A Game-Theoretic Analysis of Cyber Switching Attacks and Mitigation in Smart Grid Systems

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Abstract—We propose a framework for the analysis of cyber switching attacks and control-based mitigation in cyber-enabled power systems. Our model of the switching attack is simple, only requiring knowledge of the sign of the local relative rotor speed, which may be estimated. The controller is modeled to be resource constrained, choosing to act only during select intervals of time. We make use of an iterated game-theoretic formulation to describe the interactions of the parties and its effect on system stability. Analytic results indicate the potential of the constrained controller to achieve transient stabilization over time using zero-determinant strategies. Numerical results of the New England 39-bus power system demonstrate the potential for such a controller to increase system resilience during cyber-attacks.

Index Terms—Cyber-physical systems, game theory, iterated games, smart grid, stabilizing control, switching attacks, system resilience, zero-determinant strategies.

I. INTRODUCTION

THE RAPIDLY growing nature of the smart grid and its ever-changing threat landscape have thrust cyber-security challenges to the forefront of smart grid system development. Evolving regulations and technical alternatives provide unprecedented opportunities for stakeholders to address cyber-physical security concerns. One role research can play within this hurried environment is to provide general guidelines and strategies for identification of attack and mitigation.

In this paper, we focus on the development of an analysis framework to model the interaction of switching-based attacks on power systems and local control strategies for mitigation. Cyber switching attacks represent a growing class of cyber-physical attacks within the research community that aim to destabilize the state of a target power system component via controlled switching of circuit breakers in the proximity of the target [1], [2]. Here, an opponent would eavesdrop on local sensor readings and estimate the state of the target to compute destabilizing switching action that is implemented through false data injection of breaker control signals.

Strategies to mitigate against such cyber-physical attacks are emerging and we focus on the use of distributed control-based paradigms in this paper. Distributed controllers represent a valuable and expanding asset base from which to improve system resilience. Recent results [3]–[6] have demonstrated the usefulness of distributed control paradigms that leverage fast-acting power sources such as storage devices. The performance of the controller, however, is strongly related to its ability to receive high-granularity real-time data.

Prior art has recently begun to consider game-theoretic-based modeling of cyber-security problems within cyber-physical systems. An iterative zero-sum game is used in [7] to model security policies at the cyber-level with corresponding optimal control response at the physical layer. A zero-sum game with a mixed strategies model is developed in [8] to formulate cyber-physical systems survivability, where the attacker (defender) plays over resources being disrupted (maintained/restored). Siever et al. [9] formulated the placement and utilization of unified power flow controllers (UPFCs) in a power transmission system as an iterative game. In response to attackers tripping transmission lines, the defender optimizes the installation locations of the UPFCs to maximize the amount of power delivered over all attacks to which the system is most vulnerable. A game-theoretic formulation of the risk dynamics of false data injection attacks (through overcompensation) targeting automatic generation control is developed in [10], where it adopts a zero-sum Markov security game model and defines risk states as function of the probability of attack and the potential impact corresponding to the attack.

Thus, previous work has focused on strategic behavior of players who have full knowledge of the payoff matrix of each other and full system state knowledge. In addition, most of these models have prioritized formulating the player interactions at the decision-making level over integrating physical system models. While some results do hypothesize a response of optimal control, there is a need to study concrete examples of incorporating controller dynamics. The proposed work in this paper is the first to illustrate a practical smart grid stabilizing controller that makes no assumptions on attacker actions, nor on full system state knowledge. The only assumption required is that the attacker has more limited resources than the electric power utility (EPU), an assumption common in spirit.
to many security threat models in the literature. Another novel aspect of this paper is that it represents a first look at integrating a game-theoretic foundation with a smart grid dynamical system description to study the system behavior in mitigating various classes of cyber-physical attacks.

Specifically, this paper addresses an analysis of the dynamic interaction of switching attacks and distributed control for attack mitigation. In contrast to much of the former smart grid security related work that focuses on aggressive approaches of attack and mitigation, we aim to incorporate constraints of both attacker and EPU leading to more conservative interactions. The limited sampling rate of sensor readings leveraged by both opponent and EPU is explicitly modeled; thus, we do not assume the availability of a continuous-stream of data. The bound on available energy resources or cost for the opponent (controller) is articulated as the process of selective participation in the attack (mitigation) process; thus, either party can choose to be active using available resources or idle hence conserving. Moreover, the opponent is not required to know the entire state of the target component to apply the attack effectively. Similarly, the distributed controller is assumed to take action based on only local information during the attack.

This paper provides a dynamical model that describes the interaction between the EPU and the attacker and the impact of their strategies on system stability. Namely, this interaction is modeled as a $2 \times 2$ iterated game, where, at each round of the game, the EPU reacts to the action of the attacker in the previous round. A salient feature of such games is that players with longer memories of the history of the game have no advantage, in the long-term, over those with shorter memories. In this regard, iterated games lend themselves to Markovian strategies referred to as zero-determinant strategies [11]. With such strategies, players can control their own long-term payoff or that of their opponent based on the structure of the payoff matrix of the game. In this paper, we deduce such strategies that allow the EPU to stabilize the power system in light of such attacks.

The remainder of this paper is organized as follows. Section II introduces our framework of study. Game-theoretic analysis is presented in Section III followed by simulation results on the 39-bus New England system in Section IV. Discussion is found in Section V and the conclusion is drawn in Section VI.

