2.44

Sanjay Nair
HW 6 ECEN 314

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

$$\frac{d}{dt} y(t) = \frac{d}{dt} \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) \cdot \frac{d}{dt} h(t-\tau)\, d\tau$$

$$\frac{d}{dt} h(t-\tau) = \frac{d(t-\tau)}{dt} \cancel{dt} \frac{d}{d(t-\tau)} h(t-\tau)$$

$$= \frac{d}{d(t-\tau)} h(t-\tau)$$

$$\frac{d}{dt} y(t) = \int_{-\infty}^{\infty} x(\tau) \frac{d}{d(t-\tau)} h(t-\tau)\, d\tau$$

$$= x\cancel{(t) * \frac{d}{dt} h(t-\tau)}$$

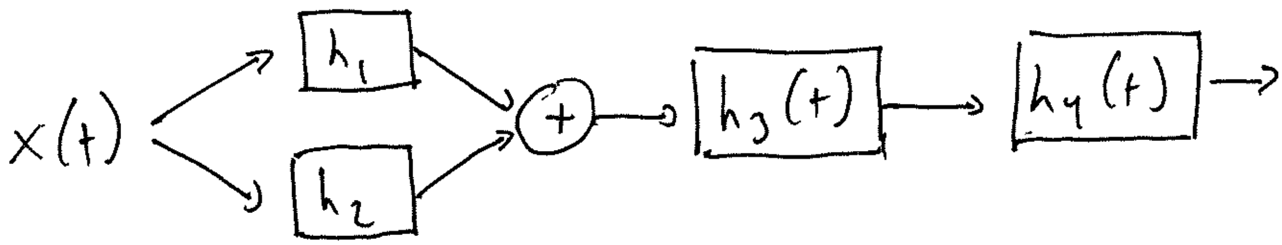
$$= x(t) * \frac{d}{dt} h(t) \qquad \text{Q.E.D.}$$

