1.17

a) \( x[n] = 5 \sin [2n] \)
non-periodic

b) \( x[n] = 5 \cos [0.2 \pi n] \)

Periodic
\( \omega = 0.2 \pi = 2 \pi f \Rightarrow f = \frac{0.2 \pi}{2 \pi} \Rightarrow \frac{1}{f} = \frac{2}{0.2} = 10 \)
here \( N = \frac{1}{f} = 10 \)

c) \( x[n] = 5 \cos [6 \pi n] \)
\( \omega = 6 \pi = 2 \pi f \Rightarrow f = \frac{6 \pi}{2 \pi} \Rightarrow \frac{1}{f} = \frac{1}{3} \)

At this moment, claiming \( \frac{1}{3} \) would be incorrect as the fundamental period for a discrete-time signal must also be discrete (integer). The fundamental period is going to be the first integer multiple of \( \frac{1}{3} \). \( \frac{1}{3} \cdot 3 = 1 \) — The fundamental period is \( N = 1 \)

d) \( x[n] = 5 \sin [6 \pi n / 35] \)
\( \omega = \frac{6 \pi}{35} = 2 \pi f \Rightarrow f = \frac{6 \pi}{2 \pi \cdot 35} \)

\( f = \frac{3}{35} \Rightarrow \frac{1}{f} = \frac{35}{3} \)

\( N \) is going to be the smallest integer that is an integer multiple of \( \frac{1}{3} \)
\( N = \frac{1}{3} \cdot 3 \cdot \frac{35}{3} \cdot 3 = 35 \)
1.18 Find the smallest angular frequencies for DT sinusoids.

a) \( N = 8 \)

Let's say \( N = 8 = \frac{\pi}{8} \)

\[
{f} = \frac{1}{{\frac{\pi}{8}}} = \frac{1}{8}
\]

\[
{\omega} = 2\pi f = \frac{2\pi}{8} = \frac{\pi}{4}
\]

b) \( N = 32 \)

\[
N = 32 = \frac{\pi}{32}
\]

\[
{f} = \frac{1}{{\frac{\pi}{32}}} = \frac{1}{32}
\]

\[
{\omega} = 2\pi f = \frac{2\pi}{32} = \frac{\pi}{16}
\]

c) \( N = 64 \)

\[
N = 64 = \frac{\pi}{64}
\]

\[
{f} = \frac{1}{{\frac{\pi}{64}}} = \frac{1}{64}
\]

\[
{\omega} = 2\pi f = \frac{2\pi}{64} = \frac{\pi}{32}
\]

d) \( N = 128 \)

\[
N = 128 = \frac{\pi}{128}
\]

\[
{f} = \frac{1}{{\frac{\pi}{128}}} = \frac{1}{128}
\]

\[
{\omega} = 2\pi f = \frac{2\pi}{128} = \frac{\pi}{64}
\]
$$x[n] = \begin{cases} 
1 & 0 \leq n \leq 9 \\
0 & \text{otherwise}
\end{cases}$$

Express \(x[n]\) as a superposition of 2 step functions.

First, sketch \(x[n]\)

Now sketch \(u[n]\)

This is almost correct except that for \(u[n]\), with \(n > 9\) we have \(u[n] = 1\) whereas for \(x[n]\) if \(n > 9\) we have \(x[n] = 0\). We need a signal that is equal to \(-1\) when \(n > 9\) and \(0\) elsewhere and then add this signal to \(u[n]\). That signal turns out to be \(-u[n-10]\)

\[x[n] = u[n] + [u[n-10]] = u[n] - u[n-10]\]
1.57 If periodic show the fundamental period for:

a) \( x[n] = \cos \left( \frac{8}{15} \pi n \right) \)

\[ \omega = \frac{8}{15} \pi \Rightarrow f = \frac{8 \pi}{2 \cdot 15} = \frac{4}{15} = \frac{15}{9} \Rightarrow T = \frac{15}{9} \]

\[ N = T \cdot 4 = 15, \text{ Periodic} \]

b) \( x[n] = \cos \left( \frac{7}{15} \pi n \right) \)

\[ \omega = \frac{7}{15} \pi n \Rightarrow f = \frac{7 \pi}{2 \cdot 15} = \frac{7}{30} \Rightarrow T = \frac{30}{7} \]

\[ N = T \cdot 7 = 30 \]

c) \( x(t) = \cos(2t) \cdot \sin(3t) \)

(\( \cos(2t) \))

\[ \omega_1 = 2 = 2 \pi f_1 \Rightarrow f_1 = \frac{2}{2 \pi} \Rightarrow T_1 = \pi \]

(\( \sin(3t) \))

\[ \omega_2 = 3 = 2 \pi f_2 \Rightarrow f_2 = \frac{3}{2 \pi} \Rightarrow T_2 = \frac{2 \pi}{3} \]

The LCM of \( T_1 \) and \( T_2 \) is \( 2 \pi \)

So, the fundamental period of \( x(t) = \cos(2t) \cdot \sin(3t) \) is \( T = 2 \pi \)
d) \( x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t-2k) \)

Remember, we can view the \( \delta \) function as:

\[ \delta(t-2k) = 1 \quad \text{if} \quad t = 2k = 0 \Rightarrow t = 2k \]

So \((-1)^k \delta(t-2k) = 0 \) unless \( t = 2k \Rightarrow k = \frac{t}{2} \)

since \( k \) is an integer value.