II. ANALYTICAL FRAMEWORK

We consider a cyber-physical smart grid system that is comprised of a physical component such as the power transmission network shown in Fig. 1(a) and a cyber-component that includes a set of measurement devices, their corresponding communication infrastructure, and a control center that processes measurements and makes decisions. We assume the existence of $N$ synchronous generators each equipped with one or more local external fast-acting storage units (such as flywheels) that can effectively inject or absorb power at the generator bus based on control signals as illustrated in Fig. 1(b). Each distributed storage unit is considered to have three possible modes of operation: 1) absorbing; 2) injecting; or 3) idle.

A. Physical Power System Model

A schematic of the New England 39-bus power system is shown in Fig. 1 for $N = 10$; the system parameters are summarized in Table I, where $M_i$ and $D_i$ are expressed in seconds, $\delta_i$ is expressed in radians, and the remaining parameters are in per unit values. We note that the relative normalized rotor speed of generator $i$ is defined as $\omega_i = (\omega_{i,\text{act}} - \omega_{i,\text{nom}})/\omega_{i,\text{nom}}$, where $\omega_{i,\text{nom}}$ is the desired nominal angular rotor speed (in radians per second) of the generator and $\omega_{i,\text{act}}$ is the actual angular rotor speed of generator $i$ (in radians per second).

In this paper we focus, in part, on the impacts of attacks and mitigation on the transient stability of the power system. Thus, we employ a swing equation model for synchronous generators. Along with a Kron-reduced representation of
physical network relationships, this model has recently demonstrated great potential for modeling issues of power system performance and stability [3], [6], [12], [13].

Let \( \dot{\delta}_i \) and \( \dot{\omega}_i \) denote time derivatives of \( \delta_i \) and \( \omega_i \), respectively. The swing equation model for generator \( i \) (assuming no power control such as a governor) within an interconnected power system can be expressed as [12], [14]

\[
\dot{\delta}_i = \omega_i,
\]
\[
M_i \dot{\omega}_i = -D_i \omega_i + (P_{m,i} - P_{e,i}),
\]

where the electrical power of generator \( i \) \( P_{e,i} \) is [15]

\[
P_{e,i} = \sum_{k=1}^{N} |E_i| |E_k| |G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)|. \tag{2}
\]

Here, it is assumed that \( G_{ik} = G_{ki} \geq 0 \) is the Kron-reduced equivalent conductance between generators \( i \) and \( k \), \( B_{ik} = B_{ki} > 0 \) is the Kron-reduced equivalent susceptance between generators \( i \) and \( k \), and \( Y_{ik} = G_{ik} + \sqrt{-1} B_{ik} \) is the Kron-reduced equivalent admittance between generators \( i \) and \( k \) (all in per unit values). Let \( \phi_{ik} = \arctan(G_{ik}/B_{ik}) \) and \( P_{ik} = |E_i| |E_k| |Y_{ik}| \), then

\[
P_{e,i} = |E_i|^2 G_{ii} + \sum_{k=1, k \neq i}^{N} P_{ik} \sin(\delta_i - \delta_k + \phi_{ik}). \tag{3}
\]

B. Switching Attack Model

Both cyber-attack and mitigation decision-making occur in the cyber-domain as presented in Fig. 1(b). In contrast to the physical system, the cyber-infrastructure is discrete-time. Thus, each party will act (or not) based on time-sample estimates of \( \dot{\omega}_i \). We let the interval between adjacent time instants employed be \( \Delta T \) (ranging from the order of milliseconds to minutes depending on the type of sensor and attacker/EPU).

We consider an attacker who aims to impact a set of target synchronous generators by gaining control of their corresponding storage units using knowledge of their local rotor speed (via cyber-intrusion); several recent publications discuss the possibility of such intrusion [16], [17]. For instance, eavesdropping of local quantities can be employed to gain estimates of generator’s rotor speed as recently demonstrated. Moreover, false data injection attacks may be imposed to control switches of storage units. The types of cyber-intrusion necessary to be able to execute such attacks are specific to the actual protocols, software and hardware architecture and is beyond the scope of this paper.

As discussed in the introduction, we consider a situation in which the opponent aims to balance the following objectives.

1) Imposing instability on the rotor speed of one or more target synchronous generators; in general, an attacker may aim to destabilize the system in a number of ways, but our focus in this paper is on rotor speed, which we believe will still shed some general insights on effective attack and mitigation.

2) Conserving storage resources to suit availability of local resources; an aggressive attack may drain storage rapidly thus limiting the ability of the opponent at an opportune time interval.

3) Remaining stealthy to discourage premature tripping by protection devices that would limit the attack scale; we assert that a desirable strategy would be for the attacker to subtly impose interarea oscillatory-type behavior to create a large-scale disturbance in contrast to aggressive short-term attacks that create high transients initiating relay protection.

Given the recent interest in switching attacks [18]–[20], we assume that the opponent actions impact the power system through a modulating sequence of possible injections. Specifically at time \( t \) an active attacker applies

\[
P_{A,i} = \begin{cases} 
+\Gamma_{i,1} & \text{if } \omega_{i,0} \geq 0 \\
-\Gamma_{i,2} & \text{if } \omega_{i,0} < 0 
\end{cases}
\]

where \( \Gamma_{i,1} \) and \( \Gamma_{i,2} \geq 0 \) are the power injected or absorbed by the storage device at generator \( i \), respectively, based on the value of the relative normalized rotor speed at time \( t_0 = t - \Delta T \) (and gleaned by the attacker through intrusion). We argue that (4) effectively models the spirit of a variety of approaches for system destabilization because (as shown in Section II-D) it promotes the deviation of \( \omega_i \) from its nominal value within the time interval \( \Delta T \).