similarly for

$$\frac{d}{dt} y(t) = \left(\frac{d}{dt} x(t)\right) * h(t)$$

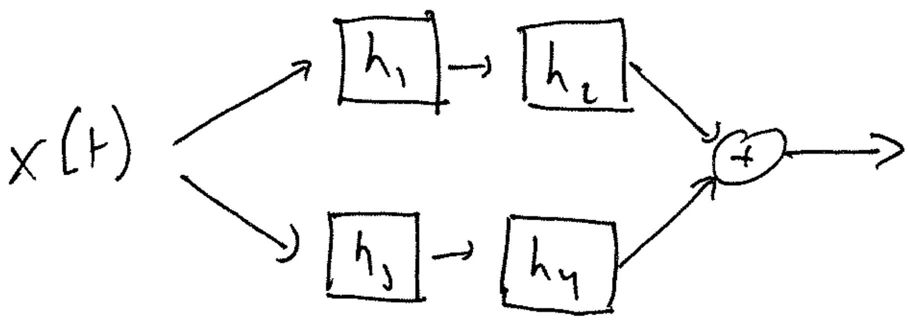
2.47

a) $h(t) = \{h_1(t) + h_2(t)\} * h_3(t) * h_4(t)$

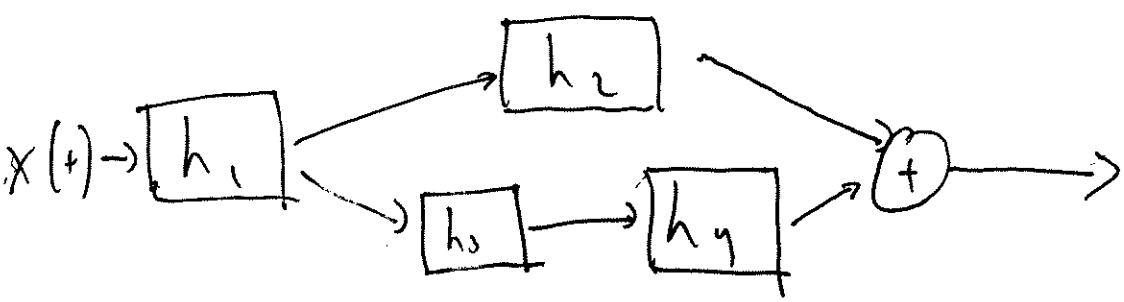


b) $h(t) = h_1(t) * h_2(t) + h_3(t) * h_4(t)$

Treating convolution as a multiplication in PEMDAS we get



c) $h(t) = h_1(t) * \{h_2(t) + h_3(t) * h_4(t)\}$



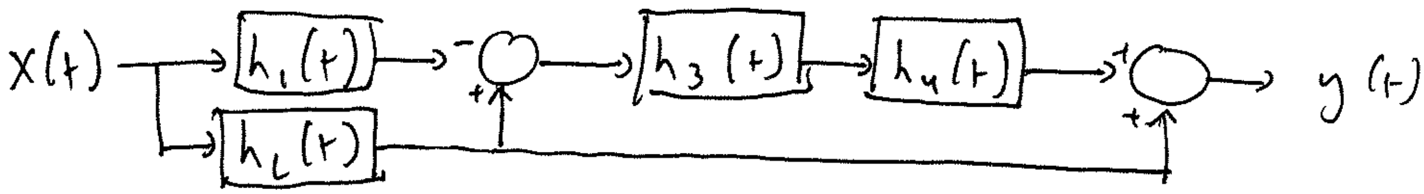
2.48

$h_1(t) = \delta(t-1)$

$h_2(t) = e^{-2t}u(t)$

$h_3(t) = \delta(t-1)$

$h_4(t) = e^{-3(t+2)}u(t+2)$



$$h(t) = [h_2(t) - h_1(t)] * [h_3(t) * h_4(t) + h_2(t)]$$

$$h_2(t) - h_1(t) = e^{-2t}u(t) - \delta(t-1)$$

$$h_3(t) * h_4(t) = \delta(t-1) * e^{-3(t+2)}u(t+2)$$

$$= e^{-3(t+1)}u(t+1)$$

$$h_3(t) * h_4(t) + h_2(t) = e^{-3(t+1)}u(t+1) + e^{-2t}u(t)$$

$$e^{-2t}u(t) * e^{-2t}u(t) = \int_{-\infty}^{\infty} e^{-2\tau}u(\tau)e^{-2(t-\tau)}u(t-\tau)$$

$\tau > 0$ $\quad$ $t-\tau \geq 0 \rightarrow \tau < t$

$t > \tau > 0$

$$= u(t)\int_0^t e^{-2\tau} \cdot e^{-2(t-\tau)} d\tau$$

$$= u(t)\int_0^t e^{-2t} d\tau = e^{-2t} \cdot t \cdot u(t)$$

$$e^{-2t}u(t) * e^{-3(t+1)}u(t+1) = \int_{-\infty}^{\infty} e^{-2\tau}\, \underset{\uparrow \atop \tau>0}{u(\tau)} \cdot e^{-3(t-\tau+1)}\, \underbrace{u(t-\tau+1)}_{t-\tau+1>0}\, d\tau$$

$$t > \tau - 1 \qquad -\tau > -t-1$$

$$t > -1 \qquad \tau < t+1$$

$$= u(t+1)\int_0^{t+1} e^{-2\tau} \cdot e^{-3(t-\tau+1)}\, d\tau$$

$$= u(t+1)\int_0^{t+1} e^{\tau-3t-3}\, d\tau$$

$$= u(t+1)\cdot e^{-3t-3}\int_0^{t+1} e^{\tau}\, d\tau$$

$$= u(t+1)\cdot e^{3t-3}\cdot\left[e^{t+1} - 1\right]$$

$$= u(t+1)\cdot\left[e^{4t-2} - e^{3t-3}\right]$$

$$h(t) = te^{-2t}u(t) + \left[e^{4t-2} - e^{3t-3}\right]u(t+1) - e^{-3t}u(t) - e^{-2(t-1)}u(t-1)$$

2.49

a) $h(t) = \cos(\pi t)$

$t < 0 \not\Rightarrow h(t) = 0$

so non-causal and not memoryless

$$\int_{-\infty}^{\infty} |\cos(\pi t)|\, dt \not< \infty$$

so it's not stable

b) $h(t) = e^{-2t} u(t-1)$

$t < 0 \Rightarrow h(t) = 0$

so causal

but

$t = 0 \not\Rightarrow h(t) = 0$

so not memoryless

$$\int_{-\infty}^{\infty} |e^{-2t} u(t-1)|\, dt = \int_{1}^{\infty} e^{-2t}\, dt < \infty$$

so it's stable

c) $h(t) = u(t+1)$

$t < 0 \not\Rightarrow h(t) = 0$ non-causal and not memoryless

$$\int_{-\infty}^{\infty} |u(t+1)|\, dt = \int_{-1}^{\infty} dt \not< \infty$$

so non-stable

2.49

d) $h(t) = 3\delta(t)$

$t \neq 0 \Rightarrow h(t) = 0$ <u>causal</u> and <u>memoryless</u>

$\int_{-\infty}^{\infty} |3\delta(t)|\,dt = 3 < \infty$ so <u>stable</u>

e) $h(t) = \cos(\pi t)\,u(t)$

$t < 0 \Rightarrow h(t) = 0$ so <u>causal</u>

$t \neq 0 \not\Rightarrow h(t) = 0$ so <u>not memoryless</u>

$\int_{-\infty}^{\infty} |\cos(\pi t)\,u(t)|\,dt = \int_{0}^{\infty} |\cos(\pi t)|\,dt \not< \infty$ so <u>not stable</u>

f) $h[n] = (-1)^n u[-n]$

$n < 0 \not\Rightarrow h(n) = 0$ <u>non-causal</u>, <u>not memoryless</u>

$\sum_{n=-\infty}^{\infty} |(-1)^n u[-n]| = \sum_{n=-\infty}^{0} |(-1)^n| = \sum_{n=-\infty}^{0} 1 \not< \infty$ <u>not stable</u>

