1.17

ECEN 314 HW 3

Solved by Sanjay Nair
sanjaynair@neo.tamu.edu

a) $x[n] = 5\sin[2n]$

non-periodic

b) $x[n] = 5\cos[0.2\pi n]$

Periodic

$\omega = 0.2\pi = 2\pi f \Rightarrow f = \frac{0.2\cancel{\pi}}{2\cancel{\pi}} \Rightarrow T = \frac{1}{f} = \frac{2}{0.2} = 10$

here $N = T = 10$

c) $x[n] = 5\cos[6\pi n]$

$\omega = 6\pi = 2\pi f \Rightarrow f = \frac{6\cancel{\pi}}{2\cancel{\pi}} \Rightarrow T = \frac{1}{f} = \frac{1}{3}$

At this moment, claiming $T = \frac{1}{3}$ would be incorrect as the fundamental period ~~h~~ for a discrete-time signal must also be discrete (integer). The fundamental period is going to be the first integer multiple of T. $T \cdot 3 = 1$ ← The fundamental period is ~~1~~ $N = 1$

d) $x[n] = 5\sin[6\pi n/35]$

~~w~~ $\omega = \frac{6\pi}{35} = 2\pi f \Rightarrow f = \frac{6\cancel{\pi}}{2\cancel{\pi}\cdot 35}$

$f = \frac{3}{35} \Rightarrow T = \frac{35}{3}$

N is going to be the smallest integer that is an integer multiple of T

$N = T \cdot 3 = \frac{35}{3} \times 3 = 35$

1.18 Find the smallest angular frequencies for DT sinusoids with fundamental periods:

a) $N = 8$

Let's say $N = 8 = T$

$$f = \frac{1}{T} = \frac{1}{8}$$

$$\omega = 2\pi f = \frac{2\pi}{8} = \frac{\pi}{4}$$

b) $N = 32$

$$N = 32 = T$$

$$f = \frac{1}{T} = \frac{1}{32}$$

$$\omega = 2\pi f = \frac{2\pi}{32} = \frac{\pi}{16}$$

c) $N = 64$

$$N = 64 = T$$

$$f = \frac{1}{T} = \frac{1}{64}$$

$$\omega = 2\pi f = \frac{2\pi}{64} = \frac{\pi}{32}$$

d) $N = 128$

$$N = 128 = T$$

$$f = \frac{1}{T} = \frac{1}{128}$$

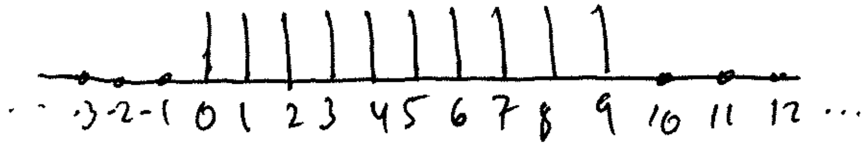
$$\omega = 2\pi f = \frac{2\pi}{128} = \frac{\pi}{64}$$

1.22

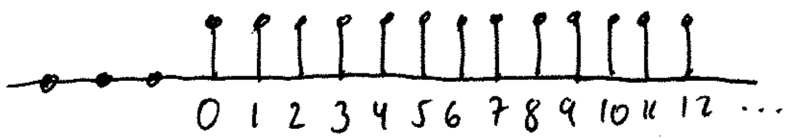
$$x[n] = \begin{cases} 1 & 0 \leq n \leq 9 \\ 0 & 0 \text{ otherwise} \end{cases}$$

Express $x[n]$ as a superposition of 2 step functions.