The reader should note that the opponent, to conserve energy or remain stealthy, may choose not to act, in which case the attacker is idle and \( P_{A,i} = 0 \).

C. EPU Stabilizing Control

An EPU can typically respond to attack impacts through a variety of technologies including fast-acting protection and distributed control. The former has the effect of power grid reduction with possible negative impacts to power delivery. The latter typically requires the attack be first detected and characterized, which may be difficult if the attack is stealthy as highlighted above.

Thus, in this paper, we consider the scenario whereby the EPU makes use of a simple local control strategy involving select (and uncorrupted) storage units that could perpetually aid in stabilization of incidental disturbances while more aggressively tackles the impacts of cyber-attack. We assume that the control is local and makes use of knowledge of the rotor speed of each associated synchronous generator as well as the attacker action (i.e., active or idle) in the previous interval; recent results have demonstrated the ability of an opponent to estimate such quantities for cyber-attack [1]. We assert that this approach provides both resilience to attack and has the potential to indicate the onset of an attack using controller statistics.

Given that the EPU control may be persistent and aims to locally push the rotor speed to its nominal value, it is important to conserve storage resources especially when being fed by a highly variable renewable resource and when operating costs are limited. Thus, the EPU control may not always choose to be active. If active, we model the impact of EPU control at time \( t \) as

\[
P_{C,i} = \begin{cases} 
-\Lambda_{i,2} & \text{if } \omega_{i,0} > 0 \\
+\Lambda_{i,1} & \text{if } \omega_{i,0} < 0 
\end{cases}
\]

\( \Lambda_{i,1} \) and \( \Lambda_{i,2} \) are the power absorbed or injected by the storage device at generator \( i \), respectively.
where $\Lambda_{i,1}$ and $\Lambda_{i,2} \geq 0$ are the amount of power the stabilizing controller is injecting (absorbing from) the power grid at generator $i$ based on the value of the rotor speed at time $t_0$ (acquired by the EPU). As shown in Section II-D, this action has the effect of helping move $\omega_i$ to its desired nominal value and thus represents in spirit the local time action of a broad class of mitigation strategies. If idle, we set $P_{C,i} = 0$.

### D. Switching Attack and Stabilizing Controller Models

We let $\Delta P_i = P_{e,i} - P_{n,i}$ and $g_i = (\Delta t / M_i) > 0$. The impact of attack and mitigation at generator $i$ can be modeled as terms on the right side of (1) [3], [6]

$$\dot{\omega}_i = \omega_i, \quad \dot{\omega}_i = \frac{1}{M_i} \left[-D_i \omega_i - \Delta P_i + P_{A,i} + P_{C,i}\right]$$

where $P_{A,i}$ and $P_{C,i}$ represent the effect of the attacker and the EPU controller, respectively. A positive value of $P_{A,i}$ ($P_{C,i}$) implies that the attacker (EPU controller) injects real power at the bus of generator $i$, and a negative value indicates absorption.

Given that the attacker and the EPU controller may each be active or idle, we consider four possibilities for action within the sampling interval $\Delta T$ and model the overall impact on relative normalized rotor speed of generator $i$. Specifically, we compute $\omega_i(t)$ where $t$ is the current sample instant relative to $\omega_i(t_0)$ for $t_0 = t - \Delta T$.

1) **Case 1 (Idle Attack and Control):** Here, we set $P_{A,i} = P_{C,i} = 0$ in (6) and integrate from $t_0$ to $t = t_0 + \Delta T$ to give

$$\omega_i(t) = \int_{t_0}^{t} \left[-D_i \omega_i - \Delta P_i\right] dt + \omega_i(t_0) = \omega_i(0). \quad (7)$$

Thus, $|\omega_i(t)| = |\omega_i(0)|$.

2) **Case 2 (Active Attack and Idle Control):** We set $P_{A,i}$ as in (4) and $P_{C,i} = 0$; integrating from $t_0$ to $t = t_0 + \Delta T$ gives

$$\omega_i(t) = \int_{t_0}^{t} \left[-D_i \omega_i - \Delta P_i + P_{A,i}\right] dt + \omega_i(t_0) = \omega_i(0) + g_i P_{A,i} \quad (8)$$

$$|\omega_i(t)| = |\omega_i(0)| + |g_i P_{A,i}| \quad (9)$$

because $\omega_i(0)$ and $P_{A,i}$ have the same sign. It is noted that the action of the attacker moves the rotor speed away from its nominal value decreasing the stability margin as intended.

3) **Case 3 (Idle Attack and Active Control):** Similarly, $P_{A,i} = 0$ and $P_{C,i}$ is represented by (5). As a result

$$\omega_i(t) = \int_{t_0}^{t} \left[-D_i \omega_i - \Delta P_i + P_{C,i}\right] dt + \omega_i(t_0) = \omega_i(0) + g_i P_{C,i} \quad (11)$$

$$|\omega_i(t)| = |\omega_i(0)| - |g_i P_{C,i}| \quad (12)$$

because $P_{C,i}$ and $\omega_i(0)$ have opposite signs in (5). Accordingly, the EPU controller drives the power system closer to stability.