When \( t \) is odd, then

\[ x(t) = 0 \]

Now if \( t \) is an integer multiple of 2 and not of 4 then \( k \) is odd

so in that case

\[ x(t) = -1 \]

if \( t \) is an integer multiple of 4 then \( k \) is even and

\[ x(t) = 1. \] The sketch of

\[ x(t) \] is

\[ \text{So } \tau = 4 \]
\[ x[n] = \sum_{k=-\infty}^{k=\infty} \{ \delta[n-3k] + \delta[n-k^2] \} \]

Look first at

\[ m = \delta[n-3k] \]

\[ \delta[n-3k] = 1 \text{ if } n-3k = 0 \Rightarrow n = 3k \]

Now look at

\[ f[n-k^2] \]

\[ \delta[n-k^2] = 1 \text{ if } n-k^2 = 0 \Rightarrow n = k^2 \]

Remember as well that the sum can be broken in two

\[ x[n] = \sum_{k=-\infty}^{k=\infty} \delta[n-3k] + \sum_{k=-\infty}^{k=\infty} \delta[n-k^2] \]

\[ \text{this sum equals} \quad \text{this sum equals} \]

\[ 1 \text{ when } n \text{ is divisible by } 3 \]

\[ 2 \text{ when } n \text{ is a power of } 2 \]

This is clearly not periodic as

\[ x[n] \text{ is the sum of two signals and one of these signals is aperiodic} \]
f) \( x(t) = \cos(t) u(t) \)

because of \( u(t) \), we have

\[
X(t) = \begin{cases} 
\cos(t) & \text{if } t > 0 \\
0 & \text{if } t \leq 0 
\end{cases}
\]

This signal is a periodic

g) \( x(t) = u(t) + u(-t) \), where

\( u(t) = \cos(t) u(t) \)

Since \( u(t) \) is undefined at \( t = 0 \)
then \( x(t) \) cannot be periodic

h) \( x(t) = u(t) + u(-t) \), where

\( u(t) = \sin(t) u(t) \)

Since \( u(t) \) is undefined at \( t = 0 \)
then \( x(t) \) cannot be periodic
1.57

i) \( x[n] = \cos\left(\frac{1}{5} \pi n\right) \sin\left(\frac{1}{3} \pi n\right) \)

Let's plot \( \cos\left(\frac{1}{5} \pi t\right) \) starting from 0 to 30

At first glance, \( x[n] = \cos\left(\frac{1}{5} \pi n\right) \sin\left(\frac{1}{3} \pi n\right) \)
looks like it should have period 30 because that's the LCM of \( T_1 = 60 \) and \( T_2 = 6 \) but let's analyze what happens from (0 to 15) and (15 to 30) from it turns out that for \( t = (15 \text{ to } 30) \)

\[
\begin{align*}
\cos\left(\frac{1}{5} \pi t\right) &= -\cos\left(\frac{1}{5} \pi (t-15)\right) \\
\sin\left(\frac{1}{3} \pi t\right) &= -\sin\left(\frac{1}{3} \pi (t-15)\right)
\end{align*}
\]
so

\[
\begin{align*}
\cos\left(\frac{1}{5} \pi (t+15)\right) \sin\left(\frac{1}{3} \pi (t+15)\right) &= -\cos\left(\frac{1}{5} \pi (t-15)\right) \sin\left(\frac{1}{3} \pi (t-15)\right)
\end{align*}
\]

so \( T = 15 \)
and since \( T \) is integer \( N = 15 \)
1.58

\[ N = 10 \]
\[ T = 10 \]
\[ f = \frac{1}{10} \Rightarrow \omega = \frac{2\pi}{10} = \frac{\pi}{10} \]

1.59: \text{complex signal} x(t)

\[ \Re\{x(t)\} = X_R(t) = A \cos(\omega t + \phi) \]
\[ \Im\{x(t)\} = X_I(t) = A \sin(\omega t + \phi) \]

amplitude \( x(t) \) = \( \sqrt{X_R^2(t) + X_I^2(t)} \)

Show that amplitude equals \( A \) despite phase angle \( \phi \)

we can substitute first by saying

\[ s = \omega t + \phi \quad \text{so} \]

\[ X_R(s) = A \cos(s) \]
\[ X_I(s) = A \sin(s) \]

amplitude \( x(t) \) = amplitude \( x(s) \) = \( \sqrt{A^2 \cos^2(s) + A^2 \sin^2(s)} \)

= \( \sqrt{A^2 \cos^2(s) + \sin^2(s)} \)

= \( A \sqrt{\cos^2(s) + \sin^2(s)} \)

= \( A \), as \( \cos^2(s) + \sin^2(s) = 1 \)

Q.E.D.
\[ x(t) = \begin{cases} 
\sqrt{\Delta} + 0.5 & -\Delta/2 \leq t \leq \Delta/2 \\
1 & t > \Delta/2 \\
0 & t < -\Delta/2 
\end{cases} \]

\( x(t) \) is applied to a differentiator and the output is \( y(t) \). Show that \( y(t) \) approaches \( s(t) \) when \( \Delta \to 0 \)

Let's look at the signal \( x(t) \) for \( \Delta \)

\( x(t) \) looks like this:

\[ \begin{array}{c}
-\Delta/2 \\
0.5 \\
0 \\
\Delta/2 \\
1
\end{array} \]

and as \( \Delta \) approaches zero \( x(t) \) approaches this Dirac delta signal

\[ \begin{array}{c}
0.5 \\
0 \\
1
\end{array} \]

which is pretty much the unit step function.

We know already that \( \frac{d}{dt} u(t) = s(t) \) so therefore \( y(t) \to s(t) \) when \( \Delta \to 0 \) Q.E.D.