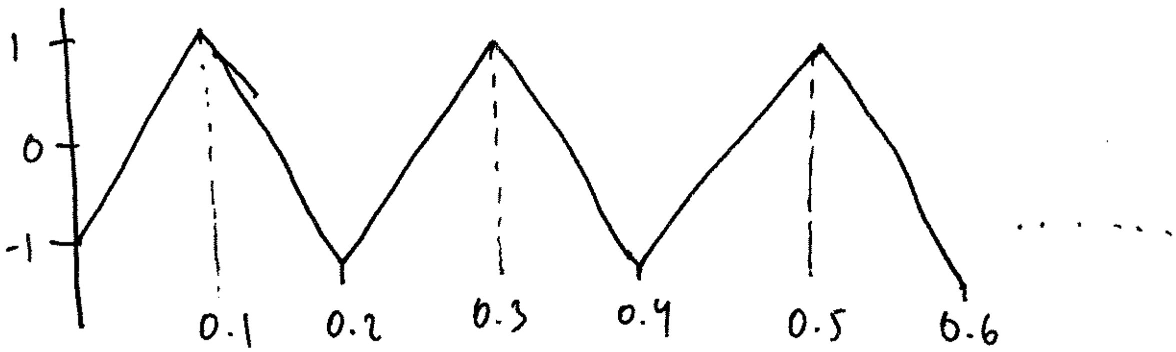
area = 0.2




$P = \frac{0.2}{T} = \frac{0.2}{0.2} = 1$

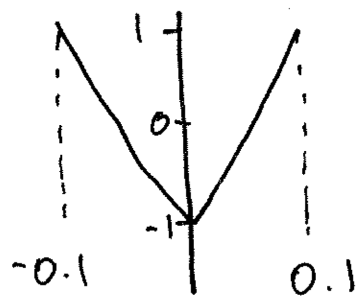
1.7 Determine the average power of the triangular wave shown in Fig 1.15

Fig. 1.15




so $T = 0.2$

look at $-0.1$ to $0.1$ interval

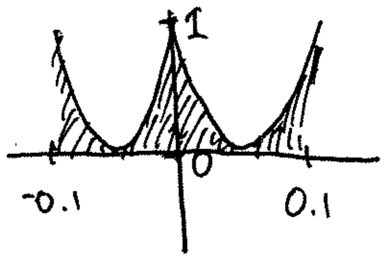



from $[0, 0.1]$ $x(t) = 20t - 1$

from $[-0.1$ to $0]$ $x(t) = -20t - 1$

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} x^2(t)\,dt$$

$x^2(t)$ looks like:




due to ~~symmetrye~~ symmetry:

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} x^2(t)\,dt = \frac{2}{T}\int_0^{T/2} x^2(t)\,dt$$

from $(0, 0.1)$

$$x^2(t) = 400t^2 - 40t + 1$$

continues on next page

$$P = \frac{2}{0.2}\int_0^{0.1} (400t^2 - 40t + 1)\,dt = 10\left( \frac{400}{3} t^3 \Big|_0^{0.1} - \frac{40}{2} t^2 \Big|_0^{0.1} + t \Big|_0^{0.1} \right)$$

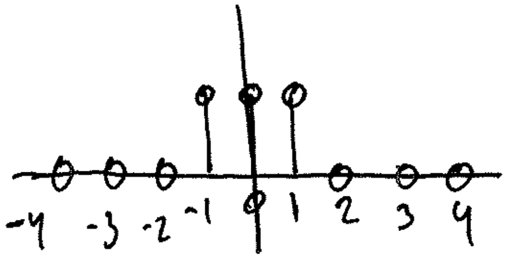
$$= 10\left( \frac{400}{3}\left(\frac{1}{10}\right)^3 - \frac{40}{2}\left(\frac{1}{10}\right)^2 + 0.1 \right)$$

$$= 10\left( \frac{4\cancel{00}}{30\cancel{00}} - \frac{4\cancel{0}}{20\cancel{0}} + 0.1 \right) = \frac{40}{30} - \frac{40}{20} + 1 = \frac{4}{3} - 2 + 1 = \frac{4}{3} - 1$$

$$= \frac{1}{3}$$

1.8: Determine the total energy of the discrete-time signal shown in Fig 1.17

Fig. 1.17:




$$E = \sum_{n=-\infty}^{\infty} x^2[n] \quad \text{(eqn. 1.18)}$$

$$E = \sum_{n=-1}^{n=1} 1^2 = 1^2 + 1^2 + 1^2 = 3$$

Problem 1.9 a, b, c, f, h

a)

$$x(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

This is an energy signal, the energy is

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_0^1 t^2\,dt + \int_1^2 (4-4t+t^2)\,dt$$

$$= \frac{t^3}{3}\Big|_0^1 + 4t\Big|_1^2 - \frac{4t^2}{2}\Big|_1^2 + \frac{t^3}{3}\Big|_1^2$$

$$= \frac{1}{3} + 4(2-1) - \frac{4}{2}(4-1) + \frac{1}{3}(8-1)$$

$$= \frac{1}{3} + 4 - 2(3) + \frac{1}{3}(7) = \frac{1}{3} + 4 - 6 + \frac{7}{3}$$

$$= \frac{8}{3} - 2 = \frac{2}{3}$$

1.9 b

$$x[n] = \begin{cases} n & 0 \le n < 5 \\ 10-n & 5 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$$

This is an energy signal with energy

$$E = \sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=0}^{4} n^2 + \sum_{n=5}^{10} 100 - 20n + n^2$$

| | | | |
|---|---|---|---|
| $n=0$ | $n^2=0$ | $n=5$ | $100-20n+n^2 = 100-100+25 = 25$ |
| $n=1$ | $n^2=1$ | $n=6$ | $100-20n+n^2 = 100-120+36 = 16$ |
| $n=2$ | $n^2=4$ | $n=7$ | $100-20n+n^2 = 100-140+49 = 9$ |
| $n=3$ | $n^2=9$ | $n=8$ | $100-20n+n^2 = 100-160+64 = 4$ |
| $n=4$ | $n^2=16$ | $n=9$ | $100-20n+n^2 = 100-180+81 = 1$ |
| | | $n=10$ | $100-20n+n^2 = 100-200+100 = 0$ |
| | 30 | | 55 |

$30 + 55 = 85$

1.9

c)

$$x(t) = 5\cos(\pi t) + \sin(5\pi t)$$

$T = 2$ $\qquad$ $\omega = 5\pi = 2\pi f \Rightarrow f = \frac{5}{2} \Rightarrow T = \frac{2}{5}$

Since 2 is an integer multiple of $\frac{2}{5}$ then the period of $x(t)$ is $T = 2$

so

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)\,dt = \frac{1}{2}\int_{-1}^{1} \left(5\cos(\pi t) + \sin(5\pi t)\right)^2 dt$$

This integral can get pretty ugly so it's ok to solve it numerically with Matlab or a calculator

$$P = \frac{1}{2} \cdot 26 = 13$$

1.9 f

$$x[n] = \begin{cases} \sin(\pi n) & -4 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$$

for integer $n$ $\quad \sin(\pi n) = 0$

so this is a zero signal and its energy and power are therefore zero as well

1.9 h

$$x[n] = \begin{cases} \cos(\pi n) & , n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\cos(\pi n) = \begin{cases} 1 & \text{if } n \text{ is even} \\ -1 & \text{if } n \text{ is odd} \end{cases}$$

This is a power signal with period $N = 2$ samples

$$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n] = \frac{1}{2} \sum_{n=0}^{1} \cos^2(\pi n) = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = 1$$

Students should check this result as it does not match with the book.

1.43 The sinusoidal signal

$$x(t) = 3\cos(200t + \pi/6)$$

is passed through a square-law device defined by the input-output relationship

$$y(t) = x^2(t)$$

Using the trig. identity

$$\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$$

Show that the output $y(t)$ consists of a dc component and a sinusoidal component

a) Specify the dc. component

b) Specify amplitude and frequency of the sinusoidal component in output $y(t)$

$$y(t) = x^2(t) = 9\cos^2(200t + \frac{\pi}{6}) = \frac{9}{2}\left(\cos(400t + \frac{\pi}{3}) + 1\right)$$

$$= \underbrace{\frac{9}{2}\cos(400t + \frac{\pi}{3})}_{\text{Sinusoid component}} + \underbrace{\frac{9}{2}}_{\text{D.C. component } = \frac{9}{2}}$$