### III. GAME-THEORETIC ANALYSIS

#### A. Zero-Determinant Strategies for Iterated Games

Consider a $2 \times 2$ iterated game with the one stage game of Fig. 2. The game has two players: 1) the EPU controller (row player); and 2) the attacker (column player). At each round of the game, a player chooses from two actions $\{1, 2\}$. Let $n_1$ and $n_2$ denote an actions of the EPU controller and attacker, respectively. A value of $n_1 = 1$ or $n_2 = 1$ refers to an active or idle EPU control in a given round of the game, respectively. We have similar interpretations for $n_2 = 1$ (active attacker) and $n_2 = 2$ (idle attacker). The value $X_{j,k}$, where $j, k \in \{1, 2\}$, denotes the relative normalized rotor speed of the power generator if the EPU controller chooses $n_1 = j$ and the attacker chooses $n_2 = k$ during the current play interval.

It is shown in [11] that, in iterated games, where the same actions and the same payoff matrices are repeated, for any strategy of the player with the longer memory, the player with the shorter memory can play any strategy that the other player can play in the long-term outcome if the opponent has played a shorter memory strategy. Thus, any history outside what is shared between the two players can be disregarded; consequently, the game can be modeled as a Markov chain taken here to be a single memory step process.

In this regard, let $n(t) = (n_1, n_2)$ denote the state of the game at round $t$ and $S = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$ be the state space of the game. Also, let $P(\cdot | \cdot)$ denote the probability operator and $k = (k_1, k_2)$, then

$$P^k = P(n_1(t + 1) = 1 | n(t) = k), \forall k \in S$$

represents the probability that the EPU controller takes action 1 ($n_1 = 1$) in round $t + 1$ if in the previous round
the controller took action \( k_1 \) and the attacker took action \( k_2 \). In the same manner, the probability that the attacker takes action \( k \) in round \( t + 1 \) is represented as

\[
\tilde{p}_A^k = \tilde{P}(n(t + 1) = 1 | n(t) = k), \quad \forall k \in S.
\]

The Markov chain has a unique stationary distribution \( \pi^T = (\pi_{1,1}, \pi_{1,2}, \pi_{2,1}, \pi_{2,2}) \), where for each \( \pi_{j,k} \), \( j \) refers to an action by the EPU controller and \( k \) refers to an action by the attacker. Further, the average long-term outcome of the game, denoted as \( u_X \), is given by [21] as \( u_X = \pi^T X \), where \( X = (X_{1,1}, X_{1,2}, X_{2,1}, X_{2,2})^T \). It is shown in [11] that if the \( p^T C \)'s are chosen such that

\[
aX + b = \begin{pmatrix} -1 + p^T_{C,1} & 1 + p^T_{C,2} \\ 1 & 1 + p^T_{C,2} \\ 1 & 1 \end{pmatrix} b
\]

where \( a \) and \( b \) are arbitrary nonzero real numbers, then the row player can fix the value of \( u_X \) regardless of the actions of the column player (the attacker) if and only if the minimum value of one in the payoff matrix of Fig. 2 exceeds the maximum value of the other row. In such case, as shown in [21] that the row player (i.e., the EPU controller) can fix the long-term average payoff \( u_X \) to any value in the range between the minimum and the maximum values representing a profound advantage to the EPU in an attack situation. To achieve a specific \( u_X \), the controller has to take an action in the current interval according to the following likelihoods [21]:

\[
p^T_{C,1} = 1 + \left( 1 - \frac{X_{1,1}}{u_X} \right) b, \quad p^T_{C,2} = 1 + \left( 1 - \frac{X_{1,2}}{u_X} \right) b
\]

\[
p^T_{C,2} = \left[ \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right] b.
\]

**B. Long-Term Mitigation Control**

Given the objectives of system destabilization (or stabilization) by the attacker (EPU controller), we make use of the value of \( |\omega_i(t)| \) to represent the payoff at the end of the interval \( \Delta T \) for each party. Thus, the EPU controller aims to minimize this payoff (i.e., drive \( |\omega_i(t)| \) to zero) while the attacker aims to increase payoff of targeted generators.

Let \( X = [X_{j,k}] \) denote the payoff matrix of generator \( i \) at the end of \( \Delta T \). Then, following the development of Section II-D, the values of \( X \) are shown in Fig. 3.

It is important to note that the payoff matrix of the game is nonstatic since the value of \( |\omega_i(t)| \) changes every \( \Delta T \); however, because all the elements of the matrix are shifted by a fixed value, \( |\omega_i(t)| \), it is shown in the Appendix that the strategy of the EPU controller, as will be subsequently derived, does not depend on the temporal value of \( |\omega_i(t)| \) and thus, the results of [21] still apply.

Let \( X_{j,\text{max}} \) and \( X_{j,\text{min}} \) denote the maximum and minimum values of row \( j \) in the payoff matrix of generator \( i \), respectively.

