A Novel Recursive Filtering Method for Blind Image Restoration

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Abstract — For classical linear image restoration, the point-spread function (PSF) of the degrading system is assumed to be given. However, in practical situations this information is often unknown and must be estimated from the blurred image itself. The task of combined PSF identification and image restoration is called *blind deconvolution.*

This paper presents a novel approach to the blind deconvolution problem for images. The technique applies to situations in which the imaged scene consists of a finite support object against a uniformly grey background. The only information required are the nonnegativity of the true image and the support size of the original object. A novel support-finding algorithm is also proposed for situations in which the exact object support is unknown.

***Keywords*— blind deconvolution; blind image restoration;**

I. Introduction

In applications such as artificial satellite imaging, astronomy, and medical imaging, improved image quality is often costly or physically impossible to obtain. In addition, little is known about the image to be restored, and it is often difficult to calculate or measure the PSF explicitly. The problem of simultaneously estimating the PSF (or its inverse) and restoring an unknown image is called *blind deconvolution.* The goal is to obtain a scaled shifted version of the original image.

Early research in blind deconvolution of images assumed that a parametric model for the PSF was known. The parameter values were estimated using the frequency domain nulls of the degraded image [1]. More recent methods estimate the image and PSF simultaneously in the restoration process [2]-[10], but are not suitable for practical imaging applications. A major drawback of existing blind deconvolution methods for images is that they suffer from poor convergence properties; the algorithms converge to local minima [2]-[7], or are so computationally demanding [8], [9] that they are impractical for real imaging applications. Another disadvantage is that some methods make restrictive assumptions on the PSF or the true image that limits the algorithm's portability to different applications [10].

This paper presents a novel technique for the class of nonparametric deterministic constraints blind image restoration methods that overcomes the limitations of existing techniques. The proposed technique is relevant to applications in which an object of finite extent is imaged against a uniformly white, grey or black background. The edges of the object are assumed to be completely or almost completely included within the observed frame. This often occurs in applications such as astronomy and medical imaging. Statistical knowledge of the original image or a parametric model of the PSF are not needed. The only information required for restoration is the nonnegativity of the true image, and the support size of the original object. The proposed method, referred to as the Nonnegativity and Support constraints Recursive Inverse Filtering (NAS-RIF) technique, involves iteratively minimizing a convex cost function. All other methods of its class incorporate the minimization of nonconvex cost functions; the advantage of the proposed NAS-RIF technique is that convergence to the global minimum is guaranteed, even in the presence of noise. In addition, the proposed technique shows faster convergence speed than existing iterative techniques and does not require heavy memory requirements. The superior performance of the NAS-RIF algorithm is demonstrated by computer simulations and comparisons with existing methods of its class.

The proposed NAS-RIF technique and the methods of [2]-[4], [9] belong the class of nonparametric deterministic constraints blind image restoration methods. They are included for comparisons and make the following assumptions to achieve blind image restoration. The degradation process is assumed to be represented by the following linear model:

$$\begin{aligned} g(x,y) &= f(x,y) * h(x,y) + n(x,y) \\ &= \sum_{\forall(m,n)} f(m,n)h(x-m,y-n) + n(x,y) \quad (1) \end{aligned}$$

where $f(x,y)$ is the true image, $h(x,y)$ is the PSF, $n(x,y)$ is the additive noise, $g(x,y)$ is the degraded image, (x,y) is the discrete pixel coordinate, and $*$ represents two-dimensional linear convolution. The true image is required to be nonnegative with known finite *support.* The

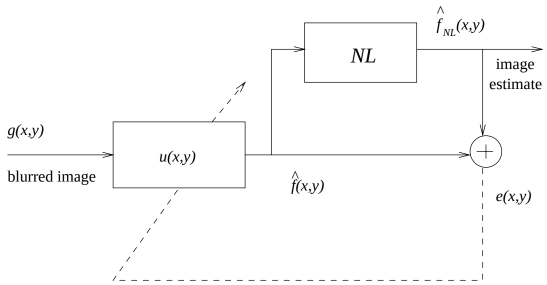

Fig. 1. Proposed Blind Deconvolution Scheme for Images

support is defined as the smallest rectangle containing the entire object. In applications such as astronomy, this information is sometimes available. In our technique, it is estimated using a novel method introduced in section II.B, if unknown. In addition to the assumptions stated above, the methods of [2]-[4], [9] require that the blur also be nonnegative with known finite support for proper restoration. In contrast, the only assumption our algorithm makes about the blur is that its inverse exists.

