ECEN 314 HW 4 Solved by Sanjay Nair

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1.26

$$y[n] = \sum_{k=0}^{\infty} \rho^k x[n-k]$$

Show that the system is BIBO unstable if

$|\rho| \geq 1$

Let's look at the sum

$$\sum_{k=0}^{\infty} \rho^k x[n-k]$$

If this were BIBO then for any bounded signal ~~y~~ $x[n]$, $y[n]$ must also be bounded. Then ~~this~~ So ~~l~~et's look at $x[n] = 1$.

~~sum~~

$$y[n] = \sum_{k=0}^{\infty} \rho^k x[n-k] = \sum_{k=0}^{\infty} \rho^k$$

this sum $\sum_{k=0}^{\infty} \rho^k$ is never bounded if

$|\rho| \geq 1$

1.64

a) $y(t) = \cos(x(t))$

i) it is memoryless because $x(t)$ is only consulted at time t

ii) The system is stable because $\cos(x(t))$ can in any case only return answers between -1 and 1

iii) It is causal because memoryless implies causality

iv) This is non-linear because

$$\cos(x_1(t) + x_2(t)) \neq \cos(x_1(t)) + \cos(x_2(t))$$

v) It is time invariant

b) $y[n] = 2x[n]u[n]$

i) memoryless

ii) It is BIBO stable because if $x[n]$ is bounded, multiplying it by 2 will still be bounded

iii) causal

iv) $$2(Ax_1[n] + Bx_2[n])u[n] =$$

$$2Ax_1[n]u[n] + 2Bx_2[n]u[n]$$

So it is linear

v) not time invariant because

$$2[x[n-n_0]]u[n] \neq 2x[n-n_0]u[n-n_0]$$

1.64

d) $y(t) = \int_{-\infty}^{t/2} x(\tau)\,d\tau$

i) This is not memoryless because $x(t)$ is consulted at times not equal to t

ii) This is not BIBO stable. If we take as an example $x(t) = 1$ then we can see that if $t \to \infty$ then $y(t) = \int_{-\infty}^{t/2} x(\tau)\,d\tau \to \infty$

iii) The system is non-causal. If you consider a t that is negative then you can clearly see that at a negative t $x(t)$ is going to be consulted at time $\frac{t}{2} > t$

iv) it is linear as integration is a famously linear operator

v) For ~~LTI~~ Time invariance we need

$$x_1(t) \xrightarrow{H} y_1(t) = \int_{-\infty}^{\frac{t}{2}} x_1(\tau)\,d\tau$$

then

$$x_1(t-t_0) \xrightarrow{H} y_1(t-t_0) = \int_{-\infty}^{\frac{t-t_0}{2}} x_1(\tau)\,d\tau$$

(continues)

1.64

d) v) continued

let's enter $x_1(t-t_0)$ into the system H

then

$$y_1(t) = \int_{-\infty}^{\frac{t}{2}} x_1(\tau - t_0)\, d\tau$$

if we transform

$\mu = \tau - t_0$, we need to change the bounds for this transform and the differential

$$\frac{d\mu}{d\tau} = \frac{d(\tau - t_0)}{d\tau} = 1 \Rightarrow d\mu = d\tau$$

lower bound

$$\mu = \tau - t_0 = \infty$$

upper bound

$$\mu = \tau - t_0 \,, \quad \tau = t/2$$

$$\mu = \frac{t}{2} - t_0 = \frac{t - 2t_0}{2}$$

we have

$$\int_{-\infty}^{\frac{t-2t_0}{2}} x_1(\mu)\, d\mu \neq \int_{-\infty}^{\frac{t-t_0}{2}} x_1(\tau)\, d\tau$$

1.64

e) $y[n] = \sum_{k-\infty}^{n} x[k+2]$

i) It is not memoryless; ~~an~~ the sum consults x at times other than n

ii) Not stable, consider if $x[n] = 1$ than if $n \to \infty$ $\sum_{k=-\infty}^{n} x[k+2] \to \infty$

iii) It is non-causal because $x[n]$ is eventually ~~const~~ ~~can~~ going to be consulted at sample $x[n+1]$ and $x[n+2]$

iv) It is linear:

$$\sum_{k=-\infty}^{n} \left(A x_1[k+2] + B x_2[k+2]\right)$$

$$= A \sum_{k=-\infty}^{n} x_1[k+2] + B \sum_{k=-\infty}^{n} x_2[k+2]$$

1.64

e) $y[n] = \sum_{k=-\infty}^{n} x[k+2]$

v) time invariant ?