Because \( |P_{C,i}| \geq |P_{A,i}| \), the maximum and minimum values of the rows of \( X \) are found in this case as

\[
X_{1,\text{max}} = |\omega_i,0| - g_i(|P_{C,i}| - |P_{A,i}|)
\]

\[
X_{1,\text{min}} = |\omega_i,0| - g_i|P_{C,i}|
\]

\[
X_{2,\text{max}} = |\omega_i,0| + g_i|P_{A,i}|
\]

\[
X_{2,\text{min}} = |\omega_i,0|.
\]

It is observed that \( X_{1,\text{max}} < X_{2,\text{min}} \) while \( X_{2,\text{max}} \neq X_{1,\text{min}} \). Consequently, because \( X_{1,\text{max}} < X_{2,\text{min}} \), the average long-term payoff attained by using the zero-determinant control strategy lies in \([X_{1,\text{max}}, X_{2,\text{min}}] = [|\omega_i,0| - g_i(|P_{C,i}| - |P_{A,i}|), 0] \). Let the specific value of the average long-term payoff at generator \( i \) be parameterized as

\[
u_X = |\omega_i,0| - g_i(|P_{C,i}| - |P_{A,i}|)
\]

where \( 0 \leq \alpha \leq 1 \) is called the persistence factor; a high value of \( \alpha \) moves \( u_X \) closer to \( |\omega_i,0| - g_i(|P_{C,i}| - |P_{A,i}|) \), indicating that the EPU controller is more aggressive in driving the rotor speed of generator \( i \) to stability.

Further, following the development in [21], the range of valid values of \( b \) in (16) can be found as

\[
\max \left( -\frac{|\omega_i,0|}{u_X - X_{1,\text{min}}}, \frac{|\omega_i,0|}{u_X - X_{2,\text{max}}} \right) \leq b < 0.
\]

Given the results of (18) and (19), and for the case of an aggressive stabilizing controller (i.e., \( \alpha \geq (1/2) \)), it can be shown that \( u_X < X_{1,\text{min}} \geq X_{2,\text{max}} - u_X \). Consequently, for \( \alpha \geq (1/2) \), \( b_{\text{min}} \leq b < 0 \), where \( b_{\text{min}} = (u_X/(u_X - X_{2,\text{max}})) \) simplifying to

\[
b_{\text{min}} = \frac{1}{g_i} \frac{|\omega_i,0| - g_i(|P_{C,i}| - |P_{A,i}|)}{|P_{A,i}| + |P_{C,i}| - |P_{A,i}| + |P_{A,i}|}.
\]

Let the specific value of \( b \) be expressed as \( b = \beta b_{\text{min}} \), where \( 0 < \beta \leq 1 \) is called the steering factor. Then

\[
b = \frac{1}{g_i} \frac{|\omega_i,0| - g_i(|P_{C,i}| - |P_{A,i}|)}{|P_{A,i}| + |P_{C,i}| - |P_{A,i}| + |P_{A,i}|}.
\]

A high value of \( \beta \) means that the EPU controller is more probable to take a stabilizing action; i.e., \( p_C \) increases with increasing \( \beta \).

Let the status of the EPU controller and attacker be \( j \) and \( k \), respectively, in the previous time interval \( \Delta T \) and be known to the EPU controller. Let \( p^T_{j,k} \) denote the probability that the controller is active at generator \( i \) in the current time interval given knowledge that \( n_1 = j \) and \( n_2 = k \) in the previous interval. Using the results of (17), (19), and (22), the probabilities of the EPU controller taking a stabilizing action are

\[
p^{1,2}_j = \frac{(1 - \alpha + \alpha \beta)(|P_{C,i}| + (1 - \alpha - \alpha \beta)|P_{A,i}|)}{|P_{A,i}| + |P_{C,i}| + (1 - \alpha)|P_{A,i}|}
\]

\[
p^{1,1}_j = \frac{|P_{C,i}| + (1 - \alpha)|P_{A,i}|}{|P_{A,i}| + |P_{C,i}| + (1 - \alpha)|P_{A,i}|}
\]

\[
p^{2,1}_j = \beta
\]

\[
p^{2,2}_j = \frac{\alpha \beta(|P_{C,i}| - |P_{A,i}|)}{|P_{C,i}| + (1 - \alpha)|P_{A,i}|}.
\]
Moreover, let \( \rho_i = (|P_{C,i}|/|P_{A,i}|) \geq 1 \) denote the ratio of the control power and the attack power at generator \( i \). Then the probabilities of action can be expressed as

\[
\begin{align*}
 p_i^{1,1} &= 1 + \beta - \beta \frac{\rho_i}{\alpha \rho_i - \alpha + 1} \\
 p_i^{1,2} &= 1 - \beta \frac{(1 - \alpha) \rho_i + \alpha}{\alpha \rho_i - \alpha + 1} \\
 p_i^{2,1} &= \beta \\
 p_i^{2,2} &= \beta \frac{\alpha \rho_i - \alpha}{\alpha \rho_i - \alpha + 1}.
\end{align*}
\]  

(24)

For the special case of \( \rho_i = \infty \); i.e., \( P_{A,i} = 0 \) and there is no attack at generator \( i \), the probabilities that the EPU controller takes a stabilizing action become

\[
\begin{align*}
 p_i^{1,1} &= 1 + \beta - \beta \frac{1}{\alpha} \\
 p_i^{1,2} &= 1 - \beta \frac{1}{\alpha} \\
 p_i^{2,1} &= \beta \\
 p_i^{2,2} &= \beta \frac{1}{\alpha}.
\end{align*}
\]  

(25)