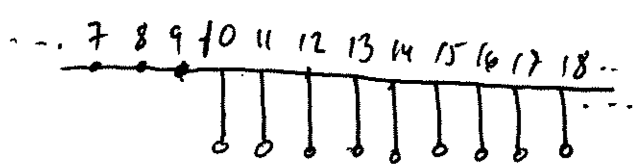
First, sketch $x[n]$



Now sketch $u[n]$



This is almost correct except that for $u[n]$, with $n > 9$ we have $u[n] = 1$ wheras for $x[n]$ if $n > 9$ we have $x[n] = 0$. We need a signal that is equal to -1 when $n > 9$ and 0 elsewhere and then add this signal to $u[n]$. That signal turns out to be $-u[n-10]$



$$x[n] = u[n] + [-u[n-10]] = u[n] - u[n-10]$$

1.57 if periodic show the fundamental period for:

a) $x[n] = \cos\left(\frac{8}{15}\pi n\right)$

$$\omega = \frac{8}{15}\pi \Rightarrow f = \frac{8\pi}{2\cdot\pi\cdot 15} = \frac{4}{15} \Rightarrow T = \frac{15}{4}$$

$N = T\cdot 4 = 15$, Periodic

b) $x[n] = \cos\left(\frac{7}{15}\pi n\right)$

$$\omega = \frac{7}{15}\pi n \Rightarrow f = \frac{7\pi}{2\cdot\pi\cdot 15} = \frac{7}{30} \Rightarrow T = \frac{30}{7}$$

$W = T\cdot 7 = 30$

c) $x(t) = \cos(2t) + \sin(3t)$

$\cos(2t)$

$$\omega_1 = 2 = 2\pi f_1 \Rightarrow f_1 = \frac{2}{2\pi} \Rightarrow T_1 = \pi$$

$\sin(3t)$

$$\omega_2 = 3 = 2\pi f_2 \Rightarrow f_2 = \frac{3}{2\pi} \Rightarrow T_2 = \frac{2\pi}{3}$$

The LCM of T_1 and T_2 is 2π

So, the fundamental period of $x(t) = \cos(2t) + \sin(3t)$ is $T = 2\pi$

d) $x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t-2k)$

Remember, we can view the δ function as

$$\delta(t-2k)=1 \quad \text{if} \quad t-2k=0 \Rightarrow t=2k$$

So $(-1)^k \delta(t-2k)=0$ unless $t=2k \Rightarrow k=\frac{t}{2}$

since k is an integer value

when t is odd then

$x(t)=0$

Now if t is an integer multiple of 2

and not of 4 then k is odd

so in that case

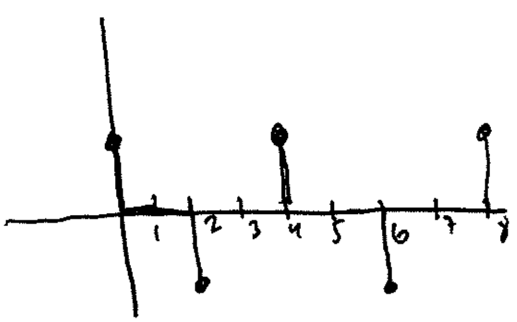
$x(t)=-1$

if t is an integer multiple of 4

then k is even and

$x(t)=1$. The sketch of

$x(t)$ is



So $T=4$

1.57

e) $x[n] = \sum_{k=-\infty}^{k=\infty} \{ \delta[n-3k] + \delta[n-k^2] \}$

Look first at

$$\delta[n-3k]$$

$$\delta[n-3k] = 1 \quad \text{if} \quad n-3k = 0 \Rightarrow n = 3k$$

Now look at

$$\delta[n-k^2]$$

$$\delta[n-k^2] = 1 \quad \text{if} \quad n-k^2 = 0 \Rightarrow n = k^2$$

Remember as well that the sum can be broken in two

$$x[n] = \sum_{k=-\infty}^{k=\infty} \delta[n-3k] + \sum_{k=-\infty}^{k=\infty} \delta[n-k^2]$$

this sume equals 1 when n is divisible by 3

this sum equals 2 ~~1~~ when n is a power of 2

This is clearly not periodic as $x[n]$ is the sum of two signals and one of these signals is aperiodic