g) $h[n] = \left(\frac{1}{2}\right)^{|n|}$

$n < 0 \not\Rightarrow h(n) = 0$ <u>non-causal</u>, <u>not memoryless</u>

$\sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^{|n|}\right| = \frac{1}{2} + 2\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n < \infty$ so stable h)

$$h[n] = \cos\left(\frac{\pi}{8}n\right)\{u[n] - u[n-10]\}$$

$n < 0 \Rightarrow h[n] = 0$ so causal

$n \neq 0 \not\Rightarrow h[n] = 0$ so not memoryless

$$\sum_{n=-\infty}^{n=\infty} |h[n]| = \sum_{n=0}^{n=9} \left|\cos\left(\frac{\pi}{8}n\right)\right| < \infty$$ so stable

i) ~~$h[n] = \cos\left(\frac{\pi}{8}n\right)\{u[n$~~

$$h[n] = 2u[n] - 2u[n-5]$$

$n < 0 \Rightarrow h[n] = 0$ causal

$n \neq 0 \not\Rightarrow h[n] = 0$ not memoryless

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{n=4} 2 < \infty$$ stable

j) $h[n] = \sin\left(\frac{\pi}{2}n\right)$

$n < 0 \not\Rightarrow h[n] = 0$ non-causal, not memoryless

$$\sum_{n=-\infty}^{\infty} \left|\sin\left(\frac{\pi}{2}n\right)\right| < \infty$$ not stable

k) $h[n] = \sum_{p=-\infty}^{\infty} \delta[n-2p]$ $\leftarrow = 1$ if $n - 2p \geq 0$

~~$n < 0$~~ $n \geq 2p \Rightarrow n \geq -2$

$n < 0 \not\Rightarrow h[n] = 0$ non-causal, not memoryless

$$\sum_{n=-\infty}^{n=\infty} |h[n]| = \sum_{n=2}^{n=\infty} \sum_{p=-\infty}^{\infty} \delta[n-2p] \not< \infty$$ not stable

2.50

a) $h[n] = \left(-\frac{1}{2}\right)^n u[n]$

$$s[n] = u[n] * h[n] = \sum u[k] \cdot \left(-\frac{1}{2}\right)^{n-k} u[n-k]$$

$k \geq 0$ $\qquad$ $n-k \geq 0$

$n \geq k \geq 0$ $\qquad$ $k \leq n$

$$s[n] = u[n] \sum_{k=0}^{n} \left(-\frac{1}{2}\right)^{n-k} = u[n] \left(-\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(-\frac{1}{2}\right)^{-k}$$

$$= u[n] \left(-\frac{1}{2}\right)^n \sum_{k=0}^{n} (-2)^k = u[n] \left(-\frac{1}{2}\right)^n \cdot \frac{1-(-2)^{n+1}}{1-(-2)}$$

$$= u[n] \cdot \left(-\frac{1}{2}\right)^n \cdot \frac{1-\left(-\frac{1}{2}\right)^{-n-1}}{3}$$

$$= \frac{1}{3} u[n] \cdot \left[\left(-\frac{1}{2}\right)^n - \left(-\frac{1}{2}\right)^{-1}\right]$$

$$= \frac{1}{3} u[n] \left[\left(-\frac{1}{2}\right)^n + 2\right]$$

2.50

b) $h[n] = \delta[n] - \delta[n-2]$

$$s[n] = u[n] * h[n] = u[n] * \delta[n] - u[n] * \delta[n-2]$$
$$= u[n] - u[n-2]$$

h) $h(t) = u(t)$

$$s(t) = u(t) * h(t) = u(t) * u(t)$$
$$= \int_{-\infty}^{\infty} u(\tau)\, u(t-\tau)\, d\tau$$

$\tau > 0$

$(t-\tau) > 0$

$t > \tau > 0$

$\tau < t$

$$s(t) = u(t) \int_0^t d\tau = t\, u(t)$$

2.51

$$y[n] = x[n] + a\,x[n-k] \qquad \text{assume } k > 0$$

h^{inv} causal

$$h[n] = \delta[n] + a\,\delta[n-k]$$

$$h[n] * h^{inv}[n] = \delta[n]$$

$$h^{inv}[n] + a h^{inv}[n-k] = \delta[n]$$

$n = 0$

$$h^{inv}[0] + a h^{inv}[-k] = 1$$

$$h^{inv}[0] = 1$$

for $n > 0$

$$h^{inv}[n] + a h^{inv}[n-k] = 0$$

$$h^{inv}[n] = -a h^{inv}[n-k]$$

$\cancel{h^{inv}[n] = (-a)^n}$ so if n is not a multiple of k $h^{inv}[n] = 0$

~~n = first multiple of~~ k $\quad n = k$

$$h^{inv}[k] = -a$$

$$h^{inv}[2k] = a^2$$

$$h^{inv}[n] = \begin{cases} 0 & \text{if } n \text{ is not multiple of } k \\ 0 & \text{if } n < 0 \\ (-a)^m & \text{where } m = \frac{n}{k} \end{cases}$$