Similarly, when \( \alpha = 1 \) (i.e., the EPU controller is very aggressive), the controller action probabilities are

\[
\begin{align*}
 p_i^{1,1} &= 1, \\
 p_i^{1,2} &= 1 - \beta \frac{1}{\rho_i} \\
 p_i^{2,1} &= \beta, \\
 p_i^{2,2} &= \beta - \beta \frac{1}{\rho_i}.
\end{align*}
\]  

(26)

In other words, \( p_i^{1,2} = p_i^{1,1} - (\beta/\rho_i) \) and \( p_i^{2,2} = p_i^{2,1} - (\beta/\rho_i) \). This means that if the attacker is not active at generator \( i \) during a specific time interval, the EPU controller will reduce the probability of taking an action to stabilize the power system by \( \beta/\rho_i \) in the subsequent interval. The controller’s behavior in this scenario reflects the case where as a result of no action from the attacker, the deviation of \( \omega_{i,0} \) is small, if existed; hence, it is not in the best interest of the EPU controller to take aggressive actions that would over-correct the deviation.

It is to be noted that if the EPU stabilizing controller chooses to act all the time, regardless of the actions of the attacker, the average long-term payoff will be in the range \( |\omega_{i,0} - g_i| P_{C,i} + 0, g_i |P_{A,i}| \), which can be better than the average long-term payoff attained by using the game-theoretic approach. However, the game-theoretic control approach gives the system operators a guarantee of an acceptable system performance while meeting constraints on the availability or the cost of the storage units.

### IV. Numerical Results

We demonstrate the validity of our analysis insights on the 39-bus New England power system of Fig. 1 with parameters of Table II and values of \( M_i \)'s and \( X_{ii} \)'s taken from [22] and [23]. To provide a conservative assessment of the attack mitigation, we do not activate governor control in our simulations.

The power system is assumed to be running normally from \( 0 \leq t < 0.5 \text{ s} \). Then, a cyber switching attack that targets both generators 2 and 9 from \( 0.5 \leq t \leq 10.5 \text{ s} \) is applied. The attacker injects or absorbs (depending on the sign of \( \omega_{i,0} \)) 0.15 p.u. of power at the external storage units of generators 2 and 9 every \( \Delta T = 50 \text{ ms} \) during the attack duration.

The EPU stabilizing controller is activated on the system generators at \( t = 0.75 \text{ s} \) (i.e., the EPU controller is activated after 250 ms from the start of the switching attack). The stabilizing control is deactivated when both cyber-attack is finished and power system is stable. The power of each storage unit is limited to 10% of the mechanical power of the corresponding generator. Further, the EPU controller can inject or absorb a real power of 0.3 p.u. at the storage unit every \( \Delta T = 50 \text{ ms} \) as well. In other words, the amount of power the EPU controller can use at any specific storage unit is limited to the minimum of 0.3 p.u. and 10% of the mechanical power of the corresponding generator.

Before the occurrence of the cyber-attack, load flow analysis of the power system yields the results tabulated in Table III; here, the mechanical and electrical powers of each generator are equal due to the steady-state nature.

Generator stability time is measured as the difference between the time after which generator’s relative normalized rotor speed is restricted to a 0.8333% threshold (i.e., the maximum rotor speed is within \( \pm 0.5 \text{ Hz} \) of the nominal value) and \( t = 0.75 \text{ s} \), the time when the stabilizing controller is activated. Further, let the \( \Delta T = 50 \text{ ms} \) interval be called the game interval, \( |P_{A,i}| = 0.15 \text{ p.u.} \), \( i \in \{2, 9\} \), and \( |P_{C,i}| = \min(0.3 \text{ p.u.}, (P_{m,i}/10)) \) \( \forall i \in \{1, \ldots, N\} \).

The activity ratio of the controller \( \xi_i \) represents the percentage of time that the EPU controller is active (\( n_1 = 1 \)) within the stabilization period. It can be shown that this measure is approximated by \( 100 \times p_i^{2,2}/(1 + p_i^{1,2} - p_i^{1,1}) \) for nonattacked generators. Using (24) and given that \( P_{A,i} = 0 \) (i.e., \( \rho_i = \infty \)), \( \forall i \not\in \{2, 9\} \), we can approximate

\[
\xi_i \approx 100 \times \alpha, \forall i \not\in \{2, 9\}.
\]  

(27)

The attacker targets fast-acting storage units at generators 2 and 9. At the end of each game interval,
TABLE IV

<table>
<thead>
<tr>
<th>Generator</th>
<th>Stability Time (second)</th>
<th>Control Power (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.1021</td>
<td>0.1881</td>
</tr>
<tr>
<td>2</td>
<td>13.6828</td>
<td>0.2000</td>
</tr>
<tr>
<td>3</td>
<td>12.2603</td>
<td>0.1255</td>
</tr>
<tr>
<td>4</td>
<td>12.2402</td>
<td>0.1721</td>
</tr>
<tr>
<td>5</td>
<td>13.3902</td>
<td>0.1488</td>
</tr>
<tr>
<td>6</td>
<td>11.5588</td>
<td>0.1684</td>
</tr>
<tr>
<td>7</td>
<td>11.2565</td>
<td>0.1543</td>
</tr>
<tr>
<td>8</td>
<td>10.6682</td>
<td>0.1854</td>
</tr>
<tr>
<td>9</td>
<td>11.8557</td>
<td>0.2007</td>
</tr>
<tr>
<td>Average</td>
<td>12.1128</td>
<td>0.2011</td>
</tr>
</tbody>
</table>

the attacker uses (4) to determine whether it needs to absorb or inject power in the next game interval; the attacker does not change the value of attack power during the 50-ms interval. In contrast, the mitigation strategy of the EPU controller relies on (23) to calculate its probability of taking action in the next game interval. If the EPU controller must be active, (5) is used to determine the specific action.