1.57

f) $x(t) = \cos(t)\,u(t)$

because of $u(t)$, we have

$$x(t) = \begin{cases} \cos(t) & \text{if } t > 0 \\ 0 & \text{if } t < 0 \end{cases}$$

This signal is aperiodic

g) $x(t) = v(t) + v(-t)$, where

$v(t) = \cos(t)\,u(t)$

Since $u(t)$ is undefined at $t=0$

then $x(t)$ cannot be periodic

h) $x(t) = v(t) + v(-t)$, where

$v(t) = \sin(t)\,u(t)$

Since $u(t)$ is undefined at $t=0$

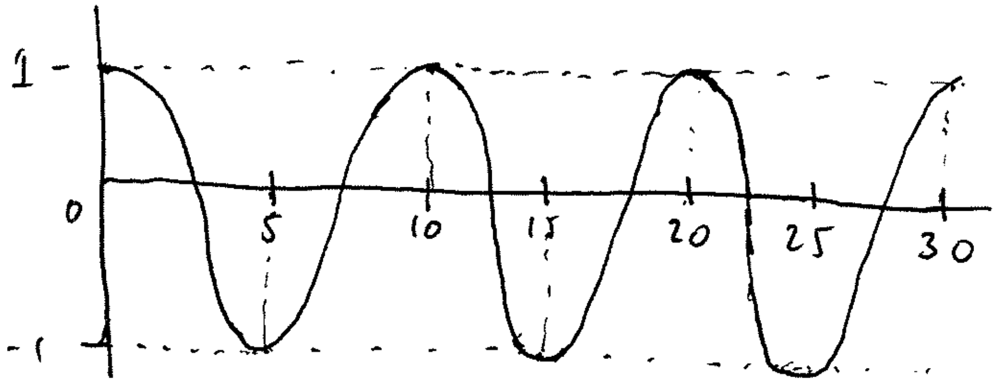
then $x(t)$ cannot be periodic

1.57

i) $x[n] = \cos\left(\frac{1}{5}\pi n\right)\sin\left(\frac{1}{3}\pi n\right)$

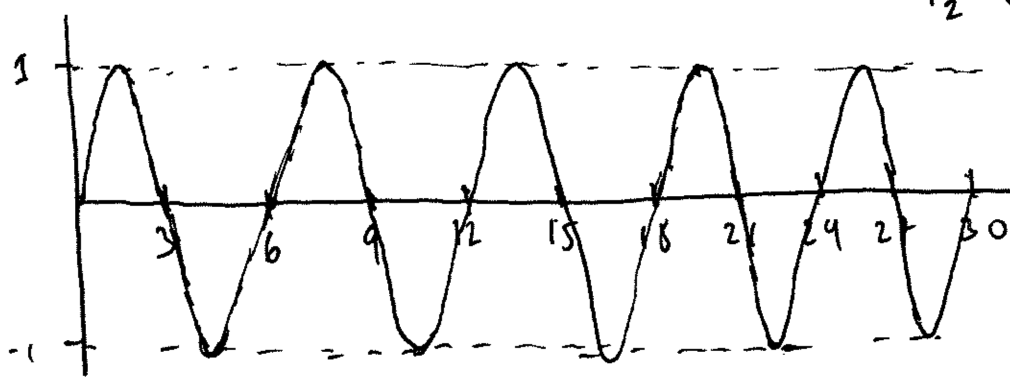
Let's plot $\cos\left(\frac{1}{5}\pi t\right)$ starting from 0 to 30

$T_1 = 10$



Let's plot $\sin\left(\frac{1}{3}\pi t\right)$ starting from 0 to 30

$T_2 = 6$



At first glance $x[n] = \cos\left(\frac{1}{5}\pi n\right)\sin\left(\frac{1}{3}\pi n\right)$ looks like it should have period 30 because that's the LCM of $T_1 = 10$ and $T_2 = 6$ but let's analyze what happens from (0 to 15) and (15 to 30)