II. Description of the Proposed Technique

A. The Proposed Blind Deconvolution Scheme

The proposed NAS-RIF technique consists of a variable FIR filter $u(x,y)$ with the blurred image $g(x,y)$ as input. The output of this filter represents an estimate of the true image $\hat{f}(x,y)$. This estimate is passed through a nonlinear filter which uses a non-expansive mapping to project the estimated image into the space representing the known characteristics of the true image. The difference between this projected image $\hat{f}_{NL}(x,y)$ and $\hat{f}(x,y)$ is used as the error signal to update the variable filter $u(x,y)$. Figure 1 gives an overview of the scheme.

For the algorithm presented, the image is assumed to be nonnegative with known support, so the *NL* block of Figure 1 represents the projection of the estimated image onto the set of images that are nonnegative with given finite support. This requires forcing the negative pixel values within the region of support to zero, and pixels values outside the region of support to the background grey-level L_B. The cost function used in the restoration procedure is defined as:

$$
\begin{aligned}
J &= \sum_{\forall(x,y)} \left[\hat{f}_{NL}(x,y) - \hat{f}(x,y)\right]^2 \\
&= \sum_{(x,y)\in D_{sup}} \hat{f}^2(x,y) \left[\frac{1-\mathrm{sgn}(\hat{f}(x,y))}{2}\right] \\
&+ \sum_{(x,y)\in \overline{D}_{sup}} [\hat{f}(x,y) - L_B]^2 + \gamma \left[\sum_{\forall(x,y)} u(x,y) - 1\right]^2 \quad (2)
\end{aligned}
$$

where $\hat{f}(x,y) = g(x,y) * u(x,y)$, and $\mathrm{sgn}(f) = -1$ if $f < 0$ and $\mathrm{sgn}(f) = 1$, if $f \geq 0$. D_{sup} is the set of all pixels inside the region of support, and $\overline{D}_{sup}$ is the set of all pixels outside the region of support. The variable γ in third term of equation 2 is nonzero only when L_B is zero, ie., the background colour is black. The third term is used to constrain the parameters away from the trivial all-zero global minimum for this situation. In many imaging applications, the mean value of the true image is preserved in the blurring process; that is, the PSF follows the constraint $\sum_{\forall(x,y)} h(x,y) = 1$. This implies that $\sum_{\forall(x,y)} h^{-1}(x,y) = 1$ from the properties of the discrete Fourier transform (DFT). Since $u(x,y)$ is an estimate of $h^{-1}(x,y)$, it can be constrained to $\sum_{\forall(x,y)} u(x,y) = 1$ to avoid this trivial all-zero solution; thus, the third term of equation 2 is a constraining penalty term.

It can be shown that equation 2 is convex, so that convergence of the algorithm to the global minimum is possible using a variety of numerical optimization routines. The conjugate gradient routine is used for the minimization of J because its speed of convergence is in general much faster than other descent routines, such as the steepest-descent method. The recursive algorithm is summarized in Table 1.

B. Determination of the Support of the True Image

For situations in which the object support size is unknown, determining it by visual inspection is often cumbersome and unreliable. A method for assessing the optimal support size automatically and objectively is proposed. It uses the hold-out (HO) method used for model validation in data analysis. The proposed support-finding algorithm is inspired by the constraint assessment algorithm of [11], but is modified for blind image restoration.

Competing assumptions on the true image, such as different support sizes, can be assessed using the hold-out method. A support size for the true image is assumed. The image estimate pixels $\hat{f}(x,y)$ outside the assumed region of support are collectively called the *estimation set*; they are used to obtain an estimate of the true image. This is accomplished by minimizing a criterion, called the *estimation error*, which incorporates only the pixels within the estimation set. Specifically, the proposed blind deconvolution algorithm is applied using the assumed support and excluding the nonnegativity constraint. The set of pixels within the assumed region of support is called the *validation set*, and is used to assess the "correctness" of the assumed support size. This is performed by computing the *validation error* which measures the energy of negative pixels of the image estimate within the assumed region of support. The assumed sup-

Table 1. Summary of the proposed NAS-RIF algorithm.

I) Definitions:

- $f_k(x,y)$: estimate of true image at kth iteration
- $u_k(x,y)$: FIR filter parameters of dimension $N_{xu} \times N_{yu}$ at iteration k
- δ: tolerance used to terminate the algorithm
- $J(\underline{u}_k)$: cost function at parameter setting $\underline{u}_k$
- $\nabla J(\underline{u}_k)$: gradient of J at $\underline{u}_k$
- $< \cdot, \cdot >$: scalar product
- Note: underlined letters represent lexicographically ordered vectors of their two-dimensional counterparts.

II) Set initial conditions ($k = 0$):

Set FIR filter $u_k(x,y)$ to all zeros with a unit spike in the middle

Set tolerance $\delta > 0$

III) At iteration (k): $k = 0, 1, 2, \ldots$

1) $\hat{f}_k(x,y) = u_k(x,y) * g(x,y)$

2) $\hat{f}_{NL}(x,y) = NL[\hat{f}(x,y)]$

3) Minimization Routine to update FIR filter parameters.