A. Stabilizing Control Without Zero-Determinant Strategies

As a benchmark, Table IV displays the performance measures of the power system when the stabilizing controller does not implement the game-theoretic approach (i.e., when the EPU controller always takes a stabilizing action regardless of the actions of the attacker in the previous game interval). This represents an extreme “best case” for stabilization irrespective of cost and energy conservation. It is noted the average stability time of the system generators is approximately 12.1 s, which means that the power system is driven to stability within about 2.36 s from the end of the cyber attack.

B. Stabilizing Control Using Zero-Determinant Strategies

The stability time of the system generators, the average control power, and the controller’s activity ratio versus the controller’s persistence factor, \( \alpha \), are shown in Fig. 4; in this case, a value of \( \beta = 0.5 \) is used. As noted in (19), the value of \( \alpha \) determines the long-term payoff of the power system during and after the cyber switching attack, and a high value of \( \alpha \) means the controller is more aggressive and so the system generators will be driven to stability faster. Furthermore, the value of the persistence factor directly affects the EPU controller’s activity as shown in (27). The tradeoff between the stability time and the average power required by the EPU controller is well demonstrated in this figure.

Fig. 5 shows the performance measures versus the steering factor of the controller, \( \beta \), for a value of \( \alpha = 0.75 \). It is observed that varying the value of \( \beta \) does not have substantial effects on the performance measures. Because \( \beta \) affects the controller’s likelihood of action as shown in (24), higher values of \( \beta \) means the EPU controller is more likely to take a stabilizing action. Moreover, results of this figure confirm that the controller meets its target long-term average payoff as long as the value of \( b = \beta b_{\text{min}} \) lies in the range \( b_{\text{min}} \leq b < 0 \). This result also suggests that the system operator has flexibility in designing the stabilizing controller.

The relation between the power ratio, \( \rho \), and the system performance is shown in Fig. 6 for \( \alpha = 0.75, \beta = 0.5, \) and \( |P_{C,i}| = \min(0.3 \text{ pu}, (P_{m,i}/10)) \forall i \in \{1, \ldots, N\} \). In this figure, the attack power is found from \( |P_{A,i}| = (|P_{C,i}|/\rho) \). Because \( \rho \) represents the ratio of the control power and the attack power, a higher value of \( \rho \) indicates a higher capability of the EPU stabilizing controller compared to that of
the attacker. Consequently, the effect of $\rho$ on the system performance resembles that of $\alpha$. It is observed that the stability time decreases steadily with increasing the power ratio. On the other hand, the average control power that is used to stabilize the power system slightly increases for the attacked generators. Consequently, a higher value of $\rho$ will drive the system quicker to stability; however, the average control power will not substantially increase.

The results of Figs. 4–6 demonstrate the tradeoff amongst the parameters of the EPU controller, the stability time, and average control power. Consequently, the system operator can choose the appropriate values of $\alpha$, $\beta$, and $\rho$, such that an acceptable stability time is achieved while the budget for external storage (cost or availability) is met.

V. DISCUSSION

The reader should note that game-theoretic analysis commonly focuses on identifying and analyzing equilibrium strategies assuming strategic rational players. Our formulation deviates from this classical formulation to include playing against a nonstrategic opponent; thus making the notion of equilibrium invalid. In essence, we assume the actions of the attacker (i.e., whether or not the attacker would access the system in the next round) to be random and not necessarily strategic. Thus, our analysis focuses on the long-term average outcome of the game from the standpoint of the controller. In this regard, we deduce reactive strategies of the controller to achieve a targeted long-term outcome without assumptions on the attacker’s strategy or the payoff matrix.

To further emphasize how the game converges to the long-term average outcome, the strategies of the defender (i.e., EPU controller) in the aforementioned game dictates that the controller takes an action to correct for any existing deviation in the system state from nominal desired state. In this framework, the controller continues to work in a series of actions to maintain system desired state. The game-theoretic formulation provides guarantees that over time the EPU is able to mitigate any set of strategies of the attacker.

The numerical results show that the attack mitigation strategy is successful in bringing the power system to stability even though the switching attack is an aggressive one. In addition, as we show that a suitable set of design parameters can lead to an acceptable performance (i.e., stability time) while meeting a budget constraint (i.e., an average control power). This type of proposed defense can be thought of as a first-line-of-defense scheme against cyber switching attacks till the system operators isolate and remove the attacked devices from the system to prevent the adversary from continuing the attack.

Our results show strategic implications for an effective control-based attack mitigation scheme. First, the formulation demonstrates how the EPU does not have to act aggressively or persistently in order to stabilize the power system. Further, because of the zero-determinant feature of the game, players with longer memories of the history of the game have no advantage in the long run over those with shorter memories; consequently, the EPU controller does not need to know the whole history of the attack in order to mitigate it efficiently, and it is actually sufficient for the EPU controller to know only the previous act of the attacker.

