4.1

ECEN 314
HW 9
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a) $x(t) = \sin(w_0 t)$

$$= \frac{1}{2j}\left(e^{jw_0 t} - e^{-jw_0 t}\right)$$

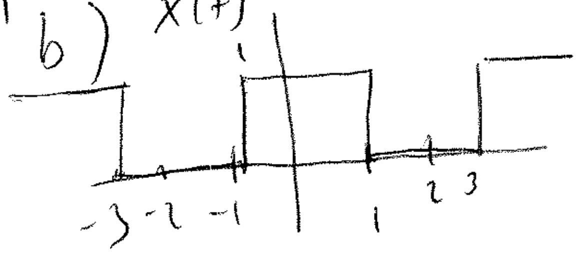
$$X[1] = \frac{1}{2j} \qquad X[-1] = -\frac{1}{2j} \qquad X[else] = 0$$

$$X(jw) = 2\pi \sum_{k=-\infty}^{\infty} X[k]\,\delta[w - kw_0]$$

$$= 2\pi \cdot \frac{1}{2j}\,\delta[w - w_0] - 2\pi \frac{1}{2j}\,\delta[w + w_0]$$

$$= \frac{\pi}{j}\left(\delta[w - w_0] - \delta[w + w_0]\right)$$

4.1 b)



$T = 4$

$\omega_0 = \frac{\pi}{2}$

$$X[k] = \frac{1}{4}\int_{-1}^{1} e^{-jk\omega_0 t}\, dt$$

$$= -\frac{1}{4jk\omega_0}\left[e^{-jk\omega_0 t}\right]_{-1}^{1}$$

$$= -\frac{1}{4jk\omega_0}\left(e^{-j\omega_0 k} - e^{j\omega_0 k}\right)$$

$$= \frac{1}{4jk\omega_0}\left(e^{j\omega_0 k} - e^{-j\omega_0 k}\right)$$

$$= \frac{1}{2k\omega_0}\sin(\omega_0 k) = \frac{1}{2k\pi/2}\sin\left(\frac{\pi k}{2}\right)$$

$$X(j\omega) = \sum_{k=-\infty}^{k=\infty} \frac{2\sin(\pi k/2)}{k}\,\delta(\omega - k\pi/2)$$

4.16

a)

$$x(t) = 2\cos(\pi t) + \sin(2\pi t)$$

$T_2 = 1$

$T = 2$

$$w_1 = 2\pi \frac{1}{T_1} = \pi \Longrightarrow T_1 = 2$$

$w_0 = \pi$

$$x(t) = \left(e^{j\pi t} + e^{-j\pi t}\right) + \frac{1}{2j}\left(e^{j2\pi t} - e^{-j2\pi t}\right)$$

$$X[1] = 1, \quad X[-1] = 1$$

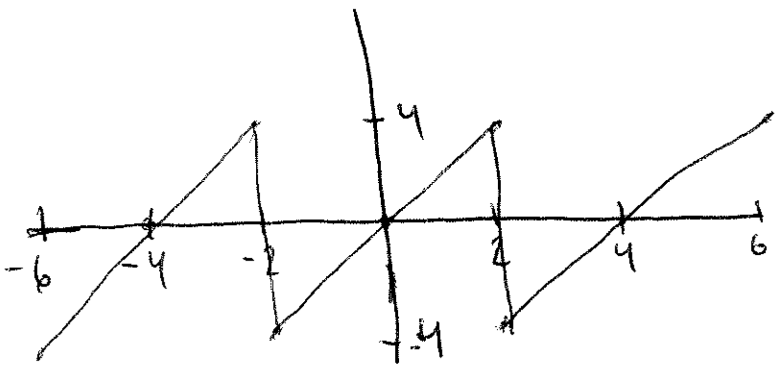
$$X[2] = \frac{1}{2j}, \quad X[-2] = -\frac{1}{2j}$$

$$X(jw) = 2\pi \sum_{k=-\infty}^{k=\infty} X[k]\,\delta(w - kw_0)$$

$$= 2\pi\Big(\delta(w - w_0) + \delta(w + w_0)\Big) + \frac{1}{j}\Big(\delta(w - 2w_0) - \delta(w + 2w_0)\Big)$$