Amplitude is $\frac{9}{2}$

Frequency is $2\pi f = 400 \Rightarrow f = \frac{400}{2\pi}$

1.44 Consider the signal

$$x(t) = A\cos(\omega t + \phi)$$

Determine the average power

Since we are only looking to get the average power of $x(t)$, we can get away with only considering

$$x'(t) = A\cos(\omega t)$$

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} \Rightarrow T = \frac{2\pi}{\omega}$$

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} x'^2(t)\,dt = \frac{\omega}{2\pi}\int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} A^2\cos^2(\omega t)\,dt$$

$$= \frac{\omega A^2}{2\pi}\int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} \cos^2(\omega t)\,dt$$

$$= \frac{\omega A^2}{2\pi}\left[\frac{t}{2} + \frac{1}{4\omega}\sin(2\omega t)\right]_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}}$$

$$= \frac{\omega A^2}{2\pi}\left[\left(\frac{\pi}{2\omega} - \left(\frac{-\pi}{2\omega}\right)\right) + \frac{1}{4\omega}\left(\sin(2\pi) - \sin(-2\pi)\right)\right]$$

$$= \frac{\omega A^2}{2\pi}\left[\frac{\pi}{\cancel{2}\omega} + \frac{1}{4\omega}\cdot 0\right] = \frac{\omega A^2}{2\pi}\cdot\frac{\pi}{\omega} = \frac{A^2}{2}$$

1.45 The angular frequency $\Omega$ of the sinusoidal signal

$$x[n] = A\cos(\Omega n + \phi)$$

satisfies the condition for $x[n]$ to be periodic. determine the average power of $x[n]$

Again because we care for average power it's equivalent to finding the average power of

$$x'[n] = A\cos(\Omega n)$$

$$P = \frac{1}{N}\sum_{n=0}^{N-1} x'^2[n] = \frac{1}{N}\sum_{n=0}^{N-1} A^2\cos^2(\Omega n)$$

$$= \frac{A^2}{N}\sum_{n=0}^{N-1}\cos^2(\Omega n) = \frac{A^2}{N}\sum_{n=0}^{N-1}\frac{1}{2}\left(\cos(2\Omega n)+1\right)$$

$$= \frac{A^2}{2N}\left(\cancel{\sum_{n=0}^{N-1}\cos(2\Omega n)}^{\,0} + \cancel{\sum_{n=0}^{N-1} 1}^{\,N}\right)$$

$$= \frac{A^2}{2N}\cdot N = \frac{A^2}{2}$$

1.49 A rectangular pulse $x(t)$ is defined by

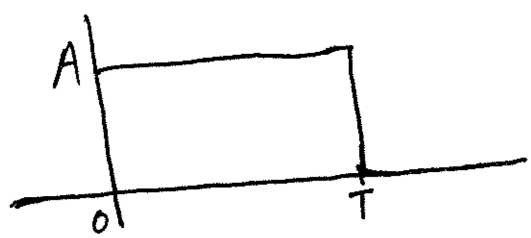
$$x(t) = \begin{cases} A, & 0 \le t \le T \\ 0 & \text{Otherwise} \end{cases}$$

The pulse $x(t)$ is applied to an integrator defined by

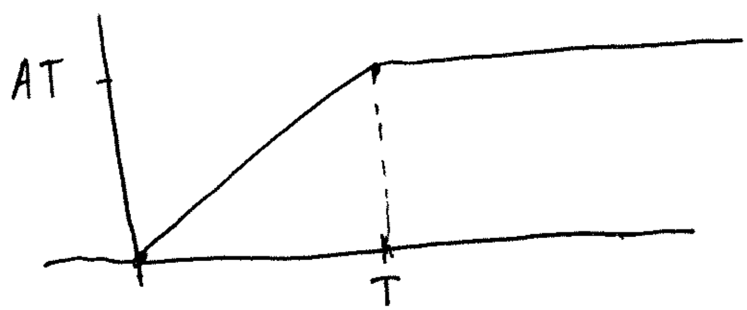
$$y(t) = \int_{0^-}^{t} x(\tau)\, d\tau$$

Find the total energy of the output $y(t)$

First let's draw $x(t)$




$y(t)$ will be the area under $x(t)$ from $0$ to $t$ so it should look like this:




Since $E = \int_{-\infty}^{\infty} y^2(t)\, dt$ then looking at the above plot of $y(t)$ $E$ is clearly infinite.