if time invariant then

$$X_1[n-n_0] \xrightarrow{H} y_1[n-n_0] = \sum_{k=-\infty}^{n-n_0} x[k+2]$$

input $x_1[n-n_0]$ into system we ~~k.~~ get

$$\sum_{k=-\infty}^{n} x[k+2-n_0]$$

consider transform $l = k - n_0$

~~if~~ we need to change the bounds

if $k = -\infty$

$l = -\infty$

if $k = n$

$l = n - n_0$

so we have

$$\sum_{l=-\infty}^{l=n-n_0} x[l+2] = \sum_{k=-\infty}^{k=n-n_0} x[k+2]$$

So it's TI

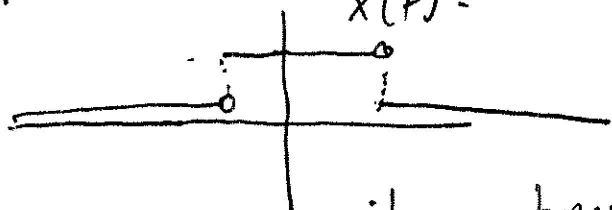
1.64

f) $y(t) = \frac{d}{dt} x(t)$

i) Technically this is memoryless because $x(t)$ is only consulted ~~when~~ at time t

ii) This system is not stable. If we input this signal:

$x(t) =$



its output $y(t)$ will not be bounded

iii) As it is technically memoryless, it is also technically causal

iv) It is linear; the derivative operator is famously linear

v) So formally speaking

$$y(t) = \lim_{\delta \to 0} \frac{x(t+\delta) - x(t)}{(t+\delta) - (t)}$$

input $x_1(t - t_0)$ into this we get

$$\lim_{\delta \to 0} \frac{x(t - t_0 + \delta) - x(t - t_0)}{(t+\delta) - t}$$

if we just shift $y_1(t - t_0) = \frac{x(t - t_0 + \delta) - x(t - t_0)}{(t - t_0 + \delta) - (t - t_0)}$ so it's time invariant

1.64

i) $x(2-t)$

i) Not memoryless

ii) This is clearly BIBO stable as the system is just a reflection and a shift

iii) non-causal; notice that if t is negative, x is consulted after time t

iv) this is linear

v) not time invariant because

$$\underbrace{x(2-t-t_0)}_{\text{output after shifted input}} \neq \underbrace{x(2-(t-t_0))}_{\text{shifted output}}$$

1.64

j) $$y[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-2k]$$

Before we do anything we should notice what the relationship between $y[n]$ and $x[n]$ is. It's easy to get overwhelmed by this sum but if we look at this we see:

$$y[n] = \begin{cases} x[n] & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

The system simply extracts the even-sampled ~~sampled~~ samples of x

So

i) it is memoryless, x is consulted only at sample n

ii) it is BIBO stable

iii) it is causal

iv) linear as well

v) NOT time invariant

1.68

Is it possible for a noncausal system to possess memory? Yes

A system is said to possess memory if its output signal depends on past or future values of the input signal.

A system is said to be causal if the present value of the output signal depends only on the present or past values of the input signal

Therefore, all non-causal systems have outputs dependent on future values of the input and by definition they all have memory

1.72

$y(t) = x^p(t)$, p integer and $p \neq 0, 1$

let's consider p positive

for ~~y~~ the system to be linear it must at least follow Superposition:

so if

$$x_1(t) \xrightarrow{H} y_1(t) = x_1^p(t)$$

and

$$x_2(t) \xrightarrow{H} y_2(t) = x_2^p(t)$$

then

$$x_1(t) + x_2(t) \xrightarrow{H} y_1(t) + y_2(t) = x_1^p(t) + x_2^p(t)$$

this is simply not true because

$$x_1(t) + x_2(t) \xrightarrow{H} y_3(t) = (x_1(t) + x_2(t))^p$$
$$\neq x_1(t)^p + x_2(t)^p$$

So the system is not linear

Q.E.D.

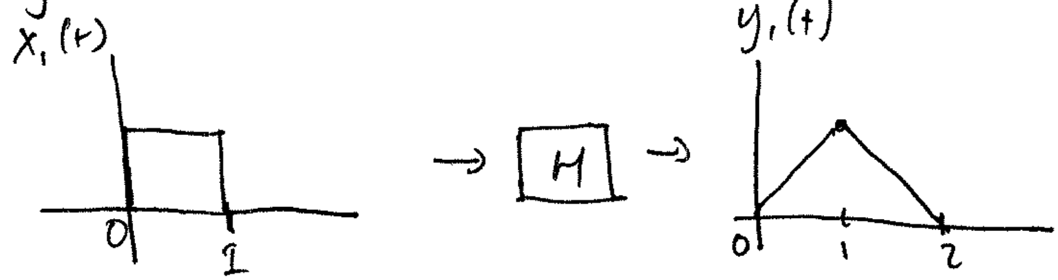
1.73 A linear time-invariant system may be causal or non-causal, Give an example of each.