4.16

d) $x(t)$



$T = 4$

$\omega_0 = \frac{\pi}{2}$

$$X[k] = \frac{1}{4}\int_{-2}^{2} 2t\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{2}\int_{-2}^{2} t\, e^{-jk\omega_0 t}\, dt$$

Use integration by parts with:

$u = t \qquad dv = e^{-jk\omega_0 t}\, dt$

$du = dt \qquad v = \frac{-1}{jk\omega_0} e^{-jk\omega_0 t}$

$$X(j\omega) = \pi \cdot 2 \sum_{k=-\infty}^{k=\infty} X[k]\, \delta[\omega - k\omega_0]$$

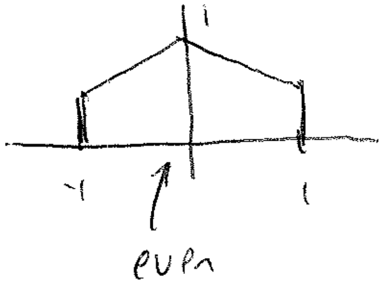
3.16 a)

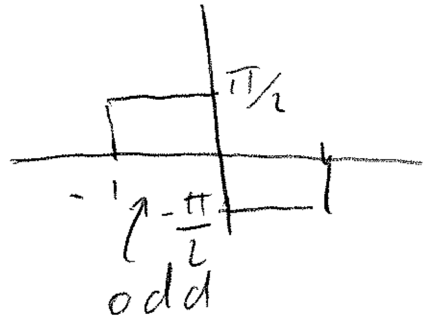
$$x(t) = 2e^{-t}u(t) - 3e^{-2t}u(t)$$

$$= \frac{2}{1+j\omega} - \frac{3}{2+j\omega}$$

3.17

a) $|X(jw)|$ $\arg(X(jw))$





$x(t)$ is real and odd as $X(jw)$ is not real

d) $X(jw) = w^{-2} + jw^{-3}$

$|X(jw)| = \sqrt{w^{-4} + w^{-6}}$ which is even

$\text{Phase}(X(jw)) = \arctan\left(\frac{w^{-3}}{w^{-2}}\right)$, which is odd

$x(t)$ is real

3.18

a) $x(t) = 3e^{-t}u(t)$ and $h(t) = 2e^{-2t}u(t)$

$y(t) = x(t) * h(t)$

$Y(j\omega) = X(j\omega)H(j\omega)$

$= \dfrac{3}{1+j\omega} \cdot \dfrac{2}{2+j\omega}$

3.20

b) $h(t) = \left(\frac{1}{\pi t}\right) \sin(\pi t)$ $\quad x(t) = \frac{3}{\pi t} \sin(2\pi t)$

$$y(t) = x(t) * h(t)$$
$$= F^{-1}\{X(j\omega) \cdot H(j\omega)\}$$

$$X(j\omega) = \begin{cases} 1, & |\omega| \le \pi \\ 0, & \text{else} \end{cases}$$

$$H(j\omega) = \begin{cases} 3, & |\omega| \le 2\pi \\ 0, & \text{else} \end{cases}$$

$$Y(j\omega) = \begin{cases} 3 & |\omega| \le \pi \\ 0, & \text{else} \end{cases}$$

$$y(t) = \frac{3}{\pi t} \sin(\pi t)$$

3.21

a) $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$

$h(t) = e^{-4t}u(t)$

$$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{\frac{1}{3+j\omega} - \frac{1}{4+j\omega}}{\frac{1}{4+j\omega}}$$

$$= \frac{4+j\omega}{3+j\omega} - 1$$

oops $\longrightarrow$

$$\cancel{= \frac{(4+j\omega)(3-j\omega)}{3^2+\omega^2} - 1}$$

$$\cancel{= \frac{2\cdot 2\cdot 3}{3^2+\omega^2} + \frac{-j\omega}{3^2+\omega^2} + \frac{\omega^2}{3^2+\omega^2} - 1}$$

$$X(j\omega) = \frac{4+j\omega}{3+j\omega} - 1 = \frac{3+j\omega}{3+j\omega} + \frac{1}{3+j\omega} - 1$$

$$= \frac{1}{3+j\omega}$$

$$x(t) = e^{-3t}u(t)$$