~~from~~ it turns out that for $t = (15 \text{ to } 30)$

$\cos\left(\frac{1}{5}\pi t\right) = -\cos\left(\frac{1}{5}\pi(t-15)\right)$

$\sin\left(\frac{1}{3}\pi t\right) = -\sin\left(\frac{1}{3}\pi(t-15)\right)$

So $\cos\left(\frac{1}{5}\pi t\right)\sin\left(\frac{1}{3}\pi t\right) =$ ~~has~~ $\cos\left(\frac{1}{5}\pi(t-15)\right)\sin\left(\frac{1}{3}\pi(t-15)\right)$

so $T = 15$

and since T is integer $N = 15$

1.58

$N = 10$

$T = 10$

$f = \frac{1}{10} \Rightarrow \omega = \frac{2\pi}{10} = \frac{\pi}{10}$

1.59: Complex signal $x(t)$

$Re\{x(t)\} = X_R(t) = A\cos(\omega t + \phi)$

$Im\{x(t)\} = X_I(t) = A\sin(\omega t + \phi)$

amplitude $(x(t)) = \sqrt{X_R^2(t) + X_I^2(t)}$

Show that amplitude equals A despite phase angle ϕ

we can substitute first by saying

$s = \omega t + \phi$ so

$X_R(s) = A\cos(s)$

$X_I(s) = A\sin(s)$

amplitude $(x(t))$ = amplitude $(x(s)) = \sqrt{A^2\cos^2(s) + A^2\sin^2(s)}$

$= \sqrt{A^2(\cos^2(s) + \sin^2(s))}$

$= A\sqrt{\cos^2(s) + \sin^2(s)}$

$= A$, as $\cos^2(s) + \sin^2(s) = 1$

Q.E.D.

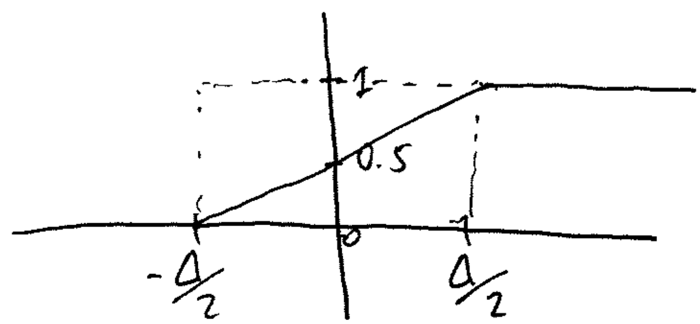
1.61.-

$$x(t) = \begin{cases} t/\Delta + 0.5 & -\Delta/2 \le t \le \Delta/2 \\ 1 & t > \Delta/2 \\ 0 & t < -\Delta/2 \end{cases}$$

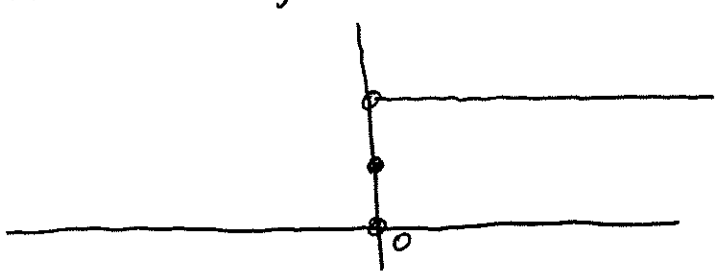
$x(t)$ is applied to a differentiator and the output is $y(t)$. show that $y(t)$ approaches $\delta(t)$ when $\Delta \to 0$

let's look at the signal $x(t)$ for Δ

$x(t)$ looks like this:



and as Δ approaches zero $x(t)$ approaches this ~~function~~ signal



~~passion~~ which is pretty ~~act~~ much the unit step function

we know already that

$\frac{d}{dt} u(t) = \delta(t)$ so therefore $y(t) \to \delta(t)$ when $\Delta \to 0$ Q.E.D.