For example: (conjugate gradient routine)

3a) $[\nabla J(\underline{u}_k)]^T = [\frac{\partial J(u_k)}{\partial u(1,1)} \; \frac{\partial J(u_k)}{\partial u(1,2)} \cdots \frac{\partial J(u_k)}{\partial u(N_{xu},N_{yu})}]$

where

$$\frac{\partial J(\underline{u}_k)}{\partial u(i,j)} = 2\sum_{(x,y)\in D_{sup}} \hat{f}_k(x,y)\left[\frac{1-\mathrm{sgn}(\hat{f}_k(x,y))}{2}\right] g(x-i+1, y-j+1) + 2\sum_{(x,y)\in \overline{D}_{sup}} [\hat{f}_k(x,y) - L_B] g(x-i+1, y-j+1) + 2\gamma\left[\sum_{\forall(x,y)} u_k(x,y) - 1\right]$$

3b) $\beta_{k-1} = (< \nabla J(\underline{u}_k) - \nabla J(\underline{u}_{k-1}), \nabla J(\underline{u}_k) >)/(< \nabla J(\underline{u}_{k-1}), J(\underline{u}_{k-1}) >)$

3c) If $k = 0$, $d_k = -\nabla J(\underline{u}_k)$
Otherwise, $\underline{d}_k = -\nabla J(\underline{u}_k) + \beta_{k-1} d_{k-1}$

3d) Perform a line minimization such as `dlinmin.c` in [12] to find t_k such that
$J(\underline{u}_k + t_k \underline{d}_k) \leq J(\underline{u}_k + t\underline{d}_k)$ for all $t \in \mathrm{R}$

3e) $\underline{u}_{k+1} = \underline{u}_k + t\underline{d}_k$

4) $k = k + 1$

5) If $J(\underline{u}_k) < \delta$, stop. Otherwise, go to **1**.

port which produces the minimum validation error is selected as the true image support. The algorithm follows in Table 2.

If the assumed support is exact or larger than the actual support a reasonable estimate of the true image can be obtained. Since the true image is nonnegative, the validation error for such an image estimate should be small. Thus, the assumed support which minimizes the validation error is intuitively a good estimate of the actual support.

Table 2. Summary of the proposed support finding algorithm.

Assume an equally spaced grid of support parameter values (L_x, L_y) from $(1,1)$ to the size of the blurred image (N_{xg}, N_{yg}).

1) Assume a rectangular support S with dimensions (L_x, L_y) from the grid. If all values in the grid have been selected before, either

1. Go to step **5** if the exhausted grid contains successive elements.
2. Form a finer grid centred about $(L_{x,min}, L_{y,min})$ (the parameters giving the minimum of the validation error found so far), and select a parameters (L_x, L_y) out of this new grid.

2) Based on the assumed support S, find the restoration filter $u^*(x,y)$ by using the conjugate gradient algorithm, to minimize the following estimation error function: $J(\underline{u}) = \sum_{(x,y)\in\overline{S}} [\hat{f}(x,y) - L_B]^2 + \gamma\left[\sum_{\forall(x,y)} u(x,y) - 1\right]^2$ where $\hat{f}(x,y) = u(x,y) * g(x,y)$ and $\overline{S}$ is the region outside the assumed support.

3) Calculate the validation error based on the minimizing filter parameters $u^*(x,y)$ of the estimation error of step **2**.

$$V(S) = \frac{1}{\| S \|} \sum_{(x,y)\in S} \hat{f^*}^2(x,y)\left[\frac{1-\mathrm{sgn}(\hat{f^*}(x,y))}{2}\right] \quad (3)$$

where $\| \cdot \|$ denotes the number of elements in the argument set, and the "restored" image estimate $\hat{f^*}(x,y) = u^*(x,y) * g(x,y)$.

4) Save the parameters $(L_{x,min}, L_{y,min})$, which give the minimum value of $V(S)$ found so far. Go to step **1**.

5) Select the support parameters that minimize $V(S)$ as the optimal support size for restoration.

III. Simulation Results and Comparisons

The results of the proposed algorithm and the IBD algorithm described in [2] and modified in [3] are shown in Figure 2.

The original toy image shown in Figure 2 (a) of support 119×81 was blurred using a 21×21 truncated Gaussian PSF; noise was added for a blurred signal-to-noise ratio (BSNR) of 70 dB. The degraded image is displayed in Figure 2 (b). The proposed support-finding algorithm estimated the support of the true image as 120×83. Based on this support, the NAS-RIF restoration (after 379 iterations) and mean square error (MSE) plots are shown in Figures 2 (c) and (e), respectively. The proposed NAS-RIF method converges to a good estimate in approximately 300 iterations. The results of the IBD method are shown in Figures 2 (d) and (f). The algorithm was unable to converge for 3500 iterations. The image estimate generated by the IBD algorithm which showed the minimum total energy of the negative pixels within the region of support and the pixels outside the region of support deviating from the background grey-level was saved as the true image estimate.