As evident from the analysis and the numerical results, the proposed framework provides a flexibility for the system operator in choosing a suitable set of values for $\alpha$, $\beta$, and $\rho$ in order to meet a specific stability time. This flexibility enriches the tunability of the attack mitigation strategy and helps the EPU to meet different resource constraints. Further, although the storage unit model is used in this paper; the model in (4) is flexible and our analysis will also hold for modulating distributed generators and loads.

As a final note, it can be deduced that if the attacker has more leverage over the stabilizing controller (i.e., $|P_{A_i}| \geq |P_{C_i}| \forall i$), then the attacker can utilize the game structure to fix the long-term outcome of the game; other defense mechanisms should be activated by the EPU in this case. Otherwise, the EPU controller will determine the long-term result of the game regardless of the actions of the attacker.

VI. CONCLUSION

This paper presents an analysis framework to determine the effects of cost or availability-constrained mitigation controllers in the presence of cyber switching attacks. We focus on a class of switching attacks and control actions that are simple making use of only the sign of the target generator’s relative normalized rotor speed. The stabilizing controller relies on receiving timely system updates in order to determine the strategy for mitigation. Our controller model makes use of
zero-determinant strategies for iterated games to determine when the controller is active. The merits of the proposed controller are investigated on the New England 39-bus power system. Numerical results detail the system performance metrics versus the parameters of the stabilizing controller. We assert that our models provide insight into a broad class of attack and mitigation strategies thus demonstrating that it is possible for stabilizing controllers that are not always active to stabilize a power grid under attack over time. Future work will include addressing mitigation in the face of multiple random attacks as well as investigating strategies for effective mitigation when $|P_{A,i}| > |P_{C,i}|$. In the latter, we will study how more aggressive attacks are conducive to early detection, notification and subsequent intervention by system operators.

**APPENDIX**

Here, we extend the work of Al Daoud et al. [21] to consider the case when the elements of the payoff matrix include a fixed bias that can change from one interval to another.

**Theorem 1:** Consider the game described in Fig. 2. Assume a fixed payoff matrix at each stage of the game and consider a set of zero-determinant strategies for this game as described in (17). If at any round of the game, the elements of the payoff matrix are shifted by a fixed value, the set of zero-determinant strategies for the game still holds.

**Proof:** Let the values of the payoff matrix, $X$, before the bias be $X = [X_{j,k}]$, where $j, k \in \{1, 2\}$. First, assume that $X_{1,\max} < X_{2,\min}$.

The average long-term payoff, $u_X$, can lie in the range of $[X_{1,\max}, X_{2,\min}]$. Let the exact value of $u_X$ be represented as $u_X = X_{1,\max} + \sigma (X_{2,\min} - X_{1,\max})$ where $0 \leq \sigma \leq 1$. Further, the minimum value of $b$ is $(u_X - X_{2,\max})$, and let the exact value of $b$ be represented as $b = \eta (u_X - X_{2,\max})$ where $0 < \eta \leq 1$. Using the definition of $p_C^{1,1}$ in (17), the following is found:

$$p_C^{1,1} = 1 + \left(1 - \frac{X_{1,1}}{u_X}\right)b = 1 + \eta \left(\frac{u_X - X_{1,1}}{u_X - X_{2,\max}}\right)u_X = 1 + \eta \left(\frac{u_X - X_{1,1}}{u_X - X_{2,\max}}\right).$$

Let $X'$ be the new payoff matrix after introducing a bias of $\Theta$ to all the elements of $X$; i.e., $X'_{j,k} = X_{j,k} + \Theta \forall j, k \in \{1, 2\}$. The value of $u_X'$ is found as

$$u_X' = X_{1,\max}' + \sigma (X_{2,\min}' - X_{1,\max}') = X_{1,\max} + \Theta + \sigma (X_{2,\min} - X_{1,\max})$$
$$= u_X + \Theta. \quad (28)$$

Further, the value of $b'$ is expressed as

$$b' = \eta \frac{u_X'}{u_X - X_{2,\max}'} = \eta \frac{u_X + \Theta}{u_X - X_{2,\max}}. \quad (29)$$

The value of $p_C^{1,1}$ is consequently found as

$$p_C^{1,1} = 1 + \left(1 - \frac{X_{1,1}'}{u_X'}\right)b' = 1 + \eta \left(\frac{u_X - X_{1,1}'}{u_X + \Theta}\right)u_X - X_{2,\max}$$
$$= 1 + \eta \left(\frac{u_X - X_{1,1}}{u_X - X_{2,\max}}\right) = p_C^{1,1}. \quad (30)$$

Similar results can be found for the values of $p_C^{1,2} , p_C^{2,1}$, and $p_C^{2,2}$. Further, analogous analysis can be conducted for the case when $X_{2,\min} < X_{1,\max}$. Accordingly, the values of the probability of taking an action do not change if all the elements of the payoff matrix undergo the same value of bias. Hence, the structure of the game does not change.

The mitigation philosophy of the controller in this game is to mitigate any resulting bias on estimated system nominal state; thus, actions of the EPU controller are relative to this value and include this as a bias in the $2 \times 2$ game. But as Theorem 1 shows, it is proven that regardless of the fluctuations of $\omega_i$ during the game, the controller will be able to stabilize the power generator as long the game is played according to (24).

**REFERENCES**


communications, and signal processing applications.


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