$y(t) = x(t)$ is LTI and causal

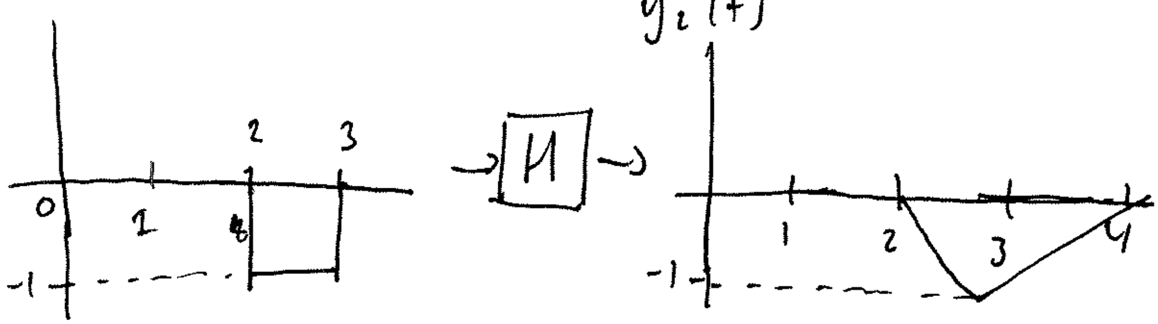
$y(t) = x(t+1)$ is LTI and noncausal

1.75

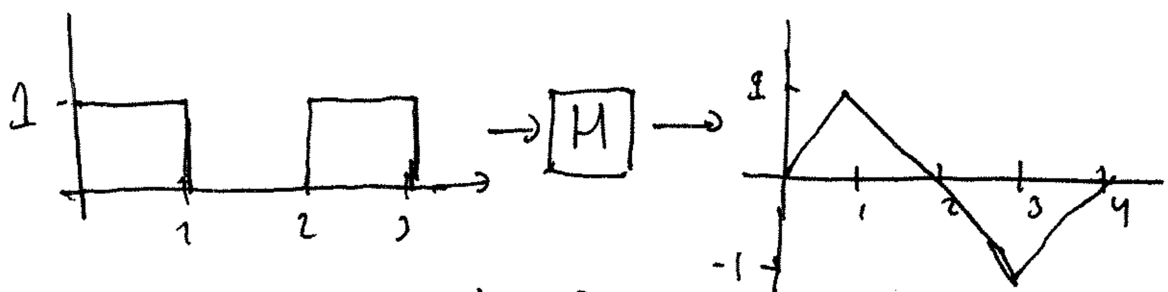
a) Fig 1.75 a



$x_2(t)$



$x_3(t)$



Could it be memoryless?

No, in the input output relationship for $x_1(t)$ show that for $x_1(t<0)=0$ $y_1(t<0)=0$ but when $x_1(t=1.5)=0$ $y_1(t=1.5)\neq 0$

Could it be causal?

Yes

Could it be time invariant? Yes

Could it be linear? No

$x_3(t)=x_1(t)-x_2(t)$ but $y_3(t)\neq y_1(t)-y_2(t)$

1.75

b) I'm going to skip rewriting the figure

Could it be memoryless?

No, it can't be because we get nonzero and zero outputs when the input is zero.

could it be causal?

No, it can't because we have non-zero outputs before the ~~outputs~~ input's non-zero values enter the system

could it be time-invariant?

No, it can't be $x_4(t)$ is just $x_2(t)$ shifted right by 1 but the output $y_4(t)$ is not $y_2(t)$ shifted to the right by 1

can it be linear?

yes it could be, we have no evidence to conclude otherwise

1.76

a) The system cannot be causal because $y_2(t)$ has non-zero values before $x_2(t)$ has non-zero values

b) No, it can't be time invariant because $X_3(t)$ is just $x_1(t)$ shifted to the right by 1. This cannot be said about $y_1(t)$ and $y_3(t)$

c) it can't be memoryless because it's noncausal

d) $x(t) = X_1(t) + 2X_3(t)$