The restoration shown in Figure 2 (d) is the true image estimate at the 1000th iteration of the IBD algorithm;

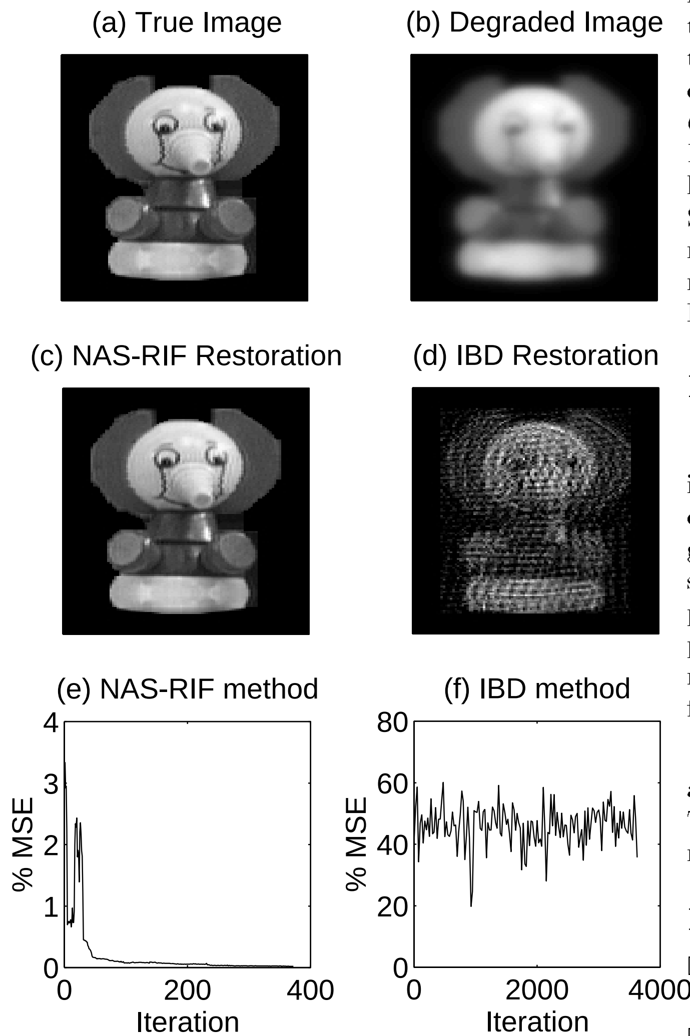

Fig. 2. Simulation Results of the NAS-RIF and IBD Restoration Methods

this estimate produced the minimum MSE as shown in the plot of Figure 2 (f). The algorithm failed to produce a reliable estimate of the true image. Simulation results of the algorithm for exact support size at various different initial conditions and noise parameter values α produced poor results as well.

Although the IBD algorithm produces comparable results to the NAS-RIF algorithm for simple binary images, it fails to converge to a reliable image estimate for more complicated grey-scale images. In addition, the algorithm often exhibits instability.

Lane's method of [4] was also simulated, but converged to local minima for all the different random initial conditions tested. The method is unsuitable for practical imaging applications as it suffers from high sensitivity to initial conditions.

McCallum's algorithm of [9] produced good results for small images; however, for the degraded toy image, it was too computationally time consuming to produce a reliable estimate. The order of the algorithm per iteration is $O(N_f^4)$, where N_f is the number of pixel values of the image estimate. In contrast, the IBD method has order $O(N_f \log_2(N_f))$, and the NAS-RIF method has order $O(N_f N_u N_{ls,k})$ per iteration, where N_u is the number of FIR filter parameters of $u(x, y)$ and $N_{ls,k}$ is the number of line searches required in `dlinmin.c` of step **3d** of Table 1. Since the number of filter parameters of $u(x, y)$ is usually much smaller than the image size, the NAS-RIF method requires much fewer computations on average than the IBD method to produce a good estimate.

IV. Conclusion

A novel method for blind deconvolution of images is presented. It is applicable to situations in which an object of finite support is imaged against a uniform background; this occurs in situations such as astronomical speckle imaging and medical imaging, among others. The proposed NAS-RIF algorithm has superior convergence properties than existing methods of its class. Simulation results demonstrate the more reliable performance and faster convergence of the method.

In situations in which the support of the true image is unknown, a support-finding algorithm is proposed. The algorithm shows promise for practical blind image restoration.

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